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Limited growth of the number of sources and decrease of inelasticity in high-energy reactions

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The limited growth of the number of sources $\langle C \rangle$ found some time ago from the analysis of multiparticle distributions is explained in terms of the interacting gluon model. It turns out to be closely related to another characteristic property of strong interactions, namely the decrease with energy of the inelasticity $\langle K \rangle$ and is due to the specific energy dependence of the gluon-gluon cross section and the form of the gluonic structure functions.

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A few years ago Ekspong [1] and Giovannini and Van Hove [2] made the following intriguing observation: if one interprets the observed broadening of the multiplicity distributions $P(n)$ with energy $\sqrt{s}$ in terms of a number C of independently produced sources (clusters, clans, fireballs, . . .) then the average number of these sources $\langle C \rangle$ increases with energy from 5 to $\simeq 8$ and saturates at that value for energies $\sqrt{s} \geq 63$ GeV. This observation, if confirmed, could constitute one of the most fundamental properties of "soft" interaction physics—at the same level as, e.g., the limitation of transverse momenta of secondaries. No explanation of this effect has been given so far. It is interesting to note that already in Ref. [3] the possibility of such an effect was discussed [4]. The purpose of this paper is to show that this property can be explained in terms of the interacting gluon model (IGM) [5–7] and that $\langle C \rangle$ is related to another characteristic property of strong interactions, namely, the inelasticity $\langle K \rangle$, in a very simple way [cf. Eq. (10)]. In particular, the limited growth of $\langle C \rangle$ is shown to be correlated with the decrease with energy of $\langle K \rangle$ [8–13].

The fact that these two observations, which were so far unrelated, are now linked in what we believe is a quite natural approach enhances considerably the importance of the concepts of number of sources and inelasticity and provides further support for the energy dependence of these observables.

To make our point we have to consider how sources (fireballs, clusters, clans, . . .) can be created in strong "soft" interactions. One answer to this question is provided by the IGM, which can be summarized in the following way (cf. Refs. [5–7] for details). When two highly energetic hadrons collide, they interact mainly through their gluon components (the $q\bar{q}$ sea pairs being converted into equivalent gluons). The valence quarks together with the gluons which escape from the interaction region then form two leading particles (or rather two leading jets containing corresponding leading particles). The interacting glue produces in a given event a number, C, of interaction centers (which we called minifireballs, MF's). Those MF's eventually form a lump of (gluonic) matter, the central fireball (CF), which results in secondaries observed in the central region of collision (the IGM itself does not specify the hadronization part of the collision process). The energy $M=\sqrt{xys}$ deposited in the central region by the incoming hadrons (x,y being fractions of their energy-momenta) determines the inelasticity $K=M/\sqrt{s}$ of the reaction. The probability to form a CF by depositing fractions x and y of the energy-momenta of the incoming hadrons is

$$\chi(x,y)=\sum_{\{C_i\}}\delta\left[x-\sum_i C_i x_i\right]\times\delta\left[y-\sum_i C_i y_i\right]\prod_{\{C_i\}}P(C_i)\ , \tag{1}$$

masses and transverse momenta are neglected; notice that

$$\chi(K)=\int_0^1 dx\int_0^1 dy\,\chi(x,y)\delta(\sqrt{K}-xy)\ .$$

Assuming now that the MF's are produced independently, i.e., their number distribution is Poissonian (the same assumption was made also in Refs. [1,2]), and expressing δ functions via Fourier integrals, one can perform all summations and arrive at the general formula [5]

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$$\chi(x,y)=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}dt\int_{-\infty}^{+\infty}du\,\exp\left[i(xt+yu)+\int_0^1 dx'\int_0^1 dy'\frac{d\langle C(x',y')\rangle}{dx'\,dy'}(e^{-i(x't+y'u)}-1)\right]. \tag{2}$$

Its central ingredient is the spectral function of produced MF's, i.e., the mean number of MF's at given x and y which is proportional to the number of gg interactions:

$$\frac{d\langle C\rangle}{dx\,dy}=\frac{\sigma_{gg}(x,y)}{\sigma_{hN}^{\text{in}}(s)}G_h(x)G_N\Theta(xy-K_{\min}^2). \tag{3}$$

From it, on the average, we have

$$\sigma_{hN}^{\text{in}}(s)\langle C\rangle=\int_0^1 dx\int_0^1 dy\,\Theta(xy-K_{\min}^2)\times\sigma_{gg}(x,y)G_h(x)G_N(y), \tag{4}$$

where $\sigma_{hN}^{\text{in}}(s)$ is the hadronic inelastic cross section for c.m. energy of reaction $\sqrt{s}$ and Θ the Heavyside function. $K_{\min}=2\mu/\sqrt{s}$ is the minimal inelasticity corresponding to the lightest MF produced, $G(x)$ is the gluonic structure function and $\sigma_{gg}(x,y)$ is the gluon-gluon cross section. This last quantity is parametrized as follows:

$$\sigma_{gg}=\frac{\alpha K_{\min}^2}{xy}+\delta\ln\left[\frac{xy}{K_{\min}^2}\right]=\frac{4\mu^2\alpha}{M^2}+\delta\ln\left[\frac{M^2}{4\mu^2}\right]. \tag{5}$$

There are only three parameters in the model: the mass of the lightest MF produced, 2μ, which defines the phase space, α which gives the σ_{gg} at the boundary of the phase space, and δ which fixes the asymptotic growth of σ_{gg} with $M^2=xys$. With such a parametrization, we can easily account for any energy dependence of $\langle C\rangle$.

Different terms in σ_{gg} determine different asymptotic energy dependences of $\langle C\rangle$:

$$\begin{aligned}&\sigma_{gg}\sim\ln\frac{M^2}{4\mu^2}\to\langle C\rangle\sim\frac{\ln^3 s}{\sigma_{hN}^{\text{in}}},\\&\sigma_{gg}\sim\text{const}\to\langle C\rangle\sim\frac{\ln^2 s}{\sigma_{hN}^{\text{in}}},\\&\sigma_{gg}\sim\frac{1}{M^2}\to\langle C\rangle\sim\frac{\ln s}{\sigma_{hN}^{\text{in}}}.\end{aligned} \tag{6}$$

To obtain Eq. (6) we have used the standard small-x behavior of the gluonic structure function, i.e., $G(x)\simeq 1/x$ (in the more detailed calculations represented in Figs. 1 and 2, the form $G(x)=[p(n+1)/x](1-x)^n$ was used with $n=5$ and $p=0.5-0.6$, cf. [7]).

Equation (2) can be approximated by the Gaussian distribution [5]

$$\chi(x,y)=\exp\left[-\frac{(x-\langle x\rangle)^2}{2\langle x^2\rangle}-\frac{(y-\langle y\rangle)^2}{2\langle y^2\rangle}\right], \tag{7}$$

where

$$\langle x^n y^m\rangle=\int_0^1 dx\,x^n\int_0^1 dy\,y^m\frac{d\langle C\rangle}{dx\,dy}. \tag{8}$$

For the inelasticity $K=M/\sqrt{s}=\sqrt{xy}$ we obtain then, taking into account that $\langle x\rangle=\langle y\rangle$ (due to symmetry of the nucleon-nucleon collision), $\langle K\rangle\simeq\langle x\rangle$. From Eqs. (8) and (3), with the same approximation as in Eq. (6), one gets

$$\langle C\rangle\sim\langle x\rangle\ln s \tag{9}$$

or

$$\langle C(s)\rangle\sim\langle K(s)\rangle\ln s. \tag{10}$$

These are our main results. The exact interrelation between the mean inelasticity $\langle K\rangle$ and the mean number of emitting sources (minifireballs) $\langle C\rangle$ for different energies is presented in Fig. 1 [14]. The parameter 2μ was kept constant ($2\mu=0.35$ GeV), while the parameters α and δ were varied in order to get different values of $\langle K\rangle$ for different energies. Notice that, because the percentage of the energy-momenta of colliding hadrons allocated to gluons is limited to 0.5–0.6, $\langle K\rangle$ is limited from above. In Fig. 2(a) we present the mean number of sources $\langle C\rangle$ obtained from the IGM using the parameters determined from estimates of the inelasticity [9] and leading particle spectra [5–7] ($\alpha=60$ mb, $\delta=0.$) and compared it with the values given by Ekspong [1]. To judge the quality of the agreement between the points from Ref. [1] and our curve, we have also calculated the mean inelasticity $\langle K\rangle$ using as input the values of $\langle C\rangle$ from Ref. [1]. The results are presented in Fig. 2(b). We see that the resulting decrease of $\langle K\rangle$ with energy is in agreement with the determination of $\langle K(s)\rangle$ as given in Ref. [9], especially if one considers the following factors.

(i) The determination of $\langle C(s)\rangle$ as performed in Refs. [1,2] assumes a negative-binomial form for the multiplici-

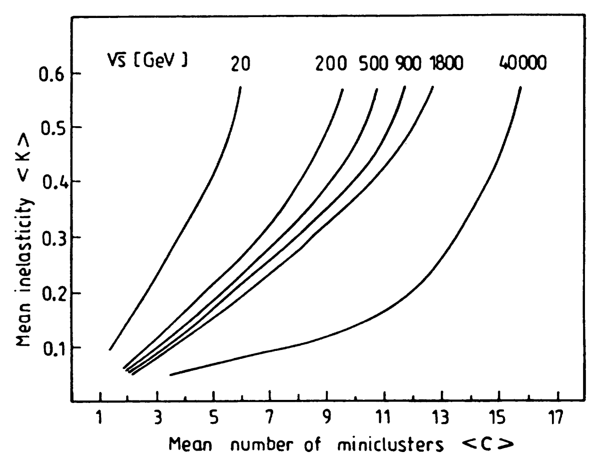

FIG. 1. The mean inelasticity $\langle K\rangle$ vs the mean number of MF's $\langle C\rangle$ for different energies.

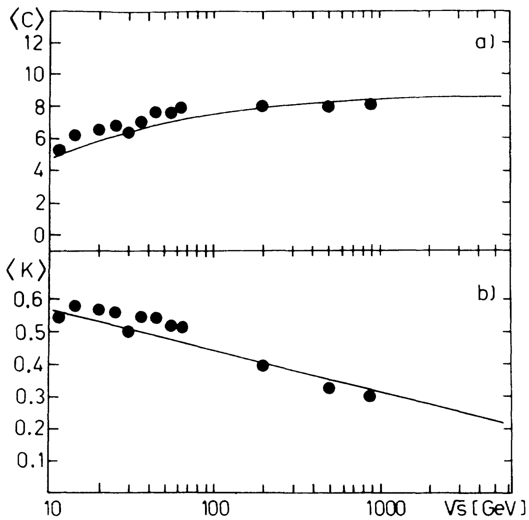

FIG. 2. (a) The energy dependence of the mean number of sources $\langle C \rangle$ as predicted by the IGM with standard parameters [5–7] (solid line) compared with values given in Ref. [1] (points). (b) The energy dependence of the mean inelasticity $\langle K \rangle$ corresponding to the mean number of sources $\langle C \rangle$ (identified with clusters or MF's in the IGM) as given in Ref. [1] [points read off from Fig. 1 and corresponding to those in Fig. 2(a)] compared with the predictions of the IGM [solid line corresponding to that in Fig. 2(a)].

ty distribution. It is known that this assumption is only approximately valid [15].

(ii) There is no direct measurement of inelasticity and therefore the estimates of $\langle K \rangle$ could vary easily within a range of $\pm 20\%$. As a matter of fact, $\langle K \rangle$ as obtained by the IGM from analyses of leading-particle spectra [5–7] starts at the value $\langle K \rangle = 0.45$ at the lowest $\sqrt{s}$ to be compared with the value $\langle K \rangle = 0.58$ as given in Fig. 2. At larger $\sqrt{s}$ the differences tend to disappear. Given these caveats, one can say that there is satisfactory agreement between $\langle C \rangle$ as given by the IGM and the corresponding values given in Ref. [1].

In all these considerations relating $\langle K \rangle$ to $\langle C \rangle$, it is essential that both these quantities refer to the same region in phase space (rapidity), which is the case for the numbers provided above [16]. At this point one should mention that the simple equation (10) may hold even if the saturation of $\langle C \rangle$ with energy would eventually break down. This is true, in particular, if one considers the prediction for $\langle C \rangle$ as given by the statistical bootstrap model [17] where it was found that $\langle C(s) \rangle \sim (\ln\sqrt{s})^b$ with $b < 1$. (The general relationships involving the energy dependence of $\langle C \rangle$ are discussed in Ref. [18].) A corollary of our finding [cf. Eq. (6)] is that the energy dependence of the gluonic cross section is limited by the inequalities: $1/M^2 < \sigma_{gg}(M^2) < \text{const}$ (modulo the assumptions about the gluonic structure functions used). This last result illustrates the possible applications of the measurement of the number of clusters or inelasticity for our understanding of the nonperturbative behavior of gluon interactions.

Finally, we would like to mention still another aspect of the result given by Eq. (10): It reduces the measurement of the inelasticity K, which is of fundamental importance in studies of quark-gluon plasma formation [19], to the measurement of the number of minifireballs C. This last quantity may be more accessible than K because it amounts to the measurement of multiplicity distributions, a subject on which much progress has been reported recently [20].

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[8] The energy dependence of inelasticity K already has a long history and has been debated again only recently. Introduced originally as a constant parameter, inelasticity K was later argued to decrease with energy—both from the analysis of cosmic-ray and accelerator experimental data [9]. The IGM [5–7] was actually the first theoretical model aimed at explaining such behavior of $\langle K \rangle$. Subsequently, other theoretical schemes also found a decreasing $\langle K \rangle$ [10]. However, there exists an opposite point of view, namely, that $\langle K \rangle$ remains constant or even increases with energy. These claims are based either on the geometrical-

model assertion that $\langle K \rangle = \langle G \rangle$, G being the overlap function, which allows one to connect increasing $\langle K \rangle$ with increasing $\sigma_{\rm el}/\sigma_{\rm tot}$ [11], or on the analysis of the hadronic cross sections $\sigma(p\text{-air})$ coming from the cosmic-ray data [12]. On the other hand, a different description of $\langle K \rangle$ within the same geometrical model leads to a decrease of inelasticity with energy [10]. Furthermore the "evidence" from cosmic-ray "data" is far from being clear-cut because these "data" already contain some assumptions on K (which are not necessarily identical with those used in [12]). Actually, one can consistently fit the same (raw) data also with an inelasticity decreasing with energy [13].

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[14] The actual calculations have been performed using a Monte Carlo event generator scheme of the IGM presented in Refs. [6,7]. It is interesting to note that, in this approach, the Poisson-like form of the MF distribution is no longer assumed but rather emerges in a natural way from a generating process itself.

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