Collective modes in dense neutrino systems

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The behavior of a collection of neutrinos, due to the "magnetic" interactions mediated by the Z boson, becomes nonperturbative in a certain kinematical domain. We give arguments that this is a manifestation of the system being dominated by a set of well-defined coherent states. We investigate the main characteristics and the symmetry properties of the corresponding new phase of dense neutrino matter.

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The physics of weakly interacting matter has attracted increasing interest during recent years. The prototype candidate is neutrino matter (at high densities and/or temperature), whose physics should reflect in a particularly clean way the short-range electroweak interactions, since it is not contaminated by strong or electromagnetic effects. Furthermore, neutrinos characterized by the above conditions constitute the most abundant form of matter in the Universe. Consequently, a better knowledge of the behavior of neutrino matter might be of importance for the understanding of various phenomena in astrophysics (solar-neutrino problem, supernovae, development of white dwarfs, . . .) and cosmology (dark-matter problem, early Universe). Of particular interest is the identification of the different phases in which neutrino matter can exist if its density is increased. Considerable work [1] has been invested in an analysis of the phase structure of spontaneously broken gauge theories including fermions under different circumstances [neutral or (weakly) charged matter, chirally coupled fermions, Abelian or non-Abelian groups]. It has been shown in particular [2] that within the electroweak standard model (SM) at very high (chiral) fermion densities ($\mu > M_Z$) gauge bosons start to condensate and fermion-number conservation will be violated. Here, we shall concentrate on the case of rather low neutrino densities ($\mu < M_Z$). We will show that even for this kinematical region a phase transition is expected to occur leading to a state which is characterized by a new collective mode and lepton-number nonconservation.

Let us consider a system of neutrinos of a given type (flavor), say electron neutrinos, at zero temperature but finite density leading to a chemical potential μ which is connected with the conserved (electron-)lepton number N. In the perturbative ground state, levels up to a Fermi surface, which is characterized by the chemical potential, are occupied. The neutrinos are considered massless, so they do not couple directly to Higgs bosons, and their interaction is exclusively due to the exchange of Z bosons. Since the latter are massive ($M_Z \approx 100$ GeV) this interaction is short ranged. The basic Lagrangian for such a system is given by

$$\mathcal{L} = i\bar{\nu}_L \partial\!\!\!/ \nu_L - \hat{g}\bar{\nu}_L \gamma^\mu \nu_L Z_\mu \tag{1}$$

($\hat{g} = g/2\cos\theta_W$). Because of the vectorial coupling (and the vanishing mass) of the fermions, this theory is invariant under the two global U(1) groups defined by

$$\nu \to e^{i\alpha}\nu \tag{2a}$$

and by

$$\nu \to e^{i\beta\gamma_5}\nu \ . \tag{2b}$$

Actually, in the present case, where only the left-handed neutrino field ν_L enters the Lagrangian, these two symmetries are the same since $\gamma_5 \nu_L = -\nu_L$. Consequently, there is only one conserved (Noether) charge $N \equiv N_5$, generically called the lepton number in the following.

We will argue now that such a sea of neutrinos, at large enough densities, has a phase that violates the lepton-number conservation. The corresponding phase transition is different from the one considered in Ref. [2] which also implies fermion-number violation but which is accompanied by the formation of a gauge-boson condensate and takes place at much higher densities ($\mu \gg M_Z$). Since the phase transition promoted in this paper occurs at lower densities ($\mu < M_Z$) it might be of greater phenomenological interest.

There are several indications leading to the conclusion that a lepton-number-violating phase occurs at moderate values of μ. We are going to list them in the following.

We first investigate the coherent states of the neutrino system. For this purpose it is necessary to calculate the Z-polarization tensor $\Pi_{\mu\nu}$ [3]. In lowest nontrivial order of (1) it is given schematically as

$$\Pi(p) \sim \int d^4k\, S_F(p+k) S_F(k) \ , \tag{3}$$

where $S_F(p)$ denotes the neutrino propagator in the (perturbative) many-body ground state. It can be decomposed into particle, antiparticle, hole, antihole contributions in the well-known manner [4]. The resulting pole structure of $S_F(p)$ is such that only particle-hole, particle-antiparticle, and antiparticle-hole correlations give nonvanishing contributions [5].

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The effects of the coherent states can be deduced from the exact two-point function G of the Z boson which is calculated from Π by means of the usual Schwinger-Dyson equation

$$G = G^0 + G^0 \Pi G = G^0 + G^0 \Pi G^0 + \cdots \tag{4}$$

(G^0 being the free Z propagator). The poles of G are therefore stemming from pairings of the particle-hole, particle-antiparticle, and antiparticle-hole type.

We are interested in the kinematical region ($\mu \gg p_0, |\mathbf{p}|$) where the matter contribution is dominant, and therefore focus on the (anti)particle-hole pairing. The corresponding contribution to Π will be denoted by $\Pi^{\rm mat}_{\mu\nu}$. The components $\Pi^{\rm mat}_{00}$ and $\Pi^{\rm mat}_{ii}$ are calculated to be [5]

$$\Pi^{\rm mat}_{00}(p) = \frac{g^2\mu^2}{8\pi^2} x \ln\left|\frac{1+x}{1-x}\right| - \frac{g^2\mu^2}{6\pi^2} \tag{5}$$

and

$$\Pi^{\rm mat}_{ii}(p) = \frac{g^2\mu^2}{8\pi^2} x^3 \ln\left|\frac{1+x}{1-x}\right| - \frac{g^2\mu^2}{6\pi^2} x^2 \quad (i \neq 0) , \tag{6}$$

where

$$x = \frac{p_0}{|\mathbf{p}|} .$$

Let us now investigate both components in some details.

It is clear from (5) that the perturbation series for $\Pi^{\rm mat}_{00}$ diverges at $x = 1$ due to the logarithm. This is as expected: it is known that precisely within this kinematical region perturbation theory will break down giving rise to a collective mode similar to the zero sound [6] in liquid ^{3}He. As has been shown there, the divergence of the perturbation expansion at $x = 1$ can be tamed by carrying out a sum of a selective set of diagrams—the ring diagrams (random phase approximation). This procedure leads to a coherent state (collective mode) with a dispersion relation analogous to the phonon.

The situation is different for $\Pi^{\rm mat}_{ii}$, the polarization due to the "magnetic" interactions. Here, the divergence at $x = 1$ does not lead to a pole of G near $x = 1$ because Π_{zz} enters in the corresponding component of G with the "wrong" sign. But there is another divergence of Π_{zz}, namely, for very large x, due to the second term in (6). This leads to a pole of G_{zz} at some large x value and, consequently, to an additional collective state, which is somewhat like the zero sound in the long-wavelength limit. This pole is determined by the root of the equation

$$p^2 - M_Z^2 - \Pi^{\rm mat}_{zz}(p) = 0 , \tag{7}$$

which satisfies $p_0 \to 0, |\mathbf{p}| \to 0$ with $x \gg 1$. By expanding the logarithm in (6) in powers of $1/x$ we thus obtain the equation

$$\frac{1}{2} x^2 + O(x) = -\frac{6\pi^2}{g^2\mu^2} M_Z^2 . \tag{8}$$

This divergence is severe, and a summing of the ring diagrams would not be sufficient to tame it [7,8]. Furthermore, the corresponding collective excitation is characterized by an imaginary dispersion relationship, since from (8) we obtain

$$\omega = c_2 |\mathbf{p}|, \quad \text{with } c_2 = \pm i \frac{\sqrt{12}\pi}{\mu g} M_Z . \tag{9}$$

Thus it is a filamentation mode which blows up with time, indicating that the neutrino system gets destabilized.

It has to be concluded, therefore, that this divergence essentially renders perturbative calculations unreliable, and the latter are to be taken as mere indications of the underlying complexity of the system. In fact, a straightforward calculation of thermodynamical quantities also leads to a nonperturbative effect, as will be mentioned in a minute. Nevertheless, we take this additional divergence as an indication of a new collective mode and are going now to investigate its possible physical signature.

The dispersion relation (9) indicates (forgetting for the moment that c_1 is imaginary) that the mode is of Goldstone type. It is plausible, therefore, to believe that some global symmetry of the system is broken and that the true ground state of the system contains these Goldstone modes.

The divergence of the theory at $x \gg 1$ in the random phase approximation may also be taken as an indication that a new scale at a lower energy is dynamically generated. The appearance of a Goldstone boson is a symptom of this phenomenon. Since these types of modes are usually found in systems interacting with short-range forces, such as in the case of ^{3}He, we assume in this work that the dynamic generation of a small scale really takes place. This is crucial in our argument for a phase transition for $\mu < m_Z$.

What is the symmetry which is expected to be broken in the new phase?

Evidently, the modes under consideration stem from pairings of (anti)particles and holes. Therefore, one is led to conclude that the new ground state is characterized by an (anti)particle-hole condensate which arises due to an attractive (magnetic) interaction between neutrinos. Indeed, the region of very large x where the new modes stem from is special for magnetic interactions. Namely, a large x implies $|\mathbf{p}|$ almost zero. Thus, a neutrino and a hole, in this state, move relative to each other with (almost) equal and opposite momenta. The corresponding parallel (weak) currents lead to a magnetic-type interaction which is attractive, very much like the Biot-Savart attraction between parallel electric currents. We believe that it is this attractive interaction between parallel-moving neutrinos which is responsible for the formation of a (anti)particle-hole condensate and thus makes the ground state unstable (filamentation instability).

The effects of the condensation can most elegantly be described by introducing a (neutrino-)hole field operator denoted by ν^h, which is correctly defined [9] as

$$\nu^h(\mathbf{r}, t) = \gamma_2 \gamma_1 \gamma_3 \nu^c(\mathbf{r}, -t) \tag{10}$$

(up to an arbitrary phase factor). Here, the superscript c denotes the ordinary charge conjugation. With the help of (10) the true ground state (containing the condensate) can be characterized by a nonvanishing value of the order parameter.

$$\langle \nu_L^\dagger \nu_L^h + \nu_L^{h\dagger} \nu_L \rangle \ . \tag{11}$$

This is a Lorentz scalar which does not vanish identically, as is shown in the Appendix. It is essentially an induced Majorana mass term. Furthermore, such a nonvanishing vacuum expectation value breaks the global lepton-number symmetry (2a) and simultaneously, of course, also its axial version (2b). Consequently, in the condensate phase, the lepton number N is not a good quantum number. And it is this spontaneous breaking of lepton number which leads to the Goldstone-boson mode mentioned above. Note that the condensate is suggested to be uniform in space since it arises due to a nonperturbative phase at large x, i.e., at small $|\mathbf{p}|$.

The observations mentioned so far indicate that ordinary (short-range interacting) neutrino matter, if its density is increased, undergoes a phase transition to a phase characterized by a (nonvanishing) neutrino-hole condensate as the true ground state. That the ordinary phase will become unstable beyond a certain density has already been observed earlier [7]. Calculations of free-energy and pressure in this case show that at some densities lower than M_Z, the correlation energy becomes dominant and the usual perturbative approximations break down. If the perturbative techniques are extrapolated to this region, one obtains negative [7] and nonmonotonic [8] behavior for pressure. This is another indication that an instability occurs and a phase transition takes place.

Unfortunately, our considerations do not allow us to specify the exact value of the critical density at which the phase will change; all we can say is that $\mu < M_Z$, so the transition might occur at a fairly low density. We generally expect such a phenomenon at the level of nuclear densities (or higher) where the effect of the weak interactions begins to be felt. Note that the condensation phenomenon in many-body electrodynamics occurs at several orders of magnitude below the usual electrodynamic scale of about an electron volt. This is due to dynamical generation of smaller scales in the theory connected to objects with phononlike dispersion. In the case of weak interactions the random phase approximation (RPA) shows that such dynamic generation of smaller scales is also possible.

The possibility that the phase transition scale could be much smaller than the weak-interaction scale leads us to speculate whether neutrino densities in supernovae or even in the center of stars might already be sufficient to allow for the neutrinos being in the new phase. Since the lepton number is not conserved in this phase, transitions between neutrinos of different type and/or between neutrinos and antineutrinos can take place and would lead to possibly spectacular effects. For instance, the neutrino flux emerging from these objects will be changed considerably if these neutrinos are originally created in a sufficiently dense surrounding. Other effects might be thought of as well. But due to our present uncertainty concerning the exact kinematical region of the phase transition we find it unreasonable to perform more detailed speculations about possible additional phenomenological consequences [whether this effect could be (partially) responsible for the "solar-neutrino puzzle," for example] at the moment.

In summary, we have found that for neutrinos there exists a phase that is characterized by a spatially uniform, lepton-number-violating particle-hole condensate. This phase appears above a critical value of the chemical potential μ_c, which is so far undetermined but is generally expected to lie below M_Z. The existence of the new phase might lead to interesting phenomena in astrophysics and cosmology. We are planning to investigate these physical consequences of the new collective state in a future paper.

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APPENDIX

We show that, for a general fermion field ψ and its hole-conjugate field ψ^h [cf. Eq. (10)], the operator expressions $\psi_L^{h\dagger}\psi_L$ and $\psi_L^\dagger \psi_L^h$ are nonvanishing Lorentz scalars.

We start from the well known behavior of a Dirac spinor under a Lorentz transformation Λ ($\Lambda^\mu_\nu = \delta^\mu_\nu + \omega^\mu_\nu$):

$$\psi(x) \to S(\Lambda)\psi(x) \ , \tag{A1a}$$

$$\bar\psi(x) \to \bar\psi(x) S^{-1}(\Lambda) \ . \tag{A1b}$$

Here, the 4×4 matrix $S(\Lambda)$ is defined by

$$S(\Lambda)\gamma^\nu S^{-1}(\Lambda) = \gamma^\mu \Lambda^\nu_\mu \ , \tag{A2}$$

which has the explicit solution

$$S(\Lambda) = \exp\left[-\frac{i}{h}\sigma^{\mu\nu}\omega_{\mu\nu}\right] \tag{A3}$$

($\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$). From (A2) we obtain

$$\gamma^0 S^\dagger \gamma^0 = S^{-1}, \quad \gamma^0 S \gamma^0 = (S^{-1})^\dagger = (S^+)^{-1} \ , \tag{A4}$$

and

$$[S, \gamma_5] = 0 \ . \tag{A5}$$

Consider now $\psi^c = \mathcal{C}\bar\psi^t$ and $\psi^h(\mathbf{r},t) = \gamma^2\gamma^1\gamma^3\psi^c(\mathbf{r},-t)$. It is straightforward to show that, under Lorentz transformations,

$$\psi^c \to S(\Lambda)\psi^c \ , \tag{A6}$$

since

$$\begin{aligned}
\mathcal{C}^{-1}S^{-1}\mathcal{C} &= \mathcal{C}^{-1}\exp\left[+\frac{i}{h}\sigma^{\mu\nu}\omega_{\mu\nu}\right]\mathcal{C} \\
&= \exp\left[+\frac{i}{h}\mathcal{C}^{-1}\sigma^{\mu\nu}\mathcal{C}\omega_{\mu\nu}\right] \\
&= \exp\left[-\frac{i}{h}(\sigma^{\mu\nu^t})\omega_{\mu\nu}\right] \\
&= S^t \ .
\end{aligned}$$

Furthermore,

$$\psi^h(\mathbf{r},t)\to\gamma^2\gamma^1\gamma^3 S(\Lambda)\psi^c(\mathbf{r},-t) . \quad \text{(A7)}$$

Now we have

$$\begin{aligned}\gamma^2\gamma^1\gamma^3 S &= i\gamma_0\gamma_5 S = i\gamma_0 S\gamma_5 \\ &= i(S^{-1})^\dagger\gamma_0\gamma_5 \\ &= (S^{-1})^\dagger\gamma^2\gamma^1\gamma^3 . \end{aligned} \quad \text{(A8)}$$

Therefore,

$$\begin{aligned}\psi^h(\mathbf{r},t) &\to (S^{-1})^\dagger\gamma^2\gamma^1\gamma^3\psi^c(\mathbf{r},-t) \\ &= (S^{-1})^\dagger\psi^h(\mathbf{r},t)\end{aligned} \quad \text{(A9a)}$$

and

$$\psi^{h\dagger}\to\psi^{h\dagger}S^{-1} . \quad \text{(A9b)}$$

As a result we see that

$$\psi^{h\dagger}\psi\to\psi^{h\dagger}S^{-1}S\psi=\psi^{h\dagger}\psi , \quad \text{(A10)}$$

i.e., $\psi^{h\dagger}\psi$ is a Lorentz scalar.

The same is true for $\psi^\dagger\psi^h$ and also for the corresponding left- (or right-) handed field components.

To demonstrate that $\psi_L^{h\dagger}\psi_L$ does not vanish identically it is sufficient to observe that

$$(\psi_L)^h=(\psi^h)_L . \quad \text{(A11)}$$

Therefore

$$\begin{aligned}\psi_L^{h\dagger}\psi_L &= ((\psi^h)_L)^\dagger\psi_L=\psi^{h\dagger}\tfrac{1}{2}(1-\gamma_5^\dagger)\psi_L \\ &= \psi^{h\dagger}\tfrac{1}{2}(1-\gamma_5)\psi .\end{aligned}$$

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