BRIEF REPORTS

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Saturation of the Drell-Hearn-Gerasimov sum rule reexamined

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The isospin decomposed Drell-Hearn-Gerasimov sum rule is examined. The single-pion photoproduction contribution is integrated out to 1.7 GeV in the laboratory photon energy. We obtain an improved estimate of the contribution of this process to the isovector and isovector-isoscalar sum rules. The qualitative features of Karliner's results remain valid. The isoscalar sum rule shows no indication of convergence.

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Interest in the Drell-Hearn-Gerasimov (DHG) sum rule [1] has recently been revived by Anselmino, Ioffe, and Leader [2]. These authors have linked the DHG sum rule to the spin crisis, finding that there should be substantial corrections from higher-twist terms. If satisfied, the DHG sum rule is important as it gives the $Q^2=0$ point to which other sum rules (for example, the Björken sum rule [3]) must extrapolate. We have reexamined the extent to which the pion-photoproduction reaction saturates the DGH sum rule, updating the analysis of Karliner [4]. We have used for the photoproduction amplitudes a recent analysis [5] to 1.8 GeV in the laboratory photon energy.

Before proceeding, we should note that the DHG sum rule for proton targets requires cross sections from all inelastic channels and not just $\gamma p \to \pi N$. The dominance of the πN channel is an assumption, as is the validity of an unsubtracted dispersion relation. We will briefly discuss these issues together with our numerical results.

As was done by Karliner [4], we have taken the original sum rule

$$\frac{2\pi^2\alpha}{M^2}(\kappa_P)^2=\int_0^\infty \frac{\sigma_{3/2}(\omega)-\sigma_{1/2}(\omega)}{\omega}d\omega\ , \tag{1}$$

and have decomposed it into scalar and vector parts via $\kappa_P=(\kappa_S+\kappa_V)/2$, κ_P being the proton anomalous magnetic moment, ω the laboratory photon energy, M the proton mass, and α the fine-structure constant. The cross sections of definite helicity are, in Walker's [6] notation,

$$\sigma_{1/2}=\frac{8\pi q}{k}\sum_{n=0}^{\infty}(n+1)(|A_{n+}|^2+|A_{(n+1)-}|^2) \tag{2}$$

and

$$\sigma_{3/2}=\frac{8\pi q}{k}\sum_{n=0}^{\infty}\frac{1}{4}[n(n+1)(n+2)] \times(|B_{n+}|^2+|B_{(n+1)-}|^2)\ , \tag{3}$$

where q and k are, respectively, the pion and photon center-of-mass momenta.

The isospin decompositions of amplitudes used in Refs. [4,5] are related through the relations

$$A^{3/2}=(\tfrac{3}{2})^{1/2}A^{(3)}\ , \tag{4}$$

$${}_pA^{1/2}=\frac{1}{\sqrt{3}}(A^{(1)}-A^{(0)})\ , \tag{5}$$

$${}_nA^{1/2}=-\frac{1}{\sqrt{3}}(A^{(1)}+A^{(0)})\ . \tag{6}$$

In addition, the helicity amplitudes used in Ref. [4] are given in terms of our multipole amplitudes [5] via

$$A_{n+}=\tfrac{1}{2}[(n+2)E_{n+}+nM_{n+}]\ , \tag{7}$$

$$A_{(n+1)-}=\tfrac{1}{2}[(n+2)M_{(n+1)-}-nE_{(n+1)-}]\ , \tag{8}$$

$$B_{n+}=E_{n+}-M_{n+}\ , \tag{9}$$

$$B_{(n+1)-}=E_{(n+1)-}+M_{(n+1)-}\ . \tag{10}$$

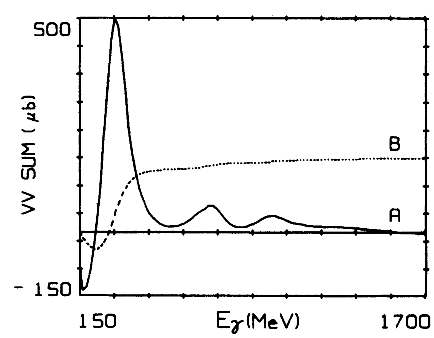

FIG. 1. Vector-vector (VV) sum-rule contributions from pion photoproduction. The quantity $\Delta\sigma$ is plotted in curve A. The integral I^{VV} is given by curve B.

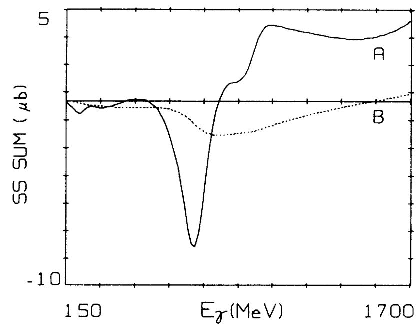

FIG. 2. Scalar-scalar (SS) sum rule. Notation as in Fig. 1.

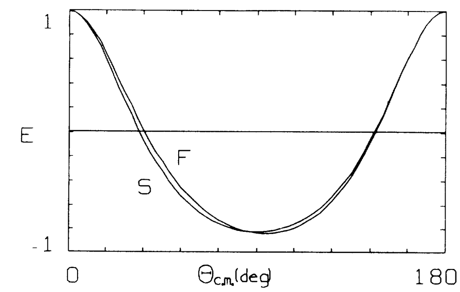

FIG. 4. Comparison of predictions from Ref. [5] (S) and Ref. [8] (F) for E in $\gamma p \rightarrow p\pi^0$ at 300 MeV.

Our results for the interference DHG sum rules

$$I^{ij}=\int_{\omega_0}^{\infty}[\sigma_{3/2}^{ij}(\omega)-\sigma_{1/2}^{ij}(\omega)]\frac{d\omega}{\omega} \tag{11}$$

are given in Figs. 1–3 for $ij=$VV, SS, and VS. Explicit relations for σ^{ij} are given in Ref. [4], along with the expected values of 219, 0.3, and -15 μb, respectively, for I^{VV}, I^{SS}, and I^{VS}. (The sum of these gives the proton sum rule.) In Figs. 1–3, we have plotted the difference $\Delta\sigma^{ij}=\sigma_{3/2}^{ij}-\sigma_{1/2}^{ij}$ as well as the integral I^{ij}. It appears that essentially all of the pion-photoproduction contribution is included in I^{VV} and I^{VS}, if the integration is extended to 1.7 GeV. The contributions to I^{VV} and I^{VS} are, respectively, 176 and 19 μb. This is to be compared with Karliner's values [4] of 170 and 24 μb, derived from analyses of photoproduction data to 1.2 GeV.

In order to estimate the uncertainty in these results, we have analyzed two data bases. The first of these was a "raw" data base to 2 GeV, described in Ref. [5]. The second was "pruned" of redundant and inconsistent measurements. The result for I^{VV} was found to be quite stable, the raw data base giving 172 μb. This data base, however, gave only 10 μb for I^{VS}. The result for I^{SS} remained small, varying from 0.4 to -0.9 μb.

Given Karliner's estimate (49 μb) for the remaining inelastic contributions to I^{VV}, this component is in good agreement with the DHG prediction. The sign of I^{VS} remains positive, in disagreement with the sum rule. Here, however, the inelastic contributions are found to be comparable to those derived from pion photoproduction. Unfortunately, Karliner's estimate for the inelastic contribution to I^{VS} is also positive (15 μb) and enhances the discrepancy. In contrast with I^{VV} and I^{VS}, the scalar-scalar sum rule shows no sign of converging. The reliability of I^{SS} is difficult to assess, particularly at higher energies, as the existing photoproduction data base may not provide sufficient constraints for the extraction of a small component.

It might be useful to recall that the $\Delta\sigma$ contribution from pion photoproduction is related to the beam-target polarization observable [7] E. In terms of Walker's amplitudes [6] we have cross sections

$$\frac{d\sigma_{3/2}}{d\Omega}=\frac{q}{k}(|H_1|^2+|H_3|^2)\ , \tag{12}$$

$$\frac{d\sigma_{1/2}}{d\Omega}=\frac{q}{k}(|H_2|^2+|H_4|^2)\ , \tag{13}$$

the difference of these being related to E where

$$E=\frac{|H_2|^2+|H_4|^2-|H_1|^2-|H_3|^2}{|H_2|^2+|H_4|^2+|H_1|^2+|H_3|^2}\ . \tag{14}$$

Measurements of E above 1 GeV, while difficult, could help to constrain the pion-photoproduction amplitudes used in the DHG sum rule. In the Δ resonance region, predictions for E are reasonably consistent. The results of two analyses [5,8] are displayed in Fig. 4. At higher energies, this consistency is lost. Figure 5 illustrates the different predictions of the above two analyses at 600 MeV. No data exist for comparison.

In summary, we have confirmed the result of Karliner for I^{VV}. We have also shown that essentially all of the pion-photoproduction contribution to I^{VS} appears [9] to

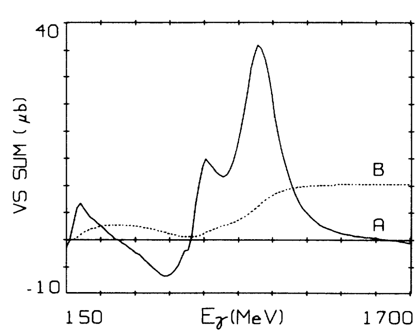

FIG. 3. Vector-scalar (VS) sum rule. Notation as in Fig. 1.

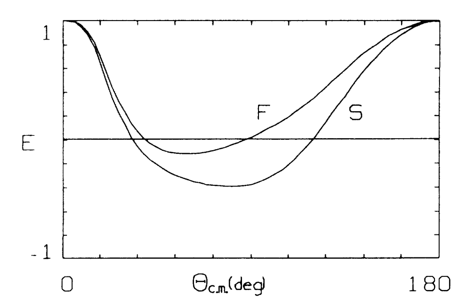

FIG. 5. Comparison of predictions from Ref. [5] (S) and Ref. [8] (F) for E in $\gamma p \rightarrow n\pi^+$ at 600 MeV.

be contained below 1.7 GeV. A large negative contribution to I^{VS} must be found if the DHG sum rule is to stand as written in Eq. (1). This component is of primary importance to the Björken sum rule, which extrapolates to the difference of the proton and neutron DHG sum rules at $Q^2=0$ (the difference being equal to $2I^{VS}$). The scalar-scalar sum rule remains a problem. The behavior of the integrand near 1.7 GeV, as displayed in Fig. 2, seems to indicate divergence rather than convergence. However, this behavior may simply reflect our inability to extract the scalar-scalar contribution near the end point of our analysis.

A direct measurement of $\sigma_{3/2}$ and $\sigma_{1/2}$ would provide the most reliable test of these sum rules. Proposals for such experimental investigations are now appearing [10]. In particular, the CEBAF proposal includes measurements of E from 0.3 to 2 GeV. Also planned is a preliminary evaluation of the $\pi\pi N$ contribution to the sum rule.

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[9] Of course the *appearance* of convergence does not constitute proof. The vector-scalar integrand could also be changing sign near 1.7 GeV.
[10] Proposal to CEBAF Physics Advisory committee 5, D. I. Sober, spokesman (unpublished). There also exists a proposal for Bates (R. Beck *et al.*) as well as a proposal from J. Soffer, CPT-90/P2443.