Photon-photon scattering contribution to the anomalous magnetic moment of the muon

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The dominant contribution to the anomalous magnetic moment of the muon in sixth order is the photon-photon scattering contribution $a_\mu^{(6)}(\gamma\gamma)$. An accurate and reliable value is needed in order to properly compare theory with the new muon $g-2$ experiment now underway at Brookhaven National Laboratory. Our result is $a_\mu^{(6)}(\gamma\gamma)=20.9469(18)(\alpha/\pi)^3$. This agrees extremely well with Kinoshita's result $a_\mu^{(6)}(\gamma\gamma)=20.9471(29)(\alpha/\pi)^3$ where the errors indicate the 90% C.L. This invalidates an old result (1975) of Samuel and Chlouber, for $a_\mu^{(6)}(\gamma\gamma)$, as well as that for the electron, Samuel and Chlouber (1978), and Samuel (1986).

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In the light of the new $g-2$ muon experiment underway at Brookhaven National Laboratory (BNL), it is essential that the theoretical prediction of quantum electrodynamics (QED) be as accurate and reliable as possible. Then, if the hadronic contribution can be determined more precisely, it will be possible to check the weak-interaction contribution predicted by the electroweak model of Weinberg, Glashow, and Salam, and thus test the standard model (SM). This will provide very tight constraints on theories "beyond the standard model," such as supersymmetry, two-Higgs-doublet models, etc. For a recent review of the experimental situation see Bailey *et al.* [1] and for a review of the theoretical situation see Kinoshita and Marciano [2]. See also Ref. [3].

In presenting our new result we wish to retract our 1975 result [4] for the light-by-light scattering contribution to $g-2$ of the muon, $a_\mu^{(6)}(\gamma\gamma)$. Moreover our earlier result [5,6(a)] in the case of the electron is also incorrect. In both cases our previous results overestimated the contribution of the integrable singularity which occurs in these 7-dimensional integrals and the errors were overly optimistic. In both cases, not enough points were used in the Monte Carlo integration routines.

The dominant contribution to the muon anomaly

$$a_\mu=\frac{g-2}{2} \tag{1}$$

in sixth order is the photon-photon scattering contribution $a_\mu^{(6)}(\gamma\gamma)$. The theoretical error in the quantum electrodynamics (QED) contribution to a_μ, a_μ(QED), is dominated by the error in the sixth-order term (41×10^{-12}) and this is dominated by the error in $a_\mu^{(6)}(\gamma\gamma)$ (36×10^{-12}). See Eq. (8). This quantity can be expressed, due to the large ratio m_μ/m_e, as

$$a_\mu^{(6)}(\gamma\gamma)=\left[A\ \ln m_\mu/m_e+B+O\left(\frac{m_e}{m_\mu}\right)\right]\left(\frac{\alpha}{\pi}\right)^3, \tag{2}$$

where A is known analytically [6(b)]:

$$A=2\pi^2/3\ . \tag{3}$$

This result has recently been corroborated by two groups from Novosibirsk [6(c)]. Unfortunately, however, B is not yet known analytically. Thus $a_\mu^{(6)}(\gamma\gamma)$ is known only numerically, in terms of a 7-dimensional integral, which may be evaluated using an adaptive Monte Carlo multidimensional integration routine. In this Brief Report, we present the results which we have obtained using the program VEGAS.

The first calculation of $a_\mu^{(6)}(\gamma\gamma)$ was made by Aldins, Brodsky, Dufner, and Kinoshita [7]. Their result was

$$a_\mu^{(6)}(\gamma\gamma)=18.4(1.1)\left(\frac{\alpha}{\pi}\right)^3 . \tag{4}$$

Subsequent calculations, which increased the accuracy, also increased the value of $a_\mu^{(6)}(\gamma\gamma)$. Chang and Levine [8] obtained

$$a_\mu^{(6)}(\gamma\gamma)=20.77(43) \tag{5}$$

and Peterman's result [9] was

$$a_\mu^{(6)}(\gamma\gamma)=19.76(16)\ . \tag{6}$$

Calmet and Peterman [10] obtained

$$a_\mu^{(6)}(\gamma\gamma)=19.79(16)\ ,$$

while Samuel and Chlouber [4] obtained the somewhat higher result

$$a_\mu^{(6)}(\gamma\gamma)=21.32(5)\ . \tag{7}$$

The best value for $a_\mu^{(6)}(\gamma\gamma)$ was obtained by Kinoshita [11] in 1989. His result is

$$a_\mu^{(6)}(\gamma\gamma)=20.9471(29)\left(\frac{\alpha}{\pi}\right)^3 . \tag{8}$$

This result was obtained using VEGAS with 10 iterations at 1.4×10^8 function calls each, followed by 20 iterations at 2.8×10^8 function calls each.

In view of the conflict between Eq. (7) and Eq. (8) and its importance in the theoretical prediction of a_μ(QED) a new, independent, more accurate calculation of $a_\mu^{(6)}(\gamma\gamma)$ was undertaken.

We first used the Monte Carlo routine SPCINT. Although our result is consistent with Eq. (8) the accuracy is very poor.

We now turn to the results obtained with VEGAS [12]. For reasons not really understood, we obtain an increase in convergence by a factor of 5 in going from SPCINT to VEGAS; i.e., to obtain the same accuracy one has to run SPCINT 25 times longer than VEGAS. This may be due to the different ways SPCINT and VEGAS adapt the integration grid. Therefore we will present our results from using VEGAS only. Our results from VEGAS are

$$a_e^{(6)}(\gamma\gamma)=0.371\,15(84) \tag{9}$$

and

$$a_\mu^{(6)}(\gamma\gamma)=20.9469(18). \tag{10}$$

Happily we have just received word [13] that the corresponding contribution for the electron, $a_e^{(6)}(\gamma\gamma)$, has just been evaluated analytically by Remiddi and Laporta.

Their result is

$$a_e^{(6)}(\gamma\gamma)=0.371\,005\,292\ . \tag{11}$$

One can see that Eq. (9) agrees with Eq. (11) extremely well within the error estimate. Our result is much more accurate than the error estimate would indicate. (We will return to the question of error estimate later.) Thus, we are able to use $a_e^{(6)}(\gamma\gamma)$ as a check on our program, by merely changing $m_\mu/m_e \rightarrow 1$, to go from the computation of $a_\mu^{(6)}(\gamma\gamma)$ to $a_e^{(6)}(\gamma\gamma)$. This gives us confidence that our program for the muon case is reliable. We use the integrand of Ref. [7].

Our result for $a_\mu^{(6)}(\gamma\gamma)$ given in Eq. (10) was obtained on our IBM 3090-200S. It required approximately 1500 h of CPU time and 5×10^{10} function calls (170 iterations at an average of 3×10^{8} function calls per iteration). We believe our answer and the error estimate is reliable; however, one is never certain that enough function calls per iteration have been used. We, of course, used double precision and tried different random-number generators.

Following the convention used by Kinoshita our error estimates are 90% C.L. It can be seen that our result in Eq. (10) agrees very well with Kinoshita's result in Eq. (8). In fact, they agree much closer than one would expect from the error estimates. This suggests that these results are much more precise than the error estimate would indicate. Thus we present our final result with a 1σ error:

$$a_\mu^{(6)}(\gamma\gamma)=20.9469(11)\ . \tag{12}$$

This reduces the error of this contribution to 14×10^{-12}. In fact the error may be as small as the difference between Eq. (12) and Eq. (8), which is 3×10^{-12}.

The new experiment at BNL will reduce the experimental error to

$$\Delta a_\mu=\pm5\times10^{-10}\ . \tag{13}$$

It can be seen from Eq. (12) that $a_\mu^{(6)}(\gamma\gamma)$ is known with sufficient precision for the comparison between theory and experiment. We have recently done [14] an analytic computation to improve the accuracy of the fourth- and sixth-order contributions. The only task remaining is to improve the accuracy of the hadronic contribution a_μ(hadronic).

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