Examination of non-$B\bar{B}$ decays of the $\Upsilon(4S)$ in hybrid models

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The large rate for the Zweig-rule-suppressed decay $\Upsilon(4S)\to\psi+X$ with a high momentum ψ is discussed and models are considered in which $b\bar{b}$ annihilation occurs in a color-octet state. Our main findings are the following. (1) The central value of the observed CLEO rate is difficult to account for in all such models. (2) Among these models, only those in which the $b\bar{b}$ (octet) is in a total spin-0 state seem to be the least inconsistent with the observed rate. (3) The spin-0 state yields a hard photon with a relatively high branching ratio which is on the verge of being ruled out by a CUSB measurement. The implications of these models for η_c, $D\bar{D}$, ϕ, and photonic final states are given. The possibility of distinguishing among the contributions of spin-1 and spin-0 color octets is also discussed.

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I. INTRODUCTION AND MOTIVATION

Recent experimental studies by CLEO [1] and ARGUS [2] of ψ-meson production from the decay of the $\Upsilon(4S)$ have found that

$$B(\Upsilon(4S)\to\psi+X,|\mathbf{P}|\geq 2\text{GeV})=0.22\pm0.06\pm0.04\% \ . \quad (1)$$

The momentum cut on the ψ ensures that the ψ mesons do not arise from the decay sequence $\Upsilon(4S)\to B\bar{B}$ followed by $B\to\psi+X$. That the observed rate (1) is puzzling can be seen by the following reasoning. Recall that the total width of $\Upsilon(4S)$ is $\sim 10^3$ times larger than the total widths of lower-lying Υ states. This is usually understood as a consequence of $\Upsilon(4S)$ being the lowest-energy $b\bar{b}$ state above the $B\bar{B}$ threshold, hence the Zweig-rule-allowed decay mode $\Upsilon\to B\bar{B}$ accounts for the increased width. It is thus surprising to observe an apparently Zweig-rule-suppressed process at such a high branching ratio. Indeed the decay $\Upsilon(1S)\to\psi+X$ has a branching ratio less than [3] $\sim 10^{-3}$ which would suggest that if the $\Upsilon(4S)$ decay were analogous, the branching ratio in (1) would be $\sim 10^{-6}$, taking into account the difference in total widths. We must conclude therefore that whatever Zweig-suppressed process accounts for the ψ mesons in (1), the corresponding decays do not occur in lower Υ states.

One class of explanations for this puzzle is that the $b\bar{b}$ annihilation takes place in a color-octet state. Explanations of this type include mixing of the conventional $\Upsilon(4S)$ (i.e., pure $b\bar{b}$ state) with either a $b\bar{b}g$ or $b\bar{b}q\bar{q}$ state. Another way octet $b\bar{b}$ annihilation may arise is in the rescattering of final-state $B\bar{B}$ mesons. In this case, the b quarks contained in the B mesons will collide with incoherent color [4–6]. The motivation for explanations of this sort is that the $b\bar{b}$ annihilation in the color-octet state can proceed through two on-shell gluons or a single off-shell gluon, potentially enhancing Zweig-suppressed modes. In contrast, the usual color-singlet $b\bar{b}$ annihilation in conventional Υ decay models must proceed through at least three gluons.

Another class of models, which is considered elsewhere [7], relies on conventional meson transitions such as $\Upsilon(4S)\to\eta h_b$ where the h_b, which is below threshold, for $B\bar{B}$, gives rise to the ψ mesons. Explanations have also been proposed which attempt to attribute (1) to extensions of the standard model [8].

There are several reasons that suggest that the origin of (1) needs to be better understood. For one thing, one expects the non-$B\bar{B}$ decays of $\Upsilon(4S)$ to contribute towards a background for the events used in determination of the Kobayashi-Maskawa (KM) parameter V_{ub} through the end-point measurements in the semileptonic transitions of B mesons. Although at present [9] the level of inaccuracy of these first determinations is appreciable enough (about 30%) that the background from non-$B\bar{B}$ decays is not serious, this situation is likely to change as efforts towards improved determination of V_{ub} continue. For that purpose a better understanding of the source(s) of non-$B\bar{B}$ decays is important.

Let us also recall that admixtures of multiquark and/or $q\bar{q}g$ states have been advocated in the past [10] in the context of lighter (u,d,s,c) quarks but a clear resolution has not been attained. In the arena of $\Upsilon(4S)$, the presence of the heavier b quarks may make this issue more tractable.

Finally, we mention the problem with the semileptonic branching ratio of the B meson [11]. Experimentally, this branching ratio is found to be $\approx 10\%$. Theoretically it is very difficult to account for a branching ratio less than about 12%. Of course the non-$B\bar{B}$ decays of the $\Upsilon(4S)$ tend to increase the true experimental branching ratio from the apparent value of about 10% but the extent of the increase cannot be ascertained reliably until we have a better understanding of the origin of (1).

In this paper, we consider the properties of such a color octet $b\bar{b}$ annihilation and find that perturbative QCD invariably leads to the formation of numerous two and three-body (quark and gluon) final states and the fraction of final states with ψ production is, in general, estimated to be so small that this mechanism can only be the dominant cause of (1) in a certain subclass of such models. In order to facilitate further experimental studies we also discuss the implications of $b\bar{b}$ octet models for various final states such as η_c, $D\bar{D}$, ϕ, and γ. In addition, we consider ψ production in $b\bar{b}g$ models through a mechanism where the $b\bar{b}$ pair annihilate in an octet state to two gluons and a subsequent collision with the constituent gluon gives rise to the ψ meson. We find that this mechanism also leads to a very small fraction of final states containing ψ mesons, not enough to explain (1).

II. THE MODEL AND INCLUSIVE DECAY RATES

Let us denote by Υ_8 the initial $b\bar{b}$ system in a color-octet state in whatever model it is generated. For our calculations we will assume weak binding and take the $b\bar{b}$ to be in an S-wave state. Thus we take the b and $\bar{b}$ quarks to be at rest and assign each of them one half of the energy of the initial $\Upsilon(4S)$ so that $p_b = p_{\bar{b}}$ and $E_b = m_b$.

If the $b\bar{b}$ in the Υ_8 are in a spin-0 state we will denote it as $\Upsilon_{8(1)}$, in a spin-1 state as $\Upsilon_{8(3)}$, while if the $b\bar{b}$ is unpolarized we will denote it as $\Upsilon_{8(4)}$. For example, in models that produce a Υ_8 as a result of $B\bar{B}$ rescattering, the annihilation will occur through a $\Upsilon_{8(4)}$ state. In models that postulate a $b\bar{b}g$ state the $b\bar{b}$ pair is in a $\Upsilon_{8(1)}$ state assuming that the relative angular momenta are all 0. Finally, models that postulate a $b\bar{b}q\bar{q}$ state may contain either $\Upsilon_{8(1)}$ or $\Upsilon_{8(3)}$ depending on the dynamics of the model.

The partial widths for the two-body decays $\Upsilon_8 \to gg$, $\Upsilon_8 \to q\bar{q}$ (where $q = u$, d, s, or c) and $\Upsilon_8 \to g\gamma$ may thus be readily calculated from the Feynman diagrams shown in Figs. 1(a)–1(c) in terms of the wave functions at the origin (ϕ_0), for the Υ_8 systems. However, in ratios between decay rates of a given Υ_8 system, the wave function will cancel avoiding the model-dependent uncertainty in ϕ_0.

For each of the Υ_8 systems, let us define the quantity $\Gamma_{2(i)}$ by

$$\Gamma_{2(i)} = \Gamma(\Upsilon_{8(i)} \to gg) + 4\Gamma(\Upsilon_{8(i)} \to q\bar{q}) , \tag{2}$$

where q is a massless quark, and $i = 1$, 3, or 4. $\Gamma_{2(i)}$ should thus be a good approximation to the total widths because, to lowest order in QCD, the possible final states are gg, $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ where all the quarks have masses much smaller than m_b.

In terms of ϕ_0, the wave function at the origin, one obtains

$$\begin{aligned} \Gamma_{2(1)} &= \frac{5}{24}\alpha_s^2 \frac{\phi_0^2}{m_b^2} , \\ \Gamma_{2(3)} &= \frac{1}{3}\alpha_s^2 \frac{\phi_0^2}{m_b^2} , \\ \Gamma_{2(4)} &= \frac{29}{96}\alpha_s^2 \frac{\phi_0^2}{m_b^2} . \end{aligned} \tag{3}$$

Note that in the nonrelativistic approximation that we are using, the sum of the amplitudes for the three graphs of Fig. 1(a), for $\Upsilon_{8(3)}$ to decay to gg, vanishes. Furthermore, $\Upsilon_{8(1)}$ cannot decay to $q\bar{q}$ at lowest order in QCD since a scalar cannot go through a single gluon.

For convenience we define $R_{2(i)}(X)$ for a given final state X to be the ratio $\Gamma(\Upsilon_{8(i)} \to X)/\Gamma_{2(i)}$. This should be

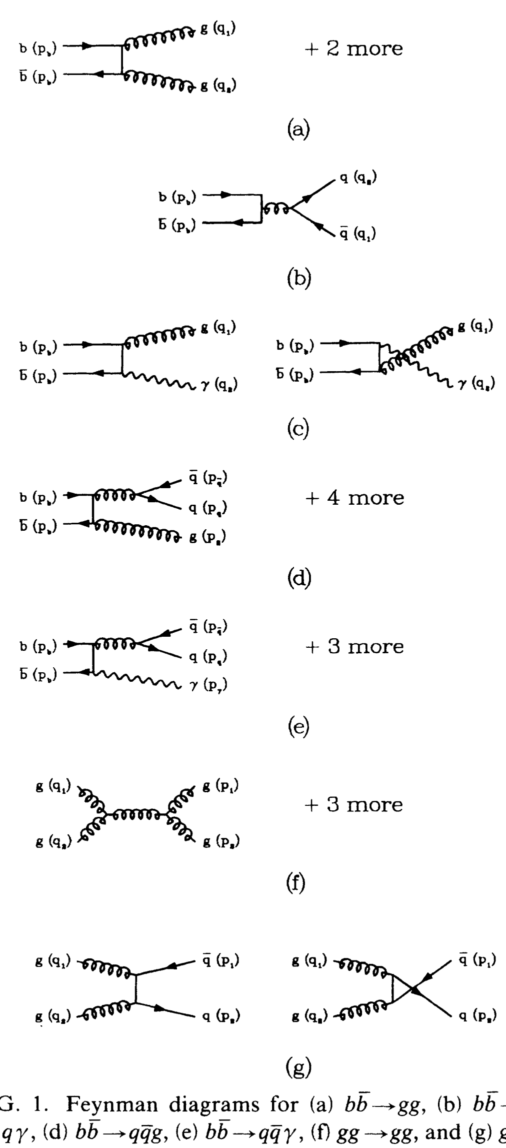


FIG. 1. Feynman diagrams for (a) $b\bar{b} \to gg$, (b) $b\bar{b} \to q\bar{q}$, (c) $b\bar{b} \to q\gamma$, (d) $b\bar{b} \to q\bar{q}g$, (e) $b\bar{b} \to q\bar{q}\gamma$, (f) $gg \to gg$, and (g) $gg \to q\bar{q}$.

a good approximation to the branching ratio of $\Upsilon_{8(i)} \to X$ to the extent that $\Gamma_{2(i)}$ is an approximation to the total width. R_2 may also be used to calculate the branching ratio of $\Upsilon(4S) \to X$ provided we assume that all the non-$B\bar{B}$ decays go through a single Υ_8 state. If we let

$$F = 1 - B(\Upsilon(4S) \to B\bar{B}) , \tag{4}$$

then under the assumptions that $R_2(X) \approx B(\Upsilon_8 \to X)$ and that all the non-$B\bar{B}$ decays go through the Υ_8 state, we may conclude that

$$B(\Upsilon(4S) \to X) = FR_2(X) . \tag{5}$$

Experimental determination of F is not presently available. The deviation between theory and experiment of the semileptonic branching ratio for B mesons may be used as one indication that $F \le 0.2$.

Table I shows expressions for R_2 for the two-body decays $\Upsilon_{8(i)} \to gg$, $\Upsilon_{8(i)} \to q\bar{q}$, where q is an essentially massless quark (i.e., u, d, or s), $\Upsilon_{8(i)} \to c\bar{c}$, and $\Upsilon_{8(i)} \to g\gamma$.

Thus the $\Upsilon_{8(i)}$, to lowest order, decays predominantly to gg and it also decays to $g\gamma$ with $R_{2(1)}(g\gamma) = 1.3 \times 10^{-2}$. In the case of $\Upsilon_{8(3)}$ the state decays predominantly to a quark pair with no $g\gamma$ while for the $\Upsilon_{8(4)}$, $R_{2(4)}(gg) = 0.17$, $R_{2(4)}(g\gamma) = 2.2 \times 10^{-3}$, and for each quark, $R_{2(4)}(q\bar{q}) = 0.21$ where we have taken $\alpha_s = 0.3$. Now let us consider the three-body decays $\Upsilon_8 \to q\bar{q}g$ and $q\bar{q}\gamma$, where $q = u$, d, s, or c. These decays proceed through the Feynman diagrams in Figs. 1(d) and 1(e). The differential rate for these is given by [12]

TABLE I. Expressions for R_2 are shown for various two-body decays of Υ_8 systems calculated to lowest order in QCD. In the case of $\Upsilon_8 \to c\bar{c}$, $x = m_c^2/4m_b^2$.

Process	$R_{2(1)}$	$R_{2(3)}$	$R_{2(4)}$
$\Upsilon_8 \to gg$	1	0	$\frac{5}{29}$
$\Upsilon_8 \to q\bar{q}$	0	$\frac{1}{4}$	$\frac{6}{29}$
$\Upsilon_8 \to c\bar{c}$	0	$\frac{1}{4}(2x+1)\sqrt{1-4x}$	$\frac{6}{29}(2x+1)\sqrt{1-4x}$
$\Upsilon_8 \to g\gamma$	$\frac{8}{15}\alpha/\alpha_s$	0	$\frac{8}{87}\alpha/\alpha_s$

$$\frac{dR_2}{dy\,dz} = \frac{NK}{\pi}\left[\left[A + B\frac{z^2}{(1-y)^2}\right] f(x,y,z) + Cg(x,y,z)\right] , \tag{6}$$

where the kinematic variables x, y, and z are defined by

$$x = \frac{m_q^2}{4m_b^2},\quad y = \frac{(p_q + p_{\bar{q}})^2}{4m_b^2},\quad z = \frac{2p_g \cdot (p_q - p_{\bar{q}})}{4m_b^2} ; \tag{7}$$

and where $4x \le y \le 1$ and $|z| \le \sqrt{(y-4x)/y}\,(1-y)$. f and g are defined to be

$$f(x,y,z) = -\frac{z^4 + 4z^2 y(2x+1) - (1-y)^2[(1+y)^2 + 8x(y-4x-2)]}{[(1-y)^2 - z^2]^2} ,$$
$$g(x,y,z) = \frac{yz^2 + (1-y)^2(y+4x)}{y^2(1-y)^2} . \tag{8}$$

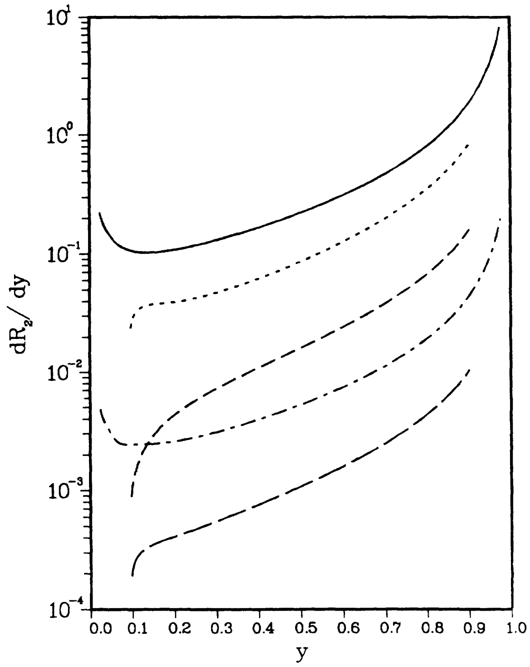
The values of the parameters N, K, A, B, and C, are given in Table II for the cases $\Upsilon_8 \to q\bar{q}g$ and $\Upsilon_8 \to q\bar{q}\gamma$ where q is a quark of charge e_q. In addition, we have considered the case denoted by $\Upsilon_8 \to (q\bar{q})_1 g$ where we project out the color singlet part of the $q\bar{q}$ system. In this case, $R_{2(1)}((c\bar{c})_1 g)$ is 0 to the relevant order in QCD because both $\Upsilon_{8(1)}$ and $(c\bar{c})_1$ must couple to at least two gluons.

Figure 2(a) shows a graph of $dR_{2(4)}/dy$ for various possible final states. $dR_{2(4)}(s\bar{s}g)/dy$ is shown as a solid line, where we have taken $m_s = 0.5$ GeV, $m_b = 5$ GeV, and $\alpha_s = 0.3$. Likewise $dR_{2(4)}(c\bar{c}g)/dy$ is shown (dots) as well as $dR_{2(4)}((c\bar{c})_1 g)/dy$ (dashes). $\sum dR_{2(4)}((q\bar{q})_1\gamma)/dy$ is shown (dot-dash) where the sum is taken over $q = u,d,s,$and c (here we have used the constituent mass $m_u = m_d = 0.3$ GeV) as well as $dR_{2(4)}(c\bar{c}\gamma)/dy$ (long dashes). Figures 2(b) and (c) show similar plots for $dR_{2(3)}/dy$ and $dR_{2(1)}/dy$ respectively.

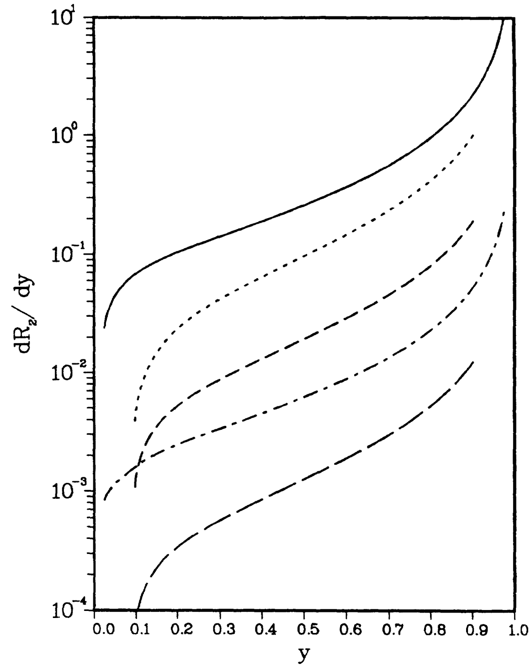
Note that in the case of $\Upsilon_{8(3)}$ and $\Upsilon_{8(4)}$ these processes become infrared divergent in the limit that $y \to 1$ corresponding to the fact that this graph is a radiative correction to $\Upsilon_{8(3)} \to q\bar{q}$. This can be seen by the sharp rise in the curves of Fig. 2 as $y \to 1$. Infrared cutoffs will therefore be necessary to get the total branching ratios. If we cut off the infrared divergence by taking the energy of the gluon, $E_g \ge 700$ MeV [which is equivalent to introducing the cut $y \le 0.86$ since $E_g = (1-y)m_b$] and $E_\gamma \ge 200$ MeV (which is equivalent to introducing the cut $y \le 0.98$), the curves in Fig. 2 may be integrated yielding the total values for R_2. In the $\Upsilon_{8(4)}$ case these are $R_{2(4)}(c\bar{c}g) = 0.10$, $R_{2(4)}(s\bar{s}g) = 0.25$, $R_{2(4)}((c\bar{c})_1 g) = 0.018$, $\sum R_{2(4)}(q\bar{q}\gamma) = 0.012$, and $R_{2(4)}(c\bar{c}\gamma) = 0.0025$.

TABLE II. The coefficients of Eq. (6), calculated using the Feynman diagrams in Figs. 1(d) and 1(e), are given.

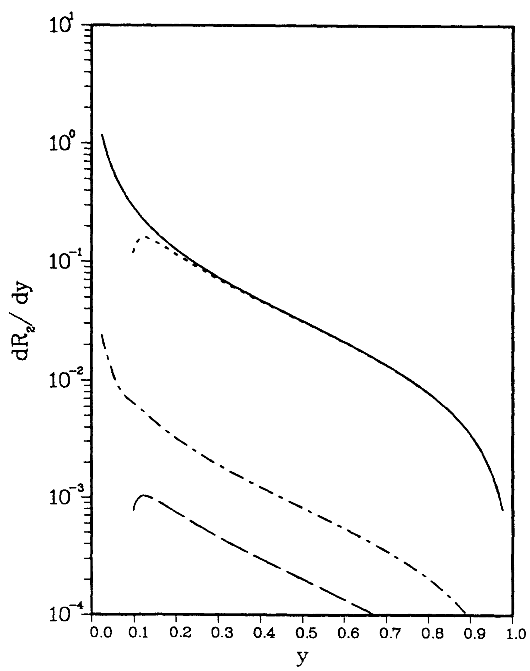
Process	N	K	A	B	C
$\Upsilon_{8(1)} \to q\bar{q}g$	$\frac{3}{40}$	α_s	0	0	$\frac{5}{3}$
$\Upsilon_{8(3)} \to q\bar{q}g$	$\frac{1}{64}$	α_s	$\frac{14}{3}$	6	0
$\Upsilon_{8(4)} \to q\bar{q}g$	$\frac{3}{232}$	α_s	$\frac{14}{3}$	6	$\frac{5}{3}$
$\Upsilon_{8(1)} \to (q\bar{q})_1 g$	0	0	0	0	0
$\Upsilon_{8(3)} \to (q\bar{q})_1 g$	$\frac{1}{64}$	α_s	$\frac{4}{3}$	0	0
$\Upsilon_{8(4)} \to (q\bar{q})_1 g$	$\frac{3}{232}$	α_2	$\frac{4}{3}$	0	0
$\Upsilon_{8(1)} \to q\bar{q}\gamma$	$\frac{3}{40}$	α	0	0	$4e_b^2$
$\Upsilon_{8(3)} \to q\bar{q}\gamma$	$\frac{1}{64}$	α	$8e_q^2$	0	0
$\Upsilon_{8(4)} \to q\bar{q}\gamma$	$\frac{3}{232}$	α	$8e_q^2$	0	$4e_b^2$



(a) $dR_{2(4)}/dy$



(b) $dR_{2(3)}/dy$



(c) $dR_{2(1)}/dy$

FIG. 2. $dR_{2(4)}/dy$ vs y for the processes $s\bar{s}g$ (solid), $c\bar{c}g$ (dots), $(c\bar{c})_1 g$ (short dashes). $\sum q\bar{q}\gamma$ (dash-dotted), $c\bar{c}\gamma$ (long dashes). (b) $dR_{2(3)}/dy$ vs y for the same final states as in (a). (c) $dR_{2(1)}/dy$ vs y for the same final states as in (a) except $(c\bar{c})_1 g$ is not shown since it is 0 to this order in QCD.

In the $\Upsilon_{8(3)}$ case these are $R_{2(3)}(c\bar{c}g)=0.11$, $R_{2(3)}(s\bar{s}g)=0.28$, $R_{2(3)}((c\bar{c})_1 g)=0.02$, $\sum R_{2(3)}(q\bar{q}\gamma)=0.013$, and $R_{2(3)}(c\bar{c}\gamma)=0.0030$; while in the $\Upsilon_{8(1)}$ case these are $R_{2(1)}(c\bar{c}g)=0.039$, $R_{2(1)}(s\bar{s}g)=0.11$, $R_{2(1)}((c\bar{c})_1 g)=0$, $\sum R_{2(1)}(q\bar{q}\gamma)=0.0028$, and $R_{2(1)}(c\bar{c}\gamma)=0.000\,28$ (see Table III).

III. IMPLICATIONS FOR SEMI-EXCLUSIVE RATES

A. Final state ψ or ϕ

In order to calculate the rate of ψ and ϕ production we will use a color evaporation model [13]. In the case of ψ for example, this model assumes that a fixed fraction of $c\bar{c}$ pairs produced by the above $c\bar{c}g$ process with center-of-mass energy below a certain threshold T are converted to ψ mesons. The conversion factor from $c\bar{c}$ pairs to ψ mesons we will denote by C_ψ. Note that the kinematics of this model imply that the ψ is always at its maximum possible energy so it will always pass the cut in (1).

To make estimates for ψ production we will take the threshold to be 3.7 GeV. Furthermore, theoretical uncertainties are taken into account by varying α_s between 0.25 and 0.3, m_c from 1.2 to 1.6 GeV, and m_b from 4.5 to 5 GeV. Also, following Ref. [12], we take C_ψ to range from $\frac{1}{3}$ to $\frac{1}{8}$.

Using these parameters, we find that for $\Upsilon_{8(4)}$, $R_{2(4)}(\psi+X)$ ranges from 1.0×10^{-4} to 1.4×10^{-3} while for $\Upsilon_{8(3)}$, $R_{2(3)}(\psi+X)$ ranges from 2.0×10^{-5} to 3.7×10^{-4}. In the $\Upsilon_{8(1)}$ case $R_{2(1)}(\psi+X)$ ranges from 6×10^{-3} to $r\times10^{-4}$.

Consider now the predictions for $\Upsilon_8\to\psi+X$ in relation to the observed experimental results. If we push F to its upper limit of 0.2 and consider the values of $R_{2(i)}(\psi+X,E_\psi\geq2$ GeV), we find that $B(\Upsilon(4S)\to\psi+X,E_\psi\geq2$ GeV) is $\leq3\times10^{-4}$ for $\Upsilon_{8(4)}$ theories, $\leq7\times10^{-5}$ for $\Upsilon_{8(3)}$ theories, and $\leq1.2\times10^{-3}$ for $\Upsilon_{8(1)}$ theories. It would thus appear that $\Upsilon_{8(1)}$ theories are the only ones that are not inconsistent with (1), as they could be within 1 to 2σ of the observed value while the other cases appear an order of magnitude or so smaller than the observation and are thus excluded.

In models involving the state $b\bar{b}g$, ψ mesons may also be produced by the constituent gluon interacting with a gluon resulting from the annihilation of the $b\bar{b}$. Thus the Υ_8 component of the system decays to gg and one of the gluons thus produced rescatters off of the constituent gluon producing a final state that could contain ψ mesons. In analyzing this model, our approach will be to calculate the branching ratio to ψ mesons in comparison to the decay width contribution of such rescattering events. This will, for instance, give an upper bound on the branching ratio to ψ mesons.

Let us consider first the rescattering process itself. To lowest order in QCD the possible processes are $g(q_1)g(q_2)\to g(p_1)g(p_2)$ and $g(q_1)g(q_2)\to q(p_1)\bar{q}(p_2)$. The differential cross sections for these 2-to-2 processes obtained from the Feynman diagrams in Figs. 1(f) and 1(g) are well known [14] but we reproduce them here purely for the sake of completion:

$$\frac{d\sigma(gg\to gg)}{d\eta}=\frac{9\pi}{16}\alpha_s^2\frac{(\eta^2+3)^3}{(1-\eta^2)^2}\,\frac{1}{s}\,, \tag{9a}$$

$$\frac{d\sigma(gg\to q\bar{q})}{d\eta}=\pi_{96}\alpha_s^2\frac{9\eta^2+7}{1-\eta^2}\left[1+\eta^2 + 8x\frac{1-4x-\eta^2}{1-\eta^2}\right]\frac{1}{s}\,, \tag{9b}$$

where $s=(q_1+q_2)^2$, $t=(q_1-p_1)^2$, $u=(q_1-p_2)^2$, $x=m_q^2/s$, and $\eta=(u-t)/s$. Note that the cross section for $gg\to gg$ has bad infrared behavior (i.e., it diverges when $|y|\to1$). This corresponds to the t- or u-channel propagator carrying low momentum. We know however that perturbative QCD is not a good description in this regime, so we will consider only events where $|t|,|u|\geq Q_{\min}^2$.

The total cross sections for these processes are

$$\sigma(gg\to gg)=\frac{9\pi\alpha_s^2}{8s}\left[\frac{1}{3}\frac{v(129-32v^2-v^4)}{1-v^2} - 16\,\mathrm{arctanh}\,v\right], \tag{10a}$$

$$\sigma(gg\to q\bar{q})=\frac{\pi\alpha_s^2}{12s}\left[8(x^2+4x+1)\mathrm{arccosh}\left[\frac{1}{4x}\right]^{1/2} -(31x+7)\sqrt{1-4x}\right] \tag{10b}$$

where $v=1-2Q_{\min}^2/8$ is the cutoff in $|y|$ corresponding to the $|t|$, $|u|\leq Q_{\min}^2$. Notice that if we take the limit $s\to\infty$ while holding $Q_{\min}$ constant, $\sigma(gg\to gg)$ approaches the constant value $\sigma_0=9\alpha_s^2/Q_{\min}^2$.

Referring to Fig. 3, which plots σ/σ_0 as a function of

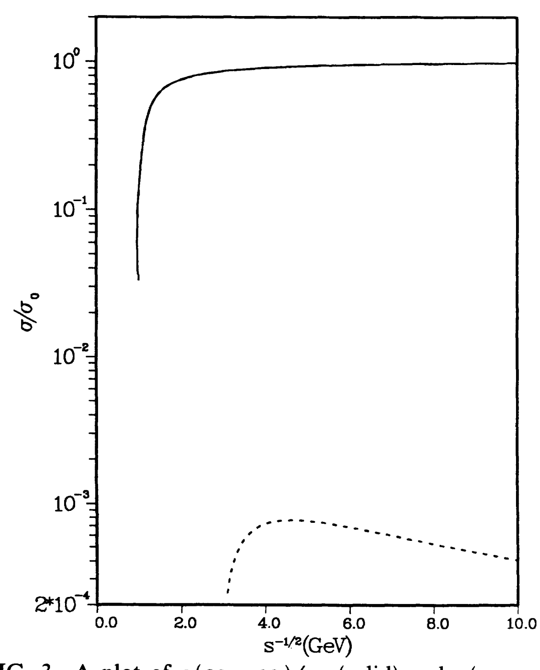


FIG. 3. A plot of $\sigma(gg\to gg)/\sigma_0$ (solid) and $\sigma(gg\to c\bar{c})/\sigma_0$ (dotted) vs $\sqrt{s}$ taking $Q_{\min}=0.7$ GeV.

TABLE III. A summary of various branching ratios given in the text are listed for $\Upsilon_{8(1)}, \Upsilon_{8(3)}$, and $\Upsilon_{8(4)}$ according to the assumptions described.

Quantity	$\Upsilon_{8(1)}$	$\Upsilon_{8(3)}$	$\Upsilon_{8(4)}$
$R_2(gg)$	1	0	0.17
$R_2(q\bar{q})$	0	0.25	0.21
$R_2(g\gamma)$	1.3×10^{-2}	0	2.2×10^{-3}
$R_2(c\bar{c}g), E_g \geq 0.7$ GeV	0.039	0.11	0.10
$R_2(s\bar{s}g), E_g \geq 0.7$ GeV	0.11	0.29	0.25
$R_2((c\bar{c})_1 g), E_g \geq 0.7$ GeV	0	0.022	0.018
$R_2(c\bar{c}\gamma), E_\gamma \geq 0.2$ GeV	0.000 26	0.0030	0.0025
$\sum R_2(q\bar{q}\gamma), E_\gamma \geq 0.2$ GeV	0.0028	0.014	0.012
$R_2(\gamma + X), E_\gamma \geq 3$ GeV	1.3×10^{2}	1.0×10^{-3}	2.2×10^{-3}
$\sum R_2(q\bar{q}\gamma)$, 0.4 GeV $\leq E_\gamma \leq 0.6$ GeV	3.7×10^{-6}	2.0×10^{-3}	1.6×10^{-3}
$R_2(c\bar{c}+X), m_{c\bar{c}} \geq 6.5$GeV	0.009	0.25	0.21
$R_2(c\bar{c}+X)$, $m_{c\bar{c}} \geq 6.5$ GeV, $E_\gamma \geq 1$ GeV	0.5×10^{-4}	8.7×10^{-4}	7.3×10^{-4}
$R_2(s\bar{s}+X)$, $m_{s\bar{s}} \geq 6.5$ GeV, $E_\gamma \geq 1$ GeV	0.5×10^{-4}	4.4×10^{-4}	3.7×10^{-4}
$R_2(\psi+X)$	4×10^{-4}–6×10^{-3}	2×10^{-5}–3.7×10^{4}	1.0×10^{-4}–1.4×10^{-3}
$R_2(\phi+X)$	8×10^{-4}–1×10^{-2}	7×10^{-6}–1×10^{-4}	1.5×10^{-4}–1.8×10^{-3}
$R_2(\phi+X)/R_2(\psi+X)$	0.2–15	0.03–3.4	$0.2-8$
$R_2(\psi+\gamma+X)/R_2(\psi+X)$	0.0064	0.016	0.0085
$R_2(\eta_c+X)/R_2(\psi+X)$	0	$0.7-2.2$	$0.53-1.5$

$\sqrt{s}$ with $Q_{\min}=0.7$ GeV, we see that the ratio between $\sigma(gg\to c\bar{c})/\sigma_0$, represented by the dashed curve, and $\sigma(gg\to gg)/\sigma_0$, shown by the solid curve, is about 1.5×10^{-3} when $\sqrt{s}\approx 2m_c$.

To further specify the model we would need to know a structure function $f_g(q)$ which gives the probability that the gluon will have four-momentum q. We attempt to take into account such effects very crudely by assuming that the constituent gluon is on shell and is equally likely to have any energy between 0 and $E_{\max}\approx 1$ GeV.

Let us label the constituent gluon 1, the gluon that collides with it 2, and the other gluon 3. As above, we use the color evaporation model which implies that the criterion for possible ψ formation is that $m_{c\bar{c}}$ be between $2m_c$ and T. This, together with the above condition, may be expressed in terms of the energies E_1 and E_3 (E_2 is constrained by energy conservation), the energies of gluon 1 and 3, respectively, by

$$0 \leq E_1 \leq E_{\max} , \tag{11a}$$

$$\frac{m_b^2-\frac{1}{4}T^2}{m_b} \leq E_3 \leq \frac{m_b^2-m_c^2}{m_b} . \tag{11b}$$

Thus the probability that a gg collision in this model will have the correct kinematics to produce a ψ is the ratio R of the area of the Dalitz plot specified by (11a) and (11b) above to the area specified by just (11a).

If we take $E_{\max}=0.7$ GeV, $m_c=1.5$ GeV, and $T=3.7$ GeV, then $R=4\times 10^{-2}$. Furthermore, for a numerical estimate we take [15] $F=0.1$ and assume that gg rescattering dominates the Υ_8 decay then

$$B(\Upsilon(4S)\to\psi+X)=RF\frac{\sigma(gg\to c\bar{c})}{\sigma(gg\to gg)}=6\times 10^{-6} , \tag{12}$$

which is well below the observed value (1); hence, this process is unlikely to make an appreciable contribution to ψ production.

Let us now use the color evaporation model to obtain R_2 for ϕ production from $\Upsilon_8\to s\bar{s}g$. We use a threshold of 1.5 GeV where we take the range for C_ϕ to be about the same as C_ψ given above. We also take m_s varying from 0.4 GeV to 0.6 GeV. In the $\Upsilon_8(4)$ case, $R_{2(4)}(\phi+X)$ ranges from 1.5×10^{-4} to 1.8×10^{-3}; in the $\Upsilon_8(3)$ case, $R_{2(3)}(\phi+X)$ ranges from 7×10^{-6} to 1×10^{-4} while $R_{2(1)}(\phi+X)$ ranges from 8×10^{-4} to 1×10^{-2}.

It is not unreasonable to assume that C_ψ and C_ϕ are not drastically different, in which case the uncertainties in the ratio $R_2(\phi+X)/R_2(\psi+X)$ would tend to cancel. Also this ratio in our model should be the same as

$$\frac{B(\Upsilon(4S)\to\phi+X, E_\phi\geq 2\ \text{GeV})}{B(\Upsilon(4S)\to\psi+X, E_\psi\geq 2\ \text{GeV})} \tag{13}$$

and has the virtue of being independent of F. For numerical estimates we take $\frac{1}{2}\leq C_\phi/C_\psi\leq 2$, and find that for $\Upsilon_{8(4)}$, $R_{2(4)}(\psi)$ ranges from 0.2 to 8, for $\Upsilon_{8(3)}$, $R_{2(3)}(\phi)/R_{2(3)}(\psi)$ from 0.03 to 3.4, and for $\Upsilon_{8(1)}$, $R_{2(1)}(\phi)/R_{2(1)}(\psi)$ ranges from 0.2 to 15. Experimental bounds [1] on this ratio show it to be ≤ 1. Thus this model suggests that ϕ production should be near the current limits in $\Upsilon_{8(4)}$ and $\Upsilon_{8(1)}$ models although, strictly speaking, considerable suppression compared to ψ cannot be ruled out.

B. Final state η_c

Turning our attention now to η_c production for Υ_8 decay, we observe that a color-singlet $c\bar{c}$ pair produced from the graphs in Fig. 1(d) will have the same (0^{-+}

quantum number as the η_c when they have relative orbital angular momentum 0. Thus we expect that a $c\bar{c}$ pair below threshold in a color-singlet state is most likely to condense into an η_c. If we assume that the probability for a color-singlet $c\bar{c}$ pair to convert to an η_c is unity, we obtain $R_{2(4)}(\eta_c+X)$ ranging from 4×10^{-5} to 2.2×10^{-4}, $R_{2(3)}(\eta_c+X)$ ranging from 5×10^{-5} to 3.7×10^{-4}, and $R_{2(1)}((cc)_1 g)=0$. So, in particular, we expect that η_c may be suppressed in $\Upsilon_{8(1)}$ models. Based on the above assumptions, we may also calculate the ratios $R_{2(4)}(\eta_c+X)/R_{2(4)}(\psi+X)$, which ranges from 0.53 to 1.5 corresponding to $\frac{1}{8}\le C_\psi \le \frac{1}{3}$, while $R_{2(3)}(\eta_c+X)/R_{2(3)}(\psi+X)$, which ranges from 0.7 to 2.2.

C. D-meson final states

We must discuss decays to charm. We suppose that the c quarks hadronize into D mesons (here we use D generically to include D, D^*, etc.) which appear in the final state. Thus since $\Upsilon_{8(3)}$ and $\Upsilon_{8(4)}$ go to $c\bar{c}$, these states will give rise to hard $D\bar{D}$ pairs. If we consider the quantity $m_{c\bar{c}}$, the invariant mass of the $c\bar{c}$ pair, this quantity will be of the same mass as the initial Υ_8 system, about $2m_b$. On the other hand, if

$$\Upsilon(4S)\to B\bar{B}\to D\bar{D}+X$$

then $m_{D\bar{D}}$, the mass of a $D\bar{D}$ pair, must satisfy

$$m_{D\bar{D}} \le \frac{m_B^2+m_D^2}{2m_B^2} m_{4S} + \frac{m_B^2-m_D^2}{2m_B^2}\sqrt{m_{4S}^2-4m_B^2} = 6.35 \text{ GeV} \quad (14)$$

so a cut of $m_{D\bar{D}} \ge 6.5$ GeV will select only the non-$B\bar{B}$ events.

Let us now discuss the expected branching ratio in Υ_8 theories for $D\bar{D}$ pairs satisfying the above condition. Taking [15] $F=0.1$ we find $B(\Upsilon(4S)\to c\bar{c}+X, m_{c\bar{c}}\ge 6.5$ GeV) to be about 2.5% in $\Upsilon_{8(3)}$ theories and 2.1% in $\Upsilon_{8(4)}$ theories. In $\Upsilon_{8(1)}$ theories the only contribution is from $c\bar{c}g$. Integrating the spectrum in Fig. 2(c) subject to $m_{c\bar{c}}\ge 6.5$ GeV (which is equivalent to $y\ge 0.42$) we get $R_{2(1)}(c\bar{c}g, m_{c\bar{c}}\ge 6.5 \text{ GeV})=9\times10^{-3}$ corresponding to $B(\Upsilon(4S)\to c\bar{c}+X, m_{c\bar{c}}\ge 6.5 \text{ GeV})=9\times10^{-4}$.

There are difficulties in interpreting these numbers as branching ratios for $m_{D\bar{D}}\ge 6.5$ GeV since one must take into account the hadronization of the c quarks into D mesons. One would in fact expect the meson branching ratios to be less than the parton branching ratios given above.

A possible way to reduce this uncertainty is to consider instead the ratio between the production of $D\bar{D}$ pairs satisfying $m_{D\bar{D}}\ge 6.5$ GeV at the continuum and on the $\Upsilon(4S)$ resonance. We expect these two hadronization processes to be analogous since in both cases the $c\bar{c}$ has an energy of about 10 GeV. Thus if we define the ratio

$$W_C = \frac{\sigma(e^+e^0\to\Upsilon(4S)\to D\bar{D}, m_{D\bar{D}}\ge 6.5 \text{ GeV})_{\text{on resonance}}}{\sigma(e+e-\to D\bar{D}, m_{D\bar{D}}\ge 6.5 \text{ GeV})_{\text{continuum}}}, \quad (15a)$$

we expect that to the extent that the hadronization effects cancel,

$$W_D \approx \frac{\sigma(e^+e^-\to\Upsilon(4S))_{\text{on resonance}}}{\sigma(e^+e^-\to c\bar{c})_{\text{continuum}}} F R_2(c\bar{c}) . \quad (15b)$$

From experiment [9] we have that $\sigma(e^+e^-\to\Upsilon(4S))_{\text{on resonance}}=1.2$ nb while $\sigma(e^+e^-\to c\bar{c})_{\text{continuum}}=16\pi\alpha^2/(9s)=1.0$ nb so that

$$\frac{\sigma(e^+e^-\to\Upsilon(4S))_{\text{on resonance}}}{\sigma(e^+e^-\to c\bar{c})_{\text{continuum}}} = 1.2 . \quad (15c)$$

Taking $F=0.1$ we get $W_D=0.030$ in the case of the $\Upsilon_{8(3)}$ and $W_D=0.025$ in the case of the $\Upsilon_{8(4)}$. While these ratios (W_D) are too small to be easily observable in Υ_8 models, an experimental signal for these at appreciably larger values could indicate a source for (1) other than the octet mechanism being considered here.

In the case of $\Upsilon_{8(1)}\to c\bar{c}g$, the $c\bar{c}$ does not carry the full mass of the Υ_8 system. Thus the assumption of the cancellation of hadronization is less likely to be valid. We therefore expect that the result we get by applying this method will only be an upper bound since softer $c\bar{c}$ pairs are unlikely to give rise to D mesons passing the cut $m_{D\bar{D}}\ge 6.5$ GeV. We therefore estimate that in this case $W_D\le 0.0010$.

By the same logic $\Upsilon_8\to s\bar{s}$ gives rise to $\Upsilon(4S)\to K\bar{K}$ with $m_{K\bar{K}}\ge 6.5$ GeV. Since the numbers calculated above are not sensitive to the mass or charge of the quark, the above estimates for branching ratios into hard $D\bar{D}$ pairs also apply to hard $K\bar{K}$ pairs.

We may also consider measuring the ratio

$$W_K = \frac{\sigma(e^+e^-\to\Upsilon(4S)\to K\bar{K}, m_{K\bar{K}}\ge 6.5 \text{ GeV})_{\text{on resonance}}}{\sigma(e+e-\to K\bar{K}, m_{K\bar{K}}\ge 6.5 \text{ GeV})_{\text{continuum}}} \quad (15d)$$

which is analogous to the quantity W_D defined above. Following the method of estimation which we used in that case, we obtain $W_K=0.12$ for $\Upsilon_{8(3)}$, $W_K=0.10$ for $\Upsilon_{8(4)}$, and $W_K\le 0.004$ for $\Upsilon_{8(1)}$. The numbers here are larger than in the D case because the charge of the s quark reduces the continuum contribution. Experimentally this case may therefore be slightly more accessible.

Another method of using D mesons to tag a non-$B\bar{B}$ decay is to look for single D mesons with energy greater than the kinematic threshold for $B\to D+X$. This has been done [1] experimentally for the specific case of D^* mesons giving

$$B(\Upsilon(4S)\to D^{*\pm}+X, \xi\ge 0.473)\le 7.4\% , \quad (16a)$$

where $\xi=|\mathbf{p}_D|/E_{\text{beam}}$ (denoted by x in Ref. [1]). The condition that $\xi\ge 0.473$ roughly corresponds on the parton level to $E_c\ge\frac{1}{2}m_b$, a condition that is always satisfied by one of the c quarks in the final state $c\bar{c}$ or $c\bar{c}g$. Let us model D^* formation by supposing that a c quark has a fixed probability F_* of forming a D^*. Thus, in our model

$$B(\Upsilon(4S)\to D^{*\pm}+X,\xi\geq 0.473)\approx F_* F R_{(2)}(c\bar{c}+c\bar{c}g)\ . \tag{16b}$$

Taking F to be about 0.1, the products $F_{(2)}(c\bar{c},E_c$ or $E_{\bar{c}}\geq\frac{1}{2}m_b)$ for $\Upsilon_{8(1)}$, $v_{8(3)}$, and $\Upsilon_{8(4)}$ respectively are 4×10^{-3}, 2.5×10^{-2}, and 2×10^{-2}. Since $F_*\leq 1$, all the cases appear to be consistent with (16a).

In passing let us mention that one can also consider hard $D\bar{D}$ pairs and $K\bar{K}$ pairs associated with a hard photon. Let us consider $m_{D\bar{D}}$ or $m_{K\bar{K}}\geq 6.5$ GeV with $E_\gamma\geq 1$ GeV, which is equivalent to taking $0.42\leq y\leq 0.8$. Under these conditions, we can calculate $R_{2(4)}(c\bar{c}\gamma)=7.3\times 10^{-4}$, $R_{2(3)}(c\bar{c}\gamma)=8.7\times 10^{-4}$, and $R_{2(1)}(c\bar{c}\gamma)=0.5\times 10^{-4}$, while $R_{2(4)}(s\bar{s}\gamma)=3.7\times 10^{-4}$, $R_{2(3)}(s\bar{s}\gamma)=4.4\times 10^{-4}$, and $R_{2(1)}(s\bar{s}\gamma)=0.5\times 10^{-4}$. Because of the hadronization difficulties we are unable to translate these parton model calculations to rates of $D\bar{D}\gamma$ ($K\bar{K}\gamma$) final states so the numbers given should be interpreted as upper bounds.

D. Final states containing photons

Since $\Upsilon_{8(1)}$ and $\Upsilon_{8(4)}$ decay to $g\gamma$, if these states are present, they should lead to the production of a hard photon via $\Upsilon(4S)\to\gamma+X$, where the photon has $E_\gamma\approx 5$ GeV. This is experimentally significant in that a photon with energy $E_\gamma\geq 2.83$ GeV cannot arise from $\Upsilon(4S)\to B\bar{B}$ and so would be another configuration of non-$B\bar{B}$ decays of $\Upsilon(4S)$. Thus again with [15] $F\approx 0.1$ the branching ratio is 1.3×10^{-3} in the case of $\Upsilon_{8(1)}$ and 2.2×10^{-4} in the case of $\Upsilon_{8(4)}$ [16]. In all of the Υ_8 states the process $\Upsilon_8\to q\bar{q}\gamma$ gives a photon spectrum which can be obtained from Fig. 2 by recalling that $E_\gamma=(1-y)m_b$. One can see that $\Upsilon_{8(3)}$ has a softer spectrum than the other cases. If one wants a hard photon from $\Upsilon_{8(3)}$, which cannot come from $B\bar{B}$, then since $\Upsilon_{8(3)}$ does not have direct (i.e., $g\gamma$) photon production, we may consider imposing the cut $E_\gamma\geq 3$ GeV. Integrating the spectra in Fig. 2(b) we thus obtain $\sum R_{2(3)}(q\bar{q}\gamma,E_\gamma\geq 3\text{ GeV})+1\times 10^{-3}$ which gives a branching ratio of 10^{-4} with $F=0.1$ as previously.

If indeed an $\Upsilon_{8(1)}$ is truly responsible for (1), we may further search for the decay $\Upsilon_{8(1)}\to g\gamma$ in the experimental data. According to Ref. [17], at 90% C.L.,

$$\begin{aligned}
&B(\Upsilon(4S)\to g\gamma,3\text{ GeV}\leq E_\gamma\leq 5.1\text{ GeV})\leq 1.4\times 10^{-3}\ ,\\
&B(\Upsilon(4S)\to g\gamma,4\text{ GeV}\leq E_\gamma\leq 5\text{ GeV})\leq 0.8\times 10^{-3}\ , \qquad (17)\\
&B(\Upsilon(4S)\to g\gamma,5.1\text{ GeV}\leq E_\gamma\leq 5.29\text{ GeV})\leq 2.3\times 10^{-3}\ ,
\end{aligned}$$

whereas in our model, $B(\Upsilon(4S)\to g\gamma)=0.013F\approx 1.3\times 10^{-3}$ for $F\approx 0.1$. The energy of the photon should be about 5 GeV though it could vary somewhat according to the detailed dynamics of the model. In any case, the branching ratio is similar to the limits above. A reduction of the experimental limit by about a factor of 3 could thus prove decisive for the $\Upsilon_{8(1)}$ possibility.

Also in Ref. [17] it was found that the branching ratio for soft photons with 0.4 GeV $\leq E_\gamma\leq 0.6$ GeV is $\leq 0.65\%$. This is well within our model for $\Upsilon_{8(1)}$ since

$$\sum R_{2(1)}(q\bar{q}\gamma,0.4\text{ GeV}\leq E_\gamma\leq 0.6\text{ GeV})=3.7\times 10^{-6}\ .$$

Another ratio that should be even more independent of theoretical uncertainties than the above is $R_2(\psi+\gamma+X)/R_2(\psi+X)$ with hard γ and ψ. If we calculate it as in the previous examples we find it is given to a good approximation by 0.085 for $\Upsilon 8(4)$, 0.016 for $\Upsilon_{8(3)}$, and 0.0064 for $\Upsilon_{8(1)}$.

IV. METHODS FOR ESTIMATING F

It would be useful at this point to have a crude estimate of the total widths that these Υ_8 states would have. To do this, let us assume that ϕ_0 is the same for Υ_8 and $\Upsilon(1S)$. Taking our cue from potential models, we will also assume that ϕ_0 will be the same for 0^{-+} and 1^{--} ground states in heavy quarkonia.

Thus

$$\begin{aligned}
&\frac{\Gamma 2(1)}{\Gamma(\eta_b)}=\frac{5}{16}\ ,\\
&\frac{\Gamma 2(3)}{\Gamma(\eta_b)}=\frac{1}{2}\ , \qquad (18)\\
&\frac{\Gamma 2(4)}{\Gamma(\eta_b)}=\frac{29}{64}\ .
\end{aligned}$$

Now,

$$\Gamma(\eta_b)\approx 6\left[\frac{\alpha_s}{\alpha}\right]^2\Gamma(\Upsilon(1S)\to e^+e^-)=16\text{ MeV}\ , \tag{19}$$

from which we conclude that $\Gamma_{2(1)}\approx 4.3$ MeV, $\Gamma_{2(3)}\approx 6.8$ MeV, and $\Gamma_{2(4)}\approx 6.2$ MeV. The total width of the $\Upsilon(4S)$ is 23.8 MeV, so if the assumptions about the ϕ_0 used to derive (18) and (19) are accurate, then the upper bound on F is about 18% (in the case of $\Upsilon_{8(1)}$).

Another relatively clean method for determining F is, of course, by measuring the branching ratio into photons, as already discussed above.

V. SUMMARY AND CONCLUSIONS

We have considered $b\bar{b}$ octet annihilation as a source of the observed rate of $\Upsilon(4S)\to\psi+X$. Despite the fact that the probability of such an admixture is not known experimentally and we also do not know how to calculate it theoretically, we show that QCD perturbation theory implies that the observed rate [1] is uncomfortably high for this class of models. Among these models, those in which a $b\bar{b}$ octet is in a spin-0 configuration appear to be the least inconsistent with the observed ψ rate. This specific configuration leads to the emission of hard photons with a relatively high branching ratio. Existing measurements by the CUSB group [17], however, seem on the verge of ruling out such photons. Improved determinations of the ψ and the photon rate are clearly highly desirable. We also give the implications of these models for many final states such as ϕ, $D\bar{D}$, $K\bar{K}$, η_c, etc. and we suggest a search for $D\bar{D}$ and $K\bar{K}$ with an invariant mass greater than 6.5 GeV. The charm-meson final states are relatively enhanced if the $b\bar{b}$ octet state has total spin 1 and the photon final state is enhanced if the state has spin 0. More experimental information on these issues will clearly elucidate the dynamics of these possible admixtures.

Note added. Since the paper was submitted for publication, CLEO has analyzed more data. The new CLEO results indicate that their original signal reported in Ref. [1] needs significant correction due to the contribution from the continuum. See, e.g., Y. Kubota *et al.*, *Psi Production in e^+e^- Annihilation in the $\Upsilon(4S)$ Energy Region* submitted to Lepton-Photon Conference, Geneva (1991). (We are particularly indebted to Ed Thorndike and Sheldon Stone for communications.) From the perspective of the current theoretical study, this development, if confirmed, would not be a complete surprise as this study suggests that the original signal was too large to explain in the model(s) under discussion in this paper. We hope that the tests and the analysis discussed in this work would still be implemented as more data become available to improve our understanding of the mixing under discussion.

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