Seesaw-model predictions for the τ-neutrino mass

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The observed deficit of solar neutrinos cannot be explained by a cooler solar model, which would suppress high-energy neutrinos more than low-energy neutrinos, contrary to observation. The small neutrino masses and flavor mixing implied by matter-amplified neutrino oscillations in the Sun are most naturally interpreted in terms of minimal grand unification theories (GUT's) incorporating the seesaw mechanism. In two such theories, SO(10) GUT and supersymmetric GUT, that are consistent with all laboratory experiments, the neutrino mixing is like quark Cabibbo-Kobayashi-Maskawa mixing and the neutrino masses are proportional to the squares of the up-quark masses. For the SO(10) GUT model, the symmetry-breaking scale is intermediate and the μ-neutrino mass is close to that observed in solar-neutrino oscillations. Although the seesaw-model mass predictions are less reliable than the mixing-angle predictions, the τ-neutrino mass may lie in the cosmologically important range 4–28 eV and be accessible to laboratory neutrino oscillation experiments or to observation in a nearby supernova.

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I. COOLER SUN CANNOT EXPLAIN OBSERVED DEFICIT OF LOW-ENERGY NEUTRINOS

The proposed solutions to the solar-neutrino problem involve changes in the solar model or new particle physics. The principal uncertainties in the standard solar model (SSM) derive from residual uncertainties in the radiative opacities and from the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ cross section and its extrapolation to solar energies. If the opacities have been overestimated, then the core of the Sun will be cooler and the production of high-energy neutrinos will be reduced.

The local neutrino production rate in the Sun is a function mainly of the local temperature. The total rates are the local rates averaged over the Sun and, once subject to the known constraints of solar and nuclear physics, are most simply expressed as power-law functions of the core temperature T_C with the approximate exponents [1]:

$$\phi(^7\mathrm{Be}) \sim T_C^8, \quad \phi(^8\mathrm{B}) \sim T_C^{18} . \tag{1}$$

T_C most simply parametrizes the effects of radiative opacities and of energy redistribution into nonthermal degrees of freedom, such as a large core magnetic field, rapid core rotation, or core accumulation of weakly-interacting magnetic fields.

The ratios r of observed to predicted rates are

$$\begin{aligned} r_{\mathrm{Cl}} &= (1\pm0.025)[(0.77)(1\pm0.09)T_C^{18} + (0.14)(1\pm0.034)T_C^8 \\ &\quad + \text{small terms}], \\ r_{\mathrm{KII}} &= (1\pm0.09)T_C^{18} , \end{aligned} \tag{2}$$

where T_C is the core temperature normalized to the SSM central temperature of 1.56×10^7 K = 1.34 keV, and we are quoting 1σ uncertainties in the ^{8}B and ^{7}Be neutrino detection rates. (The ^{8}B uncertainties are correlated between the two experiments. We also include a 2.5% uncertainty in the chlorine absorption cross section. All these 1σ uncertainties that we use derive from the 3σ uncertainties in nuclear cross sections and neutrino absorption cross sections given in Bahcall and Ulrich [1].) The effect of nonstandard solar models on neutrino production can now be simply parametrized by changing T_C.

It is easy to see that a cooler Sun would require higher-energy neutrinos to be more suppressed, in contradiction with the observations [2,3]. Thus the Kamiokande II (KII) rate $r_{\mathrm{KII}}=0.46\pm0.08$ [4] can be fit by a $4\pm1\%$ reduction in T_C ($T_C^{\mathrm{KII}}=0.958\pm0.011$), but the Homestake rate $r_{\mathrm{Cl}}=0.28\pm0.025$ [5] requires an $8\pm1\%$ reduction ($T_C^{\mathrm{Cl}}=0.918\pm0.007$). These two temperature reductions differ from one another by 3σ. The combined rates can be simultaneously fit by an $8\pm1\%$ temperature reduction ($T_C=0.923\pm0.008$), but only with $\chi^2/N_{\mathrm{DF}}=8.5$, which rejects the cooler Sun hypothesis at 99.8% confidence. (This argument is insensitive to the temperature exponents in Eq. (1). Provided the ^{7}B neutrino flux is not less temperature dependent than the ^{7}Be neutrino flux, $r_{\mathrm{Cl}}\geq r_{\mathrm{KII}}$ would be expected. Including the cross section uncertainties, this extreme hypothesis is already rejected at $\chi^2/N_{\mathrm{DF}}=4.46$ or 96.6% C.L. The enhanced temperature dependence of the ^{8}B neutrino flux only strengthens this conclusion.)

Of the particle-physics solutions of the solar-neutrino problem, only matter-amplified neutrino [Mikheyev-Smirnov-Wolfenstein (MSW)] oscillations occur naturally for a range of neutrino parameters [2]. In this interpretation of the combined Homestake, Kamiokande II, and preliminary Soviet-American Gallium Experiment (SAGE) data, either semiadiabatic or large-mixing adiabatic oscillations are taking place in the Sun because of neutrino vacuum mixing $\sin\theta > 0.03$ and the mass-squared difference $\delta m^2 = 0.08-20$ meV^2 (1 meV$\equiv 10^{-3}$ eV). We believe, from laboratory limits and from theoretical expectations in the next section, that the neutrino mixing matrix is similar to the quark [Cabibbo-Kobayashi-Maskawa (CKM)] mixing matrix. The oscillations in the Sun are therefore most likely $\nu_e \to \nu_\mu$ with the μ-neutrino mass $m_{\nu_\mu} = 0.3-4$ meV. For Cabibbo mixing, the solar neutrino oscillations are semiadiabatic with $m_{\nu_\mu} \simeq 0.5-0.7$ meV, but m_{μ_ν} could be as much as six times larger if the neutrino mixing is much smaller or much larger than Cabibbio mixing.

II. SEESAW MODEL FOR SMALL NEUTRINO MASSES

The MSW solution requires new physics beyond the electroweak standard model (SM). Unless one wants to invoke a new symmetry, the most natural and minimal extension of the SM that explains the smallness of ordinary ν_L masses is the seesaw model. This model [6] invokes a superheavy right-handed Majorana neutrino N_R, which can form a (Majorana) mass $M_N \overline{N}{}^c_L N_R$ with itself and a (Dirac) mass $m_D \overline{\nu}_L N_R$ with the ν_{iL} of the standard model, i.e., a mass term

$$m_D \overline{\nu}_L N_R + M_N \overline{N}{}^c_L N_R = \frac{1}{2}(\nu_L \overline{N}{}^c_L) \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix}$$

in the Lagrangian. Allowing for three flavors for both the R and L neutrinos, m_D and M_N are each 3×3 matrices. If the three eigenvalues of M_N are all much larger than the components of m_D, then diagonalizing the Lagrangian leads to the three light Majorana neutrinos with mass matrix $m_\nu = m_D M_N^{-1} m_D^T$, and three (unobserved) superheavy Majorana neutrinos with mass matrix M_N. The limit with M_N proportional to the identify is the *quadratic* seesaw, because the m_ν then vary as m_D^2. The eigenvalues of M_N alternatively might follow the same hierarchy as m_D (M_N proportional to m_D), leading to the *linear* seesaw, m_ν varying as m_D. Simple seesaw models assume a vanishing ν_L Majorana mass, but important terms of this type might be generated at the loop level in extended models. The matrix M_N can also be generated at loop level; in some of these cases, M_N is proportional to m_D, leading to a linear seesaw.

Quarks and charged leptons are expected to have masses of order m_D. All of these masses are evaluated at the grand-unified-theory (GUT) scale M_X, so that, neglecting family mixing for the moment, we finally have for each family $m_{\nu_i}(X) = m_{D_i}^2(X)/M_{N_i}(X)$. These masses now have to be run down to the low-energy scale by renormalization-group calculations that are model dependent [3].

If one knows m_{ν_μ} then m_{ν_τ} can be predicted in a specific model, although it clearly makes a substantial difference whether $m_{D_3} \sim m_t$ or m_τ is assumed, and whether there is a quadratic or linear seesaw. For $m_{\nu_\mu} \sim 0.5$ meV one expects [3] $m_{\nu_\tau} \sim 3-21$ eV or 0.1 eV for the quadratic seesaw with $m_{D_3} \sim m_t$ or m_τ, respectively. The corresponding predictions for the linear seesaw are 0.03–0.05 eV and 0.01 eV. For these models M_{N_2} varies from 2×10^{10} GeV ($m_D \sim$ lepton masses) to 5×10^{12} GeV ($m_D \sim$ quark masses). Such a scale may arise naturally from the breakdown of Peccei-Quinn symmetry at a scale $\sim 10^{11}-10^{12}$ GeV needed to close the Universe by invisible axions or may arise from the breakdown of hidden supergravity symmetry breaking at $\sqrt{m_{3/2} M_{\text{Planck}}} \sim 10^{11}$ GeV, where $m_{3/2} \sim 1$ TeV is the gravitino mass.

To discuss the seesaw model predictions in any greater detail requires a definite model. The most predictive are grand-unified theories, which naturally give $m_D = m_u$ at the tree level and generally predict CKM mixing; i.e., that the quark and leptonic mixings are the same. The Majorana matrix M_N and the value of the top-quark mass $m_t = 124\pm 34$ GeV [7] are the major uncertainties. If we run the tree-level relations from the unification scale to low energies, renormalization-group corrections significantly change the masses. For these reasons, seesaw calculations should be viewed as a semiquantitative guide to possible neutrino masses rather than as rigorous predictions. For a wide range of parameters neither the tree-level uncertainties nor the corrections significantly affect the mixing-angle predictions [3].

We discuss two theoretical models [7,8] that are minimal extensions of SU(5) to SO(10) and are both consistent with all present laboratory experiments. [Groups larger than SO(10) would give similar results.] Both extensions include a superheavy right-handed Majorana neutrino that induces a quadratic seesaw with up-quark masses and neutrino flavor mixings nearly equal to the quark (CKM) mixings, i.e., $m_{\nu_e}:m_{\nu_\mu}:m_{\nu_\tau} = m_u^2:m_c^2:m_t^2$ (before applying radiative corrections) and $\sin^2 2\theta_{e\mu} = 0.18$, $\sin^2 2\theta_{\mu\tau} = (0.004-0.014)$, $\sin^2 2\theta_{e\tau} = (4\times 10^{-6})-(2\times 10^{-3})$. The first model, the SO(10) minimal supersymmetric (SUSY) GUT, successfully predicts the weak angle but leads to small neutrino masses. The second model, a non-SUSY SO(10) GUT, requires intermediate scale symmetry breaking, but predicts a μ-neutrino mass near that derived from solar-neutrino oscillations and a τ-neutrino mass that may be cosmologically important.

In the SUSY GUT model, the unification scale is $M_X = 1.6\times 10^{16\pm 0.4}$ GeV and the SUSY-breaking scale is assumed ≤ 1 TeV. A reasonable Yukawa coupling of Higgs bosons to right-handed neutrinos then makes each right-neutrino mass $M_{N_i} \sim (0.01-0.1) M_X = 1.6\times 10^{15\pm 1.4}$ GeV, near the GUT scale. The radiatively corrected predictions for the neutrino masses are then given by the approximately quadratic seesaw formulas

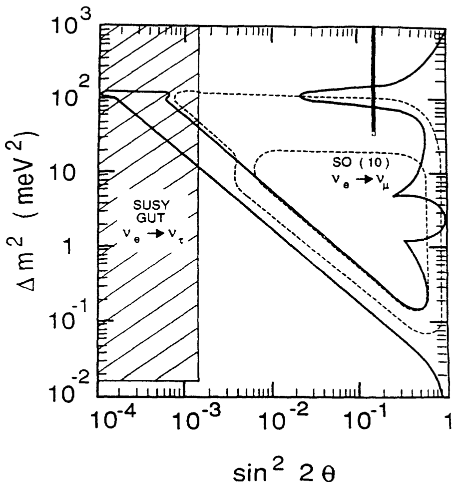

FIG. 1. Regions for the neutrino mixing and squared mass difference allowed at 90% C.L. by the Homesake (dashed lines) and Kamiokande II (solid lines) observations. Superimposed are the predictions of the SUSY and non-SUSY models described in the text. The mixing-angle predictions are much more robust than those of the neutrinos masses: the m_{ν_μ} prediction for the non-SUSY model could be lowered by reasonable changes in the intermediate scale.

$$m_{\nu_e} = (0.05)\frac{m_u^2}{M_N} < 2\times10^{-11} \text{ eV} ,$$

$$m_{\nu_\mu} = (0.09)\frac{m_c^2}{M_N} = (6\times10^{-9})-(4\times10^{-6}) \text{ eV} ,$$

$$m_{\nu_\tau} = (0.38)\frac{m_t^2}{M_N} = 0.000\,11-0.87 \text{ eV} .$$

The large uncertainties derive from the uncertainties in M_N and its possible family dependence and from the uncertainties in the top-quark mass and its running. The radiative corrections to m_{ν_τ} are quite different from those to m_{ν_μ} because of the large top-quark–Higgs-boson coupling.

The SUSY GUT model gives the μ neutrino too small a mass for $\nu_e \to \nu_\mu$ to appear in solar-neutrino oscillations. If we instead use the small $\nu_e \to \nu_\tau$ mixing, we obtain a δm^2 and mixing at the upper left corner of the MSW triangle, close to but somewhat outside the range needed to solve the solar-neutrino problem (see Fig. 1). The neutrino mass contribution to the cosmological matter density is then small, less than the baryon mass density.

In the non-SUSY SO(10) GUT, a first symmetry breaking down to a left-right-symmetric model takes place at a GUT scale $M_X = 9.6\times10^{16\pm0.4}$ GeV, followed by a second symmetry breaking down to the SM at an intermediate scale $M_R = 1.5\times10^{10\pm0.3}$ GeV. A reasonable Yukawa coupling of Higgs bosons to right-handed neutrinos then gives each right-handed neutrino a mass $M_N \approx (0.01-1)M_R = (0.7-3)\times10^{9\pm1}$ GeV, near the intermediate scale. The radiatively corrected SO(10) predictions for the neutrino masses are then given by the approximately quadratic seesaw formulas

$$m_{\nu_e} = (0.05)\frac{m_\mu^2}{M_N} < 2\times10^{-5} \text{ eV} ,$$

$$m_{\nu_\mu} = (0.07)\frac{M_c^2}{M_N} = 0.0055-2.2 \text{ eV} ,$$

$$m_{\nu_\tau} = (0.18)\frac{m_t^2}{M_N} = 60-10^5 \text{ eV} .$$

At its lower limit, the μ-neutrino mass is only eight times larger than that observed in solar-neutrino oscillations for Cabibbo-angle mixing. At its lower limit, the τ-neutrino mass is only twice the cosmological upper limit of 28 eV. Smaller neutrino masses would be obtained if the intermediate scale Higgs-boson content were altered so as to realize the intermediate scale symmetry breaking at a somewhat larger value of M_R.

III. POSSIBLE 17-keV NEUTRINO

A neutrino with a mass of 17 keV appearing with 1% mixing in β decay has been reported in some crystalline detector experiments [9], but not in magnetic spectrometer experiments [10]. If it exists, a 17-keV neutrino must impart only a small (< 1 eV) Majorana mass to the electron neutrino, in order to evade double-β-decay constraints, and needs to decay invisibly and fast, in order to evade cosmological and astrophysical constraints [11,12]. The constraint on double-β decay will be satisfied if the 17-keV neutrino is a Dirac neutrino or there are compensatory mixings among massive Majorana neutrinos [13]. The strongest evidence against a 17-keV *Dirac* neutrino comes from the 4-sec cooling time observed for the hot neutron-star remnant from supernova 1987A: conventional Z^0 exchange would convert ν_L into sterile ν_R, which would accelerate the observed cooling rate by a factor of at least 2 [14]. The double-β decay, nucleosynthesis, and supernova constraints can all be satisfied [11] if the μ and τ neutrinos are both *Majorana* with $m_{\nu_\mu} \geq m_{\nu_\tau} = 17$ keV. In that case, however, MSW resonant conversion in the Sun would require an additional sterile (singlet) neutrino with mass 0.3–4 meV.

A 17-keV neutrino appearing with 1% mixing in β decay cannot be responsible for MSW solar-neutrino oscillations and cannot arise naturally by the three-family seesaw mechanism. Nonseesaw theoretical models can be contrived, but small neutrino masses do not occur naturally and require a symmetry-breaking scale much smaller than the intermediate or GUT scales obtained above for our seesaw models. The models in Ref. [15] produce a 17-keV Dirac neutrino, which satisfies the lifetime and double-β-decay constraints, but not that from the cooling of supernova 1987A.

IV. PREDICTIONS FOR THE τ-NEUTRINO MASS

We have just seen that at least some theoretical models imply the quadratic seesaw relation $m_{\nu_\tau} = (2-4)(m_t/m_c)^2 m_{\nu_\mu} = (0.7-4)\times10^4 m_{\mu_\nu}$. If symmetry breaking transpires at an intermediate scale, so that $\nu_e \to \nu_\mu$ is taking place in the Sun with $m_{\nu_\mu} = 0.3-4$ meV, then m_{ν_τ} will lie in the cosmologically important range 2–100 eV. Regardless of theoretical motivation, this mass range will be pursued in terrestial experiments and in observations of core-collapse supernovas in our own or nearby galaxies.

Since $m_{\nu_\tau}=92\Omega_\nu h^2$ eV, the mass density in present-day neutrinos will equal that in baryons ($\Omega_B h^2 \simeq 0.01-0.02$) if $m_{\nu_\tau} \simeq 1-2$ eV. The dynamical age (> 13 Gyr) of the Universe determines an upper limit $\Omega_0 h^2 < 0.25$, if there is no cosmological constant, and $\Omega_0 h^2 < 0.38$, if we admit the maximum cosmological constant $\lambda_0 = 0.8$ permitted by observations of the frequency of multiple imaging of quasistellar objects (QSO's) [11]. [These limits depend only on the minimum age of the Universe and $h < 0.5$, and not on assumptions about a vanishing cosmological constant, a flat Universe ($\Omega_0 = 1$), or $h < 1$. For the Universe to be flat, we require $h < 1.1$, if we allow a cosmological constant, and $h < 0.5$, if there is no cosmological constant at present.]

If neutrinos constitute 80–90% of the present Universe, our age constraint by itself implies $m_{\mu_\tau} < 28-32$ eV or 19–21 eV, depending on whether or not we allow a cosmological constant. If the Universe is to be flat, the upper limits must be realized. In that case, massive neutrinos would close the Universe and provide a natural source of hot dark matter. Together with non-Gaussian primordial fluctuations, massive neutrinos might then be responsible for large-scale structure, especially if the canonical cold-dark-matter scenario proves unable to account for the very large cosmological structures now observed.

Present laboratory limits on $\nu_\mu \rightarrow \nu_\tau$ oscillations, from the E531 emulsion experiment at Fermilab [16], are $\sin^2 2\theta < \sin^2 2\theta_c = 0.18$ for $\delta m^2 > 2$ eV^2 and $\sin^2 2\theta < 0.004$ for $\delta m^2 > 20$ eV^2, at 90% C.L. These experiments apparently already exclude τ neutrinos massive enough to close the Universe if, as we expect from the CKM matrix, $\sin^2 2\theta \approx 0.008$. These experiments deserve repetition and refinement.

Atmospheric neutrino experiments [17] exclude smaller $\delta m^2 > 10^{-4\pm0.7}$ eV^2 but only for $\sin^2 2\theta > 0.4$. Atmospheric neutrino observations can confirm solar-neutrino oscillations only if, contrary to preliminary SAGE results, the large-mass large-angle MSW solution is realized in the Sun.

V. CONCLUSIONS

At least two theoretical models consistent with all laboratory data lead to a quarklike neutrino mixing and seesaw neutrino masses quadratic in the up-quark masses. For the non-SUSY GUT model, the symmetry-breaking scale is intermediate and the μ-neutrino mass is close to that observed in solar-neutrino oscillations. Although seesaw-model mass predictions are less reliable than mixing-angle predictions, the τ-neutrino mass may lie in the cosmologically important range 4–28 eV and be accessible to laboratory neutrino oscillation experiments or to observation in a nearly supernova.

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