Gluon-mediated rare decays of the top quark: Anomalous threshold and its phenomenological consequences

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Flavor-changing neutral-current decays of the top quark by one-gluon emission (e.g., $t \to cg$) and exchange (e.g., $t \to cg^* \to c\bar{q}q$) are analyzed in the three-generation standard and two-Higgs-doublet models. These rare decays provide a sensitive probe of the analytic properties of (penguin-dominated) loop-induced neutral currents. The occurrence of an anomalous threshold in the amplitude for the one-gluon exchange mode requires a careful description of unstable (or off-mass-shell) external particles, in order to obtain sensible results for physical quantities. The branching fraction $\sum_q B(t \to cg^* \to c\bar{q}q)$ in the standard model turns out to be *larger* than $B(t \to cg)$ for top quark masses $\lesssim$ 180 GeV, due in part to the effects of the anomalous threshold. This is despite a suppression of the exchange mode by a factor of order $\alpha_s(m_t^2)/12\pi \approx 3 \times 10^{-3}$, coming from coupling constants and phase space. The largest branching fractions occur in the two-Higgs-doublet models, where $B(t \to cg) \sim 10^{-5}$ and $\sum_q B(t \to cg^* \to c\bar{q}q) \sim 10^{-6}$ in some regions of parameter space.

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I. INTRODUCTION

The suppression of loop-induced flavor-changing neutral currents (FCNC's) due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [1], following from the unitarity of the Cabibbo-Kobayashi-Maskawa mixing matrix [2, 3], is a fundamental theoretical component of the standard model of electroweak interactions. The investigation of FCNC's has been pursued with renewed vigor in recent years, as the search intensifies for detailed tests of the loop structure of the standard model, and for processes that are especially sensitive to new physics. Flavor-changing neutral-current decays of B mesons have attracted particular attention, with the prospect of observing significant CP violation in this system [4].

At sufficiently high energies, it becomes possible to make reliable calculations of loop-induced neutral currents mainly from short-distance interactions. Thus much of the recent work on FCNC's, including rare decays of the B meson [5–7] and the Z boson [8], has relied on the leading short-distance contribution to the amplitude, corresponding to the so-called one-loop penguin diagram (cf. Fig. 1) [9–12].

In this paper we show that FCNC decays of the top quark by one-gluon emission (e.g., $t \to cg$) and exchange (e.g., $t \to cg^* \to c\bar{q}q$) provide an important new probe of the structure of loop-induced neutral currents. Our results go beyond earlier studies of top-quark FCNC's, which only considered decays to an on-shell external vector boson (photon [13–15], gluon [16], or Z [16, 17]). In particular, the occurrence of an anomalous threshold in the amplitude for the one-gluon-exchange mode requires a careful description of unstable (or off-mass-shell) external particles, in order to obtain sensible results for physical quantities. [Rare top-quark decays to an off-shell photon or Z boson exhibit analytic properties similar to $t \to cg^*$. We restrict our attention in this paper to

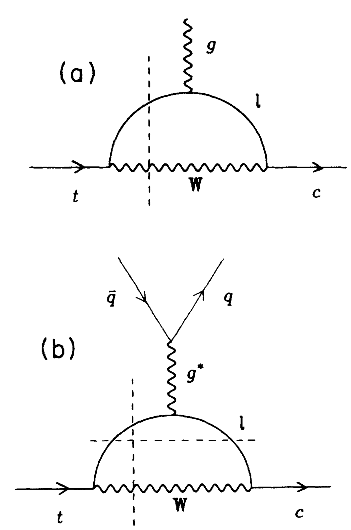

FIG. 1. Typical one-loop penguin diagrams contributing to (a) $t \to cg$, and (b) $t \to cg^* \to c\bar{q}q$. The dashed lines represent the unitarity cuts corresponding to the physical thresholds $t \to$ real$\{Wl\}$ and $g^* \to$ real$\{\bar{l}l\}$, where $l = b, s, d$ is a quark in the loop.

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gluon-mediated decays since they are favored by overall couplings.]

We consider the gluonic rare decays of the top quark in the context of the three-generation standard and two-Higgs-doublet models. In these models, FCNC decays of the top quark proceed to leading order through penguin diagrams with only light-quark flavors (b, s, and d) in the loop. One might therefore expect a substantial GIM suppression of these rare decays (especially of the on-shell mode), based on the known behavior of the penguin amplitude at low energies [11]. On the other hand, we find that the degree of cancellation between the amplitudes for the internal quark flavors, which underlies the GIM mechanism, is extremely sensitive to the analytic properties of the penguin amplitude at high energies.

We note that while the penguin diagram has been extensively analyzed in B and Z physics applications (including radiative decays such as $b \to s\gamma$ and $b \to s\gamma^* \to se^+e^-$ [5, 6] and analogous gluonic modes [7]), rare decays of the top quark are sensitive to analytic properties of the penguin that are not manifested unless the mass of an *external* quark is above ≈ 80 GeV. For example, it has been shown [13] that the occurrence of normal thresholds in the amplitude for the decay of the top quark to an on-shell photon ($t \to q\gamma$) makes it possible to "soften" the characteristic low-energy GIM suppression of a neutral current with light internal quarks. We have also shown [15] that normal thresholds generate a CP asymmetry in $t \to q\gamma$ (an asymmetry for radiative decays of a light quark to an on-shell photon, such as $b \to s\gamma$, cannot be generated from the one-loop penguin).

We present a careful analysis of the analytic structure of the one-loop penguin diagram. Gluonic modes should provide the best opportunity for experimental observation of FCNC decays of the top quark in both the standard and two-Higgs-doublet models, and we find that the occurrence of an anomalous threshold in the penguin amplitude has important consequences for phenomenology. In particular, the branching fraction for a given one-gluon-exchange mode (e.g., $t \to cg^* \to c\bar{q}q$) in the standard model turns out to be *larger* than for the corresponding decay by one-gluon emission (e.g., $t \to cg$) for top-quark masses m_t in the experimentally interesting range $m_t \lesssim 120$ GeV. This is despite a suppression of the exchange mode by a factor of order $\alpha_s(m_t^2)/12\pi \approx 3\times10^{-3}$ relative to gluon emission, coming from coupling constants and phase space. The enhancement of the off-shell gluon mode is due to a logarithmic GIM effect in a region of phase space (compared with a quadratic GIM suppression for real gluon emission), which originates in part from the effects of the anomalous threshold.

Our results demonstrate that the usual concept of an on-shell gluon, as effectively describing the hadronization of a color-octet current with unit probability, is not always reliable in the case of gluon-mediated FCNC's. Indeed we find that the branching fraction $\sum_q B(t \to cg^* \to c\bar{q}q)$, summed over all final states, is greater than $B(t \to cg)$ for $m_t \lesssim 180$ GeV. Our description of the one-gluon-exchange mode makes use of constituent masses for the final-state quarks, and therefore only gluons with momentum transfers above a minimum value of $O(\Lambda_{\rm QCD})$ contribute to this mode ($\Lambda_{\rm QCD}$ denotes the QCD scale parameter). The distinction between a gluon with a small momentum transfer and one that is on shell is crucial here because the form factors with a logarithmic GIM effect are absent for real gluons.

II. SINGULARITY STRUCTURE OF THE PENGUIN DIAGRAM

To begin with, we analyze the conditions for the occurrence of an anomalous threshold in the amplitude for the penguin diagram, Fig. 1. The singularity structure of the penguin is identical to that of the corresponding three-point function for scalar fields, which we denote by I_3. After reduction to a convenient set of Feynman parameters we have, up to irrelevant overall factors [18],

$$I_3 \equiv \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\, \delta\left(\textstyle\sum\alpha_i - 1\right)\frac{1}{\mathcal{D}}, \tag{1}$$

where

$$\mathcal{D} = \alpha_2\alpha_3 k^2 + \alpha_3\alpha_1 p^2 - [\alpha_1 m_B^2 + (\alpha_2+\alpha_3)m_l^2]\textstyle\sum\alpha_i. \tag{2}$$

We shall use B to refer to the boson in the loop (e.g., B denotes the W boson in the case of the standard model). m_B and m_l are the masses of the boson and a given quark flavor in the loop, while p and k are the momenta of the external top quark and gluon, respectively. We have ignored the squared momentum p_f^2 of the final-state quark in Eq. (2) ($p_f^2 = m_{c,u}^2 \ll p^2 = m_t^2$).

The leading singularity in the integral I_3 occurs when the masses and external momenta permit a solution to the following set of algebraic equations (called Landau equations), with all $\alpha_i \neq 0$ [18–20]:

$$\mathcal{D} = 0, \qquad \frac{\partial\mathcal{D}}{\partial\alpha_i} = 0, \quad i = 1,2,3. \tag{3}$$

The second set of equations above implies that the point α_i is "trapped" on the surface of singularity $\mathcal{D}(\alpha_i) = 0$ [18]. After some algebra, we find that a solution exists when

$$k^2 = k_s^2 \equiv \frac{1}{2m_B^2}\Big[\,p^2(m_B^2+m_l^2) - (m_B^2-m_l^2)^2 - \lambda^{1/2}(p^2,m_B^2,m_l^2)|m_B^2-m_l^2|\Big], \tag{4}$$

provided that

$$k^2 > p^2\frac{2m_l^2}{m_B^2+m_l^2}. \tag{5}$$

$\lambda(x,y,z) \equiv x^2+y^2+z^2-2xy-2yz-2xz$ is the usual kinematical function for a two-body decay. We note that Eq. (4) can also be inverted to give

$$p^2 = \frac{1}{2m_l^2}[k^2(m_B^2+m_l^2) - \lambda^{1/2}(k^2,m_l^2,m_l^2)|m_B^2-m_l^2|]. \tag{6}$$

The leading singularity in the three-point function requires all three internal propagators to be on shell [18]. This occurs in the present case above *both* physical thresholds $t \rightarrow \text{real}\{Bl\}$ (for sufficiently large m_t), and $g^* \rightarrow \text{real}\{l\bar{l}\}$ (for sufficiently large k^2), where l denotes the quark in the loop [cf. Eqs. (4) and (6)]. This situation is illustrated by the cuts in Fig. 1. Note that an anomalous threshold does not occur in the on-shell ($k^2 = 0$) mode.

Explicit integration of Eq. (1) in the neighborhood of the point α_i determined by the solution to the Landau conditions shows that the imaginary part of I_3 exhibits a logarithmic divergence [18, 20]

$$\text{Im}\, I_3(p^2, k^2 \approx k_s^2) \sim \ln |k^2 - k_s^2|, \tag{7}$$

while the real part of I_3 has a discontinuity at k_s [21]. (We note that nonleading singularities in I_3 are given by solutions to the Landau equations with one α_i equal to zero, and with the corresponding derivative of $\mathcal{D}$ omitted from Eq. (3). The nonleading singularities correspond in this case to *finite* branch cuts associated with one of the normal thresholds $t \rightarrow \text{real}\{Bl\}$ or $g^* \rightarrow \text{real}\{l\bar{l}\}$. The influence of normal thresholds on the GIM suppression of $t \rightarrow q\gamma$ was analyzed in Ref. [13].)

The divergence in the three-point function associated with the anomalous threshold is unphysical. A finite value for physical quantities is obtained when one takes proper account of the fact that the anomalous threshold occurs only if at least two of the *external* particles are either unstable, or off their mass shell (in such a way that the off-shell momentum squared is greater than the threshold for particle production; in our case, this corresponds to gluon $k^2 > 4m_l^2$). Indeed the lifetime of the top quark is orders of magnitude smaller than the time scale for its FCNC decays; Eq. (7) on the other hand assumes that the external particles are asymptotic states.

A physically reasonable "cure" for the divergence due to the anomalous threshold is obtained by averaging the squared amplitude, computed as in Eqs. (1)–(7) assuming asymptotic states, over a Breit-Wigner distribution for the invariant mass of at least one of the unstable (or off-mass-shell) external particles. Observe that the singularity is logarithmic, and is therefore integrable. The width obtained from the three-point function in this way then receives a finite contribution from the anomalous threshold, roughly of order $\Gamma_{\text{ext}} \ln^2(\Gamma_{\text{ext}}/M)$, where Γ_{ext} is the width of an external particle, and M is the mass of some internal line.

In the case of the top-quark decay by one-gluon exchange (e.g., $t \rightarrow cg^* \rightarrow c\bar{q}q$), we already integrate over the phase space of the final $\bar{q}q$ pair; hence, the width for this mode can be obtained from the penguin diagram with the top quark treated as an asymptotic state. One could also average over a Breit-Wigner distribution for the invariant mass p^2 of the top (which would result in a finite differential decay width $d\Gamma/dk^2$); however, this has a negligible effect on the total width.

We note in this connection that Hou and Stuart have recently studied the occurrence of an anomalous threshold in the decay of a fourth-generation b' quark to a Higgs boson ($b' \rightarrow bH$) [22]. These authors suggest that a physically motivated cure for the divergence due to the anomalous threshold is to include the width Γ_W for the W boson in *internal* propagators, resulting in an amplitude that goes like $\ln(\Gamma_W)$ near the threshold [cf. Eq. (7)]. However, this procedure does not deal satisfactorily with the assumption of asymptotic external states, and is therefore inadequate in general; note especially that the widths of internal lines in an arbitrary three-point function are independent of the overall amplitude (unlike the widths of the external particles), and can in fact be zero. For example, in a two-Higgs-doublet extension to the standard model (Sec. IV), a process such as $b' \rightarrow bH$ receives contributions from penguin diagrams with internal charged Higgs bosons, whose width is orders of magnitude smaller than that of the external b' quark in some regions of parameter space.

III. GLUONIC DECAYS IN THE STANDARD MODEL

We now explicitly evaluate the gluonic rare decays of the top quark in the standard model. We anticipate that the GIM suppression of the one-gluon-exchange mode may be "softened" by logarithms associated with anomalous thresholds, which depend in general on the mass of the internal quark [cf. Eqs. (4) and (5)].

The general vertex function V^μ for $t \rightarrow cg^*$ (on- or off-shell gluon) obeys current conservation to one-loop order, and can therefore be written in the form [9–11]

$$\begin{aligned} V^\mu = {} & (k^2\gamma^\mu - k^\mu \not{k})(F_1^L L + F_1^R R) \\ & + i\sigma^{\mu\nu} k_\nu (F_2^L m_c L + F_2^R m_t R), \end{aligned} \tag{8}$$

where $R, L \equiv \frac{1}{2}(1 \pm \gamma^5)$. We have suppressed color labels and a sum over internal quark flavors [with the appropriate Kobayashi-Maskawa (KM) matrix elements] in Eq. (8). We consider decays to a final-state charm quark, which are favored by the KM matrix.

We introduce dimensionless form factors $\widetilde{F}$ according to

$$F \equiv \frac{ig_s g^2}{32\pi^2 m_W^2} \widetilde{F}, \tag{9}$$

where g_s is the strong coupling ($\alpha_s \equiv g_s^2/4\pi$), g is the weak-isospin coupling, and m_W is the W mass. The form factors for $t \rightarrow cg^*$ in the standard model can be extracted from an analysis of the radiative decay $q_i \rightarrow q_j\gamma$ in Ref. [11], where contributions from penguins with the vertices $\gamma l\bar{l}$ and γW^+W^- are tabulated separately. The form factors given in Ref. [11] are expressed in terms of double integrals over Feynman parameters. We have found it useful here to do one parameter integration analytically, using a transformation due to 't Hooft and Veltman [21]. Our results can be expressed in the form

$$\widetilde{F}(\hat{m}_t^2, \hat{k}^2, \hat{m}_l^2) = \int_0^1 dz \left\{ (z-1)\frac{1}{D(z)} \sum_{n=0}^{1} a_n z^n + \ln\left[\frac{z(z-1)\hat{m}_t^2 + (1-z)\hat{m}_B^2 + z\hat{m}_l^2}{z(z-1)\hat{k}^2 + \hat{m}_l^2}\right] \frac{1}{D^2(z)} \sum_{n=0}^{3} b_n z^n \right\}, \quad (10)$$

where

$$D(z) \equiv z(\hat{m}_t^2 - \hat{k}^2) + \hat{m}_l^2 - \hat{m}_B^2. \quad (11)$$

The masses and momenta above are scaled by the W mass (e.g., $\hat{m}_t^2 \equiv m_t^2/m_W^2$), and we again use m_B to denote the mass of the boson in the loop ($m_B = m_W$ in the standard model). The above integral can be reduced to a sum of Spence functions and logarithms [21]. However, Eq. (10) provides a compact expression that is suitable for numerical evaluation. We note that the usual Feynman $i\epsilon$ prescription for internal masses (e.g., $m_l^2 \to m_l^2 - i\epsilon$) provides for the appropriate contour of integration around the branch cut in the logarithms in Eq. (10).

The coefficients for the form factors $\widetilde{F}_1^L$ and $\widetilde{F}_2^R$ in the standard model are given in Table I; we neglect $\widetilde{F}_1^R$ and $\widetilde{F}_2^L$, which make contributions to V^μ that are suppressed by factors of m_c/m_t relative to the leading terms [11]. For real-gluon emission ($k^2 = 0$), only the spin-flip term contributes, and we obtain

$$\Gamma(t \to cg) = \frac{m_t^5 G_F^2}{48\pi^3} \frac{\alpha_s(m_t^2)}{2\pi} \left|\Delta_2^R(\hat{m}_t^2, \hat{k}^2 = 0)\right|^2, \quad (12)$$

where we define a net form factor Δ_2^R according to

$$\Delta_2^R(\hat{m}_t^2, \hat{k}^2) \equiv \sum_l V_{tl} V_{lc}^* \widetilde{F}_2^R(\hat{m}_t^2, \hat{k}^2, \hat{m}_l^2), \quad (13)$$

with V_{ij} the KM matrix [3]. A color factor of $\frac{4}{3}$ is included in Eq. (12). We can keep only the b- and d-quark form factors to an excellent approximation:

$$\Delta_2^R(\hat{m}_t^2, \hat{k}^2 = 0) \simeq V_{tb} V_{bc}^* [\widetilde{F}_2^R(\hat{m}_t^2, \hat{k}^2 = 0, \hat{m}_b^2) - \widetilde{F}_2^R(\hat{m}_t^2, \hat{k}^2 = 0, \hat{m}_d^2)], \quad (14)$$

since $|V_{ts} V_{cs}| \approx |V_{tb} V_{cb}|$, and $\widetilde{F}_2^R(\hat{m}_t^2, \hat{k}^2 = 0) \propto \hat{m}_l^2$ [11, 13].

In the case of the width for one-gluon exchange, we have

$$\Gamma(t \to cg^* \to c\bar{q}q) = \frac{m_t^5 G_F^2}{288\pi^3} \left(\frac{\alpha_s(m_t^2)}{2\pi}\right)^2 \times \int_{4m_q^2/m_t^2}^{1} d\rho\,(1-\rho)^2 \left((1+2\rho)\left|\Delta_1^L(\hat{m}_t^2, \hat{k}^2 = \rho\hat{m}_t^2)\right|^2 + \frac{1}{\rho}(2+\rho)\left|\Delta_2^R(\hat{m}_t^2, \hat{k}^2 = \rho\hat{m}_t^2)\right|^2 \right), \quad (15)$$

where Δ_1^L is defined as in Eq. (13), and where a color factor of $\frac{2}{3}$ is included. We find that an excellent approximation can again be obtained by keeping only the b- and d-quark contributions to the form factors [as in Eq. (14)]. We also find in the standard model that the spin-flip term Δ_2^R makes a small contribution ($\lesssim 2\%$) compared to the anapole form factor Δ_1^L.

We compare the branching ratios $B(t \to cg)$ and $\sum_q B(t \to cg^* \to c\bar{q}q)$ in Fig. 2, where the sum is taken over the five quark flavors lighter than the top. We also show the branching ratio for the decay by one-gluon exchange into a single-quark species, $B(t \to cg^* \to c\bar{c}c)$. The branching fractions are relative to the width for the dominant semiweak decay $t \to bW$ [23, 24]:

$$\Gamma(t \to bW) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \lambda^{1/2}\left(1, \frac{m_W^2}{m_t^2}, \frac{m_b^2}{m_t^2}\right) \times \left[\left(1 - \frac{m_b^2}{m_t^2}\right)^2 + \frac{m_W^2}{m_t^2}\left(1 + \frac{m_b^2}{m_t^2}\right) - 2\frac{m_W^4}{m_t^4}\right]. \quad (16)$$

We compute $\alpha_s(m_t^2)$ for five light flavors, and with $\Lambda_{\rm QCD} = 130$ MeV (corresponding to a four-flavor scale

TABLE I. Coefficients a_n and b_n in the dominant form factors for $t \to cg^*$ (on- or off-shell gluon) in the standard model. The form factors are computed from Eq. (10) with $m_B \equiv m_W$. The masses and momenta are scaled by the W mass, e.g., $\hat{m}_t^2 \equiv m_t^2/m_W^2$ [see Eq. (17) for $\hat{m}_W^2$.]

	$\widetilde{F}_1^L$	$\widetilde{F}_2^R$
a_0	$-\hat{m}_l^2 + 2$	$-\hat{m}_l^2 + 2$
a_1	$-(\hat{m}_l^2 + 2)$	$-(\hat{m}_l^2 + 2)$
b_0	$(\hat{m}_W^2 - 2)\hat{m}_l^2$	$(\hat{m}_W^2 - 2)\hat{m}_l^2$
b_1	$-\hat{m}_t^2\hat{m}_l^2 + \hat{m}_l^4 - 2\hat{m}_t^2 + 4\hat{m}_W^2 + 2\hat{k}^2$	$-\hat{m}_t^2\hat{m}_l^2 + \hat{m}_l^4 + 2\hat{m}_t^2 + 2\hat{k}^2$
b_2	$-2(\hat{m}_W^2 - 2)\hat{m}_l^2 - 4\hat{m}_t^2 + 2\hat{m}_t^4 - 4\hat{m}_W^2$	$-4\hat{k}^2$
b_3	$(2\hat{m}_t^2 - \hat{k}^2)(\hat{m}_l^2 + 2)$	$(\hat{m}_l^2 + 2)\hat{k}^2$

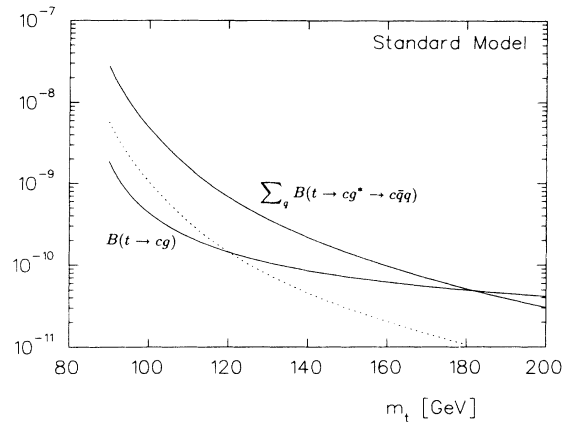

FIG. 2. Standard model branching ratios for FCNC decays of the top-quark by one-gluon exchange $[\sum_q B(t \to cg^* \to c\bar{q}q)]$, summed over all five flavors of quark lighter than the top, and by one-gluon emission $[B(t \to cg)]$, versus the top-quark mass m_t. The dashed line shows the decay by one-gluon exchange into a single quark species $[B(t \to cg^* \to c\bar{c}c)]$.

parameter of 200 MeV). The net form factors are computed as in Eq. (14), with KM matrix elements $|V_{tb}| \approx 1$ and $|V_{cb}| = 0.045$. We use loop-quark masses $m_b = 5$ GeV and $m_d = 10$ MeV. The mass m_q of the quarks produced by the virtual gluon ($g^* \to \bar{q}q$, $m_q \ll m_t$) is not neglected in the lower limit of the phase-space integral [Eq. (15)], in order to obtain a proper account of the logarithmic GIM effect at small k^2; constituent masses are appropriate in this case, and we use $m_b = 5$ GeV, $m_c = 1.6$ GeV, $m_s = 500$ MeV, and $m_u = m_d = 350$ MeV.

For completeness, we have included the width $\Gamma_W \approx 2.25$ GeV of the W boson in internal propagators, by making the substitution

$$\hat{m}_W^2 \to (m_W^2 - i\Gamma_W m_W)/m_W^2 \qquad (17)$$

in Eq. (10) [$m_W = 80.6$ GeV]. It should be clear however that the occurrence of an anomalous threshold does *not* make it necessary to include the width of an internal particle in the three-point function, as described in Sec. II. The off-shell width depends smoothly on Γ_W (singularities in the off-shell form factors in the limit $\Gamma_W \to 0$ due to the anomalous threshold are "cured" by the integration over the $\bar{q}q$ phase space); we find that the quantitative effect of Γ_W on $\Gamma(t \to cg^* \to c\bar{q}q)$ is comparable to its effect on $\Gamma(t \to cg)$.

A striking feature of our results is that the width for one-gluon exchange is larger than for one-gluon emission, for a range of top-quark masses. This is despite a suppression of the exchange mode by a factor of $\alpha_s(m_t^2)/12\pi \approx 3 \times 10^{-3}$ relative to gluon emission, coming from coupling constants and phase space [cf. Eqs. (12) and (15)]. Figure 2 shows that $B(t \to cg^* \to c\bar{c}c)$ for the decay into a single quark species falls below $B(t \to cg)$ at $m_t \approx 120$ GeV. This is due in large part to the effects of the anomalous threshold in the one-gluon-exchange channel, which moves out of the physical region for the heaviest internal (bottom) quark for $m_t \gtrsim 115$ GeV [where, as it turns out, the inequality in Eq. (5) is not satisfied]. This is illustrated by a comparison of the anapole form factor Δ_1^L at two values of the top quark mass, shown in Fig. 3.

We observe that the enchancement of the one-gluon-exchange mode comes from logarithms in the loop quark masses, in part of the phase space, compared to a quadratic GIM suppression of the decay to an on-shell gluon. We note that a logarithmic GIM effect in the anapole form factor for $k^2 \lesssim m_l^2$ is well known at low-energies [11]. However the anomalous threshold in the penguin amplitude at high energies (85 GeV $\lesssim m_t \lesssim$ 115 GeV) produces additional logarithms in the loop-

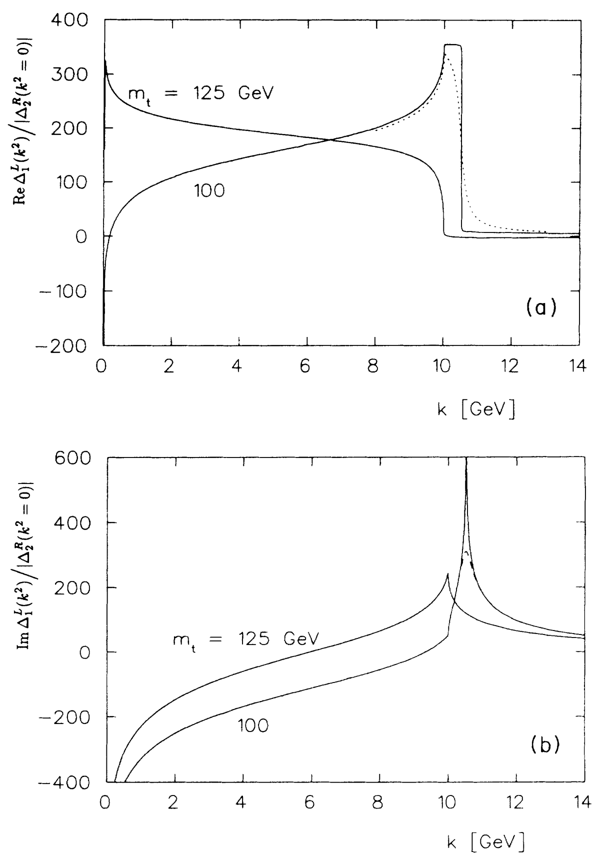

FIG. 3. Net anapole form factor $\Delta_1^L(k^2)$ for $t \to cg^*$ versus the momentum transfer $k \equiv \sqrt{k^2}$ through the virtual gluon, for two values of the top-quark mass, $m_t = 100$ and 125 GeV. The real and imaginary parts of the form factor are shown in (a) and (b), respectively. We scale $\Delta_1^L(k^2)$ by the spin-flip form factor $|\Delta_2^R(k^2 = 0)|$, in order to compare the widths to off- and on-shell gluons (cf. Fig. 2). The solid lines are calculated with $\Gamma_W = 0$ in internal propagators. The dashed lines show the effect of including $\Gamma_W = 2.25$ GeV in the form factors for $m_t = 100$ GeV (the effect is negligible for $m_t = 125$ GeV). An anomalous threshold produces a discontinuity in $\mathrm{Re}\,\Delta_1^L$ and a logarithmic divergence in $\mathrm{Im}\,\Delta_1^L$ in the curve for $m_t = 100$ GeV, but moves out of the physical region for $m_t \gtrsim 115$ GeV.

quark masses for k^2 in the neighborhood of $4m_l^2$ (cf. Fig. 3), and this provides for a further enhancement of the exchange mode.

Within overall factors, $|\Delta_2^R(k^2 = 0)|^2 \sim \hat{m}_l^4$, and $\langle|\Delta_1^L(k^2)|^2\rangle \sim (m_l/m_t)^2 \ln^2(\hat{m}_l)$, where the factor of $(m_l/m_t)^2$ roughly accounts for the fraction of the phase-space integration (represented by the angular brackets) in which a logarithmic GIM effect occurs. The bottom quark contribution to the net form factors dominates because of KM matrix elements [cf. Eq. (14)]. The width for one-gluon exchange is therefore enhanced relative to the on-shell mode by a factor of order $\ln^2(m_b/m_W)\, m_W^4/(m_b^2 m_t^2)$, which balances against the suppression by $\alpha_s(m_t^2)/12\pi$ due to phase-space and coupling constants (Ref. [25]).

IV. TWO-HIGGS-DOUBLET MODELS

We now broaden our analysis to include a two-Higgs-doublet extension to the standard model. The radiative decay $t \to q\gamma$ in a four-generation version of the two-Higgs-doublet model was analyzed in earlier work, in connection with Primakoff production of the top quark as a probe of new physics [14]. Here we consider a more conservative three-generation scenario (the two-Higgs-doublet model has been applied more recently to rare top-quark decays in Refs. [16, 17]).

Flavor-changing neutral-current decays in these models receive contributions from penguin diagrams with charged Higgs bosons $H^\pm$ in the loop, in addition to the penguin diagrams of the standard model. The coupling of the $H^\pm$ to quarks is of two types, given by the Lagrangian [26]

$$\mathcal{L}_H = \frac{g}{\sqrt{2}m_W} H^+ \overline{U}\, V_{\rm KM}\, (M_U \mathcal{F}_U L + M_D \mathcal{F}_D R)\, D + \text{H.c.}, \qquad (18)$$

where

$$\mathcal{F}_U = \cot\beta, \quad \mathcal{F}_D = \begin{cases} -\cot\beta & \text{(model I)}\ , \\ \tan\beta & \text{(model II)}. \end{cases} \qquad (19)$$

$U \equiv (u,c,t)$ and $D \equiv (d,s,b)$, M_U and M_D are the corresponding diagonal mass matrices, and $V_{\rm KM}$ is the KM matrix. $\tan\beta$ is the ratio of vacuum expectation values of the two Higgs doublets [26].

The amplitudes for gluonic penguin diagrams with an internal $H^\pm$ can be obtained from results in Ref. [11] for the radiative decay $q_i \to q_j\gamma$ in the standard model, by applying some simple substitution rules to diagrams with internal charged Goldstone bosons (Ref. [27]). We express our results for the extra contributions to the form factors due to penguins with an internal $H^\pm$ as one-dimensional integrals of the form in Eq. (10), with $m_B = m_H$, where m_H is the mass of the charged Higgs boson.

For the sake of illustration, we consider model II for large $\tan\beta$, where the biggest FCNC decays of the top quark occur. Model II couplings are suggested by the minimal supersymmetric extension to the standard model [26]. Comparable branching ratios arise in model I for small $\tan\beta$, but are excluded by a bound from K and B physics, $\tan\beta \gtrsim 0.1$ [28]. A constraint on large $\tan\beta$ in model II comes from $B \to e\bar{\nu}X$ [29]:

$$\tan\beta < 0.9 \frac{m_H}{1\ \text{GeV}}. \qquad (20)$$

A "perturbative" tbH^+ coupling in Model II requires $\tan\beta \lesssim 600\ \text{GeV}/m_b$.

The amplitude for penguins with an internal $H^\pm$ can be simplified for $\tan\beta \gtrsim 1$, where all terms proportional to the outgoing quark mass can be neglected. The form factors $\widetilde{F}_1^L$ and $\widetilde{F}_2^R$ then dominate, as in the standard model, and the widths for the gluonic modes can be computed from Eqs. (10)–(15), with the coefficients listed in Tables I and II. We include the width of the H^+ in internal propagators [cf. Eq. (17)]

$$\hat{m}_H^2 \equiv (m_H^2 - i m_H \Gamma_H)/m_W^2. \qquad (21)$$

Decays of a charged Higgs boson with $m_H < m_t + m_b$ and $\tan\beta \gtrsim 2$ are dominated by $H^+ \to \tau\bar{\nu}_\tau$, where [26]

$$\Gamma_H = \frac{G_F}{4\sqrt{2}\pi} m_H m_\tau^2 \tan^2\beta. \qquad (22)$$

We observe that Γ_H is less than a few MeV in some regions of parameter space, even for large m_H. For ex-

TABLE II. Contributions to the form factors for $t \to cg^*$ in the two-Higgs-doublet model II, due to penguin diagrams with an internal charged-Higgs-boson line. These coefficients are used in Eq. (10) with $m_B \equiv m_H$ [see Eq. (21) for $\hat{m}_H^2$]. The total amplitude is obtained by adding these terms to the standard model form factors given in Table I.

	$\widetilde{F}_1^L$	$\widetilde{F}_2^R$
a_0	$\hat{m}_l^2$	$\hat{m}_l^2$
a_1	$-\hat{m}_l^2 \tan^2\beta$	$-\hat{m}_l^2 \tan^2\beta$
b_0	$-\hat{m}_H^2 \hat{m}_l^2$	$-\hat{m}_H^2 \hat{m}_l^2$
b_1	$-(\hat{m}_l^2 - 2\hat{m}_H^2)\hat{m}_l^2 \tan^2\beta + (\hat{m}_t^2 - 2\hat{m}_l^2 + 2\hat{m}_H^2)\hat{m}_l^2$	$\hat{m}_t^2 \hat{m}_l^2 + \hat{m}_l^4 \tan^2\beta$
b_2	$-(2\hat{m}_H^2 + 2\hat{m}_t^2 - \hat{k}^2 - 2\hat{m}_l^2)\hat{m}_l^2 \tan^2\beta - (2\hat{m}_t^2 - \hat{k}^2)\hat{m}_l^2$	$-\hat{k}^2 \hat{m}_l^2 (\tan^2\beta + 1)$
b_3	$(2\hat{m}_t^2 - \hat{k}^2)\hat{m}_l^2 \tan^2\beta$	$\hat{k}^2 \hat{m}_l^2 \tan^2\beta$

ample, $\Gamma_H \approx 4$ MeV for $m_H = m_W$ and $\tan\beta = 5$ (to be compared with $\Gamma_W \approx 2$ GeV). It is therefore important to recognize that the divergence associated with an anomalous threshold is "cured" by a proper account of the unstable (or off-mass-shell) *external* particles (see Sec. II).

The branching ratios for the gluonic decays in the two-Higgs-doublet model are taken relative to the semiweak widths for $t \to bW$ [Eq. (16)] plus $t \to bH^+$, where [26]

$$\Gamma(t \to bH^+) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \lambda^{1/2}\left(1, \frac{m_H^2}{m_t^2}, \frac{m_b^2}{m_t^2}\right) \left[\left(1 + \frac{m_b^2}{m_t^2} - \frac{m_H^2}{m_t^2}\right)\left(\frac{m_b^2}{m_t^2}\tan^2\beta + \cot^2\beta\right) + 4\frac{m_b^2}{m_t^2}\right]. \tag{23}$$

The total width is illustrated in Fig. 4.

We compare the branching ratios $B(t \to cg)$ and $\sum_q B(t \to cg^* \to c\bar{q}q)$ as functions of m_t and $\tan\beta$ in Figs. 5 and 6. Observe the significant increase over the standard-model branching fractions at large $\tan\beta$, as expected [30]. The quadratic GIM suppression of the on-shell amplitude is compensated by the fact that the loop-quark mass m_l in some terms is multiplied by $\tan\beta$. Note that while the one-gluon-exchange mode also increases with $\tan\beta$, it eventually falls well below the decay into an on-shell gluon. The reason is that the form factors for both on- and off-shell gluons have terms $\propto \hat{m}_l^2 \tan^2\beta$ in the two-Higgs-doublet model—the logarithms in $\hat{m}_l$ that enchance the off-shell mode in the standard model become negligible compared to these quadratic terms at sufficiently large $\tan\beta$. Hence the ratio of $B(t \to cg^* \to c\bar{q}q)$ to $B(t \to cg)$ for very large $\tan\beta$ is determined essentially by coupling constants and phase space factors, becoming of order $\alpha_s(m_t^2)/12\pi$. Nevertheless, we observe that the exchange mode receives some enhancement from logarithms in the loop-quark masses, coming in part from the anomalous threshold, even for $\tan\beta$ as large as the upper bound of Eq. (20).

V. CONCLUSIONS

We have shown that gluon-mediated rare decays of the top quark provide a sensitive new probe of the analytic structure of loop-induced FCNC's. We found an anomalous threshold in the penguin amplitude for the decay by one-gluon exchange, which requires a proper account of unstable (or off-mass-shell) external particles, in order to obtain finite values for physical quantities. A logarithmic GIM effect in the anapole form factor, due in part to the anomalous threshold, makes $\sum_q B(t \to cg^* \to c\bar{q}q) > B(t \to cg)$ in the standard model for $m_t \lesssim 180$ GeV. This is despite a suppression of the exchange mode by a factor of order $\alpha_s(m_t^2)/12\pi \approx 3 \times 10^{-3}$, coming from coupling constants and phase space. Our results demonstrate that an on-shell gluon cannot always be used to describe the hadronization of a color-octet current in the case of gluon-mediated FCNC's. The largest branching fractions occur in the two-Higgs-doublet model. For model II couplings, favored by the minimal supersymmetric extension to the standard model, we find $B(t \to cg) \sim 10^{-5}$ and $\sum_q B(t \to cg^* \to c\bar{q}q) \sim 10^{-6}$, for large $\tan\beta$ (the ratio of vacuum expectation values of the two Higgs doublets).

We finally mention some results that we have obtained for the CP asymmetry $\mathcal{A}$:

$$\mathcal{A} \equiv \frac{\Gamma_{\bar{t}} - \Gamma_t}{\Gamma_{\bar{t}} + \Gamma_t}, \tag{24}$$

where Γ_t and $\Gamma_{\bar{t}}$ denote the widths for the mode of interest [e.g., $\Gamma_t = \Gamma(t \to cg)$ compared to $\Gamma_{\bar{t}} = \Gamma(\bar{t} \to \bar{c}g)$]. We find that the asymmetries are quantitatively similar to those for top-quark radiative decays, which we analyzed in Ref. [15]. The asymmetries are largest for final states with an outgoing u quark (e.g., $t \to ug^*$), because of KM angles which suppress the width: $\mathcal{A}(t \to q'g^*) \propto 1/|V_{bq'}|^2$ (on- or off-shell gluon). We observe that the

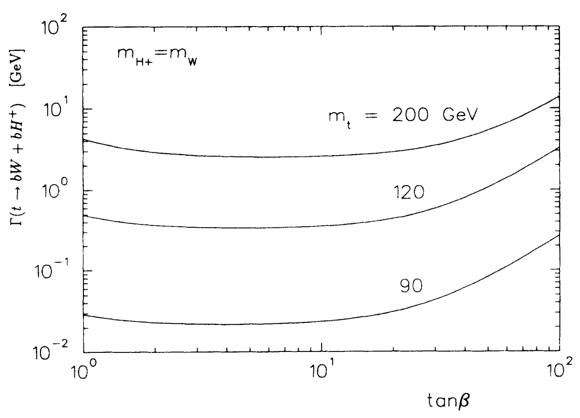

FIG. 4. Total width $\Gamma(t \to bW + bH^+)$ for the dominant semi-weak decays of the top quark in the two-Higgs-doublet model II for $m_{H+} = m_W$, as a function of $\tan\beta$, for three values of the top-quark mass.

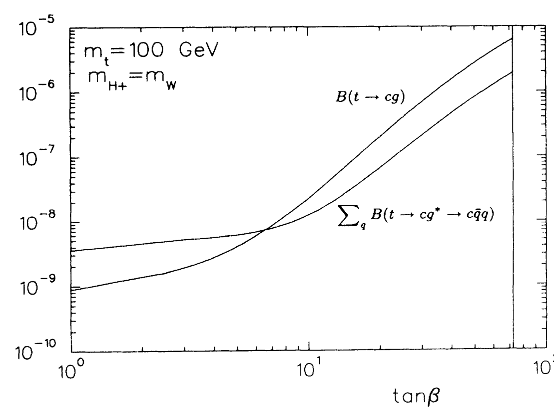

FIG. 5. Branching ratios $\sum_q B(t \to cg^* \to c\bar{q}q)$ and $B(t \to cg)$ in the two-Higgs-doublet model II as functions of $\tan\beta$, for $m_t = 100$ GeV and $m_{H+} = m_W$.

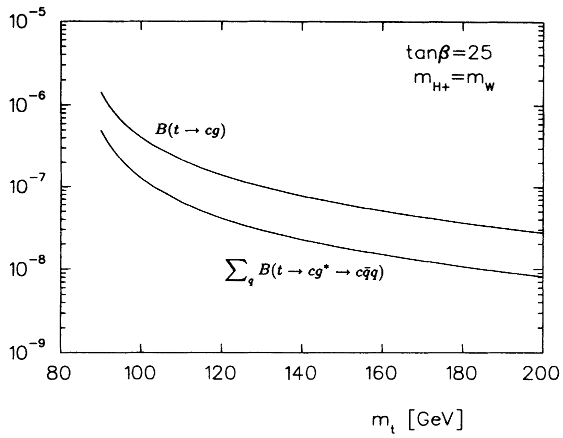

FIG. 6. Branching ratios $\sum_q B(t \to cg^* \to c\bar{q}q)$ and $B(t \to cg)$ in the two-Higgs-doublet model II as functions of m_t, for $\tan\beta = 25$ and $m_{H^+} = m_W$.

product $\mathcal{A}(t \to q'g^*)\, B(t \to q'g^*)$ is approximately equal in either case $q' = c, u$ (on- or off-shell gluon); hence the asymmetry for $q' = u$ is favored, despite the smaller width [15].

The asymmetry is strongly dependent on the (internal) strange-quark mass, $\mathcal{A} = O((m_s/m_b)^2)$ [15]; we use $m_s = 150$ MeV. We take $|V_{ub}| \approx 0.006$, and we saturate the (approximate) experimental bound on the rephasing-invariant measure J of CP violation, $|J| \lesssim 10^{-4}$ (where $J \equiv c_1c_2c_3s_1^2s_2s_3 \sin\delta$ in the KM notation). Some results in the standard model are $|\mathcal{A}|(t \to ug^* \to u\bar{u}u) \approx 0.3$–$1.1\%$ for $m_t \approx 100$–200 GeV, and $|\mathcal{A}|(t \to ug) \approx 0.2\%$ (roughly constant for m_t in the above range). The asymmetries in the two-Higgs-doublet model II are comparable to those in the standard model for $\tan\beta \approx 1$, and decrease with increasing $\tan\beta$ [$|\mathcal{A}| \lesssim 0.2\%$ for $\tan\beta \gtrsim 10$ and m_t in the above range]. Although the CP asymmetries are reasonably large, the phenomenological applications are rather limited, due to the small branching ratios for these rare decays.

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