PHYSICAL REVIEW D VOLUME 45, NUMBER 5 1 MARCH 1992

Nonlocal condensates and QCD sum rules for the pion wave function

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(Received 17 September 1991)

The QCD-sum-rule calculation of the pion wave function by Chernyak and Zhitnitsky (CZ) implicitly assumes that the correlation length of vacuum fluctuations is large compared to the typical hadronic scale $\sim 1/m_\rho$, so that one can substitute the original nonlocal objects such as $\langle \bar{q}(0)q(z)\rangle$ by constant $\langle \bar{q}(0)q(0)\rangle$-type values. We outline a formalism enabling one to work directly with the nonlocal condensates, and construct a modified sum rule for the moments $\langle \xi^N \rangle$ of the pion wave function. The results are rather sensitive to the value of the parameter $\lambda_q^2 = \langle \bar{q}D^2 q\rangle/\langle \bar{q}q\rangle$ specifying the average virtuality of the vacuum quarks. Varying it from the most popular value $\lambda_q^2 = 0.4$ GeV^2 up to the value $\lambda_q^2 = 1.2$ GeV^2 suggested by the instanton-liquid model, we obtain $\langle \xi^2 \rangle = 0.25$–$0.20$, to be compared to the CZ value $\langle \xi^2 \rangle = 0.43$ obtained with $\lambda_q^2 = 0$.

PACS number(s): 12.38.Bx, 11.50.Li, 12.38.Lg, 14.40.Aq

I. INTRODUCTION

The standard trick incorporated in all the approaches based on the asymptotic freedom of QCD and factorization is the introduction of some phenomenological functions and/or numbers accumulating necessary information about nonperturbative long-distance dynamics of the theory. The most important examples are parton distribution functions $f_{p/H}(x)$ used in the perturbative-QCD approaches to hard inclusive processes [1], hadronic wave functions $\varphi_\pi(x)$, $\varphi_N(x_1, x_2, x_3)$, etc., which naturally emerge in the asymptotic QCD analyses of hard exclusive processes [2–6], and quark and gluon condensates $\langle \bar{q}(0)q(0)\rangle$, $\langle G(0)G(0)\rangle$, the basic parameters of the QCD-sum-rule approach [7], describing the nonperturbative nature of the QCD vacuum.

The hope is that in some future approach they all will be calculated from the first principles of QCD without any model and/or *ad hoc* assumptions. A less ambitious program is to calculate the hadronic functions $f(x), \varphi(\{x\})$ using the QCD sum rules [7], with only the condensate values treated as input parameters.

While the parton distribution functions can be extracted rather reliably from experimental data, the situation with the hadronic wave functions is much more complicated. Normally, they appear only in an integrated form. Furthermore, the very applicability of the perturbative-QCD formulas at accessible energies is questionable [8, 9]. In this situation, the QCD-sum-rule approach and lattice calculations are the only reliable way to get information about the form of the hadronic wave functions. In particular, the most popular set of hadronic wave functions [10], due to Chernyak, A. R. Zhitnitsky, and I. R. Zhitnitsky (CZ), was produced with the help of QCD sum rules.

One should remember, however, that the operator-product expansion (OPE), the starting point of any QCD-sum-rule analysis, has different forms depending on the situation. The presence of a large (or small) extra parameter might essentially modify the expansion. The most well-studied example is the modification of the OPE for the form factors at small momentum transfer q [11, 12]. In that case a simple-minded extrapolation from the region of moderately large q is completely unjustified: one cannot reproduce in that way even the normalization conditions such as $F_\pi(0) = 1$. Our goal in the present paper is to show that in calculating the $N \geq 2$ moments of the pion wave function one faces another situation requiring a modification of the underlying expansion. We construct a modified sum rule and show that, for a standard choice of the condensate values, it produces a pion wave function that strongly differs from the CZ form.

II. PION WAVE FUNCTION AND QCD SUM RULES: CRITICISM OF THE CZ APPROACH

The first application of the QCD sum rules to the pion wave function $\varphi_\pi(x)$ was the calculation of its zero moment, i.e., the pion decay constant f_π , in the pioneering paper of Shifman, Vainshtein, and Zakharov [7]. It was calculated there within 5% accuracy. This success inspired Chernyak and A. R. Zhitnitsky [13] to calculate the whole pion wave function by reconstructing it from the next moments $\langle \xi^N \rangle$ (where $\xi = 2x - 1$). They extracted $\langle \xi^2 \rangle$ and $\langle \xi^4 \rangle$ from the relevant sum rule precisely in the same way as the f_π value. However, the nonperturbative terms in their sum rule

$$f_\pi^2\langle\xi^N\rangle = \frac{M^2}{4\pi^2}\frac{3}{(N+1)(N+3)}(1-e^{-s_0/M^2}) + \frac{\alpha_s\langle GG\rangle}{12\pi M^2} + \frac{16}{81}\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4}(11+4N) \quad (1)$$

have a completely different N dependence compared to the perturbative one and, *a priori,* it is not clear whether a straightforward use of the $N=0$ technology can be justified for higher N. The scale determining the magnitude of all the hadronic parameters including s_0 (the "continuum threshold" [7]) is eventually settled by the ratios of the condensate contributions to the perturbative term. If the condensate contributions in the CZ sum rule (1) would have the same N behavior as the perturbative term, then the N dependence of $\langle\xi^N\rangle$ would be determined by the overall factor $3/[(N+1)(N+3)]$ and the resulting wave function $\varphi_\pi(x)$ would coincide with the "asymptotic" form [4, 6]

$$\varphi_\pi^{\rm as}(x) = f_\pi\phi(x), \quad (2)$$

where $\phi(x) = 6x(1-x)$. However, the ratios of the $\langle\bar{q}q\rangle$ and $\langle GG\rangle$ corrections to the perturbative term in Eq. (1) are growing functions of N. In particular, in the $\langle\bar{q}q\rangle$ case, the above-mentioned ratio for $N=2$ is, by a factor 95/11, larger than that in the $N=0$ case. For $N=4$ the enhancement factor is 315/11 . As a result, the effective vacuum scales of (mass)^2 dimension are, by factors $(95/11)^{1/3}\approx 2.1$ and $(315/11)^{1/3}\approx 3.1$, larger than that for the $N=0$ case. Approximately the same factors [$5^{1/2}\approx 2.2$ and $(35/3)^{1/2}\approx 3.4$] one obtains also for the gluon-condensate term. Hence, the parameters $s_0^{(N)}$ and the combinations $f_\pi^2\langle\xi^N\rangle$ straightforwardly extracted from the sum rule (1) must be larger than the "asymptotic" values $s_0^{N=0}\approx 0.75$ GeV^2 and $f_\pi^2\langle\xi^N\rangle^{\rm as} = 3f_\pi^2/[(N+1)(N+3)]$ just by the factors 2 (for $N=2$) and 3 (for $N=4$). These are just the results obtained in Ref. [13].

To better understand the structure of the relevant power series it is instructive to rewrite the sum rule for the pion wave function $\varphi_\pi(x)$ itself [14]:

$$f_\pi\varphi_\pi(x) = \frac{M^2}{4\pi^2}(1-e^{-s_0/M^2})\phi(x) + \frac{\alpha_s\langle GG\rangle}{24\pi M^2}[\delta(x)+\delta(1-x)] + \frac{8}{81}\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4}\{11[\delta(x)+\delta(1-x)] + 2[\delta'(x)+\delta'(1-x)]\}. \quad (3)$$

The $O(1)$ and $O(N)$ terms in Eq. (1) correspond to the $\delta(x)$ and $\delta'(x)$ terms in Eq. (3). In its turn, the presence of the $\delta(x)$ functions in Eq. (3) is evidently indicating that the vacuum fields are treated as carrying zero fraction of the pion momentum. This can be easily understood by observing that the operator product expansion [underlying Eqs. (1),(3)] is, in fact, a power-series expansion over small momenta k of vacuum quarks and gluons. Retaining only the $\langle\bar{q}q\rangle$ and $\langle GG\rangle$ terms [as in Eqs. (1), (3)] is just equivalent to the assumption that k is not simply small but exactly zero.

However, it is much more reasonable to expect that the vacuum quanta have a smooth distribution with a finite width μ. In configuration space, this means that vacuum fluctuations have a finite correlation length of the order of $1/\mu$, so that the two-point condensates like $\langle\bar{q}(0)q(z)\rangle$ die away for $|z|$ large compared to $1/\mu$. Of course, one can always expand $\langle\bar{q}(0)q(z)\rangle$ in powers of z starting with the constant term $\langle\bar{q}(0)q(0)\rangle$ that produces eventually the $\delta(x)$ term. The question is, whether it is reasonable to do this, since the expansion resulting from such a Taylor series will not necessarily behave well.

According to the standard estimate [15], the average virtuality of the vacuum quarks

$$\lambda_q^2 \equiv \langle\bar{q}D^2q\rangle/\langle\bar{q}q\rangle = 0.4\pm 0.1 \text{ GeV}^2 \quad (4)$$

(here D is the covariant derivative) is not small compared to the relevant hadronic scale

$$s_0^{N=0}\approx 4\pi^2 f_\pi^2 = 0.7 \text{ GeV}^2.$$

Even a larger value (by a factor of 3) was obtained for λ_q^2 in the instanton-liquid model by Shuryak [16]. Thus, the correlation length of vacuum fluctuations is not much larger than the hadronic size, and the constant-field approximation for the vacuum fields might not work, i.e., the higher-power corrections might well ruin the conclusions derived from the sum rule (1).

In what follows, we outline a formalism (its preliminary version can be found in Ref. [17]) that enables one to take into account the effects due to the k (or x) distribution of vacuum fluctuations. To this end we note that in all standard calculations of the power corrections *via* the OPE one starts with some *nonlocal condensates* such as $\langle\bar{q}(0)q(z)\rangle$, $\langle\bar{q}(0)\gamma A(y)q(z)\rangle$, etc. (such objects have been discussed for almost 10 years now, see, e.g., [18]), which are subsequently expanded over the local condensates (LC) $\langle\bar{q}q\rangle$, $\langle\bar{q}D^2q\rangle$, etc. Our strategy is to avoid such an expansion and deal directly with the nonlocal condensates (NLC).

III. NONLOCAL CONDENSATES

The simplest bilocal condensate $M(z)\equiv\langle\bar{q}(0)q(z)\rangle$ is just the nonperturbative part of the quark propagator. So, it is convenient to parametrize it in the manner of the well-known α representation for a propagator:

$$\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle\int_0^\infty e^{\nu z^2/4} f_S(\nu)\,d\nu. \quad (5)$$

Of course, because of gauge invariance, the quark fields $q(z)$ are always accompanied by an appropriate Wilson-line operator. But here and in the following we take quark and gluon fields in the Fock-Schwinger gauge $z^\mu A_\mu(z) = 0$ where the path-ordered exponentials are equal to 1 and the covariant derivatives are converted into the ordinary ones. Another comment concerning Eq. (5) is that in deriving a QCD sum rule one can always perform a Wick rotation $z_0 \to iz_0$ and treat all the coordinates as Euclidean, with $z^2 < 0$.

The functions such as $f_S(\nu)$ describe the distribution of the vacuum fields in virtuality. Note, that the moments of $f_S(\nu)$ are proportional to the vacuum matrix elements of the local operators

$$\langle \bar{q}(0)q(0)\rangle \int_0^\infty f_S(\nu)\,\nu^N\,d\nu \sim \frac{1}{(N+1)!}\langle \bar{q}(0)(D^2)^N q(0)\rangle \quad (6)$$

with increasing number of derivatives. By analogy with the hadronic distribution functions, one can call $f(\nu)$ the "vacuum distribution functions."

The expansion of the condensate $M(z^2)$ over the local condensates corresponds to that of the distribution function $f_S(\nu)$ over the $\delta^{(n)}$ functions:

$$f_S(\nu) = \delta(\nu) - L_S\delta'(\nu) + \cdots, \quad (7)$$

with L_S fixed just by the average virtuality of the vacuum quarks [Eq. (4)]: $L_S = \lambda_q^2/2$.

There is another (vector) bilocal condensate $M_\mu \equiv \langle \bar{q}(0)\gamma_\mu q(z)\rangle$, containing a γ matrix:

$$\langle \bar{q}(0)\gamma_\mu q(z)\rangle = iz_\mu A \int_0^\infty e^{\nu z^2/4} f_V(\nu)\,d\nu, \quad (8)$$

where $A = \frac{2}{81}\pi\alpha_s\langle\bar{q}q\rangle^2$. The zeroth moment of $f_V(\nu)$ is zero in the limit of massless quarks, and that is why the $\delta^{(n)}(\nu)$ expansion for $f_V(\nu)$ starts with the $\delta'(\nu)$ term :

$$f_V(\nu) = \delta'(\nu) - L_V\,\delta''(\nu) + \cdots, \quad (9)$$

with the parameter L_V determined by the magnitude of the condensates of dimension eight.

For the gluonic nonlocal condensate, in the Fock-Schwinger gauge, one has

$$\langle A_\mu^a(z)A_\nu^b(y)\rangle = \delta^{ab}\,[y_\mu z_\nu - g_{\mu\nu}(z\cdot y)]\frac{\langle GG\rangle}{384} \times M_G((z-y)^2, z^2, y^2) + \cdots, \quad (10)$$

where the M_G function depends not only on the interval $(z-y)^2$, but also on z^2 and y^2. However, since the coefficients in front of z^2 and y^2 in the expansion

$$M_G = 1 - \frac{\langle GD^2G\rangle - \frac{2}{3}\langle j^2\rangle}{18\langle GG\rangle}\left((y-z)^2 + \frac{y^2+z^2}{8}\right) + \cdots \quad (11)$$

are rather small, one can start with the approximation

$$M_G(z^2, y^2, (z-y)^2) \simeq \int_0^\infty e^{\nu(z-y)^2/4} f_G(\nu)\,d\nu \quad (12)$$

introducing the distribution function $f_G(\nu)$.

There are three simplest trilocal quark-gluon condensates

$$M_{\mu\nu}(y,z) \equiv \langle \bar{q}(0)\gamma_\nu A_\mu(y)q(z)\rangle = [z_\mu y_\nu - g_{\mu\nu}(z\cdot y)]M_1 + (y_\mu y_\nu - g_{\mu\nu}y^2)M_2 + \cdots, \quad (13)$$

$$\tilde{M}_{\mu\nu}(y,z) \equiv \langle \bar{q}(0)\gamma_\nu\gamma_5 A_\mu(y)q(z)\rangle = \epsilon_{\mu\nu\rho\sigma}y^\rho z^\sigma M_3 + \cdots. \quad (14)$$

The functions M_{1-3} can be parametrized by a triple-integral representation of the same type:

$$M_i(z^2, y^2, (z-y)^2) = \alpha_s\langle\bar{q}q\rangle^2 \int_0^\infty e^{\nu_1 z^2/4 + \nu_2 y^2/4 + \nu_3(z-y)^2/4} f_i(\nu_1,\nu_2,\nu_3)\,d\nu_1 d\nu_2 d\nu_3. \quad (15)$$

The limiting case of the standard local condensates (corresponding to $\lambda_q^2 \to 0$) is obtained by the substitution $f_i(\nu_1,\nu_2,\nu_3) \to A_i\delta(\nu_1)\delta(\nu_2)\delta(\nu_3)$, with $A_i = \{-\frac{3}{2}, 2, \frac{3}{2}\}$.

Incorporating the nonlocal condensates as described above, one arrives at a modified diagram technique, with some lines and vertices being the ordinary perturbative ones, and some corresponding to the nonlocal condensates. Increasing the number of loops, one should consider the condensates containing more and more fields. We restrict our analysis here to the two-loop level. Then, in addition to those already listed, one encounters the four-quark condensate. To simplify the calculation, we apply the vacuum-dominance hypothesis and factorize it into a product of two bilocal ones.

IV. SUM RULE

Using the representations (4)–(7), and calculating the coefficient functions we obtain a modified QCD sum rule, with the δ functions of Eq. (3) substituted by the functionals $\delta\Phi_i(x)$ of six vacuum distribution functions:

$$f_\pi\varphi_\pi(x) = \frac{M^2}{4\pi^2}(1 - e^{-s_0/M^2})\Phi^{\text{pert}}(x) + \left(4\bar{x}f_V(xM^2) + \sum_{i=1}^4 \delta\Phi_i(x) + \delta\Phi_G(x)\right) + (x \to \bar{x}), \quad (16)$$

where $\bar{x} = 1 - x$, M^2 is the Borel parameter and

$$\Phi^{\rm pert}(x) = 6x\bar{x}\left\{1 + C_F\frac{\alpha_s}{4\pi}\left[5 - \frac{\pi^2}{3} + \ln^2\left(\frac{\bar{x}}{x}\right)\right]\right\}$$

is the "perturbative" contribution [free-quark loop plus $O(\alpha_s)$ radiative corrections].

The simplest contribution, proportional to the f_V function taken at $\nu = xM^2$, is displayed explicitly in Eq. (16). Other contributions have a more involved form (see Ref. [17]).

The most intriguing conclusion to be drawn from Eq. (16) is that $\varphi_\pi(x)$, *the longitudinal-momentum distribution of quarks inside the pion, is directly related to* $f(\nu)$, *the virtuality distribution of quarks and gluons in the vacuum.* Therefore, it is very important to know the form of the latter to estimate the moments $\langle\xi^N\rangle_\pi$ for $N > 0$.

V. MODELING $f_i(\nu)$

To obtain the original sum rule (3), one should take the first term of the $\delta^{(n)}$ expansion for the $f(\nu)$'s. It should be understood that this approximation is really the simplest model for the distribution functions $f(\nu)$. However, such a model (used, as a matter of fact, by CZ [13]) is evidently too crude if the L_i parameters characterizing the width of $f_i(\nu)$ are comparable in magnitude to the relevant hadronic scale. In this situation, instead of the standard expansion over the local condensates we propose to use an expansion in which the (relatively) large average virtuality of the vacuum fields is taken into account just in the first term. For the functions $M(z^2)$ having finite widths of order μ^2, it is much more preferable to use the expansion of $f(\nu)$ over $\delta^{(n)}(\nu - \mu^2)$. The first term of this expansion

$$M(z^2) = M(0)[e^{z^2\mu^2/4} + \cdots] \tag{17}$$

takes into account the main effect caused by the finite width of the function $M(z^2)$, while subsequent terms describe effects due to the deviation of its form from the Gaussian one.

To construct the Gaussian *Ansätze* one should know the second term of the z^2 expansion of the relevant nonlocal condensates, e.g., incorporating Eq. (4) we take $f_S(\nu) = \delta(\nu - \lambda_q^2/2)$.

For M_μ the situation is more complicated: L_V is determined by five different LC, the values of which are poorly known. The simplest model is to assume that all the nonlocal distributions have the same width. So, we take $f_V^{(1)}(\nu) = \delta'(\nu - \lambda_q^2/2)$. Of course, it is more reasonable to expect that the shift parameters L_i, though all of the same order of magnitude, are still numerically different. Another model for L_V is to extract the part proportional to $\langle\bar{q}D^2q\rangle\langle\bar{q}q\rangle$ from all the relevant LC of dimension eight and neglect the remaining contributions. This gives the value $L_V = \frac{7}{20}\lambda_q^2$, rather close to the naive estimate.

In a similar way we construct the model for the trilocal functions:

$$f_i(\nu_1,\nu_2,\nu_3) = A_i\delta(\nu_1 - L_i^{(1)})\delta(\nu_2 - L_i^{(2)})\delta(\nu_3 - L_i^{(3)}). \tag{18}$$

One can try to determine $L_i^{(j)}$'s from the expansion of the relevant NLC by retaining only the $\langle\bar{q}D^2q\rangle\langle\bar{q}q\rangle$ part of the coefficients in front of z^2, y^2 or $(z - y)^2$, respectively. This gives $L_1^j = \{\frac{53}{288}, -\frac{1}{144}, \frac{2}{9}\}\lambda_q^2$ for the f_1 function, $L_2^j = \{\frac{1}{192}, \frac{517}{960}, \frac{31}{192}\}\lambda_q^2$ for the f_2 function and $L_3^j = \{-\frac{1}{32}, \frac{19}{72}, \frac{1}{6}\}\lambda_q^2$ for the f_3 function [17]. According to these estimates, the trilocal condensates in some directions decrease much slower than in the others, and sometimes even increase when the distance between the quarks increases, which is completely unrealistic. Hence, it is not safe to neglect other LC in estimating the width parameters and, in the absence of a reliable model of the QCD vacuum, we simply assume that the trilocals decrease at the same rate in all directions and take $L_i^{(j)} = \lambda_q^2/2$.

To model the nonlocality effects for the gluonic contribution, we assume, by analogy with the quark case, that the $\delta(x)$ terms of the $O(\langle GG\rangle)$ contribution [Eq. (3)] should be substituted by $\delta(x - L_G/M^2)$ in Eq. (16), with $L_G = \frac{2}{9}\lambda_q^2$, as suggested by Eq. (11).

VI. NUMERICAL ESTIMATES

Within the simplified version of our Gaussian ("delta-function") model for the nonlocal condensates, the pion wave-function sum rule has the following form:

$$\begin{aligned} f_\pi^2\varphi_\pi(x) = {}& \frac{M^2}{4\pi^2}(1 - e^{-s_0/M^2})\Phi^{\rm pert}(x) + \frac{1}{24\pi}\alpha_s\langle GG\rangle\delta(x - \tfrac{2}{9}\Delta) \\ & + \frac{8}{81M^4}\pi\alpha_s\langle\bar{q}q\rangle^2\Bigg\{\bar{x}\delta'(x - \Delta) + 18\frac{\theta(x < \Delta)}{\Delta^2(1 - \Delta)}\bar{x}[x + (\Delta - x)\ln(\bar{x})] \\ & + \frac{3}{1 - \Delta}\left(\frac{\delta(x - \Delta) - \delta(x - 2\Delta)}{\Delta} - (1 - \Delta)\delta(x - \Delta) + \frac{2}{3}(1 - 2\Delta)\frac{(2 + \Delta)}{\Delta}\delta(x - 2\Delta)\right) \\ & - 2\bar{x}\frac{\theta(\Delta < x < 2\Delta)}{\Delta}\left[\frac{3x}{1 - \Delta} + \frac{2}{\Delta}\left(3 - \frac{\Delta + 2\bar{x}}{1 - \Delta}\right)\right]\Bigg\} + (x \to \bar{x}), \end{aligned} \tag{19}$$

where $\Delta = \lambda_q^2/2M^2$.

The main observation is that in place of the $\delta(x)$-type contributions we have now either the δ functions with the shifted arguments or the functions that are smooth at $x = 0$. In both cases, the moments of such terms decrease as N increases. Hence, for sufficiently large values of λ_q, there is no dramatic increase in the ratios of the condensate contributions to the perturbative term. Taking $\lambda_q^2 = 0.4$ GeV^2 [15], we obtain for the lowest moments

$$\langle\xi^2\rangle = 0.25, \quad \langle\xi^4\rangle = 0.12, \quad \langle\xi^6\rangle = 0.07. \qquad (20)$$

These values do not differ strongly from those corresponding to the asymptotic wave function. Therefore, it is not surprising that the model wave function

$$\varphi_\pi^{\text{mod},1}(x) = \frac{8}{\pi} f_\pi \sqrt{x(1-x)},$$
$$\varphi_\pi^{\text{mod},2}(x) = 6f_\pi x(1-x)\{1 + \tfrac{8}{9}[1 - 5x(1-x)]\} \qquad (21)$$

reproducing these values (20), are also close to the asymptotic wave function. The second model corresponds to the expansion over the Gegenbauer polynomials $C_n^{3/2}(\xi)$ (the eigenfunctions of the evolution equation [4, 6]).

Thus, the moments of the pion wave function are rather sensitive to the functional form of the nonlocal condensates. The faster the NLC decrease with distance, the faster is the decrease with N of the relevant contribution to the $\langle\xi^N\rangle$ sum rule. Of course, in the $\lambda_q \to 0$ limit, Eq. (19) reduces to the original CZ sum rule (3),(1), and one obtains large CZ values for the moments. With $\lambda_q^2 = 0.4$ GeV^2, the condensate terms still decrease more slowly with N than the perturbative contribution, and the $\langle\xi^N\rangle$ values (20) are still larger than $\langle\xi^N\rangle^{\text{as}}$. To get the asymptotic value for $\langle\xi^2\rangle$, one should take $\lambda_q^2 = 1.2$ GeV^2. Surprisingly enough, it is this huge value of λ_q^2 that is favored by a calculation within a rather realistic QCD-vacuum model developed by Shuryak [16]. The recent lattice result $\langle\xi^2\rangle = 0.11$ [19], is still rather far from these values, but the disagreement might be essentially reduced by a renormalization factor (of order of 1.5) not included in the quoted lattice value.

Our results depend on the models we accepted for the nonlocal condensates. However, the sum rule is dominated by a single contribution [the second term in the large curly brackets in Eq. (19)] which is due to the four-quark condensate $\langle\bar{q}(0)q(x)\bar{q}(y)q(z)\rangle$, factorized *via* the vacuum-dominance hypothesis to the product of the simplest $\langle\bar{q}(0)q(z)\rangle$-type condensates. This factorization amounts to neglecting the dependence on the distance between the two $\bar{q}q$ pairs. If one takes this dependence into account, then the dominant term of Eq. (19) will produce the contributions that will decrease faster with N, and the resulting $\langle\xi^N\rangle$ will be even farther from the CZ values.

VII. CONCLUSIONS

Our basic idea in the present paper is that the nonperturbative information about the QCD vacuum structure should be accumulated in the functions describing the momentum distribution of the vacuum quark and gluonic fields. For the vacuum, these functions play the role analogous to that of the parton distributions in the case of the hadrons. Ideally, the vacuum distribution functions should be calculated from the theory of the QCD vacuum. In the absence of such a theory, one can incorporate the fact that the same vacuum distribution functions appear in different NLC-modified QCD sum rules for hadronic wave functions, parton distribution functions, hadronic form factors etc. This opens a possibility of finding the vacuum distribution functions (universal for all the hadrons) from the experimentally known hadronic functions.

ACKNOWLEDGMENTS

We are most grateful to I.I. Balitsky, V.L. Chernyak, A.V. Efremov, A.G. Grozin, B.L. Ioffe, V.A. Nesterenko, M.A. Shifman, and I.R. Zhitnitsky for stimulating discussions. One of us (A.R.) is most grateful to N. Isgur for the warm hospitality at CEBAF. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC05-84ER40150.

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