Time machine and self-consistent evolution in problems with self-interaction

I. D. Novikov
Nordisk Institute for Teoretisk Fysik (NORDITA), Copenhagen, Denmark;
Copenhagen University Observatory Øster Voldgade 3, DK-1350 Copenhagen K, Denmark;
Astro Space Center of P. N. Lebedev Physical Institute, Moscow, Russia
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Physical processes with self-interactions in spacetimes with closed timelike curves are discussed. Examples of self-consistent solutions of the corresponding problems are obtained.

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I. INTRODUCTION

In Refs. [1–3] the possibility was discussed of creating, in principle, time machines [closed timelike curves (CTC's)], allowing one to travel into the past (or of CTC's having existed from the very beginning of the expansion of the Universe). It is not clear whether the laws of physics permit the existence of the CTC's, see [4–7]. In this paper we suppose that the creation of a time machine is possible.

We discuss a time machine, which is (after completion of its creation) a static wormhole. In this construction there are two spherical holes (mouths) A and B in a three-dimensional space, connected with each other by a short handle, and there are CTC's, which pass through the wormhole. The length l of the handle can be arbitrarily small and it does not depend on the distance R between A and B in external space. We suppose that $l \ll R$ and that l is negligible, $l \approx 0$. In our model treatment the spacetime outside the mouths is a practically flat Minkowski spacetime. If somebody (or something) enters mouth B and moves through the short handle he (or it) exits mouth A practically immediately, according to his (or its) proper time, but with the shift into the past by a period δt, according to the time t of the reference frame in which mouths A and B are at rest. Traveling through the wormhole in the opposite direction (from A to B) would be to travel into the future (with a shift by a period δt). The period δt with the length R (and l if it is not negligible) are the main parameters of the time machine.

The assumption of the possibility of the existence of a time machine creates a lot of questions. One of the most important among them is the problem of causality. The existence of CTC's allows one to travel into the past. At first sight it inevitably leads to the possibility of changing the past, thereby producing causality violations. But it is not so.

In Ref. [8] (see also [9]) the principle of self-consistency was briefly discussed. According to this principle all events on CTC's are self-consistent; that is, they influence each other around the closed timelike lines in a self-adjusted way. In the case of an open timelike curve, any event X divides other events on this curve into two parts: future events and past events with respect to X. All past events can influence X, but future events cannot. On a CTC the choice of the event X divides other events on the curve into future events and past ones only locally. In this case events which locally are in the future with respect to X can influence the event X circularly around the CTC. There is no global division of events on the CTC into future and past. The future influences the present as well as the past. Not only is the future the result of evolution of the past, but the past is the result of the future also. All events in a spacetime with CTC's must be self-consistent.

In Ref. [10] we gave a new formulation and discussed the principle of self-consistency (PSC) which states that the only solutions to the laws of physics that can occur locally in the real Universe are those which are globally self-consistent. The PSC by fiat forbids changing the past. All events happen only once, and cannot be changed.

We discussed in [6] the self-consistent solutions to the so-called "billiard ball problem," which is the following: a solid perfectly elastic ball moves relative to the mouths of the wormhole. Its speed is assumed to be small compared with the speed of light, so it can be treated nonrelativistically. The ball enters the wormhole through mouth B, appears from A in the past and continuing its motion, it can encounter and collide with itself.

At first glance there is a "paradox" in this problem (the so-called "Polchinski paradox" [11]). The initial position and velocity of the ball are chosen in such a way that the ball moves along the trajectory α_1 (see Fig. 1), enters mouth B, and exits from mouth A before it entered into B. The ball continues its motion along the trajectory α_2.[1] The timing is just right for the ball to hit itself at the point Z, knocking its "younger" self along trajectory α_3 and thereby preventing itself from ever reaching mouth B. Such an evolution is self-inconsistent and impossible. It is not the solution of the evolution equations.

The mistake (the reason for the "paradox") is obvious: when at the beginning of our discussion we continued the trajectory α_1 after point Z, we did not take into account the influence of the impact and considered the motion of

[1]The trajectory α_2 is well defined if the trajectory α_1 is given (see [12]).

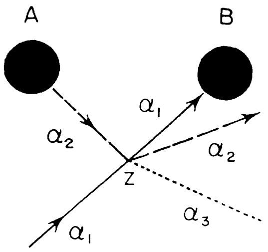

FIG. 1. The self-inconsistent evolution in the billiard ball problem.

this ball along the trajectory α_2 without taking into account this impact. This means that we did not take into account the influence of the future on the past.

In Ref. [12] the authors demonstrated that for initial data which give self-inconsistent "solutions" there are also self-consistent solutions. The self-consistent solution is shown in Fig. 2. The initial data (initial position and velocity of the ball) are the same as in Fig. 1. The part of trajectory α_1 before the collision with the "older" self coming from the future is the same. This "older" ball moves along trajectory β_1 which is a little different from the one α_2 in Fig. 1. The "older" ball on β_2 strikes itself on α_2 gently, deflecting itself into a slightly altered trajectory β_1. This altered trajectory β_1 takes the ball into the mouth B at a slightly altered point compared to the point in Fig. 1. The ball exits from the mouth A before it went into mouth B, and moves along the trajectory β_2 to the collision event. This solution is self-consistent.

In Ref. [12] it has been demonstrated that there are infinite numbers of the self-consistent solutions in the general case of the billiard ball problem and the quantum-mechanical interpretation of the multiplicity was made, see also [13]. The self-consistent solutions for an inelastic billiard ball with friction are discussed in [14].

In this paper some new examples of self-consistent evolutions in problems which are more complicated than the motions and the collisions of billiard balls are discussed.

II. A PISTON IN A TUBE

Let us discuss the following problem. It involves a time machine such as that of Fig. 1, but with a tube, as illustrated in Fig. 3. A piston can move in the tube. We suppose for simplicity that the walls of the tube are frictionless and do not influence the velocity of the piston. The initial position and velocity of the piston are chosen

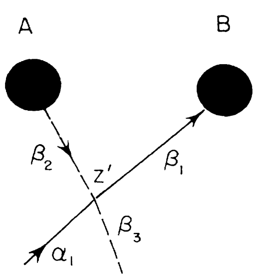

FIG. 2. The self-consistent evolution in the same problem as in Fig. 1.

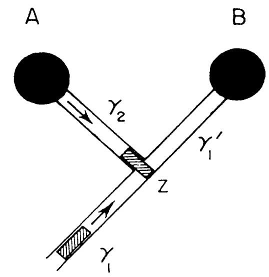

FIG. 3. The self-inconsistent motion of a piston in a tube.

in such a way that the piston moves along the part γ_1 and γ_2 into mouth B, passes through the wormhole, exits from mouth A in the past (with a shift δt into the past), continues its motion in the part of γ_2 of the tube and arrives at the junction Z of the parts γ_1, γ'_1, and γ_2 of the tube some time δt_1 before the "younger" piston arrives at the junction. After moving into part γ_1 the "older" piston blocks the junction and the "younger" one cannot travel into mouth B. The evolution is self-inconsistent.

There is a self-consistent solution of the problem with the qualitative form shown in Fig. 4. The piston starts in the part γ_1 of the tube with the same initial velocity and position as in Fig. 3. While passing the junction of the two parts of the tube the piston is subject to friction with the front part of its "older" self, which just appeared from the end of the part γ_2 of the tube. The velocity of the "younger" one decreases because of the friction. After that the "younger" piston moves along the parts γ'_1 and γ_2 of the tube with a smaller velocity and arrives at the junction Z at the moment when the "younger" one moving along part γ_1, arrives at the same place. Now the evolution is self-consistent.

Let us give a quantitative treatment of the problem. We use the following notation: L_1 is the length of γ'_1 (Fig. 4), L_2 is the length of γ_2 (Fig. 4) (L_2 could be equal L_1 in the simplest case), v_1 is the velocity of the piston before interaction with "older" self, and v_2 is the velocity of the piston after interaction with "older" self and it is therefore the velocity during the motion along γ'_1 and γ_2. δt_1 is the difference between the moments of arrival at the junction of the "younger" piston and the "older" one *without taking into account the interaction between them* (friction). δt_2 is the very small difference between the moments of arrivals of the front ends of the "younger" and the "older" pistons at the junction (taking into account the interaction between the "younger" and the "older" versions of the piston). $\delta v = \delta v(\delta t_2, v_2)$ is the

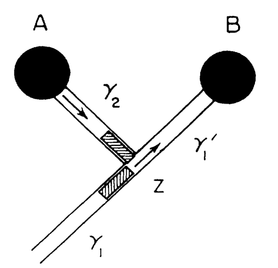

FIG. 4. The self-consistent motion of a piston in a tube.

difference between v_1 and v_2, $\delta v \equiv v_1 - v_2$; it describes the change of the velocity because of friction between the two versions of the piston, and this function is known and it depends crucially on δt_2.

We denote by $t=0$ the moment when the front end of the "younger" piston arrives at the junction and shall suppose that the size of the piston is much smaller than L_1 and L_2.

Now one can write the relations

$$(L_1+L_2)/v_1+\delta t_1-\delta t=0 , \quad (1)$$

$$(L_1+L_2)/v_2+\delta t_2-\delta t=0 , \quad (2)$$

$$v_2=v_1-\delta v(\delta t_2, v_2) . \quad (3)$$

Equation (1) determines δt_1 from the initial condition (the velocity v_1). Equations (2) and (3) should be combined and solved simultaneously to determine δt_2 and v_2. The second term in Eq. (2) is much smaller than the other two, which means that in the first approximation the solution for v_2 is

$$(v_2)_0=(L_1+L_2)/\delta t . \quad (4)$$

One can write the exact solution in the form

$$v_2=(v_2)_0+\delta v_2 , \quad (5)$$

and rewrite the set (2) and (3) in the form

$$\delta t_2[(v_2)_0/\delta v_2+1]=\delta t , \quad (6)$$

$$\delta v_2=[v_1-(v_2)_0]-\delta v(\delta t_2,(v_2)_0+\delta v_2) . \quad (7)$$

According to the physical point of view, the function $\delta v(\delta t_2,(v_2)+\delta v_2)$ is an increasing function of the two variables δt_2 and δv_2 and its dependence on δt_2 is very steep.[2] Using these properties one can demonstrate that there is a reasonable solution of the set (6) and (7) for any reasonable δv. Actually one can solve (6) with respect to δt_2 and substitute this expression in δv. Now δv is a very fast increasing function of one variable δv_2. This function tends to zero at $\delta v_2 \to 0$. The left-hand side of (7) is a linearly increasing function of δv_2 and it is equal to zero when $\delta v_2=0$. The right-hand side of (7) is a very fast decreasing function, which tends to $[v_1-(v_2)_0]>0$ when $\delta v_2 \to 0$, and is equal to zero at some (small) positive value of $\delta v_2=(\delta v_2)_1$. From the last two sentences it follows that (7) has a solution $\delta v_2=(\delta v_2)_2$ and $0<(\delta v_2)_2<(\delta v_2)_1$. After that one can calculate δt_2 from (6). The solution exists even in the case of an arbitrary small friction coefficient, and thus it exists for a case which is arbitrary close to the ideal one.

Thus we have demonstrated that for any initial data which give a paradoxical self-inconsistent "solution" of the form shown in Fig. 3 there are self-consistent solutions with the form shown in Fig. 4.

[2]We do not need to know either the details of this dependence or the physics of the friction process.

III. A BALL WITH A BOMB

In this section we discuss another example of self-consistent evolutions. This involves a time machine such as that of Fig. 1 and a single ball. This ball contains a charge of dynamite (a bomb) and a fuse. The fuse explodes the bomb when any external body touches the surface of the ball.[3]

The self-consistent evolution is shown in Fig. 5. The initial data are arranged in such a way that the ball enters mouth B, emerges from mouth A in the past, continues the motion and arrives at the point Z just in time to collide with the "younger" version itself. This encounter leads to the explosion. We did not take into account the influence of the future on the past before the ball entered the mouth B, and this is the reason for the "paradox."

But there is a self-consistent evolution, as shown in Fig. 6. The initial data are the same as in Fig. 5, but before reaching the point Z it meets the fragment of the explosion of itself. This fragment hits the ball and it is the cause of the explosion, the fragments of the ball fly in all directions with velocities much larger than the velocity of the ball. Some of them fly into mouth B and emerge from mouth A in the past. One can show that they will continue to fly in practically all directions from mouth A, because they have different impact parameters when they flew into mouth B. One of the fragments from mouth A crosses the trajectory of the ball at the point Z^1 exactly at that moment when the ball arrives at the same point Z^1. This fragment is the cause of the explosion of the ball. The consequence of the explosion (the fragment) is the cause of the explosion.

Now we give the quantitative description of the evolution. Let us suppose for simplicity that the sizes of the mouths are negligible compared with all distances in the problem (see below). The notation for the lengths of the segments and the angle Φ are clear from Fig. 6. We denote the velocity of the ball before the explosion by v_1, and the velocities of the fragments by v_2. We suppose that $v_2 \gg v_1$ and assume that velocity v_2 is constant and isotropic and that it does not depend on v_1 and the parameters of the collision of the ball and the fragment.

Now we can write the equations

$$L_2^2=R^2+L_1^2-2RL_1\cos\Phi , \quad (8)$$

$$(L_1+L_2)/v_1=\delta t . \quad (9)$$

These formulas determine the parameters of the self-inconsistent "solution" L_2 and v_1, if the parameters δt, R, L_1, and Φ are given. Now if we add the velocity v_2 to given parameters, one can write down the following set (10) and (11) to determine the parameters of the self-consistent solution L_3 and L_4:

$$(L_3+L_4)/v_2=\delta t , \quad (10)$$

[3]For simplicity we suppose that even a smallest touch leads to explosion. The case when an external body carries too little energy to cause an explosion is discussed at the end of this section.

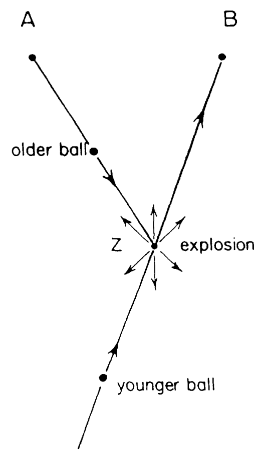

FIG. 5. The self-inconsistent evolution in the problem of a ball with a bomb.

$$L_3^2=R^2+L_4^2-2RL_4\cos\Phi \ . \quad (11)$$

The solution of the set is

$$L_3=\frac{\delta t^2 v_2^2-2\delta t v_2 \cos\Phi+R^2}{2(\delta t v_2-R\cos\Phi)} \ , \quad (12)$$

$$L_4=\frac{\delta t^2 v_2^2-R^2}{2(\delta t v_2-R\cos\Phi)} \ . \quad (13)$$

One can show that L_3 and L_4 are always positive. Thus we have demonstrated the existence of a self-consistent solution to the problem.

If in order to trigger the explosion the external body must carry the amount of energy above some threshold, then there could be also a self-consistent solution which describes a gentle strike by the old and young versions of the ball without explosion (see Sec. I).

IV. OTHER EXAMPLES

In Sec. II we discussed the problem of collisions of a perfectly elastic ball with itself from the future. In a separate paper we shall demonstrate that the same conclusion—the existence of a self-consistent solution—is correct in the case of inelastic collisions also (see [14]).

Now let us consider the problem which is a more complicated version of the problem of the preceding section. The problem is the following (see Figs. 7–9). Let us suppose that there is the ball with a bomb and a radio transmitter (see Fig. 7), which gives a directed beam. The fuse explodes the bomb if, and only if, it is irradiated by the beam of such a radio transmitter from a distance of, say, 30 m (see Fig. 7).

The self-inconsistent evolution is shown in Fig. 8. The "younger" ball explodes, on being irradiated by the radio transmitter of the "older" ball after it comes from the future.[4]

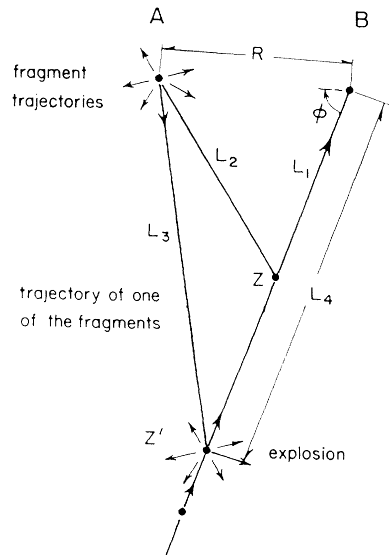

FIG. 6. The self-consistent evolution in the same problem as in Fig. 5.

Now a fragment of the explosion cannot be the cause of the explosion and at first glance, the problem of constructing a self-consistent evolution looks insoluble, but that is not the case.

In Fig. 9 one can see the self-consistent evolution. Before reaching the mouth B the "younger" ball encounters its "older" self from the future but with a change orientation of the radio transmitter (in fact the "younger" ball with the radio transmitter rotates after the point Z, and the "older" one rotates also). Now the fuse is not irradiated by the radio transmitter and there is no reason for the explosion. The inelastic collision of the "older" and the "younger" versions of the ball leads to a change in the orientation of the radio transmitters of both balls (rotation of the balls[5]) and drives both of them into slightly altered trajectories. Self-consistent evolution without an explosion is possible.[6]

Analogously one can construct a self-consistent evolution of the following problem.[7] In self-consistent evolution, one imagines that a mass of uranium slightly less than the critical mass enters mouth B, exits from mouth A and collides with its "younger" self. The new total mass of both pieces is greater than the critical one and the collision leads to an explosion.

[4]This problem was proposed to me by Chiminello Francesco, who heard my lecture in Padova in 1989.

[5]We suppose that dynamic friction coefficient is not equal to zero, which is sufficient condition for the existence of the solution.

[6]If there is some probability for the fuse to explode the bomb because of the collision, then probably another self-consistent evolution is possible: evolution with the explosion caused by the collision of the "younger" ball with a fragment from the explosion, as described in Sec. III. In the case of the existence of two or more self-consistent evolutions for the fixed initial data, it seems likely that in an experiment each of them could happen experimentally with some final probability (see papers [4,10]).

[7]The problem was proposed to me by Dr. Kurt Stokbro, a participant of the NORDITA Colloquium, 1990.

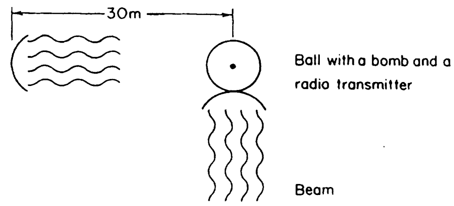

FIG. 7. A billiard ball with a bomb and a radio transmitter.

A self-consistent evolution: the mass of uranium is on the same trajectory, but analogously to the problem of Sec. I, it strikes its "older" self a gentle, glazing blow.[8] The bad contact of the two pieces does not lead to a real explosion, but changes their trajectories. The altered trajectory takes the mass to the collision with the bad contact.

V. DISCUSSION AND CONCLUSIONS

In the previous section we gave self-consistent evolutions of some problems. For some of these problems as well as others that we have not discussed the methods for finding self-consistent solutions were not obvious.[9]

Of course it is not difficult to imagine a more complicated problem similar to those discussed (for example, to introduce gyroscopes to preserve the direction of a radar beam and so on), or to propose other ones. The proof of the possibility of constructing self-consistent evolutions in each of them is unknown, but the following remarks may be useful in attempting to find general methods of constructing self-consistent evolutions if self-inconsistent evolutions are known.

In some problem the evolution of the system is a smooth function of the parameters of the problem (of the parameters of the initial data and the parameters of the interaction with itself coming from the future or with some signals etc. from itself, the signals which passed through the time machine and return into the past). One example of such a problem is the perfect elastic collision of a ball with itself from the future (see Sec. I). In this case it is not difficult (as a rule) to write the self-consistent equation for the problem, taking into account the influence of the future on the past.[10]

Another problem, the interaction with itself or with some signals from itself which passed through the time machine and return from the future, can result in one or more states which are sharply different from the result of the evolution without the interaction. Such an example was given in Sec. II (the open and closed junction). In this case the self-consistent evolution should be the result of the existence of possible intermediate states between these extreme states.[11] In the example in Sec. II the self-consistent solution is the result of interactions of the two versions of the piston as the "older" piston begins to block the junction.

Intermediate states could be unimportant when we do not consider the influence of the future on the past (only "open" or "closed" states could be important in this case but not the short period when the piston enters the junction and locks it). But in the case of self-consistent evolution these intermediate states are the subject of an automatic very fine self-tuning.

In the third category of problems the character of the interactions in the self-consistent solutions could be qualitatively completely different from the interactions in the self-inconsistent solutions. An example is the collision of a ball with itself or with the fragment of itself, as discussed in Sec. III.

Finally, in the fourth category of problems, not only interactions but also the results of them are absolutely (qualitatively) different in the self-consistent and self-inconsistent evolutions. An example is the evolutions in the problem of the ball with a bomb and a radio transmitter in Sec. IV.

In the paper [10] we gave a model example of the "completely sticky" ball with very artificial properties. In this problem there are probably initial data which give no self-consistent evolutions. One can propose other model examples of that kind. We would like to emphasize that if there is no global self-consistent evolution for some initial data of a problem, then the PSC will prohibit these initial conditions.

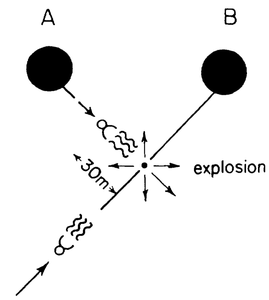

FIG. 8. The self-inconsistent evolution in the problem of a ball with a bomb and a radio transmitter.

[8]Of course, one can construct a system in which a gentle strike is impossible. The general discussion of such cases is given in the next section.

[9]Some of them (see Sec. III, for example) are closer to the "paradox of killing one's younger self" (which was discussed in [10]), then the "paradox" of the collision of the elastic ball with itself from the future.

[10]Strictly speaking in the case of the existence of the time machine there is no global division on events on the future and the past with respect to some event even on one CTC. We used this inaccurate but figurative expression to simplify the explanation and for brevity.

[11]Dr. A. Illarionov has pointed out (in the discussion of this paper) that in the case of the existence only of extreme states (without intermediate ones) self-consistent solutions could be impossible. But we emphasize that in classical physics the intermediate states exist always; jumps without them are impossible. With regard to the self-consistent solution in quantum mechanics see [4,10,12].

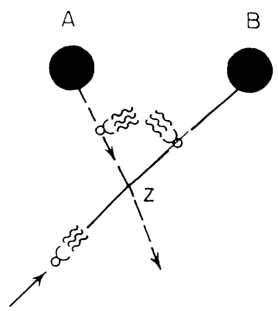

FIG. 9. The self-consistent evolution in the same problem as in Fig. 8.

For our discussions the following point is important. In paper [10] we argued that for any quantum system (and therefore for a classical system as a limit of a quantum one) in a spacetime with a time machine quantum mechanics gives unique, self-consistent probabilities for the outcome of all sets of measurements that one might choose to make.

In conclusion we wish to say that new investigations of the problem of the time machine and its consequences are needed and we have not yet strict proof of the possibility of the existence of a time machine.

ACKNOWLEDGMENTS

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[1] M. S. Morris, K. S. Thorne, and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).

[2] I. D. Novikov, Zh. Eksp. Teor. Fiz. **95**, 769 (1989) [Sov. Phys. JETP **68**, 439 (1989)].

[3] V. P. Frolov and I. D. Novikov, Phys. Rev. D **42**, 1057 (1990).

[4] S.-W. Kim and K. S. Thorne, Phys. Rev. D **43**, 3929 (1991).

[5] V. P. Frolov, Phys. Rev. D **43**, 3878 (1991).

[6] S. W. Hawking (unpublished).

[7] K. S. Thorne, Ann. N.Y. Acad. Sci. (to be published).

[8] I. D. Novikov, *Evolution of the Universe* (Cambridge University Press, Cambridge, England, 1983), p. 169; Ya. B. Zeldovich and I. D. Novikov, *Stroenie i Evolutsia Vselennoi* (Nauka, Moscow, 1975), p. 679.

[9] I. D. Novikov and V. P. Frolov, *Physics of Black Holes* (Kluwer, New York, 1989).

[10] J. Friedman, M. S. Morris, I. D. Novikov, F. Echeverria, G. Klinkhammer, K. S. Thorne, and U. Yurtsever, Phys. Rev. D **42**, 1915 (1990).

[11] J. Polchinski (unpublished); also see [10].

[12] F. Echeverria. G. Klinkhammer, and K. S. Thorne, Phys. Rev. D **44**, 1077 (1991).

[13] J. L. Friedman (unpublished).

[14] E. L. Mikheeva and I. D. Novikov (in preparation).