Extended weak isospin and fermion masses in a unified model

G. C. Joshi and R. R. Volkas
Research Centre for High Energy Physics, School of Physics, University of Melbourne, Parkville 3052 Australia
(Received 10 September 1991)

It is interesting to ask whether the weak-isospin gauge group $SU(2)_L$ is embedded in a larger symmetry group. From a model-building perspective there are usually too many possible answers to this question, because extensions invariably introduce new gauge anomalies. New fermions then have to be postulated, usually in an *ad hoc* manner, to cancel these anomalies. However, the quark-lepton symmetric models of Foot and Lew allow one to extend the weak-isospin group in a disciplined way, because the new anomalies can now cancel between quarks and generalized leptons. The basics of a model with gauge group $G = [SU(3)]^4$ are presented in this paper. We find that, through left-right symmetry, a partial unification of the gauge coupling constants of the theory naturally suggests itself. We also find that the symmetries of the model can impose restrictions on fermion masses and mixing angles at the tree level, which renormalization effects and mixing phenomena may modify in a predictive manner.

PACS number(s): 12.10.Dm; 12.15.Cc; 12.15.Ff

I. INTRODUCTION

Much rests on determining the theoretical status of the standard model (SM). The overwhelming majority of people who have thought much about it agree that it must be a "low-energy" effective field theory derivable from some more satisfactory construction. The reason for this strong feeling is, of course, a desire for greater understanding and predictivity rather than because of a phenomenological crisis.

Opinion appears, however, to be divided on whether we are likely to be able to figure out just what this "more satisfactory construction" is without an attendant phenomenological crisis to guide our impoverished imaginations. We hold the view that it is worth expending the effort to fathom out possibilities for physics beyond the SM. A major argument in favor of this stance is that, although almost certainly incomplete, the SM nonetheless embodies a great deal of systematic knowledge gained about particle physics in a theoretically consistent way. It imposes a discipline on the model builder by providing well-defined tools (local gauge invariance, spontaneous symmetry breaking, and so on) with which to work. In other words, it is quite likely that the next effective field theory approximation to be phenomenologically sanctioned will be built from similar components to the SM, components we may well be able to assemble at our desks before the experiments are performed.

There is another motivation for studying extensions of the SM: by doing so we also learn about the structure of the SM itself. In particular, we discover which aspects of the SM are robust, and which can be easily replaced by some sort of generalization (we will call such aspects "fragile").

An important area of the SM to look at, from both of these points of view, is its gauge group $G_{\rm SM}$ where $G_{\rm SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. In this article, we wish to address one aspect of the problem of whether, and how, it is desirable to extend $G_{\rm SM}$. Of course, several celebrated extensions of the SM gauge group have already been extensively studied in the literature. These include the left-right symmetric model [1], the color SU(5) scenario [2], Pati-Salam partial-unification theory (PUT) [3] and grand-unified theories (GUT's) [4]. More recently, a novel extension of $G_{\rm SM}$ called the "quark-lepton symmetric model" was proposed by Foot and Lew [5] as a way of introducing an exact discrete symmetry between quarks and (generalized) leptons. We will use both left-right symmetry and quark-lepton symmetry extensively in this paper.

There are two main reasons why left-right symmetry is an interesting idea. First, the standard quark-lepton spectrum is much more elegantly described in left-right symmetric models than in the minimal SM. Under the gauge group $G_{LR} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ the fermion spectrum is

$$q_L \sim (3,2,1)(1/3), \qquad q_R \sim (3,1,2)(1/3),$$
$$\ell_L \sim (1,2,1)(-1), \qquad \ell_R \sim (1,1,2)(-1). \tag{1}$$

This spectrum clearly has fewer arbitrary features than the SM spectrum. The second reason for being interested in left-right symmetry is that it is compatible with a parity-invariant Lagrangian.

The most pertinent observation to make from our point of view about the left-right symmetric gauge group is that it is an extension of only the $U(1)_Y$ factor of $G_{\rm SM}$. In fact, $U(1)_Y$ is a subgroup of $SU(2)_R \otimes U(1)_{B-L}$. Weak hypercharge is therefore fragile, because it can be easily extended. The resulting theory is elegant, because gauge anomaly cancellation is achieved without the introduction of any *ad hoc* exotic fermion states. (Of course, one has to add a right-handed neutrino to the spectrum. However, this particle is necessary in order to define fermion representations under G_{LR}. Its introduction is therefore extremely well motivated.)

The standard way to perform G_{LR} symmetry breaking is through nonzero vacuum expectation values (VEV's) for a Higgs bidoublet $\phi \sim (1,2,2)(0)$ together with a pair of left-right symmetric Higgs triplets. In this paper, however, we will be led to using the alternative (minimal) set of Higgs multiplets given by

$$\rho_L \sim (1,2,1)(1), \qquad \rho_R \sim (1,1,2)(-1), \qquad \rho_L \leftrightarrow \rho_R^c. \tag{2}$$

These multiplets are usually not used because they cannot couple to quarks and leptons at tree level. However, if the exotic fermions

$$\begin{aligned} &E_{L,R} \sim (1,1,1)(-2), \quad N_{L,R} \sim (1,1,1)(0), \\ &U_{L,R} \sim (3,1,1)(4/3), \quad D_{L,R} \sim (3,1,1)(-2/3), \end{aligned} \tag{3}$$

are added to the theory, an interesting scenario for charged-fermion masses results, which goes by the name of the "universal seesaw mechanism [6]." These vectorlike exotic fermions can couple to ordinary fermions through $\rho_{L,R}$ and they can also have bare masses. We will assume that these bare masses are much larger than $\langle \rho_{L,R} \rangle$. Since the normal Dirac mass terms for the standard fermions are absent due to the absence of ϕ, seesaw forms are generated for the charged-fermion mass matrices. Thus a qualitative understanding of why fermion masses are much smaller than the weak scale is achieved. (The top quark is a special case. See Davidson and Wali in the second paper of Ref. [6] for a discussion.) If one does not wish to use the universal seesaw mechansim, then one may introduce the bidoublet ϕ together with $\rho_{L,R}$. If the fermions in Eq. (3) are also introduced, then these will still mix with the ordinary quarks and leptons through nonzero VEV's for $\rho_{L,R}$. Analogues of the fermions $N_{L,R}$ and $D_{L,R}$ will be used later on in this paper.

It has recently been shown that the color group $SU(3)_c$ is also fragile. The SM can be elegantly extended to the gauge group $SU(5)_c \otimes SU(2)_L \otimes U(1)_{Y'}$ [2]. Again, from an anomaly-cancellation perspective this extension is smooth, because no exotic fermions beyond those necessary to define representations of the gauge group need be introduced.

So, by exhibiting two examples, we have shown that both $U(1)_Y$ and $SU(3)_c$ can be extended very easily. The fermion spectra are either just as complicated [color SU(5)], or simpler (left-right symmetry), than the SM. Anomalies cancel within a family, just as for the SM, and all exotic fermions are very well motivated given the gauge group extension employed. Furthermore, the left-right symmetric model has as a bonus the prospect of a parity-invariant Lagrangian. We therefore describe both $U(1)_Y$ and $SU(3)_c$ as fragile; they can be easily enlarged. (Needless to say, both of these extensions have an acceptable, and even interesting, phenomenology.)

The weak-isospin group $SU(2)_L$ is quite a contrast. Suppose we wish to embed it in a larger group $SU(3)_L$. Let us consider the group $G_{\text{attempt}} = SU(3)_c \otimes SU(3)_L \otimes U(1)_{Y''}$. A candidate fermion spectrum might be

$$\begin{aligned} &Q_L \sim (3,3)(y_Q), \quad u_R \sim (3,1)(y_u), \\ &d_R \sim (3,1)(y_d), \quad q'_R \sim (3,1)(y_q), \\ &F_L \sim (1,3)(y_F), \quad e_R \sim (1,1)(y_e), \\ &f'_R \sim (1,1)(y_f). \end{aligned} \tag{4}$$

One could also try variations on this pattern, by making the quarks a $\bar{3}$ of $SU(3)_L$ and/or introducing right-handed neutrinos. At any rate, it is certainly possible to choose the Y'' quantum numbers so that the standard hypercharge generator is a linear combination of Y'' and T_L, where $T_L \equiv \text{diag}(-2,1,1)$ is a diagonal generator of $SU(3)_L$. One again finds that exotic fermions have to be introduced in order to fill out the representations of weak isospin. However, the crucial observation is that no matter how hard one tries, one cannot ever cancel anomalies within a fermion family. Cancellation cannot occur between left- and right-handed sectors, because $SU(3)_L$ has to be chiral. Cancellation also cannot occur between quarks and leptons, because of the mismatch in degrees of freedom: quarks are colored but leptons are not. Remember that $[SU(2)_L]^3$ anomalies are always zero because SU(2) only has real representations. The trouble with $SU(3)_L$ is that $[SU(3)_L]^3$ anomalies are not automatically zero. A similar problem would occur with most other extensions of weak isospin one might contemplate [7].

Since we have good reason to believe that anomalous gauge theories are sick, we now need to introduce exotic fermions for the sole purpose of cancelling this inevitable gauge anomaly. This of course is a technically acceptable course of action [8]. Unfortunately, there are always an infinite number of exotic fermion spectra that will do the job. So unless one has some other consideration that would allow the choice of fermion representations to be considerably reduced [9], one is led to either making an *ad hoc* selection, or to living with an ill-defined sector in one's theory.

One might call the above theory an attempt at constructing an extension of $SU(2)_L$ which was "once removed" from the SM. This attempt essentially fails; $SU(2)_L$ is robust. However, the quark-lepton symmetric models referred to earlier provide a framework for constructing a sensible extension of weak isospin which is "twice removed" from the SM. This possibility arises because the gauge group in quark-lepton symmetric models forces the number of leptonic degrees of freedom to equal the number of quark degrees of freedom. The barrier to nontrivial anomaly cancellation is destroyed.

We refer readers to the original papers on quark-lepton symmetry [5] for a complete description of this idea. We present only a quick sketch below.

The basic model has gauge group $G_{q\ell}$ where

$$G_{q\ell} = SU(3)_\ell \otimes SU(3)_q \otimes SU(2)_L \otimes U(1)_X, \tag{5}$$

under which the fermions form the spectrum,

$$\begin{aligned} &Q_L \sim (1,3,2)(1/3), \quad u_R \sim (1,3,1)(4/3), \\ &d_R \sim (1,3,1)(-2/3), \quad F_L \sim (3,1,2)(-1/3), \\ &E_R \sim (3,1,1)(-4/3), \quad N_R \sim (3,1,1)(2/3). \end{aligned} \tag{6}$$

The group $SU(3)_q$ is just the usual color group given a different label, while $SU(3)_\ell$ is leptonic color. Standard hypercharge is given by $Y = X + T_\ell/3$ where $T_\ell \equiv \text{diag}(-2,1,1)$ in $SU(3)_\ell$ space. Thus the basic quark-lepton symmetric model is an extension of $U(1)_Y$, just like the left-right symmetric model. In order to define a representation of leptonic color, the number of leptonic degrees of freedom have been tripled. Also, a generalized right-handed neutrino multiplet (N_R) has been added so that there is an exact correspondence between quark and lepton degrees of freedom. An exact interchange symmetry can now be defined [10]:

$$\begin{aligned} &Q_L \leftrightarrow F_L, \quad u_R \leftrightarrow E_R, \quad d_R \leftrightarrow N_R, \\ &G_q^\mu \leftrightarrow G_\ell^\mu, \quad W^\mu \leftrightarrow W^\mu, \quad C^\mu \leftrightarrow -C^\mu, \end{aligned} \tag{7}$$

where $G^\mu_{q,\ell}$, W^μ, and C^μ are gauge fields for $SU(3)_{q,\ell}$, $SU(2)_L$, and $U(1)_X$, respectively. Gauge anomaly cancellation is manifest for each fermion family.

The first stage of symmetry breaking leads to

$$G_{q\ell} \to SU(2)' \otimes SU(3)_q \otimes SU(2)_L \otimes U(1)_Y, \tag{8}$$

where $SU(2)'$ is an unbroken subgroup of leptonic color. The generalized lepton multiplets decompose into the standard leptons together with very heavy charge $\pm 1/2$ $SU(2)'$ doublet fermions called "liptons." These exotic states are confined into heavy unstable integrally charged bound states by the asymptotically free $SU(2)'$ interaction [11].

Electroweak symmetry breaking proceeds through a standard Higgs doublet $\phi \sim (1,1,2)(1)$ which transforms into its charge-conjugate field ϕ^c under the quark-lepton discrete symmetry. The field ϕ couples in the usual way to the fermions:

$$\begin{aligned} \mathcal{L}^{(2)}_{\rm Yuk} = h_1(\overline{F}_L E_R \phi + \overline{Q}_L u_R \phi^c) \\ + h_2(\overline{F}_L N_R \phi^c + \overline{Q}_L d_R \phi) + \text{H.c.} \end{aligned} \tag{9}$$

Note, however, that quark-lepton symmetry has imposed relations that are not present in the SM between the Yukawa coupling constants.

The standard leptons and the quarks gain tree-level mass matrices given by

$$m_u = m_e = h_1 \langle \phi \rangle \quad \text{and} \quad m_d = m_\nu = h_2 \langle \phi \rangle. \tag{10}$$

These mass relations obviously require further comment. As in GUT's or the Pati-Salam PUT, these tree-level results are due to the *raison d' être* of the theory: a relationship between quarks and leptons. As such they are to be welcomed, because after all we ultimately *do* want to understand how fermion masses are interrelated (assuming there is a solution to this problem). The hard work is to find a theory in which untenable but suggestive tree-level relations are renormalized into predictive and correct results. The basic quark-lepton symmetric model probably fails to do this [12], particularly if the quark-lepton symmetry breaking scale is relatively low (as it can be phenomenologically). Therefore, one should view the fermion mass sector of a theory such as this as a starting point for constructing a more satisfactory scheme. Indeed, the extended weak-isospin theory we will introduce in the next section will have more varieties of physics in the fermion mass generation sector, and we hope the qualitative observations we will make will have some value in the continuing struggle to understand quark and lepton masses. Note also that these mass relations may be avoided by complicating the Higgs sector. If, for instance, a second Higgs doublet is introduced then a successful fermion mass spectrum can be arranged at the expense of predictivity. This shows that it is not inevitable for quark-lepton symmetry to yield mass relations, although the fact that it *can* is potentially very important.

The extended weak-isospin model we will introduce shortly will actually be based on the combination of quark-lepton and left-right symmetry. The gauge group for this combined theory is $G_{q\ell LR} = SU(3)_\ell \otimes SU(3)_q \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_V$, under which the fermions transform as

$$\begin{aligned} &Q_L \sim (1,3,2,1)(1/3), \qquad Q_R \sim (1,3,1,2)(1/3), \\ &F_L \sim (3,1,2,1)(-1/3), \qquad F_R \sim (3,1,1,2)(-1/3). \end{aligned} \tag{11}$$

Symmetry breaking is achieved through generalizations of the Higgs multiplets used in the minimal quark-lepton symmetric model described above.

In this paper we will present an outline for a theory of extended weak isospin, predicated upon the quark-lepton symmetry idea. We will concentrate on the left-right symmetric version of this scheme, because its gauge group and fermion spectrum turn out to be quite elegant. As a bonus we will discover that a reduction in the number of independent gauge coupling constants from three to two naturally suggests itself; that is, the theory will exhibit partial unification, although in a different way from the model of Pati and Salam [3]. We will discuss possible patterns of symmetry breakdown, and consequent effects on fermion mass generation, although a rigorous analysis of the Higgs potential and other more complex issues that will arise will be beyond the scope of this introductory article.

The remaining two sections of this paper decribe the extended weak-isospin model, and furnish concluding remarks.

II. EXTENDED WEAK ISOSPIN

A. Introduction

We are now ready to present our model [13]. The gauge group is G where

$$G = SU(3)_\ell \otimes SU(3)_q \otimes SU(3)_L \otimes SU(3)_R. \tag{12}$$

The generalized quark and lepton spectrum is

$$\begin{aligned} &Q_L \sim (1,3,\overline{3},1), \qquad Q_R \sim (1,3,1,\overline{3}), \\ &F_L \sim (3,1,3,1), \qquad F_R \sim (3,1,1,3). \end{aligned} \tag{13}$$

We impose two discrete symmetries on the Lagrangian. First there is quark-lepton symmetry, which is defined by the interchanges [14]

$$\begin{aligned} &Q_L \leftrightarrow F_L, \qquad Q_R \leftrightarrow F_R, \\ &G_q^\mu \leftrightarrow G_\ell^\mu, \qquad W_L^\mu \leftrightarrow -W_L^{\mu *}, \qquad W_R^\mu \leftrightarrow -W_R^{\mu *}. \end{aligned} \tag{14}$$

Second, there is left-right symmetry given by

$$Q_L \leftrightarrow Q_R, \qquad F_L \leftrightarrow F_R,$$
$$W_L^\mu \leftrightarrow W_R^\mu, \qquad G_q^\mu \leftrightarrow G_q^\mu, \qquad G_\ell^\mu \leftrightarrow G_\ell^\mu. \tag{15}$$

The existence of these two discrete symmetries means there are only two independent coupling constants in the theory, thus making it a model of partial unification. Later on we will analyze the implications of this through the renormalization-group evolution of the gauge coupling constants.

As promised, we have used quark-lepton symmetry to cancel $[\mathrm{SU}(3)_{L,R}]^3$ gauge anomalies between quarks and leptons. The fermion spectrum contains a number of nonstandard states, but all of these have to be there to define representations of the gauge group. None of these fermions have been introduced for the sole purpose of cancelling gauge anomalies.

Observe that there are no U(1) factors in the gauge group G. That the weak hypercharge generator of the SM can be embedded in the non-Abelian group arising from the desire to extend weak isospin is a rather surprising and pleasing result. We find the gauge group structure and the fermion spectrum of this model to be very symmetric and aesthetically satisfying. We want to emphasize this here, because the details of symmetry breaking and fermion mass generation in this model will turn out to be a little complicated. However, the low-energy debris should not detract from the elegance of the fully symmetric Lagrangian. After all, ultimately we have a rather complicated low-energy world to explain.

B. First stage of symmetry breaking

The first stage of symmetry breaking proceeds through the analogue of the χ Higgs bosons of quark-lepton symmetric models [5]. There are four multiplets which are all interrelated through the two discrete symmetries. They are introduced through the Yukawa Lagrangian $\mathcal{L}_{\rm Yuk}^{(1)}$ where

$$\mathcal{L}_{\rm Yuk}^{(1)} = \lambda[\,\overline{(F_L)^c}F_L\chi_{1L} + \overline{(F_R)^c}F_R\chi_{1R} + \overline{(Q_L)^c}Q_L\chi_{2L} + \overline{(Q_R)^c}Q_R\chi_{2R}] + \text{H.c.} \tag{16}$$

The quantum numbers of the χ fields are

$$\chi_{1L} \sim (3,1,3,1), \qquad \chi_{1R} \sim (3,1,1,3),$$
$$\chi_{2L} \sim (1,3,\overline{3},1), \qquad \chi_{2R} \sim (1,3,1,\overline{3}). \tag{17}$$

The transformation laws for these fields under the discrete symmetries are completely obvious, so we will not display them. It is clearly also possible to introduce more than one copy of the multiplets in Eq. (17). How many copies will ultimately be necessary to make our model fully realistic will remain an open question.

Note that the χ fields transform under both the color and the weak-isospin parts of the gauge group, unlike their simpler cousins in the basic left-right symmetric version of the quark-lepton symmetric model [5]. The new χ's are nevertheless a direct generalization of the old χ's because the weak-isospin singlet in Eq. (16) is antisymmetric in both cases. When nonzero VEV's develop for $\chi_{1L,R}$ both quark-lepton symmetry and extended weak isospin break down at the same scale. Whether parity also breaks down at this scale depends on whether $\langle\chi_{1L}\rangle = \langle\chi_{1R}\rangle$ or not. This question can only be answered by examining the Higgs potential in detail, which is beyond the scope of this paper. The complexities inherent in fermion mass generation in this model will render uncertain the optimal form of the Higgs-boson spectrum. Given this we prefer in this introductory article to adopt an open-minded position on the exact mode of symmetry breaking; we will confine ourselves to some general observations, which we think are interesting in themselves. Therefore, the reader should keep in mind that after the first stage of symmetry breaking, parity may or may not be spontaneously broken.

We postulate that the $T_\ell = T_L = -2$ component of χ_{1L}, and the $T_\ell = T_R = -2$ component of χ_{1R} develop nonzero VEV's. This breaks G down to $\mathrm{SU}(2)' \otimes G_{LR}$:

$$G \xrightarrow{\langle\chi_{1L,R}\rangle} \mathrm{SU}(2)'\otimes\mathrm{SU}(3)_q\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\otimes\mathrm{U}(1)_{B-L}, \tag{18}$$

where $B-L$ is defined to be the generator left unbroken by these VEV's. It is given by

$$B-L = \frac{1}{3}(T_\ell - T_L - T_R)\,. \tag{19}$$

The branching rules for the fermion representations verify that this quantity is indeed $B-L$. Under $\mathrm{SU}(2)'\otimes G_{LR}$ the lepton multiplets break up as follows:

$$F_L \to (2,1,2,1)(0)\oplus(1,1,2,1)(-1)\oplus(2,1,1,1)(1) \oplus(1,1,1,1)(0), \tag{20}$$

$$F_L \to f_L \oplus \ell_L \oplus k_L \oplus N_L, \tag{21}$$

$$F_R \to (2,1,1,2)(0)\oplus(1,1,1,2)(-1)\oplus(2,1,1,1)(1) \oplus(1,1,1,1)(0), \tag{22}$$

$$F_R \to f_R \oplus \ell_R \oplus k_R \oplus N_R. \tag{23}$$

The line following the branching rule establishes the nomenclature we will use for the fermions. The multiplets $f_{L,R}$ are the liptons of the quark-lepton symmetric model; $\ell_{L,R}$ are the standard leptons; $k_{L,R}$ are charge $+1/2$ fermions which are the extended weak-isospin partners of the liptons, while $N_{L,R}$ are neutral fermion partners of the standard leptons. The quark fields are given by

$$Q_L \to (1,3,2,1)(1/3)\oplus(1,3,1,1)(-2/3), \tag{24}$$

$$Q_L \to q_L \oplus D_L, \tag{25}$$

$$Q_R \to (1,3,1,2)(1/3)\oplus(1,3,1,1)(-2/3), \tag{26}$$

$$Q_R \to q_R \oplus D_R, \tag{27}$$

where $q_{L,R}$ are the standard quarks and $D_{L,R}$ are their charge $-1/3$ extended weak-isospin partners. As one can see, Eq. (19) does indeed define $B-L$.

Since the χ fields have the same quantum numbers under G as the fermions we also know how they break up under $SU(2)' \otimes G_{LR}$. We will denote the various components by χ_H^f, χ_H^ℓ, χ_H^k, χ_H^N, χ_H^q, and χ_H^D where $H = L, R$ in an obvious notation. The fields χ_H^N are the ones which develop nonzero VEV's.

The Yukawa Lagrangian Eq. (16) in terms of these fields is

$$\begin{aligned}\mathcal{L}_{\rm Yuk}^{(1)} = \lambda\{ & \overline{(f_L)^c} f_L \chi_L^N + 2[\overline{(\ell_L)^c} f_L \chi_L^k + \overline{(k_L)^c} f_L \chi_L^\ell \\ & + \overline{(f_L)^c} N_L \chi_L^f + \overline{(k_L)^c} \ell_L \chi_L^f] \\ & + \overline{(q_L)^c} q_L \chi_L^D + 2\overline{(D_L)^c} q_L \chi_L^q \\ & + (L \to R)\} + \text{H.c.} \end{aligned} \tag{28}$$

We see from this that only the liptons pick up mass at tree level from $\langle \chi_{L,R}^N \rangle \neq 0$. An interesting question at this point is whether the vectorlike fermions k, N, and D pick up mass through radiative corrections or are kept exactly massless by a chiral symmetry. We need to know this, because if there is a chiral symmetry keeping these particles massless, then they may be given controllably small masses once this chiral symmetry is broken in some manner. This is important from a phenomenological perspective because (relatively) light exotic k, N, and D fermions would be interesting things to look for in the 100 GeV to 1 TeV region in collider experiments. Also, we may want N and D to play a role in quark and lepton mass generation through mixing, so they should not be too massive. In addition, the mass spectrum of the fermions affects the renormalization group analysis to be performed shortly.

In fact it is easy to show that there *is* a global chiral symmetry which maintains the masslessness of k, N and D to all orders. An analysis of the global symmetries of the theory (as so far constructed) shows that there are four unbroken U(1) invariances.

We notice first of all that there are three global symmetries in the Lagrangian, given by

$$\begin{aligned} \text{U}(1)_{C_L}: \quad & F_L \to e^{i\alpha} F_L, \quad \chi_{1L} \to e^{-2i\alpha}\chi_{1L}, \\ & Q_R \to e^{i\alpha} Q_R, \quad \chi_{2R} \to e^{-2i\alpha}\chi_{2R}, \end{aligned} \tag{29}$$

$$\begin{aligned} \text{U}(1)_{C_R}: \quad & F_R \to e^{i\beta} F_R, \quad \chi_{1R} \to e^{-2i\beta}\chi_{1R}, \\ & Q_R \to e^{-i\beta} Q_R, \quad \chi_{2R} \to e^{2i\beta}\chi_{2R}, \end{aligned} \tag{30}$$

$$\text{U}(1)_{B'}: \quad Q_{L,R} \to e^{i\gamma/3} Q_{L,R}, \quad \chi_{2R} \to e^{-2i\gamma/3}\chi_{2R}. \tag{31}$$

(There are three rather than four independent global symmetries because of the Higgs potential term $\chi_{1L}^\dagger \chi_{1R} \chi_{2L}^\dagger \chi_{2R}$.) Nonzero VEV's for $\chi_{L,R}^N$ leave the generators

$$T_L - C_L, \quad T_R - C_R, \quad \text{and} \quad B' \tag{32}$$

unbroken [15]. The symmetry $\text{U}(1)_{B'}$ is an extended baryon number, while the other two unbroken global symmetries are completely new invariances associated with the exotic fermions. (Note that no physical Goldstone bosons are generated from this spontaneous symmetry breaking, because the $C_{L,R}$ charges are proportional to $T_{L,R}$ for the Higgs fields which actually develop the nonzero VEV's. Because the latter generators are coupled to gauge fields, the would-be Goldstone bosons are eaten. The reader may recall that the same principle applies to the usual left-right symmetric model, where the putative Majoron is eaten by the $B-L$ gauge field.)

If we add to the generators of Eq. (32) the unbroken purely gauge generator $B-L$ then four unbroken global symmetries emerge. By taking appropriate linear combinations, we may write the action of these symmetries on the $\text{SU}(2)' \otimes G_{LR}$ multiplets in a simple manner. Two of these are the vectorlike extended baryon and extended lepton number groups given by

$$\begin{aligned} B': \quad & q_{L,R} \to e^{i\alpha/3} q_{L,R}, \quad D_{L,R} \to e^{i\alpha/3} D_{L,R}, \\ & \chi_{L,R}^{q,D} \to e^{-2i\alpha/3} \chi_{L,R}^{q,D}; \end{aligned} \tag{33}$$

$$\begin{aligned} L': \quad & \ell_{L,R} \to e^{i\beta} \ell_{L,R}, \quad N_{L,R} \to e^{i\beta} N_{L,R}, \\ & \chi_{L,R}^{f,k} \to e^{-i\beta} \chi_{L,R}^{f,k}. \end{aligned} \tag{34}$$

There is another vector-like symmetry $U(1)_{\mathcal{F}}$ which acts only on the exotic fermions. Explicitly, it is given by

$$\begin{aligned} \mathcal{F}: \quad & k_{L,R} \to e^{-i\gamma} k_{L,R}, \quad N_{L,R} \to e^{-i\gamma} N_{L,R}, \\ & \chi_{L,R}^{f,\ell} \to e^{i\gamma} \chi_{L,R}^{f,\ell}, \\ & \chi_{L,R}^{q} \to e^{-i\gamma} \chi_{L,R}^{q}, \quad D_{L,R} \to e^{i\gamma} D_{L,R}. \end{aligned} \tag{35}$$

Finally, there is the axial analogue $\mathcal{A}$ of $\mathcal{F}$ acting on the exotic fermions:

$$\begin{aligned} \mathcal{A}: \quad & k_{L,R} \to e^{\mp i\delta} k_{L,R}, \quad N_{L,R} \to e^{\mp i\delta} N_{L,R}, \\ & D_{L,R} \to e^{\pm i\delta} D_{L,R}, \chi_{L,R}^{f} \to e^{\pm i\delta} \chi_{L,R}^{f}, \\ & \chi_{L,R}^{\ell} \to e^{\pm i\delta} \chi_{L,R}^{\ell}, \quad \chi_{L,R}^{q} \to e^{\mp i\delta} \chi_{L,R}^{q}. \end{aligned} \tag{36}$$

Clearly, $\text{U}(1)_{\mathcal{A}}$ maintains k, N, and D as massless fermions. Note that we have not yet completed specifying the Higgs fields in the theory. It will turn out that this global axial symmetry is explicitly broken in the full theory. Nevertheless, it is useful to introduce it because we can discuss k, N, and D mass generation in terms of how it is broken. Another relevant observation is that exact $\text{U}(1)_{\mathcal{F}}$, together of course with $\text{SU}(2)_L \otimes \text{SU}(2)_R$ weak isospin, prevents d-D and ν-N mixing (note also that this mixing is allowed by both L' and B').

Before introducing the next stage of symmetry breaking, we have to comment on the role of the $\chi_{L,R}^\ell$ Higgs fields. The reader will note that they have the quantum numbers of the minimal left-right symmetric Higgs bosons which can perform $\text{SU}(2)_R$ and left-right symmetry breaking. Also, the D and N fields have the correct quantum numbers to be the exotic fermions necessary for mixing with d quarks and neutrinos, respectively. However, we prefer not to use $\chi_{L,R}^\ell$ for this role. There are two reasons for this. First, for reasons of phenomenological interest we prefer to induce $SU(2)_R$ breaking at a much lower scale than extended weak isospin. Since $\chi^{\ell}_{L,R}$ are in the same multiplet as $\chi^{N}_{L,R}$ we would expect that if they were to develop nonzero VEV's they would be of the same high scale as $\langle\chi^{\ell}_{L,R}\rangle$. Second, as one can see from Eq. (28), nonzero VEV's for $\chi^{\ell}_{L,R}$ would only mix the liptons with the k's, rather than the ν's with the N's and/or the d's with the D's.

C. Second stage of symmetry breaking

We have two more symmetry breaking transitions to induce: $SU(2)_R$ and electroweak symmetry breaking. To this end we introduce either one copy or several of the extended weak-isospin generalization of the standard Higgs bidoublet Φ of left-right symmetric models. The Yukawa Lagrangian is $\mathcal{L}^{(2)}_{\rm Yuk}$ where

$$\mathcal{L}^{(2)}_{\rm Yuk} = h(\overline{F}_L F_R \Phi + \overline{F}_R F_L \Phi^c + \overline{Q}_L Q_R \Phi^c + \overline{Q}_R Q_L \Phi) + \text{H.c.} \tag{37}$$

Note that quark-lepton symmetry imposes equality of quark and lepton Yukawa coupling constants. Under G, Φ transforms like,

$$\Phi \sim (1,1,3,\overline{3}), \tag{38}$$

while under the subgroup $SU(2)' \otimes G_{\rm SM}$ it breaks up as follows:

$$\Phi \to (1,1,2,2)(0) \oplus (1,1,2,1)(-1) \oplus (1,1,1,2)(1) \oplus (1,1,1,1)(0), \tag{39}$$

$$\Phi \to \phi \oplus \rho_L \oplus \rho_R \oplus \sigma. \tag{40}$$

Under both quark-lepton and left-right symmetries $\Phi \leftrightarrow \Phi^c$.

The Yukawa Lagrangian may be rewritten as

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm Yuk} = h(&\overline{f}_L f_R \phi + \overline{k}_L f_R \rho_R + \overline{f}_L k_R \rho_L + \overline{k}_L k_R \sigma \\ &+ \overline{\ell}_L \ell_R \phi + \overline{N}_L \ell_R \rho_R + \overline{\ell}_L N_R \rho_L + \overline{N}_L N_R \sigma \\ &+ \overline{q}_L q_R \phi^c + \overline{D}_L q_R \rho^c_R + \overline{q}_L D_R \rho^c_L + \overline{D}_L D_R \sigma^c) \\ &+\text{H.c.}\end{aligned} \tag{41}$$

We would like to identify ϕ with the usual Higgs bidoublet in left-right symmetric models. Nonzero VEV's for ϕ will give ordinary quarks and leptons a mass. As is usual in quark-lepton symmetric models equal masses are induced for quarks and leptons. There is, however, an additional constraint due to extended weak isospin: the mass matrix for down quarks is proportional to the mass matrix for up quarks. Similarly, the charged lepton mass matrix is proportional to the neutrino Dirac mass matrix. This implies that the Kobayashi-Maskawa (KM) matrices are just equal to unity in both sectors. This constraint occurs because SU(3) weak isospin forbids the charge conjugate of ϕ (given by $\phi^c = \tau_2 \phi \tau_2$) from coupling to ordinary fermions.

We reiterate our attitude to results such as these: it is probably a *good* thing that these constraints exist in the theory. We ultimately want a predictive theory of fermion masses. It makes sense to first construct a model which provides incorrect but suggestive mass and mixing-angle relations, and then to see whether radiative corrections in the model itself, or in some development of it, can perturb the wrong tree-level results into correct renormalized results. Remember also our previous remark that relations such as these may be evaded at the expense of predictivity by enlarging the Higgs sector.

The Higgs bosons ρ_L and ρ_R are the appropriate fields to serve as the left-right symmetry breaking doublets. If the left-right symmetry breaking scale is not too much higher than the electroweak scale, then only a relatively small hierarchy in VEV's need be induced between the extended weak-isospin partners ϕ, σ, and $\rho_{L,R}$. Furthermore, nonzero values for $\langle\rho_{L,R}\rangle$ will induce mixing between ordinary down quarks and D's, and ordinary neutrinos and N's.

Finally, the singlet field σ will give tree-level masses to the k, N, and D exotic fermions if it acquires a nonzero VEV. For phenomenological reasons we require this VEV to be fairly large, so that the exotic fermion masses cause no experimental problems. However, it need not be much larger than the VEV for ρ_R. Note that a nonzero $\langle\sigma\rangle$ also breaks extended weak isospin. (We choose not to use it as the primary source of extended weak-isospin breaking because it is in the same multiplet as ϕ and $\rho_{L,R}$.)

Since the VEV for σ induces k, N, and D masses, it evidently has to spontaneously break $U(1)_{\mathcal{A}}$. However, with the introduction of Φ it is natural to break this axial symmetry explicitly in the Lagrangian. This is accomplished through the gauge invariant Higgs potential terms

$$\chi^{\dagger}_{1L}\chi_{1R}\Phi + \chi^{\dagger}_{2L}\chi_{2R}\Phi^c + \text{H.c.} \tag{42}$$

These terms explicitly break $U(1)_{C_L} \otimes U(1)_{C_R}$ down to the diagonal subgroup. This reduces the number of exact global symmetries after the first stage of symmetry breaking by one. It is straightforward to check that $U(1)_{\mathcal{A}}$ is the explicitly broken symmetry. Furthermore, since the absence of the Higgs-boson trilinear term in Eq. (42) increases the symmetry of the theory, its coefficient may be kept small in a technically natural manner. This protects k, N, and D from obtaining large radiative masses from the heavy sector of the theory.

The vectorlike symmetry $U(1)_{\mathcal{F}}$ is spontaneously broken through nonzero VEV's for $\rho_{L,R}$. This is correlated with the fact that d-D and ν-N mixing is induced through these Higgs fields [remember that exact $U(1)_{\mathcal{F}}$ forbids this mixing]. There is however a remnant exact global symmetry given by $U(1)_{\mathcal{F}'}$ where

$$\mathcal{F}' = (B-L) + \mathcal{F} = B' - L'. \tag{43}$$

This is a fermion-number–like symmetry which does not distinguish between d's and D's, or between ν's and N's. Again, there is no Goldstone boson produced by this spontaneous breaking.

D. Fermion mass and mixing-angle generation

Let us summarize the symmetry breaking and fermion mass generation physics in the model. The gauge-symmetry breaking pattern is

$$\begin{aligned}
&SU(3)_\ell \otimes SU(3)_q \otimes SU(3)_L \otimes SU(3)_R \\
&\quad\downarrow \langle \chi^N_{L,R}\rangle \\
&SU(2)' \otimes SU(3)_q \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \\
&\quad\downarrow \langle \rho_R\rangle, \langle\sigma\rangle \\
&SU(2)' \otimes SU(3)_q \otimes SU(2)_L \otimes U(1)_Y \\
&\quad\downarrow \langle \phi\rangle, \langle\rho_L\rangle \\
&SU(2)' \otimes SU(3)_q \otimes U(1)_Q .
\end{aligned} \tag{44}$$

The various types of fermions gain mass as follows: (a) liptons gain tree-level masses from $\langle\chi^N_{L,R}\rangle$; (b) the k, N, and D exotic fermions gain tree-level masses from $\langle\sigma\rangle$, while (c) ordinary quarks and leptons gain masses from ϕ, and through the mixing with D and N which is induced by $\langle\rho_{L,R}\rangle$. The symmetries of the model (both exact and approximate) ensure that (i) the k, N, and D fermions do not get large radiative masses induced by the heavy sector, and (ii) that d-D and ν-N mixing is absent until $\rho_{L,R}$ develop nonzero VEV's.

Given the hierarchy between the $\langle\chi\rangle$ and $\langle\Phi\rangle$ scales necessary phenomenologically, we see that in general the liptons should be unobservably heavy, while the k, N, and D fermions may have masses as low as 100 GeV or so.

There are two other interesting observations: (i) The KM matrix is diagonal at tree level due to constraints from extended weak isospin if the minimal Higgs sector of only one Φ is assumed. (ii) If there is only one Φ then its VEV induces the mass relations $m_u = m_e$ and $m_d = m_\nu$ at tree level. Nonzero VEV's for $\rho_{L,R}$ induce d-D and ν-N mixing, but quark-lepton symmetry ensures that the mass relations remain nevertheless.

It is clear that the combination of quark-lepton symmetry and extended weak isospin can be used to constrain fermion masses and mixing angles in an interesting way. It is tantalizing to think that radiative corrections in the theory may be sufficient to correct the tree-level results quoted above into renormalized quantities that are both correct and predictive. One would expect, for instance, that the spontaneous breaking of quark-lepton symmetry at the $\langle\chi^N_{L,R}\rangle$ scale would induce, through radiative effects, a correction to the mass relations $m_u = m_e$ and $m_d = m_\nu$. Also, the breakdown of extended weak isospin at the same high scale should generate off-diagonal KM matrix elements.

Of course, it would be remarkable if these radiative corrections yielded correct masses and mixing angles. The authors feel that the scenario studied here is sufficiently different from conventional GUT models to be worth some further study. We know that conventional GUT models can succesfully predict the ratio m_b/m_τ, while failing to correctly acount for all of the other mass relations. We hope the scheme presented above will prove useful in developing new ideas in the continuing effort to understand the origin of fermion masses and mixing angles.

E. Constraints from partial unification

We have fairly laboriously developed a scenario above where: (i) extended weak isospin together with quark-lepton symmetry is broken, and liptons become massive, at a high scale; (ii) $SU(2)_R$ and electroweak breaking, together with mass generation for the other fermions, occurs at a much lower mass scale. We now have to check that this qualitative pattern is consistent with the partial unification of gauge coupling constants enforced by the two discrete symmetries of the model [16].

The analysis is quite straightforward (we assume three generations of fermions). Denote the three principal scales in the theory as

$$\Lambda_1 \sim \langle\chi_{1L,R}\rangle; \quad \Lambda_2 \sim \langle\rho_R\rangle,\ \langle\sigma\rangle; \quad M_W \sim \langle\rho_L\rangle,\ \langle\phi\rangle. \tag{45}$$

We will ignore all the fine structure in this hierarchy as, for example, may exist between $\langle\chi_{1R}\rangle$ and $\langle\chi_{1L}\rangle$ if parity is spontaneously broken at the high scale of the theory rather than the intermediate scale. Consequently, a single coupling constant g_2 will be assumed for both $SU(2)_L$ and $SU(2)_R$ between Λ_1 and Λ_2. The one-loop renormalization group equations for the fine-structure constants α_3, α_2, α_{BL} and α_Y (where these refer to color, the weak SU(2)'s, $B-L$, and Y, respectively) will be used. (The coupling constants of $U(1)_Y$ and $U(1)_{B-L}$ are chosen to be $g_Y/2$ and $g_{BL}/2$, respectively.) The generic form of these equations is

$$\frac{1}{\alpha(M_1)} = \frac{1}{\alpha(M_2)} + \frac{b}{2\pi}\ln\left(\frac{M_1}{M_2}\right), \tag{46}$$

where $M_{1,2}$ represent two mass scales in the theory, and b is given by

$$b = -\frac{11}{3}T(\text{gauge boson}) + \frac{2}{3}T(\text{Weyl fermion}) + \frac{1}{3}T(\text{complex scalar}). \tag{47}$$

The group-theoretic quantities T are defined by $\mathrm{Tr}(X^a X^b) = T(R)\delta^{ab}$ where the X^a's are the generators of the group in the representation R.

By matching up the running coupling constants at each symmetry breaking scale, one may relate the values of the fine-structure constants at M_W to the scales Λ_1 and Λ_2. The equation is

$$\frac{1}{\alpha_Y} - \frac{1}{3\alpha_3} - \frac{5}{3\alpha_2} = \frac{\hat{b}_3 - b_3 + 2\hat{b}_2 - 5b_2 - 3\hat{b}_{BL} + 3b_Y}{6\pi}\ln\left(\frac{\Lambda_2}{M_W}\right) + \frac{3\hat{b}_{BL} - \hat{b}_3 - 2\hat{b}_2}{6\pi}\ln\left(\frac{\Lambda_1}{M_W}\right), \tag{48}$$

where the $\hat{b}$ quantities refer to running between Λ_1 and Λ_2, while the b's without carets refer to running between Λ_2 and M_W. For three generations of fermions, these quantities are

$$\hat{b}_3 = -5, \quad \hat{b}_2 = -10/3 + n/2, \quad \hat{b}_{BL} = 6 + n/3, \tag{49}$$
$$b_3 = -7, \quad b_2 = -10/3 + n/2, \quad b_Y = 20/3 + n/2,$$

where n is the number of Φ multiplets. These numbers differ from those in the usual left-right symmetric model because of the effect of the exotic fermions k and D between Λ_1 and Λ_2. Note that we have assumed all of the degrees of freedom in the χ Higgs bosons become massive at Λ_1 and thus do not contribute to the running of the coupling constants in the region of interest.

Substituting these numbers into Eq. (48) yields

$$\frac{1}{\alpha_Y} - \frac{1}{3\alpha_3} - \frac{5}{3\alpha_2} = \left(\frac{14-n}{6\pi}\right) \ln\left(\frac{\Lambda_2}{M_W}\right) + \frac{89}{18\pi} \ln\left(\frac{\Lambda_1}{M_W}\right). \tag{50}$$

The central values of the allowed ranges for the fine-structure constants at M_W are [17]

$$\alpha_3 = 0.108, \quad \alpha_2 = 0.03322, \quad \text{and} \quad \alpha_Y = \frac{3}{5} \times 0.016\,887. \tag{51}$$

Interesting representative solutions of Eq. (50) with these inputs are

$$\Lambda_2 = M_W \quad \Longrightarrow \quad \Lambda_1 \sim 3 \times 10^{14} \text{ GeV}, \tag{52}$$

$$\Lambda_1 = \Lambda_2 \equiv \Lambda, \; n = 1 \quad \Longrightarrow \quad \Lambda \sim 4 \times 10^{10} \text{ GeV}. \tag{53}$$

These results show that Λ_2 can be anywhere between a few 100 and 10^{10} GeV, while Λ_1 ranges between about 10^{14} and 10^{10} GeV. Therefore, a low $SU(2)_R$ breaking scale, together with low masses for the k, N, and D fermions, is compatible with partial unification in this model. Note that the high scale Λ_1 is always comfortably below the Planck mass.

F. Possible low-energy phenomenological signatures

The most interesting prediction of our extended weak-isospin model is the existence of the class of exotic fermion we call k. These are charge $+1/2$ particles that transform as doublets under the unbroken remnant group $SU(2)'$. Since they receive mass at the second stage of symmetry breaking, they could well be as light as 100 GeV or so. Their experimental signature is quite striking: they emerge as integrally charged exotic hadrons of the $SU(2)'$ sector. Their properties are quite similar to those of liptons; the main difference is that they do not couple to $W_{L,R}$ bosons, unlike the liptons. Since k masses are expected to be much higher than the $SU(2)'$ confinement scale Λ', they will form nonrelativistic bound states. A simple calculation shows, in fact, that Λ' is given by

$$\Lambda' = M_W \left(\frac{\Lambda_1}{M_W}\right)^{(\hat{b}'_2 - \hat{b}_3)/b'_2} \left(\frac{\Lambda_2}{M_W}\right)^{(\hat{b}_3 - b_3 + b'_2 - \hat{b}'_2)/b'_2} \times \exp\left(\frac{2\pi}{b'_2 \alpha_3}\right). \tag{54}$$

The quantities $\hat{b}'_2$ and b'_2 refer to the running of the $SU(2)'$ fine-structure constant for $\Lambda_1 \leftrightarrow \Lambda_2$ and $\Lambda_2 \leftrightarrow \Lambda'$, respectively. They are given by

$$\hat{b}'_2 = -16/3 \quad \text{and} \quad b'_2 = -22/3. \tag{55}$$

The exotic fermions k contribute to this running between Λ_1 and Λ_2, whereas the running below Λ_2 is due entirely to the $SU(2)'$ gauge bosons. Substituting these numbers into Eq. (54) yields

$$\Lambda' = M_W \left(\frac{\Lambda_1}{M_W}\right)^{1/22} \exp\left(\frac{-3\pi}{11\alpha_3}\right). \tag{56}$$

Note that Λ' turns out not to depend on the intermediate scale Λ_2. Using $\Lambda_1 = 3 \times 10^{14}$ one finds that

$$\Lambda' \approx 110 \text{ MeV}, \tag{57}$$

which is of the order of the color scale. These exotic nonrelativistic bound states will decay via electromagnetic, Z and $SU(2)'$ interactions. We will not consider their phenomenology in detail here, because of its similarity to lipton physics (which has been studied elsewhere [5]). Although our model also has lipton states, they are expected to be extremely heavy and thus irrelevant for collider experiments.

Another interesting prediction of the model is that of $SU(2)_R$ gauge forces which are spontaneously broken by doublet Higgs bosons, rather than the more standard choice of triplets. Allied with this is the existence of the exotic fermions N and D which in general will mix with neutrinos and down quarks once the aforementioned Higgs doublets develop nonzero VEV's. These exotic fermions form half of the states required to institute a universal seesaw mechanism. It is possible that an extension of our model could be constructed which contains all of the fermions required for the universal seesaw mechanism.

III. CONCLUSION

It is important to analyze the standard model in a rigorous way, to see which features of it have a degree of permanence, and which may be enlarged without catastrophic consequences. We have argued in this article that both the color group $SU(3)_c$ and the weak hypercharge group $U(1)_Y$ are fragile — they can be replaced by $SU(5)_c$ and $SU(2)_R \otimes U(1)_{B-L}$, respectively without greatly disturbing the structure of the theory. The weak-isospin group $SU(2)_L$ is in a different category. It is robust because [weak isospin]^3 anomalies only cancel automatically when real representations are used.

However, a sensible and interesting extension of weak-isospin from SU(2) to SU(3) can be achieved provided one also introduces a spontaneously broken leptonic color group. This led us to construct an extended weak-isospin model which had as its starting point the quark-lepton symmetric model of Foot and Lew [5] rather than the standard model. In this way a theory "twice removed" from the SM emerged.

It turned out that elegance demanded the introduction also of left-right symmetry, leading to the gauge group $SU(3)_\ell \otimes SU(3)_q \otimes SU(3)_L \otimes SU(3)_R$. As well as providing an aesthetically pleasing spectrum of quark and lepton quantum numbers, this gauge group with the two imposed discrete symmetries implied a partial unification of gauge coupling constants. A renormalization-group analysis of the running coupling constants then showed that a low $SU(2)_R$ breaking scale was possible, and that the extended weak-isospin breaking scale had to be in the range 10^{10} to 10^{14} GeV.

A number of testable low-energy phenomenological consequences are possible. The most striking prediction is that of charge $+1/2$ fermions confined into integrally charged nonrelativistic bound states by an unbroken $SU(2)'$ gauge force. Also, $SU(2)_R$ is broken in an unconventional way through Higgs doublets rather than the more familiar triplets. This breaking in general also leads to neutrino and down-quark mixing with the exotic fermions N and D.

The quark-lepton discrete symmetry and extended weak isospin lead to a number of interesting fermion mass and mixing-angle relations. An important open question is whether radiative corrections in the model as is, or in some development of it, can lead to correct mass and mixing-angle predictions.

ACKNOWLEDGMENTS

This research was supported by the Australian Research Council. R.R.V. would like to thank Robert Foot, Xiao-Gang He, and Henry Lew for discussions.

[1] For a review, see R. N. Mohapatra, *Unification and Supersymmetry* (Springer-Verlag, New York, 1986).

[2] R. Foot and O. F. Hernández, Phys. Rev. D **41**, 2283 (1990); **42**, 948(E) (1990); for further phenomenological analyses, see S. L. Glashow and U. Sarid, Phys. Lett. B **246**, 188 (1990); R. Foot, O. F. Hernández, and T. G. Rizzo, *ibid.* **246**, 183 (1990); **261**, 153 (1991); E. D. Carlson, L. J. Hall, U. Sarid, and J. W. Burton, Phys. Rev. D **44**, 1555 (1991); T. G. Rizzo, University of Wisconsin at Madison Report No. MAD/PH/626 (unpublished); O. F. Hernández, Phys. Rev. D **44**, 1997 (1991).

[3] J. C. Pati and A. Salam, Phys. Rev. D **8**, 1240 (1973); **10**, 275 (1974).

[4] For a review, see Mohapatra [1].

[5] R. Foot and H. Lew, Phys. Rev. D **41**, 3502 (1990); Nuovo Cimento A **104**, 167 (1991); for a phenomenological analysis, see R. Foot, H. Lew, and R. R. Volkas, Phys. Rev. D **44**, 1531 (1991).

[6] A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1988); **60**, 1813 (1988); S. Rajpoot, Phys. Lett. B **191**, 122 (1987); see also R. N. Mohapatra, *ibid.* **201**, 517 (1988).

[7] It is possible to extend $SU(2)_L$ in an anomaly-free manner using $Sp(2N)$. This idea has been used (see references below) to unify weak isospin and horizontal symmetry. Since our motivation for extending weak isospin is different, we will not pursue this possibility here. It is also possible to use $Sp(2N)$ as an extension of weak isospin independent of any unification with horizontal symmetry, but to the best of our knowledge such models are not as elegant as those which also incorporate horizontal symmetry. One of us (R.R.V.) would like to thank R. Foot for some correspondence on this point. The relevant references are T. K. Kuo and N. Nakagawa, Phys. Rev. D **30**, 2011 (1984); Nucl. Phys. **B250**, 641 (1985); A. Bagneid, T. K. Kuo, and N. Nakagawa, Int. J. Mod. Phys. A **2**, 1351 (1987); V. Barger *et al.*, *ibid.* **2**, 1327 (1987); T. K. Kuo, U. Mahanta, and G. T. Park, Phys. Lett. B **248**, 119 (1990).

[8] For a recent study, see K. T. Mahanthappa and P. K. Mohapatra, Phys. Rev. D **42**, 1732 (1990).

[9] For instance, in a supersymmetric theory the exotic fermions which cancel the quark and lepton anomaly can be the superpartners of Higgs bosons. See X.-G. He, G. C. Joshi, B. H. J. McKellar, and R. R. Volkas, Phys. Lett. B **222**, 86 (1989).

[10] It is possible to interpret various different forms of quark-lepton symmetry as nonstandard C, P, and T symmetries. In this paper we will concentrate on the nonstandard C version. For a discussion of this issue, see R. Foot, H. Lew, and R. R. Volkas, University of Melbourne Report No. UM-P-91/46 and University of Southampton Report No. SHEP-90/91-25 (unpublished).

[11] The phenomenology of liptons is similar though not identical to the phenomenology of "quirks" in the SU(5)-color model. For an account of the latter, see Carlson *et al.* [2].

[12] However, recent work has shown that radiative corrections in one version of a quark-lepton-symmetric model with left-right symmetry are sufficient to evade the quark-lepton mass relations. See R. Foot and H. Lew, Purdue University Report No. PURD-TH-91-12 (unpublished).

[13] The gauge group and fermion spectrum of our model were also independently created earlier by R. Foot and H. Lew in unpublished work. The gist of this idea was previously mentioned in Footnote 6 of Foot, Lew, and Volkas [5].

[14] As in the basic quark-lepton-symmetric model, various versions of quark-lepton symmetry may be employed here (see [12]). We concentrate on one particular version in this article.

[15] The linear combination $C_L + C_R - 3B'$ is free of all gauge anomalies, and can therefore be made a local symmetry. Note that it is compatible with both quark-lepton and left-right symmetry. If this group is gauged, then its gauge boson will acquire a large mass from the first stage of symmetry breaking. We will not comment on this possible gauge group any further because its existence is irrelevant for physics below $\langle \chi^N_{L,R} \rangle$.

[16] For another partial unification model involving quark-lepton symmetry, see R. Foot, H. Lew, and R. R. Volkas, Phys. Rev. D **44**, 859 (1991).

[17] U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. B **260**, 447 (1991).