Nature of direct CP violation in $K^0 \to \pi\pi$ decays

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Galic's relation between $(\phi_{+-} - \phi_{00})$ and $|\eta_{00}/\eta_{+-}|$ is generalized to include possible CPT noninvariance. Future data can improve the limits on the CPT-noninvariant contributions that are allowed at present.

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A nonzero value of the observables $O_1 = 1 - |(\eta_{00}/\eta_{+-})|^2$ and/or $O_2 = \tan(\phi_{+-} - \phi_{00})$ indicates the presence of direct CP violation in $K^0 \to \pi\pi$ decays. Galic [1] deduced a relation between O_1 and O_2 under the assumption of CPT invariance. We derive the corresponding relation without assuming CPT invariance. Use of recent data [2,3] in our relation shows that the relative strength of the CPT-noninvariant amplitude responsible for direct CP violation can be appreciable, though of course it is allowed to vanish altogether.

We express various symmetry violations in $K \to \pi\pi$ decay amplitudes as [4]

$$\langle I|\mathcal{T}|K^0\rangle = a_I e^{i\delta_I}(1 + i\theta_I + i\alpha_I + \beta_I) \ , \tag{1a}$$

$$\langle I|\mathcal{T}|\bar{K}^0\rangle = a_I e^{i\delta_I}(1 - i\theta_I + i\alpha_I - \beta_I) \tag{1b}$$

where I indicates the $\pi\pi$ state of total isospin I ($I = 0$, 2); a_I, θ_I, α_I, and β_I are real parameters; δ_I is the S-wave $\pi\pi$ scattering phase shift (at an energy equal to the kaon mass) in the state with isospin I. Nonzero

$$\theta = (\theta_2 - \theta_0)$$

implies T noninvariance and CP noninvariance, but is allowed by CPT invariance. Nonzero α_I implies CPT noninvariance and T noninvariance, but is allowed by CP invariance. Nonzero β_I indicates CPT noninvariance and CP noninvariance, but is allowed by T invariance. We take $|\bar{K}^0\rangle = CP|K^0\rangle$. The decaying states are

$$|K_S\rangle = |K_1\rangle + \varepsilon_S|K_2\rangle \ , \tag{2a}$$

$$|K_L\rangle = |K_2\rangle + \varepsilon_L|K_1\rangle \ , \tag{2b}$$

where the CP eigenstates $|K_{1,2}\rangle$ are

$$|K_{1/2}\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \pm |\bar{K}^0\rangle) \ , \tag{3}$$

and $\varepsilon_{S,L}$ are complex parameters; CPT invariance requires $\varepsilon_S = \varepsilon_L$. While $\varepsilon_{S,L}$ describe CP violation in the structure of the kaon states, θ_I, α_I, and β_I describe "direct" violations of various symmetries.

We drop terms of second and higher order in $\varepsilon_{S,L}$ throughout since the observed CP violation is small. Similarly we shall retain θ_I, α_I, and β_I only to the lowest significant order. Since $(a_2/a_0)^2$ is known (see, e.g., Refs. [4,5]) to have a value comparable to $|\eta_{+-}|$, we shall keep terms of only low order of $A \equiv (a_2/a_0)$.

Expressing $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^0\rangle$ in terms of the $|I = 0,2\rangle$ states, one obtains the usual relations [4,5]

$$\eta_{+-} \equiv |\eta_{+-}|e^{i\phi_{+-}} \equiv \frac{\langle \pi^+\pi^-|\mathcal{T}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{T}|K_S\rangle} \ , \tag{4a}$$

$$= E + \frac{a}{\sqrt{2}}(\beta + i\theta) \ , \tag{4b}$$

$$\eta_{00} \equiv |\eta_{00}|e^{i\phi_{00}} \equiv \frac{\langle \pi^0\pi^0|\mathcal{T}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{T}|K_S\rangle} \ , \tag{5a}$$

$$= E - \sqrt{2}a(\beta + i\theta) \ , \tag{5b}$$

where

$$E = \beta_0 + i\theta_0 + \varepsilon_L \ , \quad E \equiv |E|e^{i\sigma} \ , \tag{6a}$$

$$a = Ae^{i\Delta} \ , \quad \Delta = \delta_2 - \delta_0 \ , \tag{6b}$$

$$\beta = \beta_2 - \beta_0 \ . \tag{6c}$$

We need not restrict ourselves to any particular choice [6] for the relative phase between $|K^0\rangle$ and $|\bar{K}^0\rangle$. Introducing the real parameters

$$\Psi = \Delta - \sigma \ , \quad c = \cos\Psi \ , \quad s = \sin\Psi \ ,$$

$$r = \frac{A}{|E|\sqrt{2}}(\beta c - \theta s) \ , \tag{7}$$

$$z = \frac{A}{|E|\sqrt{2}}(\beta s + \theta c) \ ,$$

one gets

$$O_1 \equiv \frac{|\eta_{+-}|^2 - |\eta_{00}|^2}{|\eta_{+-}|^2} = 6r \ , \tag{8a}$$

$$O_2 = \tan(\phi_{+-} - \phi_{00}) = 3z \ . \tag{8b}$$

The desired generalization of Galic's relation follows from Eqs. (8a) and (8b) as

$$2rO_2 = zO_1 \ . \tag{8c}$$

Since β and θ are independent unknowns, the ratio z/r is unknown. Thus the relation (8c) cannot be used to predict O_2/O_1 in general. If, however, the source of direct CP violation is purely CPT noninvariance, one has $\theta = 0$, $\beta \neq 0$, and $z = r\tan\Psi$. If the source of direct CP violation is purely T noninvariance, $\beta=0$, $\theta\neq 0$, and $z=-r\cot\Psi$; this is the case considered by Galic [1].

In order to know Ψ, occurring in Eqs. (8), we need Δ and σ. Equations (4)–(6) lead to

$$E=\tfrac{1}{3}(\eta_{00}+2\eta_{+-}) , \tag{9a}$$

$$\tan\sigma=\frac{b\sin\phi_{00}+\sin\phi_{+-}}{b\cos\phi_{00}+\cos\phi_{+-}} \tag{9b}$$

where $b=\frac{1}{2}|\eta_{00}/\eta_{+-}|$.

Since the ratio of the amplitude parameters β and θ associated with direct CP violation is not known, we rewrite Eq. (8c) as

$$\frac{\beta}{\theta}=\frac{(2O_2/O_1)+\cot\Psi}{(2O_2/O_1)\cot\Psi-1} , \tag{10}$$

and evaluate the right-hand side of Eq. (10) for two typical values of Δ, using CERN data [2] for O_1 and O_2. Given the large uncertainty in Δ, the accuracy of σ [see Eq. (9b)] is not important. We take [2,7]

$$O_1=0.020\pm0.0064 ,$$
$$O_2=-0.0035\pm0.05 , \tag{11}$$
$$\sigma=(44.6\pm1.2)^0 .$$

If one further takes [8]

$$\Delta=(-41.4\pm8.1)^0 , \tag{12a}$$

one gets, from Eq. (10),

$$\frac{\beta}{\theta}=0.4\pm5 . \tag{13a}$$

If the value of Δ is changed to [9]

$$\Delta=(-29.2\pm3)^0 \tag{12b}$$

the result of Eq. (13a) changes to

$$\frac{\beta}{\theta}=0.7\pm6 . \tag{13b}$$

The main source of the large errors in Eqs. (13a) and (13b) is O_2. If data from Ref. [3] (instead of Ref. [2]) were used, the result for (β/θ) would continue to be inconclusive.

The amplitude parameter β refers to CPT noninvariance (and T invariance), and θ refers to T noninvariance (and CPT invariance). Better data on $O_{1,2}$ and on Δ would help to improve the limits on (β/θ) which is at present allowed to be large; see Eqs. (13a) and (13b).

Note added in proof. If our β were conclusively nonzero, it would mean a nonzero value for one or both of β_0 and β_2. The CPT-noninvariant contribution to the neutral kaon mass matrix was recently bounded by Carosi *et al.* [2] by assuming $\beta_0=0$. See also L. Lavoura, Carnegie-Mellon University Report No. CMU-HEP 91-21, 1991 (unpublished). While Lavoura analyzes the combination E of our Eq. (9a), we analyze contributions to $(\eta_{+-}-\eta_{00})$.

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