Small-scale structure on cosmic strings and galaxy formation

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The density perturbations produced by cosmic strings in cold dark matter are examined using the Zel'dovich approximation. We use the results from recent numerical simulations which show that the strings have significant small-scale structure. It is shown that the string network produces wakelike overdensities which may be able to account for the observed large-scale structure of the Universe. In the process of producing wakes the strings also produce large-scale peculiar velocity fields. It is shown that these velocities are coherent over distances which are too small to account for the observed large-scale streaming motions. It is also shown that the small-scale structure on the strings can fragment the wakes into pieces which have the mass of a galaxy.

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INTRODUCTION

Over the past decade the possibility that cosmic strings produced the density perturbations required for the formation of galaxies has been extensively studied. In the original scenario for string-seeded galaxy formation [1–7] it was assumed that the strings had essentially no structure on scales smaller than the horizon and that the typical separation between the strings was approximately equal to the horizon. The loops produced by the string network would then typically be on the size of the horizon. It was then assumed that these loops would fragment into a small number of non-self-intersecting loops which would exist for long periods of time decaying only through the emission of gravitational radiation. It was these long-lived loops that would produce the density fluctuations required for galaxy formation. Early work on this scenario seemed quite promising. For example, Turok [4] found that this scenario gives the correct correlation function for Abell clusters.

The above assumptions upon which the original scenario for galaxy formation rests have recently come into conflict with numerical simulations of the evolution of cosmic-string networks performed by Bennett and Bouchet [9–12] and by Allen and Shellard [8] (the simulations of Albrecht and Turok [13] disagree with these simulations). These simulations show that there is a significant amount of small-scale structure on scales from the horizon down to the limit of resolution of the simulation. In fact Bennett and Bouchet [9–11] find that the energy of the small-scale structure is about 45% of the total energy of the strings in the radiation-dominated era and about 28% in the matter-dominated era. This percentage remains approximately constant as the string network evolves. Most of this structure is due to kinks which form when strings cross and intercommute. Numerical studies [14] indicate that strings will always intercommute when they cross. Allen and Caldwell [16] estimate that there will be on the order of 10^6 kinks on a horizon-size segment of string in the radiation-dominated era and about 5×10^3 kinks on a horizon-size segment of string in the matter-dominated era. In these simulations the coherence length of the strings is about one-half of the horizon and the typical interstring separation is about one-third of the horizon. It is also found that the velocity of the string averaged over a coherence length is only about $0.15c$ [8,15]. These recent simulations also contradict the assumptions that the scale of loops produced by the string network is set by the horizon size. The stable loops produced in the simulations are very small ($<10^{-3}ct$) and their size is set by the scale of the small-scale structure on the long strings. From these simulations we can conclude that the loops produced during the evolution of the cosmic-string network are probably too small to play a significant role in the formation of galaxies [9]. Therefore if strings are relevant for galaxy formation it is the density fluctuations generated by the long strings which will have acted as the seeds for galaxy formation.

The density perturbations produced by a moving straight string have been extensively studied [3,17,18]. The effect of a straight string moving through a medium of cold dark matter in a flat background space-time can be seen quite easily since the space-time about the string is that of a flat space-time minus a wedge. As the matter flows by the string it forms a wedge of overdensity $\delta\rho/\rho=1$ behind the string. The opening angle of the wedge is $8\pi G\mu$ ($c=1$). For $G\mu\simeq10^{-6}$ the string leaves a nearly plane wake of overdensity behind it. Analysis [17,18] shows that for cold collisionless matter in a Robertson-Walker (RW) space-time the wakes which formed at $z\simeq2z_{eq}$ (z_{eq} = redshift at which matter and radiation densities are equal) will be the most dominant (i.e., have the largest surface density).

Since recent numerical simulations show that the long strings have significant small-scale structure it is important to understand the effect that this structure will have on the density perturbations produced by the string network. In this paper we will examine the density pertur-

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bations produced, in a medium of cold dark matter, by cosmic strings with small-scale structure. Since the typical distance between the strings is approximately one-third of the horizon this will also be the typical separation between the wakes. For wakes formed at $t_{\rm eq}$ the average separation between the wakes today is $\simeq 5.3h^{-2}\Omega_0^{-1}$ Mpc (h is defined by $H=100h$ km/s Mpc). Wakes formed before $t_{\rm eq}$ can accrete most of the matter in the Universe, but will be swept up into wakes formed at $\sim t_{\rm eq}$ or later. Wakes which form at later times have larger interwake separations but accrete a much smaller fraction of the matter in the Universe. We will assume that it is the last wakes which accrete almost all of the matter in the Universe which set the size of the large-scale structure. We find that the interwake separation for these wakes is about $\sim 16\sqrt{\mu_6}h^{-1}$ Mpc. The time at which these wakes formed will depend on μ_6, h, and Ω_0 but will be larger than $t_{\rm eq}$ for $\Omega_0=1$. For wakes formed at $t_{\rm eq}$ we require $h\Omega_0 \lesssim \frac{1}{4}$ to match the observed large-scale structure. There will also be larger voids with much smaller underdensities. Since it is expected that the string velocity is coherent over about an expansion time (except when the long strings intercommute) we can approximate the structure of the wake generated by a string as a series of connected sheets whose surface dimensions (for wakes formed at $t_{\rm eq}$) are about $\simeq 4\times 9h^{-2}\Omega_0^{-1}$ Mpc. We will find that the thickness of the sheet is $\sim 11\mu_6\Omega_0$ Mpc. Between these sheets we expect there to be regions of low density. According to earlier ideas, galaxies would form about the loops and some fraction of the loops would be swept up into the wakes. Thus the loops would accrete the galaxies and the wakes may produce the large-scale structure. But as we have seen the loops are probably too small and are moving too rapidly to be effective in forming galaxies in this fashion. Hence the fragmentation of these sheets into galaxies and clusters of galaxies will likely occur by a different mechanism, which will be discussed later in this paper.

Recent surveys [19] indicate that galaxies may lie on the surfaces of bubblelike structures whose sizes are $\approx 25-50h^{-1}$ Mpc and whose interiors are low-density voids. The density of these voids is only about 20% of the mean density. Thus the above scenario for galaxy formation may be able to account for the large-scale structure of the Universe for $\mu_6 \gtrsim 3$.

We will also find that traveling wave pulses on the string can fragment the wake into galaxy mass objects. Throughout this paper we will take the cosmological constant to be zero.

STRING DYNAMICS

Since we will be dealing with small-scale structure on cosmic strings later in this paper it will be important to review some of the properties of the dynamics of strings in a flat space-time. We will also examine the gravitational field, in the weak-field limit, produced by cosmic strings.

The motion of a string in Minkowski space generates a two-dimensional surface which can be parametrized by two variables τ and σ. The action for the string is the area of this surface, i.e.,

$$S=-\mu\int\sqrt{-g^{(2)}}\,d\tau\,d\sigma\ , \tag{1}$$

where $g^{(2)}$ is the determinant of the induced metric on the surface and μ is the linear mass density of the string. In Minkowski space the equations of motion which follow from this action are

$$\frac{\partial}{\partial\tau}\left[\frac{x'^2\dot{x}^\mu-(\dot{x}\cdot x')x'^\mu}{[(\dot{x}\cdot x')^2-\dot{x}^2x'^2]^{1/2}}\right]+\frac{\partial}{\partial\sigma}\left[\frac{\dot{x}^2x'^\mu-(\dot{x}\cdot x')\dot{x}^\mu}{[(\dot{x}\cdot x')^2-\dot{x}^2x'^2]^{1/2}}\right]=0\ , \tag{2}$$

where

$$\dot{x}^\mu=\frac{\partial x^\mu}{\partial\tau},\quad x'^\mu=\frac{\partial x^\mu}{\partial\sigma}\ . \tag{3}$$

By performing a coordinate transformation on the surface it is always possible to find a gauge in which

$$\tau=t,\quad \dot{x}^\mu x'_\mu=0,\quad \dot{x}^2+x'^2=0\ . \tag{4}$$

In this gauge the equations of motion are

$$\ddot{\mathbf{x}}-\mathbf{x}''=0 \tag{5}$$

with the constraints

$$\dot{\mathbf{x}}\cdot\mathbf{x}'=0,\quad \dot{\mathbf{x}}^2+\mathbf{x}'^2=1\ , \tag{6}$$

where we have taken $x^\mu=(t,\mathbf{x}(t,\sigma))$. The constraint $\dot{\mathbf{x}}\cdot\mathbf{x}'=0$ tells us that the velocity of the string is perpendicular to the string.

The energy-momentum tensor of the string can be found from the action and is given by [7]

$$T^{\mu\nu}=\mu\int d\sigma(\dot{x}^\mu\dot{x}^\nu-x'^\mu x'^\nu)\delta^3(\mathbf{x}-\mathbf{x}(t,\sigma))\ . \tag{7}$$

The metric in the weak-field limit is written as $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$, where $h_{\mu\nu}$ is given by

$$h_{\mu\nu}=4G\int d^3x'\frac{S_{\mu\nu}(\mathbf{x}',\tau_R)}{|\mathbf{x}-\mathbf{x}'|}\ . \tag{8}$$

τ_R is the retarded time and $S_{\mu\nu}=T_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}T$. Substituting Eq. (7) into Eq. (8) gives

$$h_{\mu\nu}(\mathbf{x},t)=4G\mu\int d\sigma\left[\frac{\dot{x}_\mu\dot{x}_\nu-x'_\mu x'_\nu-\eta_{\mu\nu}\dot{x}^2}{R-\mathbf{R}\cdot\dot{\mathbf{x}}}\right]\ , \tag{9}$$

where $\mathbf{R}=\mathbf{x}-\mathbf{x}(\sigma,\tau_R)$ and all quantities in the integral are evaluated at the retarded time τ_R.

Another solution to the equations of motion which does not in general satisfy the gauge conditions (6) is

$$t=\tau,\quad x=\sigma,\quad y=f(\sigma\pm\tau),\quad z=g(\sigma\pm\tau)\ , \tag{10}$$

where f and g are arbitrary functions of the same argument (i.e., either $\sigma+\tau$ or $\sigma-\tau$). These solutions represent waves propagating along a string which is situated on the x axis. It is important to note that the sum of two waves propagating in opposite directions is not, in general, a solution to the equations of motion. The energy-momentum tensor for these solutions has been found by Vachaspati [20] and for f and g being functions of $\sigma-\tau$ is given by

$$T^{\mu\nu}=\mu\delta(y-f)\delta(z-g)\begin{bmatrix} 1+f'^2+g'^2 & f'^2+g'^2 & -f' & -g' \\ f'^2+g'^2 & -1+f'^2+g'^2 & -f' & -g' \\ -f' & -f' & 0 & 0 \\ -g' & -g' & 0 & 0 \end{bmatrix},$$

where $f'=df(u)/du$. In the weak-field limit the metric is

$$g_{\mu\nu}=\begin{bmatrix} -[1+h(f'^2+g'^2)] & h(f'^2+g'^2) & -hf' & -hg' \\ h(f'^2+g'^2) & 1-h(f'^2+g'^2) & hf' & hg' \\ -hf' & hf' & 1-h & 0 \\ -hg' & hg' & 0 & 1-h \end{bmatrix},$$

where

$$h=4G\mu\ln\{\rho_0^{-2}[(y-f)^2+(z-g)^2]\} \tag{11}$$

and ρ_0 is a constant of integration. Garfinkle [21] has found the exact generalization of this metric. It is of the same form as the metric in the weak-field limit with h replaced by

$$h=1-\left[\frac{\rho}{\rho_0}\right]^{-8G\mu}, \tag{12}$$

where $\rho^2=(y-f)^2+(z-g)^2$. This reduces to (11) for

$$e^{-1/8G\mu}\ll\frac{\rho}{\rho_0}\ll e^{1/8G\mu}. \tag{13}$$

Since the radius of a cosmic string is much larger than $e^{-1/8G\mu}$ and since $e^{1/8G\mu}$ is an extremely large number we see that the weak-field limit is valid in any region of interest exterior to the string ($\rho_0\sim$ radius of cosmic string). Thus from now on we will use the weak-field approximation (11) for traveling waves. We will also use the weak-field approximation (9) for general string motions.

THE ZEL'DOVICH APPROXIMATION AND STRAIGHT STRINGS

In this section we will review the Zel'dovich [22,23] approximation for the growth of density and velocity perturbations and apply it to the velocity perturbations generated by a moving straight string. We take the Universe to contain both matter and radiation.

We begin by writing the trajectory of a cold-dark-matter particle as

$$\mathbf{r}(\mathbf{q},t)=\frac{a(t)}{a(t_i)}[\mathbf{q}+\boldsymbol{\Psi}(\mathbf{q},t)], \tag{14}$$

where t_i is some initial time, $\mathbf{q}$ are the comoving coordinates of the matter and $\boldsymbol{\Psi}$ is the perturbation to the Hubble expansion. Once we have picked an origin for our coordinates the Newtonian approximation will be valid as long as the particle velocities do not approach the speed of light and as long as we deal with particles which are well within the horizon. For particles which satisfy these conditions we have

$$\frac{\partial^2\mathbf{r}}{\partial t^2}=-\nabla_{\mathbf{r}}\Phi, \tag{15}$$

where Φ is the gravitational potential which satisfies

$$\nabla_{\mathbf{r}}^2\Phi(\mathbf{r},t)=4\pi G[\rho_b(t)+3P_b(t)+\delta\rho(\mathbf{r},t)], \tag{16}$$

$\rho_b(t)$ is the background density, $P_b(t)$ is the background pressure, and $\delta\rho(\mathbf{r},t)$ is the density perturbation in the matter. Now transform to the comoving coordinates. This transformation is well behaved as long as $\mathbf{q}(\mathbf{r})$ is a single-valued function. When $\mathbf{q}(\mathbf{r})$ becomes a multiple-valued function we say that shell crossing has occurred. From now on we shall only consider regions in which shell crossing has not occurred. In the comoving coordinates the matter density (ρ_c) is constant. Transforming back to the $\mathbf{r}$ coordinates gives

$$\rho_m(\mathbf{r},t)-\frac{a(t)^{-3}\rho_c}{\left|\det\left[\delta_{ij}+\frac{\partial\Psi_i}{\partial q_j}\right]\right|}, \tag{17}$$

where $\rho_b^m(t)=a(t)^{-3}\rho_c$ is the background matter density. For $|\partial\Psi_i/\partial q_j|\ll 1$ we can expand the determinant to linear order in $\partial\Psi_i/\partial q_j$ to get

$$\rho_m(\mathbf{r},t)\simeq\rho_b^m(t)[1-\nabla_{\mathbf{q}}\cdot\boldsymbol{\Psi}(\mathbf{q},t)]. \tag{18}$$

Therefore the density perturbation to linear order is given by

$$\delta\rho(\mathbf{r},t)=-\rho_b^m(t)\nabla_{\mathbf{q}}\cdot\boldsymbol{\Psi}(\mathbf{q},t). \tag{19}$$

Equation (16) then becomes

$$\nabla_{\mathbf{r}}^2\Phi(\mathbf{r},t)=4\pi G\left[\rho_b(t)+3P_b(t)-\frac{a(t)}{a(t_i)}\rho_b^m(t)\nabla_{\mathbf{r}}\cdot\boldsymbol{\Psi}(\mathbf{q},t)\right]. \tag{20}$$

The solution to this equation is

$$\nabla_{\mathbf{r}}\Phi(\mathbf{r},t)=\frac{4\pi}{3}G\left[[\rho_b(t)+3P_b(t)]\mathbf{r}-3\frac{a(t)}{a(t_i)}\rho_b^m(t)\boldsymbol{\Psi}(\mathbf{q},t)\right]. \tag{21}$$

Here we have assumed that $\nabla_r \times \Psi = 0$. It can be shown that this will be satisfied by the density perturbations produced by cosmic strings.

The equation of motion for Ψ can be obtained by substituting (21) and (14) into (15). This gives

$$\left[\frac{\partial^2}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial}{\partial t} - 4\pi G \rho_b^m(t)\right]\Psi = 0 . \qquad (22)$$

The solution to this equation is well known [24,25] and for an Einstein–de Sitter space-time (i.e., $\Omega_0 = 1$) is given by

$$\Psi(\mathbf{q},t) = \alpha(\mathbf{q})D_1(t) + \beta(\mathbf{q})D_2(t) , \qquad (23)$$

where

$$D_1(t) = 1 + \tfrac{3}{2}a(t) , \qquad (24)$$

$$D_2(t) = [1 + \tfrac{3}{2}a(t)] \ln \left[\frac{[1+a(t)]^{1/2}+1}{[1+a(t)]^{1/2}-1}\right] - 3[1+a(t)]^{1/2} \qquad (25)$$

and α and β are arbitrary functions of $\mathbf{q}$. For $a(t)$ we use the scale factor for a universe filled with both radiation and matter and take $a(t_{eq}) = 1$. In this section we will only consider wakes formed at t_{eq}. The first term in (23) is referred to as the growth solution and the second term is referred to as the decay solution. As we shall see below the effect of the cosmic string can be approximated by a pure velocity perturbation. That is as the string passes by a given particle we can approximate its effect as a velocity perturbation at time t_{eq}. We will find that it will be a reasonable approximation to give every particle its velocity perturbation at the same time. Hence the initial conditions for (22) are

$$\Psi(\mathbf{q},t_{eq}) = 0, \quad \frac{\partial \Psi}{\partial t}(\mathbf{q},t_{eq}) = \mathbf{v}_i(\mathbf{q}) . \qquad (26)$$

For $t \gg t_{eq}$ the decay solution becomes negligible and the solution to (22) satisfying the initial conditions (26) is

$$\Psi(\mathbf{q},t) \simeq 0.63 t_{eq} \mathbf{v}_i a(t) . \qquad (27)$$

For $\Omega_0 \le 1, \Psi \simeq 0.63 t_{eq} \mathbf{v}_i (1+z_{eq})\Omega_0^{1/2}$ where $(1+z_{eq}) = a(t_0)$ and t_0 is the present time. The perturbation $\delta\rho/\rho$ is given by

$$\frac{\delta\rho}{\rho_m}(\mathbf{r},t) \simeq 0.63 t_{eq} a(t) \nabla_q \cdot \mathbf{v}_i \Omega_0^{1/2} . \qquad (28)$$

As we shall see the velocity perturbations produced by the string satisfy $\nabla_q \cdot \mathbf{v}_i(\mathbf{q}) = 0$ (except on the surface swept out by the string) so that to linear order the string produces no density perturbations.

We now calculate the velocity perturbations generated by a straight string [17]. The gravitational field produced by the string at distances which are much smaller than the horizon will be closely approximated by the gravitational field of a string in Minkowski space-time. To calculate the effect of the string on its surroundings we will consider a straight string moving in a medium of cold collisionless matter. As the string sweeps by a given particle it will attract that particle towards the surface generated by the motion of the string. For small $G\mu$ the velocity of the particle will always be nonrelativistic. Since, as can be seen below, the influence of the string is largest when it is closest to the particle, it will be a reasonable approximation to give each particle its impulse at some time close to when the string passed by. Since the velocity of the string is $\sim 0.15c$ and the motion of the particles is nonrelativistic it will also be a reasonable approximation to give to each particle its impulse at the same initial time t_{eq}. The problem has now been reduced to one of planar geometry, for which the Zel'dovich approximation turns out to give the exact results up to the time of shell crossing. For particles with nonrelativistic velocities the equations of motion in the weak field are

$$\frac{d^2 x^i}{dt^2} = -\frac{\partial h_{0i}}{\partial t} + \frac{1}{2}\frac{\partial h_{00}}{\partial x^i} . \qquad (29)$$

The total impulse given to the particle, taken to be at a fixed position, is then

$$\Delta v^i = -\int_{-\infty}^{\infty} \frac{\partial h_{0i}}{\partial t} dt + \frac{1}{2}\int_{-\infty}^{\infty} \frac{\partial h_{00}}{\partial x^i} dt . \qquad (30)$$

We will now take the string to lie along the x axis at $t=0$ and have a velocity $\mathbf{v} = \beta c \hat{\jmath}$. Since the problem has planar symmetry we only need to calculate Δv_z. The first term in (30) involves h_{03} evaluated at $\pm\infty$, which is zero. Therefore

$$\Delta \mathbf{v} = \frac{1}{2}\int_{-\infty}^{\infty} \nabla h_{00} dt . \qquad (31)$$

Substituting h_{00} from (9) into (31) and using the relation

$$dt = \left[1 - \frac{[\mathbf{x} - \mathbf{x}(\sigma,\tau)] \cdot \dot{\mathbf{x}}(\sigma,\tau_R)}{|\mathbf{x} - \mathbf{x}(\sigma,\tau_R)|}\right] d\tau_R \qquad (32)$$

gives

$$\Delta v_z = 2G\mu \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\sigma \nabla_z \frac{|\dot{\mathbf{x}}(\sigma,\tau)|^2}{|\mathbf{x} - \mathbf{x}(\sigma,\tau)|} . \qquad (33)$$

Substituting $\mathbf{x}(\sigma,\tau) = \gamma^{-1}\sigma\hat{\imath} + \beta c \tau \hat{\jmath}$ into the above expression for Δv_z and integrating gives

$$\Delta v_z = -4\pi G\mu\gamma\beta c \,\mathrm{sgn}(z) . \qquad (34)$$

To get an idea of the magnitude of Δv_z take $G\mu = 10^{-6}$ and $\gamma\beta = 1$. Then $|\Delta v_z| \simeq 10^{-5}c$, which is certainly nonrelativistic. It is important to notice that $|\Delta v_z|$ is independent of z. Bertschinger [22] has calculated the impulse assuming that the particles have an initial velocity given by the Hubble flow. This introduces a Δv_y component and a correction to Δv_z. These changes are quite small if the particles are well within the horizon and will therefore be ignored. Thus Δv_z is to be identified as v_{zi} in Eq. (27). From (28) we see that to linear order the density perturbation generated by the string is zero (except at $z=0$ where it is singular). The velocity perturbations produced by the string create two surfaces of interest. The first surface, known as the turnaround surface, consists of those particles whose velocity in the z direction, at the present time, is zero. The particles which define this surface are in the process of turning around from the Hubble flow and heading back towards the plane traced out by the string. To find this surface we look for the coordinates **q** which satisfy

$$\dot{z}=\dot{a}(t)[q_z+\Psi_z(\mathbf{q},t)]+a(t)\dot{\Psi}_z(\mathbf{q},t)=0 \ . \quad (35)$$

For $\mathbf{\Psi}(\mathbf{q},t)$ given in (27) this becomes

$$q_z=-2\Psi_z(\mathbf{q})\simeq -1.3t_{\rm eq}(1+z_{\rm eq})v_{zi}(\mathbf{q})\Omega_0^{1/2} \ . \quad (36)$$

For the velocity perturbation of a straight string q_z is given by

$$q_z\simeq \pm 5.0\pi t_{\rm eq}(1+z_{\rm eq})\frac{G\mu}{c^2}\gamma\beta c\Omega_0^{1/2} \ . \quad (37)$$

We take $t_{\rm eq}=3\times 10^{10}h^{-4}\Omega_0^{-3/2}$ s, $1+z_{\rm eq}=2.5\times 10^4h^2\Omega_0$ and $G\mu/c^2=\mu_6\times 10^{-6}$. The coordinate distance to the turnaround surface is then given by

$$q_z\simeq \pm 1.2\times 10^{-4}\gamma\beta\mu_6h^{-2}\ \text{Mpc} \ . \quad (38)$$

The physical distance to this surface $[z=(1+z_{\rm eq})(q_z+\Psi_z)]$ is $\simeq 1.5\gamma\beta\mu_6\Omega_0$ Mpc. The coordinate distance to the horizon at this time is $\simeq 6.4\times 10^{-4}h^{-4}\Omega_0^{-2}$ Mpc. The surface density contained within the turnabout surfaces is

$$\sigma\simeq 2(1+z_{\rm eq})q_z\rho_0\simeq 1.7\times 10^2\gamma\beta\mu_6h^2\Omega_0^2M_\odot/\text{Mpc}^2 \ , \quad (39)$$

where

$$\rho_0=\frac{1}{6\pi G t_0^2}\Omega_0 \quad (40)$$

and t_0 is the present time. The other surface of interest is the surface which has turned around and reached $z=0$ at the present time. This surface is defined by $z=(1+z_{\rm eq})(q_z+\Psi_z)=0$. The coordinate distance to this surface is one-half of the coordinate distance to the turnaround surface. Therefore the surface density of the matter that has fallen back onto the wake is $\sigma\simeq 8.7\times 10^{11}\gamma\beta\mu_6h^2\Omega_0^2M_\odot/$Mpc.

We define the amount of matter which has accreted by the wake to be the amount of matter contained within the turnaround surfaces. This is an overestimate of the amount of matter accreted by the wake. A lower bound on the mass accreted is given by the mass which has collapsed to $z=0$. This mass is one-half of the mass within the turnaround surfaces. From (38) the coordinate thickness of the wake is $\sim 2.4\times 10^{-4}\gamma\beta\mu_6h^{-2}$ Mpc. We will take the ratio of the present coordinate thickness of the wake to the interstring separation ($\sim\frac{1}{3}$ of the horizon) at $t_{\rm eq}$ as an estimate of the fraction of the matter in the Universe that has been accreted by the wakes. Therefore wakes formed by straight strings could have accreted about $110\mu_6\beta\gamma h^2\Omega_0^2$ percent of the cold collisionless matter in the Universe. The actual fraction of matter accreted by the wakes will depend on the geometry of the wakes and will be larger than our estimate if the wakes form closed surfaces. To match the observed large-scale structure $h\Omega_0\lesssim\frac{1}{4}$. Thus the amount of matter accreted by the wakes is $\lesssim 7\mu_6\beta\gamma$ percent of the matter in the Universe. So far we have discussed the density perturbations in the cold collisionless matter, which we have taken to be the dominant form of matter in the Universe. There is of course a baryonic component to the matter density. This baryonic component cannot begin to collapse onto the wakes until after recombination. We will assume that the distribution of luminous matter mirrors the distribution of the cold collisionless matter. Therefore to be in reasonable agreement with the Center for Astrophysics (CFA) survey [19] which found that the density of the luminous matter in the voids is $\simeq 20\%$ of the mean density of luminous matter we would require $\mu_6\gg 1$ (for $\beta\gamma<1$). Since the loops produced by the string network are very small it can be shown that they do not disrupt the wakes [9]. It can also be shown [18] that the wakes formed by the string network will not collapse under their own gravitational attraction.

VELOCITY PERTURBATIONS 1

In this section we examine the velocity perturbations produced by a string carrying traveling waves of the form (10) (i.e., waves propagating in one direction only). The velocity perturbations produced by strings carrying waves propagating in both directions will be examined in the next section. Solutions of the form (10) are not actually valid in an expanding universe. Let $y=f$ be the displacement of the string from its equilibrium position. In a radiation-dominated universe with $|f'|,|\dot{f}|\ll 1$ the string satisfies the equation of motion [7]

$$\ddot{f}+\frac{2}{\tau}\dot{f}-f''=0 \ , \quad (41)$$

where τ is the conformal time defined by $\tau\propto t^{1/2}$ and $a(\tau)\propto\tau$. This equation has the plane-wave solution

$$f(x,\tau)=\frac{A}{\tau}\sin[k(x-\tau)] \ , \quad (42)$$

where x is a comoving coordinate and $\lambda=2\pi/k$ is the comoving wavelength. Thus the physical amplitude of the wave is constant but the physical wavelength grows with time. For particles which are well within the horizon it will be a reasonable approximation to neglect the smoothing of the string. Thus to calculate the impulse we will use the metric for a string in flat space-time and take f and g to be the wave forms on the string as the string passes the particle. For this metric to be a good approximation we also require that the particle be much closer to the string than the horizon and that the amplitude of the waves on the string be much smaller than the horizon. To simplify the calculations we will also assume that the amplitude of the waves is much smaller than the coordinate distance from the string to the particles of interest (i.e., particles on the turnaround surfaces). As before we will take the string to pass through the x axis at $t=0$ and to have the velocity $\mathbf{v}=\beta c\hat{\jmath}$. As in the case of a straight string we are interested in Δv_z. The z component of the total impulse given to a particle, taken to be at rest, is

$$\Delta v_z = -\int_{-\infty}^{\infty} \frac{\partial h_{03}}{\partial t} dt + \frac{1}{2}\int_{-\infty}^{\infty} \frac{\partial h_{00}}{\partial z} dt \ . \tag{43}$$

The first term is just h_{03} evaluated at the end points of integration. Since $h_{03} = (1 - r^{-8G\mu})g'$ the impulse given to the particle in this coordinate system will oscillate with g' no matter how far away the string is. But since the curvature tensor behaves as [21] $\simeq 1/r$ we can see that this is just a coordinate effect. Thus there will exist a coordinate system (S) which in the vicinity of the particle becomes Minkowski when the string is far away. It is in this coordinate system that we need to calculate Δv_z. In (S) Δv_z will be essentially unchanged if the amplitude of the infinite wave train is set to zero at very large distances. For the truncated wave in the coordinate system we have used the particle will begin and end up in a coordinate system which can be made locally Minkowski by a simple coordinate transformation. This transformation will change the velocity by order μ^2, which we can neglect. Therefore to order μ, Δv_z in the coordinates we have used with $h_{03} = 0$ at $t = \pm\infty$ gives the correct impulse. Therefore

$$\Delta v_z = \frac{1}{2}\int_{-\infty}^{\infty} \frac{\partial h_{00}}{\partial z} dt \ . \tag{44}$$

The metric (11) is for a string at rest on the x axis. The h_{00} component of the metric for a string with velocity $v\hat{\jmath}$ is given by

$$h_{00} = -4G\mu\gamma^2 \ln\{[\gamma(y - \beta ct) - f]^2 + (z - g)^2\} \times [(g')^2 + (\beta - f')^2] \ , \tag{45}$$

where

$$f = f[x - \gamma(ct - \beta y)], \quad g = g[x - \gamma(ct - \beta y)] \ . \tag{46}$$

The integration constant (ρ_0) in (11) has been dropped since it does not appear in Δv_z. Substituting this into (44) gives

$$\Delta v_z = -4G\mu\gamma^2 \int_{-\infty}^{\infty} \frac{[z - g][(g')^2 + (\beta - f')^2]}{[\gamma(y - \beta ct) - f]^2 + (z - g)^2} dt \ . \tag{47}$$

Now consider particles with $x = y = 0$. Particles with $x \neq 0$, $y \neq 0$ can be considered by shifting the wave up or down the string. As stated before we are only interested in particles with $z \gg f, g$. Δv_z then becomes

$$\Delta v_z = -4G\mu\gamma^2 z \int_{-\infty}^{\infty} \frac{(g')^2 + (\beta - f')^2}{(\gamma\beta ct)^2 + z^2} dt. \tag{48}$$

Now $g(-\gamma ct)$ and $f(-\gamma ct)$ can be written as

$$g(-\gamma ct) = \int_{-\infty}^{\infty} g_k e^{i\gamma ckt} dk, \quad f(-\gamma ct) = \int_{-\infty}^{\infty} f_k e^{i\gamma ckt} dk \ . \tag{49}$$

Substituting (49) into (48) and integrating gives

$$\Delta v_z = -4\pi \frac{G\mu}{c^2}\gamma c \left[\beta + 2i\int_{-\infty}^{\infty} k f_k e^{-|kz/\beta|} dk - \frac{1}{\beta}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} kk'(g_k g_{k'} + f_k f_{k'}) e^{-|(k+k')z/\beta|} dk\, dk'\right] \mathrm{sgn}(z) \ . \tag{50}$$

The first term in the large parentheses is the velocity change produced by a straight string while the last two terms are the velocity change produced by the waves on the string. An important property of the last term is the dependence on $1/\beta$. The reason for this dependence on β is that as β gets smaller the string spends more time in the vicinity of the particle and hence gives the particle a larger impulse.

It will be instructive to consider Δv_z for a monochromatic wave on the string. Consider

$$g = a \sin\{k_0[x - \gamma(ct - \beta y)]\} \ . \tag{51}$$

The Fourier transform of g is

$$g_k = \frac{a}{2i}[\delta(k + k_0) - \delta(k - k_0)] \ . \tag{52}$$

At $x = y = 0$, Δv_z is given by

$$\Delta v_z = -4\pi\gamma \frac{G\mu}{c^2}\left[\beta + \frac{2\pi^2}{\beta}\left[\frac{a}{\lambda}\right]^2 \times (1 + e^{-4\pi|z|/\lambda\beta})\right] c\ \mathrm{sgn}(z) \ , \tag{53}$$

where $\lambda = 2\pi/k_0$. For $|z| > \lambda$, Δv_z is essentially independent of z. An important property of Δv_z to notice is that in the limit $a \to 0$ the wave will still have an effect if (a/λ) remains nonzero. Therefore small-scale structure on any scale can influence the velocity perturbations produced by the string. This could be important for predictions of density perturbations which come from numerical simulations of cosmic string evolution. In these simulations there is a lower limit of resolution for the small-scale structure. Therefore if there is significant small-scale structure on scales smaller than the limit of resolution the predictions of density perturbations from these simulations will be too small. To get an idea of the effect of the sine wave on Δv_z we can take the result from the Bennet and Bouchet [9–11] simulation that in the radiation-dominated era about 50% of the energy of the string resides in the small-scale structure. The energy of a moving string can be found from its rest energy and momentum via a Lorentz transformation. Actually the energy and momentum of a segment of string will not in general transform as a four-vector if f is nonzero at the end points. If we are interested in the energy of a length Δx of string this effect will be small if $f/\Delta x$ is small. The energy of a length Δx of string with $f/\Delta x \ll 1$ is thus

$$E=\gamma\mu\left[\Delta x+\int_{x_0}^{x_0+\Delta x}[(f')^2+(g')^2-\beta f']dx\right], \quad (54)$$

where f' and g' are the wave forms in the rest frame of the string. For $f=0$ and g given by (51) we find

$$E\simeq\gamma\mu\Delta x\left[1+2\pi^2\left[\frac{a}{\lambda}\right]^2\right] \quad (55)$$

for $\Delta x \gg \lambda$. Therefore the ratio of the energy in the wave (E_w) to the energy of the string is

$$\frac{E_w}{E}=\frac{2\pi^2(a/\lambda)^2}{1+2\pi^2(a/\lambda)^2} . \quad (56)$$

Here we have defined the energy in the waves as the total energy of the string segment minus the energy of a straight piece of string connecting the end points of the segment. Solving (56) for a/λ and substituting it into (53) gives

$$\Delta v_z=-4\pi\gamma\frac{G\mu}{c^2}\left[\beta+\frac{1}{\beta}\left[\frac{E_w/E}{1-E_w/E}\right]\times(1+e^{-4\pi|z|/\lambda\beta})\right]c\ \mathrm{sgn}(z) . \quad (57)$$

For waves whose wavelength is much smaller than the z coordinate of interest and for $E_w/E=\frac{1}{2}$ we find

$$\Delta v_z=-4\pi\gamma\frac{G\mu}{c^2}\left[\beta+\frac{1}{\beta}\right]c\ \mathrm{sgn}(z) . \quad (58)$$

If there is significant small-scale structure on scales smaller than the resolution limit of the simulation the effect of the wave on the velocity perturbation will be larger.

We now consider the average impulse generated by a statistical ensemble of waves. The ensemble average of $f'(t)$ and $g'(t)$ will be taken to satisfy

$$\langle f'(t)\rangle=\langle g'(t)\rangle=0 . \quad (59)$$

Taking the ensemble average of (48) gives

$$\langle v_z\rangle=-4\frac{G\mu}{c^2}\gamma^2 zc\int_{-\infty}^{\infty}\frac{\beta^2+\langle f'^2\rangle+\langle g'^2\rangle}{(\gamma vt)^2+z^2}dt , \quad (60)$$

where we have taken each string to have the same velocity. This gives the average velocity perturbation at some z distance away from a given string. The energy in expression (54) is the energy of a length Δx of string. If we consider a length Δx for each member of the ensemble the average energy will be

$$\langle E\rangle=\gamma\mu\left[\Delta x+\int_{x_0}^{x_0+\Delta x}(\langle f'^2\rangle+\langle g'^2\rangle)dx\right] . \quad (61)$$

We now consider ensembles which in addition to satisfying (59) also satisfy the requirement that $\langle f'^2\rangle+\langle g'^2\rangle$ be independent of x (and hence independent of t). For such ensembles we have

$$\langle E\rangle=\mu\gamma(1+\langle f'^2\rangle+\langle g'^2\rangle)\Delta x . \quad (62)$$

The average energy in the waves is then

$$\langle E_w\rangle=\mu\gamma(\langle f'^2\rangle+\langle g'^2\rangle)\Delta x . \quad (63)$$

In simulations of string network evolution $E_w/E=\langle E_w\rangle/\langle E\rangle$ is found to be essentially time independent in both the radiation and matter-dominated eras. But since E_w/E has a different value in each era it will not be constant in the transition era. Since we are interested in wakes formed near t_{eq}, E_w/E will not be strictly constant at this time. But since E_w/E changes from about 0.5 in the radiation-dominated era to about 0.3 in the matter-dominated era it is a reasonable approximation to have E_w/E constant. Substituting the above results into (60) gives

$$\langle\Delta v_z\rangle=-4\pi\frac{G\mu}{c^2}\gamma c\left[\beta+\left[\frac{E_w/E}{1-E_w/E}\right]\frac{1}{\beta}\right]\mathrm{sgn}(z) . \quad (64)$$

The above expression for $\langle\Delta v_z\rangle$ is identical to expression (58) for Δv_z if $|z|>\lambda$.

Now consider solutions to the equations of motion (2) of the form

$$x^\mu=[t,\sigma,f(x,t),g(x,t)] \quad (65)$$

with

$$|f'|,\quad |\dot f|,\quad |g'|,\quad |\dot g|\ll 1 . \quad (66)$$

To linear order in f and g the equations of motion become linear wave equations. Thus for f and g satisfying (66) the general solution will be

$$x^\mu=[t,\sigma,f_u(x-t)+f_v(x+t),g_u(x-t)+g_v(x+t)] , \quad (67)$$

where f_u,f_v,g_u,g_v are arbitrary functions of their respective arguments. This solution satisfies the equations of motion up to second order in f and g. Hence the next-order correction to (67) is cubic in f and g. To second order in f and g the energy-momentum tensor can be written as

$$T^{\mu\nu}=T^{\mu\nu}_{(0)}+T^{\mu\nu}_{(u)}+T^{\mu\nu}_{(v)}+T^{\mu\nu}_{(I)} , \quad (68)$$

where

$$T^{\mu\nu}_{(0)}=\mu\delta(y-f)\delta(z-g)\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},$$

$$T^{\mu\nu}_{(u)}=\mu\delta(y-f)\delta(z-g)\begin{bmatrix}\left[\frac{\partial f_u}{\partial u}\right]^2+\left[\frac{\partial g_u}{\partial u}\right]^2 & \left[\frac{\partial f_u}{\partial u}\right]^2+\left[\frac{\partial g_u}{\partial u}\right]^2 & -\frac{\partial f_u}{\partial u} & -\frac{\partial g_u}{\partial u}\\ \left[\frac{\partial f_u}{\partial u}\right]^2+\left[\frac{\partial g_u}{\partial u}\right]^2 & \left[\frac{\partial f_u}{\partial u}\right]^2+\left[\frac{\partial g_u}{\partial u}\right]^2 & -\frac{\partial f_u}{\partial u} & -\frac{\partial g_u}{\partial u}\\ -\frac{\partial f_u}{\partial u} & -\frac{\partial f_u}{\partial u} & 0 & 0\\ -\frac{\partial g_u}{\partial u} & -\frac{\partial g_u}{\partial u} & 0 & 0\end{bmatrix},$$

$$T^{\mu\nu}_{(v)}=\mu\delta(y-f)\delta(z-g)\begin{bmatrix}\left[\frac{\partial f_v}{\partial v}\right]^2+\left[\frac{\partial g_v}{\partial v}\right]^2 & -\left[\frac{\partial f_v}{\partial v}\right]^2-\left[\frac{\partial g_v}{\partial v}\right]^2 & \frac{\partial f_v}{\partial v} & \frac{\partial g_v}{\partial v}\\ -\left[\frac{\partial f_v}{\partial v}\right]^2-\left[\frac{\partial g_v}{\partial v}\right]^2 & \left[\frac{\partial f_v}{\partial v}\right]^2+\left[\frac{\partial g_v}{\partial v}\right]^2 & -\frac{\partial f_v}{\partial v} & -\frac{\partial g_v}{\partial v}\\ \frac{\partial f_v}{\partial v} & -\frac{\partial f_v}{\partial v} & 0 & 0\\ \frac{\partial g_v}{\partial v} & -\frac{\partial g_v}{\partial v} & 0 & 0\end{bmatrix},$$

and

$$T^{\mu\nu}_{(I)}=-2\mu\delta(y-f)\delta(z-g)$$
$$\times\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&2\left[\frac{\partial f_u}{\partial u}\right]\left[\frac{\partial f_v}{\partial v}\right] & \left[\frac{\partial f_u}{\partial u}\right]\left[\frac{\partial g_v}{\partial v}\right]+\left[\frac{\partial f_v}{\partial v}\right]\left[\frac{\partial g_u}{\partial u}\right]\\ 0&0&\left[\frac{\partial f_u}{\partial u}\right]\left[\frac{\partial g_v}{\partial v}\right]+\left[\frac{\partial f_v}{\partial v}\right]\left[\frac{\partial g_u}{\partial u}\right] & 2\left[\frac{\partial g_u}{\partial u}\right]\left[\frac{\partial g_v}{\partial v}\right]\end{bmatrix}.$$

Thus $T^{\mu\nu}_{(u)}$ and $T^{\mu\nu}_{(v)}$ are the energy-momentum tensors for the u and v waves, respectively, and $T^{\mu\nu}_{(I)}$ is the interaction energy-momentum tensor. If the waves propagating in opposite directions have different polarizations then only $T^{23}_{(I)}=T^{32}_{(I)}$ will be nonzero. This component is not used in the calculation of the velocity perturbation produced by the string. Therefore if the waves propagating in opposite directions have different polarizations the total velocity impulse, to lowest order, will be the sum of the velocity perturbations produced by the individual waves. Consider an ensemble of strings in which the waves propagating in opposite directions are uncorrelated and hence satisfy

$$\left\langle\left[\frac{\partial f_u}{\partial u}\right]\left[\frac{\partial f_v}{\partial v}\right]\right\rangle=\left\langle\left[\frac{\partial f_u}{\partial u}\right]\left[\frac{\partial g_v}{\partial v}\right]\right\rangle=\left\langle\left[\frac{\partial g_u}{\partial u}\right]\left[\frac{\partial f_v}{\partial v}\right]\right\rangle=\left\langle\left[\frac{\partial g_u}{\partial u}\right]\left[\frac{\partial g_v}{\partial v}\right]\right\rangle=0\ . \tag{69}$$

It is easy to see from (8) that for $y^2+z^2\gg f^2+g^2$ (i.e., the particle is much further from the string than the amplitude of the wave) the average impulse given to the particle can be written as

$$\langle \Delta v_z \rangle = \langle \Delta v_z^{(u)} \rangle + \langle \Delta v_z^{(v)} \rangle \ , \tag{70}$$

where $\langle \Delta v_z^{(u)} \rangle$ is the average impulse produced by the u wave and $\langle \Delta v_z^{(v)} \rangle$ is the average impulse produced by the v wave. Thus on average, to lowest order in E_w/E, the impulse produced by the two waves adds linearly.

VELOCITY PERTURBATIONS 2

We now examine the velocity perturbations generated by a string which is described in the gauge given in (6). In this gauge we can have waves propagating in both directions. We begin by writing the position of the string as

$$\mathbf{x}(\sigma,t) = \alpha\sigma\hat{\mathbf{i}} + \beta ct\hat{\mathbf{j}} + \mathbf{h}(\sigma,t) \ , \tag{71}$$

where $\mathbf{h}$ represents the waves propagating along the string and α is a constant. For $\mathbf{x}(\sigma,t)$ to satisfy the constraints (6) we must have

$$\beta^2 + 2c^{-1}\beta\dot{h}_y + \alpha^2 + 2\alpha h_x' + \mathbf{h}'^2 + c^{-2}\dot{\mathbf{h}}^2 = 1 \tag{72}$$

and

$$v h_y' + \alpha\dot{h}_x + \dot{\mathbf{h}}\cdot\mathbf{h}' = 0 \ . \tag{73}$$

We now define the energy in the waves as we did previously for the traveling waves. The energy of a length $\Delta\sigma$ of string is

$$E = \mu\Delta\sigma \ . \tag{74}$$

The length $\Delta\sigma$ corresponds to a length Δx via

$$\Delta x = \alpha\Delta\sigma + \Delta h_x \ . \tag{75}$$

For a long segment of string with h_x not too large the last term in (75) will be negligible. The energy of a straight string with length Δx is $\mu\gamma\Delta x \simeq \alpha\mu\gamma\Delta\sigma$. The energy in the waves is therefore given by

$$E_w = \mu(1-\alpha\gamma)\Delta\sigma \ . \tag{76}$$

Thus α can be written as

$$\alpha = \gamma^{-1}(1 - E_w/E) \ . \tag{77}$$

We now consider an ensemble of strings which satisfies

$$|\mathbf{h}| \ll z_{\text{ta}}, \quad \langle \dot{\mathbf{h}} \rangle = \langle \mathbf{h}' \rangle = 0 \ , \tag{78}$$

where z_{ta} is the z coordinate of the turnaround surface. We also take each string to have the same E_w/E and the same velocity. This means that each string has the same α. Taking the ensemble average of (33) for $x=y=0$ gives

$$\langle \Delta v_z \rangle = -2\frac{G\mu}{c^2}z \times \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} dt \frac{\beta^2c^2 + \langle \dot{\mathbf{h}}^2 \rangle}{(\alpha^2\sigma^2 + \beta^2c^2t^2 + z^2)^{3/2}} \ . \tag{79}$$

By integrating by parts twice and using the equations of motion (5) we get

$$\int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} dt \frac{|\dot{\mathbf{h}}|^2}{(\alpha^2\sigma^2+\beta^2c^2t^2+z^2)^{3/2}} = \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} dt \frac{c^2|\mathbf{h}'|^2}{(\alpha^2\sigma^2+\beta^2c^2t^2+z^2)^{3/2}} + \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} dt \frac{\frac{\partial}{\partial t}[\mathbf{h}\cdot\dot{\mathbf{h}}] - c^2\frac{\partial}{\partial\sigma}[\mathbf{h}\cdot\mathbf{h}']}{(\alpha^2\sigma^2+\beta^2c^2t^2+z^2)^{3/2}} \tag{80}$$

If we require that the ensemble satisfy $(\partial/\partial t)\langle \mathbf{h}\cdot\dot{\mathbf{h}} \rangle = (\partial/\partial\sigma)\langle \mathbf{h}\cdot\mathbf{h}' \rangle = 0$ then $\langle \Delta v_z \rangle$ can be written as

$$\langle \Delta v_z \rangle = -2\frac{G\mu}{c^2}z \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} \times dt \frac{\beta^2c^2 + \frac{1}{2}(\langle \dot{\mathbf{h}}^2 \rangle + c^2\langle \mathbf{h}'^2 \rangle)}{(\alpha^2\sigma^2+\beta^2c^2t^2+z^2)^{3/2}} \ . \tag{81}$$

We now require that $\langle \dot{\mathbf{h}}^2 \rangle + c^2\langle \mathbf{h}'^2 \rangle$ be independent of σ and t. We then have

$$\langle \Delta v_z \rangle = -\frac{4\pi G\mu}{\alpha\beta c}[\beta^2 + \tfrac{1}{2}(c^{-2}\langle \dot{\mathbf{h}}^2 \rangle + \langle \mathbf{h}'^2 \rangle)]\,\mathrm{sgn}(z) \ . \tag{82}$$

Taking the ensemble average of (72) gives

$$c^{-2}\langle \dot{\mathbf{h}}^2 \rangle + \langle \mathbf{h}'^2 \rangle = (1-\alpha^2-\beta^2) \ . \tag{83}$$

$\langle \Delta v_z \rangle$ can then be written as

$$\langle \Delta v_z \rangle = -4\pi\frac{G\mu}{c^2} \times \left[\beta + \left[\frac{E_w/E - \frac{1}{2}\gamma^{-2}E_w^2/E^2}{1-E_w/E}\right]\frac{1}{\beta}\right]\mathrm{sgn}(z)c \ . \tag{84}$$

This is the same as (64) to lowest order in E_w/E, as expected. The reason for the difference between (64) and (84) is that if we convert the Vachaspati traveling-wave solutions into this gauge they violate $\langle h_x \rangle = \langle h_x' \rangle = 0$. Expression (84) is the velocity perturbation we will use in subsequent calculations (expression (84) is equivalent to the expression for the velocity perturbation derived by Vachaspati and Vilenkin [26]).

ACCRETION WAKES

In this section we examine the accretion wakes produced by strings with small-scale structure. We first consider wakes formed at t_{eq}. To find the average distance to the turnaround surface we would need to calculate

$$\langle q_z \rangle = -1.3t_{\text{eq}}(1+z_{\text{eq}})\langle v_{zi} \rangle \Omega_0^{1/2} \ , \tag{85}$$

where $\langle v_{zi} \rangle$ is the average of v_i evaluated not at fixed q_z but at the various q_z's of the turnaround surfaces. Let the q_z's from the turnaround surfaces lie in the interval $[q_z^{(\min)}, q_z^{(\max)}]$. $\langle v_{zi} \rangle$ will in general be different from the $\langle \Delta v_z \rangle$ we have calculated unless each Δv_z is essentially independent of q_z for $q_z \in [q_z^{(\min)}, q_z^{(\max)}]$. For example, Δv_z for a monochromatic wave of the form (51) becomes independent of q_z for $q_z > \lambda$. Consider an ensemble of strings carrying monochromatic waves with $\lambda \in [\lambda^{\min}, \lambda^{\max}]$. If $q_z^{(\min)} > \lambda^{(\max)}$ then $\langle \Delta v_z \rangle \simeq \langle v_{zi} \rangle$. From (50) we expect $\langle \Delta v_z \rangle \simeq \langle v_{zi} \rangle$ if the wavelengths of the Fourier modes on the strings are less than $q_z^{(\min)}$. For ensembles which satisfy $\langle \Delta v_z \rangle = \langle v_{zi} \rangle$ the average comoving coordinate of the turnaround surface is given by

$$\langle q_z \rangle = \pm 5.0 \pi t_{\rm eq}(1+z_{\rm eq}) \frac{G\mu}{c^2} \gamma \left[\beta + \left[\frac{E_w/E - \frac{1}{2}\gamma^{-2}E_w^2/E^2}{1-E_w/E} \right] \frac{1}{\beta} \right] c\Omega_0^{1/2} \tag{86}$$

$$\simeq \pm 1.2 \times 10^{-4} \gamma \left[\beta + \left[\frac{E_w/E - \frac{1}{2}\gamma^{-2}E_w^2/E^2}{1-E_w/E} \right] \frac{1}{\beta} \right] \mu_6 h^{-2} \text{ Mpc} . \tag{87}$$

We now take $\beta = 0.15$ and $E_w/E = 0.4$ (average of E_w/E for radiation- and matter-dominated eras). The coordinate distance to the turnaround surface is

$$q_z \simeq 4.5 \times 10^{-4} \mu_6 h^{-2} \text{ Mpc} . \tag{88}$$

The physical distance to the surface is $\simeq 5.7 \mu_6 \Omega_0$ Mpc and the surface density contained within the turnaround surface is

$$\sigma \simeq 6.4 \times 10^{12} \mu_6 h^2 \Omega_0^2 M_\odot / \text{Mpc}^2 . \tag{89}$$

One must be careful in applying the above results if the turnabout surfaces get close to the horizon. Since the coordinate interwake separation at $t_{\rm eq}$ is $\simeq 2.1 \times 10^{-4} h^{-4} \Omega_0^{-2}$ Mpc we see that the wakes could have accreted about

$$110 \mu_6 \gamma \left[\beta + \left[\frac{E_w/E - \frac{1}{2}\gamma^{-2}E_w^2/E^2}{1-E_w/E} \right] \frac{1}{\beta} \right] h^2 \Omega_0^2$$

percent of the matter between the wakes. The actual fraction of matter accreted by the wakes will depend on the geometry of the wakes and will be larger than our estimate if the wakes form closed surfaces. For $E_w/E = 0.4$, $\beta = 0.15$, and $h\Omega_0 = 0.25$ (about the largest value of $h\Omega_0$ consistent with the size of the large-scale structure) we find that the wakes could have accreted about $25\mu_6$ percent of the matter in the Universe. These wakes produce voids with the observed densities for $\mu_6 \sim 3$.

We now examine how the interwake separation (Δ), the fraction of matter accreted by the wakes (f), and the surface density (σ) vary with t_i and h for $\Omega_0 = 1$. We will initially use the approximation that the Universe is matter dominated. Corrections to this approximation will then be discussed. A simple calculation shows that $\Delta \sim t_i^{1/3}, f \sim t_i^{-2/3}$, and $\sigma \sim t_i^{-1/3}$ (this is actually an underestimate of Δ, f, and σ). Therefore $\Delta \sim 5.3(t_i/t_{\rm eq})^{1/3} h^{-2}$ Mpc, $f \sim 4.2\mu_6(t_i/t_{\rm eq})^{-2/3} h^2$, and $\sigma \sim 6.4 \times 10^{12} \mu_6 (t_i/t_{\rm eq})^{-1/3} h^2 M_\odot/\text{Mpc}^2$. If we want the interwake separation to be Sh^{-1} Mpc (observations indicate that $S \sim 25-50$) then $\Delta \sim Sh^{-1}$. This implies that $(t_i/t_{\rm eq})^{1/3} = (S/5.3)h$. Substituting this into f and σ gives $f \simeq (120/S^2)\mu_6$ and $\sigma \sim (3.4 \times 10^{13}/S)\mu_6 h$. To produce structure on $25h^{-1}$ Mpc (i.e., $S = 25$) with $h = \frac{1}{2}$ then requires $t_i \sim 13 t_{\rm eq}$. We find that $f \sim 0.20\mu_6$ and $\sigma \sim 6.8 \times 10^{11} \mu_6 M_\odot/\text{Mpc}^2$. For $S = 25$ and $h = 1$ we have $t_i \sim 100 t_{\rm eq}$, $f \sim 0.20\mu_6$, and $\sigma \sim 1.4 \times 10^{12} \mu_6 M_\odot/\text{Mpc}^2$. Therefore as h increases the wakes required to produce scales of order $25h^{-1}$ form at later times. The fraction of matter accreted by these wakes depends on time only through the time dependence of S. Wakes with $S = 50$ only accrete $\sim 5\mu_6$ percent of the matter in the Universe. We will assume that it is the last wakes which accrete almost all of the matter in the Universe (i.e., $f \lesssim 1$) which set the size of the large-scale structure. Since $f \sim 1/S^2$, the interwake separation of these wakes is not too sensitive to the actual value of f used ($0.5 \leq f \leq 1$). It is important to remember that the wakes with the largest surface densities are produced before the wakes we are considering. Here we have taken the simple assumption that these early wakes are accreted by the wakes we are considering. The wakes we are considering will not generally be accreted by later wakes since these later wakes only accrete a small fraction of the matter in the Universe. If this assumption turns out to be incorrect then the higher surface density wakes will probably be responsible for the large-scale structure. If this is the case then the size of the voids ($\sim 4h^{-2}\Omega_0^{-1}$) is too small to account for the observed large-scale structure ($\Omega \simeq 1$). Wakes which accrete 80% of the matter (to be in agreement with the CFA survey) have an interwake separation of $\sim 12\sqrt{\mu_6} h^{-1}$ Mpc which compares favorably with observations, for $\mu_6 \sim 4$. We also expect the string network to produce larger voids, but with considerably smaller underdensities since $f \sim 1/S^2$. We now discuss the corrections to the approximation used. For $t_i \sim 10 t_{\rm eq}$, f is underestimated by a factor of $\simeq 1.2$ and Δ is underestimated by a factor of $\simeq 1.3$. For $t_i \sim 100 t_{\rm eq}$, f is underestimated by a factor of $\simeq 1.2$ and Δ is underestimated by a factor of $\simeq 1.5$. We also have to correct for the change in the energy in the small-scale structure. E_w/E changes from about $\simeq 0.4$ in the transition era to about $\simeq 0.3$ in the matter-dominated era. The net effect of these corrections is to increase S by a factor of about 1.3 for $10 t_{\rm eq} \lesssim t_i \lesssim 100 t_{\rm eq}$. Thus the interwake spacing becomes $\sim 16\sqrt{\mu_6} h^{-1}$ which does not differ significantly from the previous value. We also get an interwake separation of $\sim 12\sqrt{\mu_6}h^{-1}$ Mpc for wakes formed at $t_{\rm eq}$ if we take $h\Omega_0$ so that $f=0.8$.

The two best bounds on $G\mu$ come from the millisecond pulsar and from the isotropy of the cosmic microwave background. Bouchet and Bennett [27] find that measurements of the timing of the millisecond pulsar constrains $G\mu$ to be $\lesssim 4\times 10^{-6}$. Bouchet, Bennett, and Stebbins [28] find that the isotropy of the cosmic microwave background constrains $G\mu$ to be $\lesssim 5\times 10^{-6}$. We see that strings can produce cosmologically significant density perturbations for $G\mu$ near but below the above constraints.

Recent surveys [29–32] indicate that regions of size $\simeq 45h^{-1}$ Mpc may have coherent streaming velocities of up to 10^3 km/s. The local group of galaxies is also known to be moving with respect to the microwave background radiation with a velocity of $\simeq 600$ km/s. It is therefore of interest to calculate the peculiar velocities generated by cosmic strings. The peculiar velocity in the Zel'dovich approximation for structure formed at $t_{\rm eq}$ is given by

$$\mathbf{v}_p=(1+z_{\rm eq})\dot{\mathbf{\Psi}}\simeq 0.25(1+z_{\rm eq})^{1/2}\mathbf{v}_i\Omega_0^{1/2} \tag{90}$$

for $(1+z_{\rm eq})\gg 1$ [recall that we have used $a(t)$ for a universe containing both matter and radiation]. From Eq. (84) we can expect peculiar velocities of magnitude

$$v_p\simeq 150\gamma\left[\beta+\left(\frac{E/E_w-\frac{1}{2}\gamma^{-2}E_w^2/E^2}{1-E/E_w}\right)\frac{1}{\beta}\right]\mu_6 h\Omega_0 \text{ km/s} . \tag{91}$$

For $E_w/E=0.4$ and $\beta=0.15$

$$v_0\simeq 560\mu_6 h\Omega_0 \text{ km/s} . \tag{92}$$

We expect these velocities to be coherent over some fraction of the interwake separation. If we consider wakes with an interwake separation of Sh^{-1} we find $v_p\sim(3.4\times 10^3/S)\mu_6$ km/s $\sim 75\mu_6$ km/s (for $S=45$). For an average interwake spacing of $20h^{-1}$ we find $v_p\sim 170\mu_6$ km/s which is close to the observed value for $\mu_6=4$. But the distance over which this velocity is coherent is smaller than observed.

We now consider the effect of small-scale structure on the fragmentation of a wake formed at $t_{\rm eq}$. Consider a wave pulse propagating along the string. As the wave propagates it will form a tube if the string velocity is nonzero (see Fig. 1). As we will see below, the surrounding matter will be attracted to this tube. The effect of this wave pulse will then be to generate a tubelike overdensity within the accretion wake of the string. The x and y impulse generated by a moving string ($f=0$) can be found from

$$\Delta\mathbf{v}=\tfrac{1}{2}\int_{-\infty}^{\infty}\nabla h_{00}\,dt . \tag{93}$$

Differentiating (45) and integrating by parts taking $g=0$ at $t=\pm\infty$ gives

$$\Delta v_y=-4\gamma G\mu\int_{-\infty}^{\infty}\frac{\beta g'(z-g)+\gamma g'^2(y-vt)}{\gamma^2(y-vt)^2+(z-g)^2} \tag{94}$$

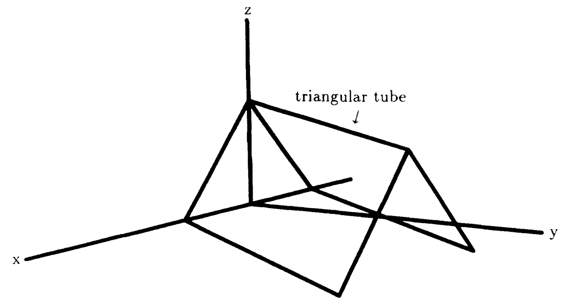

FIG. 1. The triangular tube formed by a triangular wave pulse propagating in the positive x direction. The velocity of the string is in the positive y direction.

and

$$\Delta v_x=-\beta\gamma\Delta v_y . \tag{95}$$

Since the velocity of the wave pulse in the (x,y) plane is $(c/\gamma,v)$ we see that the impulse given to the particles is at right angles to the tube generated by the wave pulse and is towards the tube. Thus there will exist a turnaround surface surrounding the tube. Particles on this surface are in the process of turning around from the Hubble flow and heading back towards the tube. Because of the symmetry we only need to examine the cross section of the turnaround surface on the plane $x=-\beta\gamma y$. Let us denote the rectangular coordinates on this plane by (l,z). To find the turnaround surface we look for particles whose (x,y) velocity orthogonal to the tube is equal to zero. This condition is $v_y=\beta\gamma v_x$. Substituting

$$\mathbf{v}=(1+z_{\rm eq})(\mathbf{q}+2\mathbf{\Psi}) \tag{96}$$

into $v_y=\beta\gamma v_x$ and using $\Delta v_x=-\beta\gamma\Delta v_y$ and $q_x=-\beta\gamma q_y$ gives

$$v_y=q_y+1.3t_{\rm eq}(1+z_{\rm eq})\Omega_0^{1/2}\Delta v_y=0 . \tag{97}$$

We take the wave pulse, in the rest frame of the string, to be the triangular wave

$$g(u)=\begin{cases}0, & u\le -a,\\ a+u, & -a\le u\le a,\\ a-u, & 0\le u\le a,\\ 0, & u\ge a.\end{cases}$$

The amplitude of the wave pulse will be written as $a=\alpha d_H\simeq 6.4\times 10^2\alpha h^{-4}\Omega_0^{-2}pc$ (d_H is the horizon distance at time $t_{\rm eq}$). α is then the ratio of the wave amplitude to the horizon at time $t_{\rm eq}$. We also write $\Delta v_y=-4G\mu\tilde{v}_y(q_y,q_z,\alpha,\beta)$. Equation (97) then becomes

$$q_y=38.0\mu_6 h^{-2}\tilde{v}_y(q_y,q_z,\alpha,\beta)pc . \tag{98}$$

Transforming $g(u)$ into a frame in which the string is moving with a velocity β and integrating (93) gives

$$\Delta v_y = -4G\mu \left\{ \mathrm{sgn}[y-\beta\gamma(z+x-a)] \left[\arctan\left[\frac{-\beta y+\gamma(a-z+\beta^2 x)}{|y-\beta\gamma(z+x-a)|} \right] - \arctan\left[\frac{-\beta y-\gamma[z-\beta^2(x-a)]}{|y-\beta\gamma(z-x+a)|} \right] \right] + \mathrm{sgn}[y+\beta\gamma(z-x-a) \left[\arctan\left[\frac{-\beta y+\gamma[z-\beta^2(x+a)]}{|y+\beta\gamma(z-x-a)|} \right] - \arctan\left[\frac{-\beta y+\gamma(z-a+\beta^2 x)}{|y+\beta\gamma(z-x-a)|} \right] \right] \right\} . \tag{99}$$

The solutions to (98) for the triangular wave pulse form two surfaces. For $\alpha \ll 1$ ($\mu_6 \sim 1$) the outer surface is the usual turnaround surface. The inner surface can be divided into two subsurfaces. For $z>0$ the velocity changes sign discontinuously across the surface. This part of the surface is generated by the discontinuous change in the velocity perturbation as we cross the string. For $z<0$ the velocity changes sign continuously. These two surfaces are shown in Fig. 2 for $\beta=0.15$, $\alpha=10^{-3}\mu_6=1$, $h=\frac{1}{2}$, and $\Omega_0=\frac{1}{2}$. In the interior of the inner surface the l-component of the velocity is outwards. In the region between the inner and outer surfaces the l-component of the velocity is directed inwards. In the region exterior to the outer surface the l-component of the velocity is outwards. Linear perturbation theory will break down near the surface of discontinuity since shell crossing will have occurred if the matter is collisionless. If the matter is collisional a shock wave will form which will prevent shell crossing. For the collisionless matter we are considering the discontinuity in the velocity stays at the same comoving coordinates as times evolves. Numerically we find that dimensions of the outer turnaround surface scale as $\sim\alpha^{1/2}\gamma^{1/2}\mu_6^{1/2}h^{-3}\Omega_0^{-1}$. This behavior can easily be seen for $\beta=0$, $|q_y| \gg |q_z|, a$. Equation (97) becomes

$$q_y = 76.1\mu_6 h^{-2}\mathrm{sgn}(y) \left[\arctan\left[\frac{6.4\times 10^2 \alpha h^{-4}\Omega_0^{-2} - q_z}{|q_y|} \right] + \arctan\left[\frac{q_z}{|q_y|} \right] \right] \tag{100}$$

$$\sim \frac{4.9\times 10^4 \alpha\mu_6 h^{-6}\Omega_0^{-2}}{q_y} \tag{101}$$

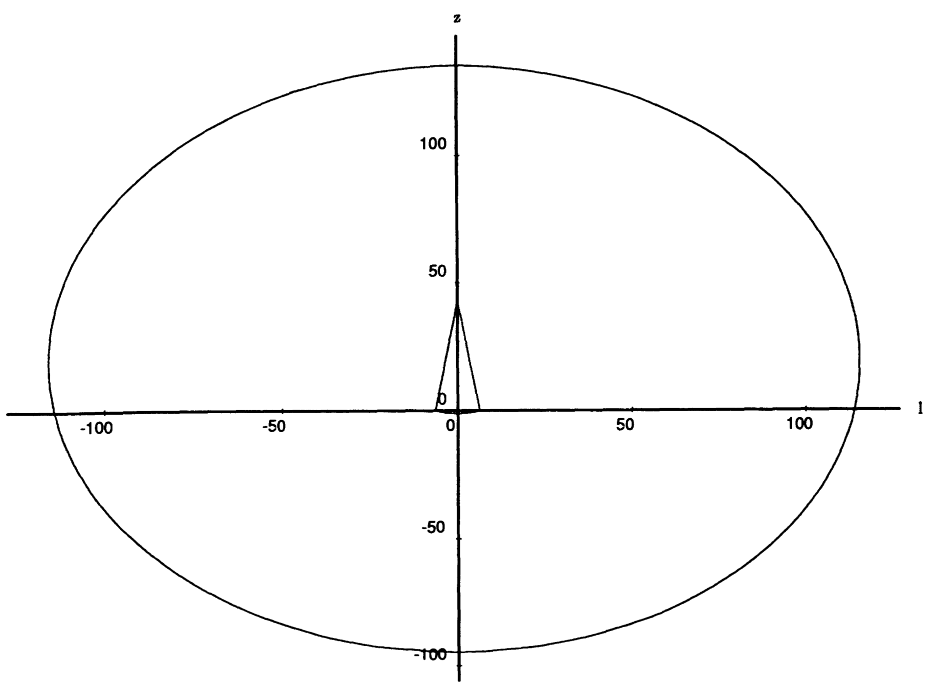

FIG. 2. A cross section of the turnaround surfaces surrounding the tube produced by the triangular wave pulse with $\beta=0.15, h=\frac{1}{2}$, $\Omega_0=\frac{1}{2}$, $\mu_6=1$, and $\alpha=10^{-3}$ (α is the ratio of the amplitude of the pulse to the horizon at t_{eq}). The outer turnaround surface scales as $\sim\alpha^{1/2}\gamma^{1/2}\mu_6^{1/2}h^{-3}\,\Omega_0^{-1}$. The cross-sectional plane is orthogonal to the tube. In the region interior to the inner surface the l-component of the velocity is outwards. In the region between the inner and outer surfaces the l-component of the velocity is inwards. In the region exterior to the outer surface the l-component of the velocity is outwards.

which has the scaling stated above for $\beta=0$. Since the inner surface scales as α these two surfaces will meet at $\alpha \sim 0.1\mu_6 h^2\Omega_0^2$.

We also need to know how the turnaround surface changes as the shape of the triangular wave is changed. If we denote the height of the wave by a and its base half-width by w, we find that the dimensions of the outer turnaround surface increase as $\sim(a/w)^{1/2}$. So as the wave pulse becomes more peaked the turnaround surface becomes larger. This behavior of the turnaround surface can be seen in the following way. The impulse given to a particle close to the triangular wave pulse, but not too close to the kinks, will be the same as the impulse produced by an infinitely long string. This impulse is proportional to $\beta_\perp\gamma_\perp$ where $\beta_\perp$ is the component of the string velocity which is orthogonal to the string. A simple calculation for the triangular wave pulse shows that the orthogonal velocity for the sides of the triangle is

$$\beta_\perp^2 \simeq 1-\gamma_s^{-2}\left[\frac{w}{a}\right]^2 \tag{102}$$

for $w/a \ll 1$, where β_s is the velocity of the string as a whole. Thus the impulse $\propto \beta_\perp\gamma_\perp \simeq \gamma_s(a/w)$. Therefore the velocity impulse grows as a/w for particles near the string. For $\beta_s=0$ and for $|q_y| \gg a, |q_z|$ we see from (101) that the distance to the turnaround surface goes as the square root of the impulse. Hence the result that the dimensions of the turnaround surface go as $(a/w)^{1/2}$ is not surprising. Since we expect the majority of waves on the string to be not too sharply peaked we will take $a/w=1$ for the subsequent calculations. If we have two waves on the string following each other too closely the velocity perturbations between the two tubes formed by these waves will tend to cancel and the outer turnaround surface will surround the tubes produced by both waves. If we have two wave pulses propagating in opposite directions on the string we expect the intersection of the two tubes to be the center of a region of large mass accretion. From (70) we see that for an ensemble of strings the average of the total velocity perturbation, to lowest order in E_w/E, will be the sum of the average velocity perturbations produced by the oppositely propagating waves. We have also found that the velocity perturbation produced by oppositely polarized waves propagating in opposite directions is, to lowest order, the sum of the perturbations produced by the individual waves. We therefore expect there to be a turnaround surface surrounding the intersection region which contains a coordinate volume of order

$$\sim \frac{4\pi}{3}(8)^3\left[\frac{\alpha}{10^{-3}}\right]^{3/2}\gamma^{3/2}\mu_6^{3/2}h^{-9}\Omega_0^{-3}pc^3 .$$

The mass contained in this volume is

$$\sim 10^{10}\left[\frac{\alpha}{10^{-3}}\right]^{3/2}\gamma^{3/2}\mu_6^{3/2}h^{-1}\Omega_0 M_\odot .$$

For

$$\left[\frac{\alpha}{10^{-3}}\right]^{3/2}\gamma^{3/2}\mu_6^{3/2}h^{-1}\Omega_0>1$$

this is of order of the mass of a galaxy. For $\alpha=10^{-2}$, $\mu_6=4$, $\gamma=1$, and $\Omega_0 h^{-1}=1$ the mass is $\sim 2.5\times10^{12}M_\odot$. Hence the intersection region of the two wave pulses can accrete a galactic mass. Since wakes which form earlier accrete most of the mass they may also assist in fragmenting the matter accreted by later wakes.

CONCLUSION

In this paper we have examined the velocity and density perturbations produced by a cosmic string with small-scale structure moving through a medium of cold collisionless matter. We find that the average velocity perturbation generated by an ensemble of strings is

$$\langle \Delta v_z\rangle = -4\pi\frac{G\mu}{c^2}\left[\beta+\left[\frac{E_w/E-\frac{1}{2}\gamma^{-2}E_w^2/E^3}{1-E_w/E}\right]\frac{1}{\beta}\right]\mathrm{sgn}(z)c , \tag{103}$$

where E_w/E is the ratio of energy of the waves on the strings to the total energy of the strings. For $E_w/E=0.4$, $\beta=0.15$ and an average interwake separation of Sh^{-1}Mpc the wakes produced by the strings have accreted $\sim(2.0\times10^4/S^2)\mu_6$ percent of the matter in the Universe. We assume that it is the last wakes which accrete almost all of the matter in the Universe (i.e., $f \lesssim 1$) which set the size of the large-scale structure. Wakes which accrete 80% of the matter in the Universe have an average interwake spacing of $\sim 16\sqrt{\mu_6}h^{-1}$ Mpc. We assume that the distribution of luminous matter mirrors the distribution of the cold collisionless matter so that the wakes will contain the same fraction of luminous matter as cold collisionless matter. A recent survey by the CFA indicates that galaxies may lie on the surfaces of bubblelike structures which contain voids of density $\sim 20\%$ of the mean density. Thus for $\mu_6 \gtrsim 3$ strings may be able to account for the observed large-scale structure. It has also been observed that large regions $\sim 45h^{-1}$ Mpc may be streaming coherently with peak velocities of up to 10^3 km/s. Strings can produce peculiar velocities of magnitude $v_p \sim (3.4\times10^3/S)\mu_6$ km/s over distances less than $\sim Sh^{-1}$. For $S=45$, $v_p \sim 75\mu_6$ km/s which is a bit small. For $S=20$, $v_p \sim 170\mu_6$ km/s which is close to the observed value for $\mu_6=4$. But the distance over which this velocity is coherent is smaller than observed.

We also find that wave packets propagating on the string will produce tubelike overdensities within the accretion wake of the string. For a triangular wave pulse the coordinate cross section of this tube has dimensions

$$\sim 8\left[\frac{\alpha}{10^{-3}}\right]^{1/2}\gamma^{1/2}\mu_6^{1/2}h^{-3}\Omega_0^{-1}pc,$$

where α is the ratio of the amplitude of the pulse to the horizon at the time of formation of the wake. We also find that the region of collision between two wave pulses propagating in opposite directions can accrete

$$\sim 10^{10}\left[\frac{\alpha}{10^{-3}}\right]^{3/2}\gamma^{3/2}\mu_6^{3/2}h^{-1}\Omega_0 M_\odot.$$

Thus structure on the strings can fragment the wake into galaxy mass objects.

After the completion of this work I received a paper by Tanmay Vachaspati and Alexander Vilenkin [26] which also discusses the role of small-scale structure in the formation of galaxies.

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