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Charge quantization in the standard model with three generations of fermions

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We analyze the problem of charge dequantization in the standard model (SM) with three generations. The extensions of the SM with more than one Higgs doublet and/or at least one right-handed neutrino with a Majorana mass term uniquely fix the fermion electric charge. Nothing but properties of the SM Lagrangian are used here to find the weak hypercharges of particles. The mixed gauge-gravitational anomaly cancellation takes place automatically.

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I. INTRODUCTION

One of the most important problems in elementary particle physics is to understand electric-charge quantization. Several attempts to explain this problem were made over the years. The first was made by Klein in the context of Kaluza-Klein theories [1]. The second interesting proposal, due to Dirac [2], postulated the existence of magnetic monopoles. Considering the motion of a charged particle in the field of a magnetic monopole, Dirac found that quantum consistency implies electric-charge quantization. However, up to now, magnetic monopoles have not been found. Another attempt to understand charge quantization was based on the assumption that weak, electromagnetic, and strong interactions are described by a non-Abelian gauge group G which contains the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ standard-model (SM) symmetry [3]. The electric-charge operator is given by a linear combination of diagonal operators of G, which remains unbroken after spontaneous symmetry breaking of G to the electromagnetic symmetry U(1). Generators of a non-Abelian group satisfy nonlinear commutation relations, so the ratios of electric charges in these theories are rational numbers. However, in all the above-mentioned approaches, it is generally impossible to understand why only the sequence of electric charges $(-1,-\frac{1}{3},0,\frac{2}{3})$ is observed. For a long time the presence of the U(1) symmetry group, with a continuous hypercharge Y, was considered to be the reason why the SM does not predict charge quantization [4]. In Ref. [5] the authors have made the observation that using the conditions of vanishing of the (i) triangle chiral gauge anomalies [6], (ii) mixed chiral-gauge–gravitational anomaly [7], and (iii) global SU(2) chiral gauge anomaly [8], it is possible to find the minimal set of particles in $SU(3)\otimes SU(2)\otimes U(1)$ and their hypercharges. After this observation a dozen or so papers have appeared [9–20] where different approaches to electric-charge quantization were presented, mostly for one quark and lepton family. In Refs. [21,22] the authors observed that within the framework of the SM with three generations of fermions there is no charge quantization, even if for the one-family version the electric charge is quantized. This is easy to understand, because in the three-generation SM there is an additional "hidden" U(1) symmetry, connected with lepton-number conservation. The main purpose of our paper is to find the minimum amount of "new physics" needed to explain electric-charge quantization. We consider the SM with more than one Higgs doublet and/or at least one right-handed neutrino (not necessarily one for each family) with a Majorana-mass term. All these small extensions of the SM are generally sufficient to fix uniquely the fermion electric charge. We prefer the "Lagrangian" approach to the charge-quantization problem. This means that only properties of a given Lagrangian (on the classical and quantum level) are used to find the fermion charges. In the next section we discuss various approaches to the one-family charge-quantization problem in the Lagrangian analysis. In Sec. III the extensions of the SM with more than one Higgs doublet and/or Majorana neutrino are considered. Section IV contains some concluding remarks.

II. CHARGE QUANTIZATION FOR ONE GENERATION OF QUARKS AND LEPTONS

In a Lagrangian field theory we would like, using only properties of the Lagrangian, to explain the charges of all particles present. Another problem is how we choose the Lagrangian. In the SM this question is equivalent to (i) why is the gauge symmetry given by the $SU(3)\otimes SU(2) \otimes U(1)$ group, (ii) what determines the fact that the left-handed (right-handed) quarks and leptons are isodoublets (singlets) of SU(2), and (iii) why are the particle masses given by the Higgs mechanism and the Yukawa term for fermions? We will not discuss these questions here. Instead, we will focus on the problem of determining the weak hypercharges Y of the quarks and leptons by using constraints from the Lagrangian. If, using the properties of the Lagrangian on the classical level (tree approximation) and in higher order (triangle-anomaly cancellation) we are able to find all the particle hypercharges, then we say that the theory "explains" charge quantization. This property is possessed by the minimal standard model

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(MSM) without the right-handed neutrino and one Higgs doublet in the case of one fermion family [5,9,12,17]. Let us describe the typical way of thinking in the "Lagrangian analysis."

A. Definition of charge operator

After spontaneous symmetry breaking of $SU(2)_L \otimes U(1)_Y$ to $U(1)_{em}$, the massless photon field A, which is a combination of neutral weak-isospin (W_3) and weak-hypercharge (Y) fields ($A^\mu = \sin\theta_W W_3^\mu + \cos\theta_W Y^\mu$), couples to the proper combination of currents J_3^μ and J_Y^μ. This combination,

$$gJ_3^\mu \sin\theta_W + g' J_Y^\mu \cos\theta_W = eJ_{em}^\mu \ , \tag{2.1}$$

is considered as the electromagnetic current eJ_{em}^μ, so the charge operator is given by

$$Q = aT_3 + b\frac{Y}{2} \ , \tag{2.2a}$$

where

$$a = \frac{g\sin\theta_W}{e}, \quad b = \frac{g'\cos\theta_W}{e}. \tag{2.2b}$$

In order to break symmetry spontaneously the Higgs field, which is a color singlet, an SU(2) doublet, and has weak hypercharge Y_H, is introduced by

$$\phi \sim (1,2,Y_H) \ . \tag{2.3}$$

It has a nonzero vacuum expectation value

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\ v\end{bmatrix} .$$

As the charge operator (2.2b) should annihilate the vacuum $Q\langle\phi\rangle = 0$, the Higgs hypercharge Y_H should be equal to a/b ($Y_H = a/b$). We can use the freedom in assigning the scale of the electric charge Q to set $a = 1$. Then

$$Q = T_3 + \frac{1}{2}\frac{Y}{Y_H} \ , \tag{2.4a}$$

and the freedom in hypercharge normalization allows us to set $Y_H = 1$, so that

$$Q = T_3 + \frac{Y}{2} \ . \tag{2.4b}$$

B. Constraints from Lagrangian in tree approximation

In agreement with our previous assumption, the quark and lepton fields form the next representation of the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$:

$$L_L = (1,2,Y_l), \quad e_R = (1,1,Y_e) \ , \tag{2.5a}$$

$$Q_L = (3,2,Y_q), \quad u_R = (3,1,Y_u), \quad d_R = (3,1,Y_d) \ . \tag{2.5b}$$

The quark and charged-lepton masses are introduced by spontaneous symmetry breaking ($\langle\phi\rangle \neq 0$). The Yukawa Lagrangian has the form

$$\mathcal{L}_Y = -h_e \bar{L}_L \phi e_R - h_d \bar{Q}_L \phi d_R - h_u \bar{Q}_L \tilde{\phi} u_R + \text{H.c.}, \tag{2.6}$$

where $\tilde{\phi} = i\tau^2\phi^*$. The U(1) invariance of the Yukawa Lagrangian implies

$$Y_e = Y_l - 1, \quad Y_d = Y_q - 1, \quad Y_u = Y_q + 1 \ . \tag{2.7}$$

So only two hypercharges are independent. Let us take them as Y_1 and Y_q. Various parts of the SM Lagrangian were used to find relations among hypercharges. The law of charge conservation for massive fermions was applied [15,18], to obtain the charge equality of the left-handed and right-handed components of the chiral fermions [e.g., $Q(e_L) = Q(e_R)$]. The same relation can be obtained from the requirement of the vectorlike nature of the electromagnetic current [19,20,22]. Unfortunately, all these properties of the SM Lagrangian give only the relation (2.7). On the tree level there are no other relations among the hypercharges in (2.5). To find new ones we have to go to higher orders.

C. Constraints from anomaly cancellation

In constructing renormalizable gauge theories of weak interactions one must ensure that the triangle anomalies are canceled [6]. This requirement gives three relations connecting the weak hypercharges in the SM [5,12,17,18]:

$$\begin{aligned}
&\text{(i)} \ [U(1)_Y]^3 \Longrightarrow 6Y_q^3 + 2Y_l^3 - 3Y_u^3 - 3Y_d^3 - Y_e^3 = 0 \ ,\\
&\text{(ii)} \ [SU(3)_c]^2 U(1)_Y \Longrightarrow 2Y_q - Y_u - Y_d = 0 \ ,\\
&\text{(iii)} \ [SU(2)_L]^2 U(1)_Y \Longrightarrow 3Y_q + Y_l = 0 \ .
\end{aligned} \tag{2.8}$$

Relation (ii) follows from (2.7) and only (i) and (iii) give independent equations:

$$\text{(iii)} \Longrightarrow Y_q = -\tfrac{1}{3}Y_l \ , \tag{2.9}$$

$$\begin{aligned}
\text{(i), (2.7), and (2.9)} &\Longrightarrow (Y_l+1)^3 = 0\\
&\Longrightarrow Y_l = -1 \ .
\end{aligned} \tag{2.10}$$

The quark and lepton hypercharges are determined uniquely,

$$Y_l = -1, \quad Y_e = -2, \quad Y_q = \tfrac{1}{3}, \quad Y_u = \tfrac{4}{3}, \quad Y_d = -\tfrac{2}{3} \ , \tag{2.11a}$$

and from (2.4b) the charges are

$$Q_\nu = 0, \quad Q_e = -1, \quad Q_u = \tfrac{2}{3}, \quad Q_d = -\tfrac{1}{3} \ . \tag{2.11b}$$

We see that the SM predicts charge quantization in the sense explained above. The fermion charges given by (2.11) result from the SM Lagrangian—the mixed chiral-gauge—gravitational anomaly cancellation is not necessary and follows from (2.7) and (2.8). To explain the results (2.11) further let us ask the question: Is it possible to build a renormalizable theory based on the SM gauge group and the SM fermion representation (2.5) in which the fermion charges are not given by (2.11)? The answer is, of course, yes but the obtained theory will have unusual features. If we require renormalizability, Eqs. (2.8) must be satisfied. Choosing Y_l and Y_e to be two independent parameters, for the other three we get [23]

$$Y_q=-\tfrac{1}{3}Y_l\ ,$$

$$Y_u=-\frac{1}{3}Y_l\left\{1+\left[\frac{3}{2}\left[\frac{Y_e}{Y_l}\right]^3-3\right]^{1/2}\right\}\ , \qquad (2.12)$$

and

$$Y_d=-\frac{1}{3}Y_l\left\{1-\left[\frac{3}{2}\left[\frac{Y_e}{Y_l}\right]^3-3\right]^{1/2}\right\}\ .$$

Choosing definition (2.4b) of the charge operator [in the case of broken $U(1)_{em}$ symmetry the definition of the charge operator is not obvious], we have

$$Q_\nu=\tfrac{1}{2}(1+Y_l)\ ,$$

$$\Delta Q_e\equiv Q_{e_L}-Q_{e_R}=\tfrac{1}{2}(Y_l-Y_e-1)\ , \qquad (2.13)$$

and

$$\Delta Q_u=-\Delta Q_d=\frac{1}{6}\left\{3+Y_l\left[\frac{2}{3}\left[\frac{Y_e}{Y_l}\right]^3-3\right]^{1/2}\right\}\ .$$

If $Y_l\neq-1$ then the neutrino can have a tiny charge. If $\Delta Q\neq 0$ then the electromagnetic current has an axial part and the charge is not conserved for massive fermions. The condition

$$\Delta Q_e=\Delta Q_u=\Delta Q_d=0 \qquad (2.14)$$

is satisfied only for the SM values of $Y_l=-1$, $Y_e=-2$. It is impossible to build a renormalizable theory based on the $SU(3)\otimes SU(2)\otimes U(1)$ gauge group with the standard fermion representation (2.5) and other than the fermion charge assignment (2.11) with the photon coupled only to the vector electromagnetic current, which gives charge conservation for massive fermions. A particular model, where hypercharges satisfy relation (2.12), was considered in Refs. [19,20,24]. The authors assume that the mixed gauge-gravitational anomaly is also canceled,

$$\text{(iv)}\ \ (\text{gravity})^2U(1)\Longrightarrow 3(2Y_q-Y_u-Y_d)+2Y_l-Y_e=0\ . \qquad (2.15)$$

Taking the neutrino charge $Q_\nu=\varepsilon/2$ as a free parameter, from (2.12) and (2.15) we get the hypercharges

$$Y_l=-1+\varepsilon,\quad Y_e=-2(1-\varepsilon)\ , \qquad (2.16a)$$

$$Y_q=\tfrac{1}{3}(1-\varepsilon),\quad Y_u=\tfrac{4}{3}(1-\varepsilon),\quad Y_d=-\tfrac{2}{3}(1-\varepsilon)\ , \qquad (2.16b)$$

the same as in Ref. [19].

The simplest generalization of the SM consists of introducing a right-handed neutrino $\nu_R\sim(1,1,Y_\nu)$. From the $U(1)_Y$ invariance of the neutrino Yukawa Lagrangian

$$-h_\nu\bar{L}_L\tilde{\phi}\nu_R+\text{H.c.}, \qquad (2.17)$$

one has

$$Y_\nu=1+Y_l\ . \qquad (2.18)$$

But, unfortunately, relation (i) in (2.8) changes,

$$\text{(i)}\rightarrow\text{(i}'\text{)}:\ \ 3(2Y_q^3-Y_u^3-Y_d^3)+(2Y_l^3-Y_e^3-Y_\nu^3)=0\ ,$$

and is now equivalent to formula (iii). So we have five equations for six hypercharges. Taking $Q_\nu=\eta$ the other charges are given by [17]

$$Q_e=-1+\eta,\quad Q_u=\tfrac{2}{3}-\tfrac{1}{3}\eta,\quad Q_d=-\tfrac{1}{3}-\tfrac{1}{3}\eta\ . \qquad (2.19)$$

Now, we have a renormalizable theory with conserved charge and massless photons. There is no axial part in the electromagnetic current, but, in spite of this, the charge given by (2.19) is not quantized. The reason for this is the existence of a new local anomaly-free gauge symmetry $U(1)_{Y'}$, where $Y'=B-L$. In this case the hypercharge Y is not uniquely defined, because any combination

$$Y\rightarrow Y\cos\theta+(B+L)\sin\theta \qquad (2.20)$$

can be considered as a new hypercharge [12,17]. If we introduce into the SM Lagrangian the Majorana-mass term $\nu_R^TC^{-1}M\nu_R$, the $B-L$ symmetry is broken and charge is quantized [12,17].

III. CHARGE QUANTIZATION FOR THREE GENERATIONS OF FERMIONS

In the previous section we have summarized various approaches to the charge-quantization problem in one-family models. In reality three generations of quarks and leptons exist and they are mixed. The total SM Lagrangian is not a simple sum of the Lagrangians for each family, and there are no circumstances for which to supplement the one-generation analysis with the rule that the second and third generation are to be viewed as copies of the first generation. We have to consider the full three-generation Lagrangian in order to prove quantization. As before the quarks and leptons form the $SU(3)\otimes SU(2)\otimes U(1)$ representations

$$L_L=(L_{L_i})\sim(1,2,Y_{l_i}),\quad e_R=(e_{R_i})\sim(1,1,Y_{e_i})\ , \qquad (3.1)$$

$$Q_L=(Q_{L_i})\sim(3,2,Y_{q_i}),\quad u_R=(u_{R_i})\sim(3,1,Y_{u_i})\ ,$$

$$d_R=(d_{R_i})\sim(3,1,Y_{d_i})\ ,$$

where $i=1,2,3$. To find the fermion charges we have to determine the fifteen hypercharges ($Y_{q_i},Y_{l_i},Y_{e_i},Y_{d_i},Y_{u_i}$; $i=1,2,3$). The first relations between them result from the requirement of U(1) invariance of the Yukawa Lagrangian

$$\mathcal{L}_Y=-(\bar{L}_R\phi\Gamma_e e_R+\bar{Q}_L\phi\Gamma_d d_R+\bar{Q}_L\tilde{\phi}\Gamma_u u_R)+\text{H.c.}, \qquad (3.2)$$

where Γ_e,Γ_d, and Γ_u are 3×3 matrices. Not all relations among lepton hypercharges that follow from (3.2) are really operative. The reason is that only some combinations of weak particle states form the physical massive eigenstates. To obtain the mass eigenfields $\hat{u},\hat{d},\hat{e}$, we perform a unitary transformation

$$u_{L,R}=U_{L,R}^u\hat{u}_{L,R},\quad d_{L,R}=U_{L,R}^d\hat{d}_{L,R},\quad e_{L,R}=U_{L,R}^e\hat{e}_{L,R}\ . \qquad (3.3)$$

Then, in the unitary gauge, without the would-be Goldstone bosons, the Yukawa Lagrangian is given by

$$\left[\phi=\frac{1}{\sqrt{2}}\begin{bmatrix}\phi^+\\ v+H+i\xi\end{bmatrix}\right]$$

$$\mathcal{L}_Y\to-\bar{\hat{e}}_L\left[m_e+\frac{m_e}{v}H\right]\hat{e}_R-\bar{\hat{u}}_L\left[m_u+\frac{m_v}{v}H\right]\hat{u}_R+\bar{\hat{d}}_L\left[m_d+\frac{m_d}{v}H\right]\hat{d}_R+\text{H.c.},\tag{3.4}$$

where

$$m_f\equiv\begin{bmatrix}m_{f_1}&0&0\\0&m_{f_2}&0\\0&0&m_{f_3}\end{bmatrix}=U_L^{f\dagger}\Gamma_f U_R^f\frac{v}{\sqrt{2}},\quad f=e,u,d\ .$$

At the same time the charged-current interaction $\mathcal{L}_{cc}$ is given by

$$\mathcal{L}_{cc}=-\frac{g}{\sqrt{2}}[\bar{\hat{\nu}}_L\gamma_\mu\hat{e}_L+\bar{\hat{u}}_L\gamma_\mu(U_L^{u\dagger}U_L^d)\hat{d}_L]W_\mu^++\text{H.c.},\tag{3.5}$$

where, for all massless neutrinos, the physical neutrino states $\hat{\nu}_L$ can be taken as

$$\hat{\nu}_L=U_L^e\nu_L\ .\tag{3.6}$$

In the quark sector the information about off-diagonal elements of the quark mass matrices $M_u=(v/\sqrt{2})\Gamma_u$ and $M_d=(v/\sqrt{2})\Gamma_d$ disappears in the Higgs sector but is present in the charged-current interaction because of the Kobayashi-Maskawa matrix $U_R^{\dagger u}U_L^d$. The information about off-diagonal elements of the charged-lepton mass matrix $M_e=(v/\sqrt{2})\Gamma_e$ is lost, and the three lepton numbers L_e, L_μ, and L_τ are conserved separately. This means that only those relations among hypercharges which result from the $U(1)_Y$ invariance of the Yukawa Lagrangian

$$\mathcal{L}_Y=-\frac{\sqrt{2}}{v}(\bar{L}_L\phi m_e e_R+\bar{Q}_L\phi M_d d_R+\bar{Q}_L\tilde{\phi}M_u u_R)+\text{H.c.}\tag{3.7}$$

are operative. As the charged-lepton masses are not equal to zero, we have three relations for lepton hypercharges,

$$-Y_{l_i}+1+Y_{e_i}=0,\quad i=1,2,3\ .\tag{3.8}$$

If all the elements of the quark mass matrices are nonzero, $(M_d)_{ij}\neq0$ and $(M_u)_{ij}\neq0$, then we have

$$-Y_{q_i}+1+Y_{d_j}=0,\quad -Y_{q_i}-1+Y_{u_j}=0\ ,\quad i,j=1,2,3\ .\tag{3.9}$$

From these relations we see that the hypercharges for different quark generations are equal.

$$Y_{q_1}=Y_{q_2}=Y_{q_3}\equiv Y_q,\quad Y_{d_1}=Y_{d_2}=Y_{d_3}\equiv Y_d\ ,$$
$$Y_{u_1}=Y_{u_2}=Y_{u_3}\equiv Y_u\ ,\tag{3.10}$$

and

$$Y_d=Y_q-1,\quad Y_u=Y_q+1\ .$$

We can prove Eqs. (3.10) even if not all elements of the quark mass matrices M_u and M_d are different from zero. It is sufficient that five elements (at least one in each row and column) do not vanish. For example, the Fritzsch mass matrix [25]

$$M_{\text{Fritzsch}}=\begin{bmatrix}0&a&0\\a&0&b\\0&b&c\end{bmatrix}\ ,\tag{3.11}$$

satisfies this requirement. We would like to stress once more that to find relations between the hypercharges we have to use the Lagrangian (3.7) with the physical fields and physical symmetries, and not the one given in (3.2). The transformation (3.6) (for massless neutrino fields) reveals the physical Lagrangian symmetry. As in the one-family case there are no more relations between the hypercharges on the tree level. The next are given by the anomaly cancellations which, in the case of three generations, take the form

$$\text{(i)}\quad\sum_{i=1}^{3}(6Y_{q_i}^3+2Y_{l_i}^3-3Y_{u_i}^3-3Y_{d_i}^3-Y_{e_i}^3)=0\ ,$$
$$\text{(ii)}\quad\sum_{i=1}^{3}(2Y_{g_i}-Y_{u_i}-Y_{d_i})=0\ ,\tag{3.12}$$

and

$$\text{(iii)}\quad\sum_{i=1}^{3}(3Y_{q_i}+Y_{l_i})=0\ .$$

Relation (ii) results from (3.10), so there are two equations for four hypercharges (e.g., Y_q, Y_{l_1}, Y_{l_2}, Y_{l_3}), and they are not fixed. In the SM with three fermion generations, charge is not quantized. This effect of dequantization has been observed before [21,22]. It is caused by the existence of local anomaly-free $U(1)_{Y'}$ symmetries, where $Y'\equiv L_e-L_\mu$, L_e-L_τ or $L_\mu-L_\tau$ and any combination $Y\cos\theta+Y'\sin\theta$ can be considered as a new hypercharge. What is now the minimal extension of the SM needed to get charge quantization? We should generalize the SM to break the hidden $U(1)_{Y'}$ symmetries without introducing other ones. We will discuss two possibilities: (i) Conservation of the separate lepton numbers L_e, L_μ, and L_τ is broken; $L=L_e+L_\mu+L_\tau$ is still conserved but $B-L$ is not a hidden anomaly-free symmetry. (ii) Conservation of total lepton number L is broken.

The simplest way to break L_e, L_μ, and L_τ while conserving total lepton number L, is to introduce right-handed neutrinos with a Dirac mass term. But, unfortunately, then $B-L$ is a hidden anomaly-free symmetry (see, however, the comment at the end of this section). Now, we would like to show that introducing more than one Higgs doublet to the SM is a way to realize (i). Let us assume that there are two Higgs doublets

$$\phi_1=\begin{bmatrix}\phi_1^+\\ \phi_1^0\end{bmatrix},\quad \phi_2=\begin{bmatrix}\phi_2^+\\ \phi_2^0\end{bmatrix}, \tag{3.13}$$

which spontaneously break the $SU(2)_L\otimes U(1)_Y$ symmetry because

$$\langle\phi_1\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}0\\ v_1\end{bmatrix},\quad \langle\phi_2\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}0\\ v_2\end{bmatrix}. \tag{3.14}$$

The Yukawa Lagrangian is now

$$\mathcal{L}_Y=-\sum_{\alpha=1}^{2}(\bar{L}_L\phi_\alpha\Gamma_e^\alpha e_R+\bar{Q}_L\phi_\alpha\Gamma_d^\alpha d_R+\bar{Q}_L\tilde{\phi}_\alpha\Gamma_u^\alpha u_R)+\text{H.c.} \tag{3.15}$$

The mass matrices

$$M^f=\frac{1}{\sqrt{2}}(v_1\Gamma_f^1+v_2\Gamma_f^2),\quad f=e,u,d, \tag{3.16}$$

are diagonalized by the transformation (3.3). The particle content of (3.13) is the following: three combinations of fields form the would-be Goldstone boson for $W^\pm$ and Z^0; the other five give one charged physical Higgs $H^\pm$ and three neutral H_1^0, H_2^0, and H_3^0 [26]. For the same reason as in the SM, the information about the off-diagonal elements of the charged-lepton mass matrix M^e does not appear in the charged-current interaction

$$\mathcal{L}_{cc}^e=-\frac{g}{\sqrt{2}}\bar{\hat{\nu}}_L\gamma^\mu\hat{e}_L W_\mu^+ +\text{H.c.}, \tag{3.17}$$

where $\bar{\hat{\nu}}_L=\bar{\nu}_L U_L^e$. The one charged $H^\pm$ and the three neutral H_i^0 $(i=1,2,3)$ Higgs particles interact with leptons:

$$\mathcal{L}_{\text{Higgs-lepton}}=-\bar{\hat{\nu}}_L A_R^e\hat{e}_R H^+ -\sum_{i=1}^{3}\bar{\hat{e}}_L B_i^e\hat{e}_R H_i^0+\text{H.c.}, \tag{3.18}$$

where

$$A_R^e=U_L^{e\dagger}(\Gamma_e^1 a_1+\Gamma_e^2 a_2)U_R^e, \tag{3.19}$$

$$B_i^e=U_L^{e\dagger}(\Gamma_e^1 b_1^{(i)}+\Gamma_e^2 b_2^{(i)}), \tag{3.20}$$

and

$$a_1=b_1^{(1)}=-iv_2^*/v,\quad a_2=b_2^{(1)}=iv_1^*/v_1,$$

$$b_1^{(3)}=-b_2^{(2)*}=v_1/v,\quad b_2^{(3)}=b_1^{(2)*}=v_2/v,$$

where $v=(|v_1|^2+|v_2|^2)^{1/2}$. If $\Gamma_e^1\neq 0$ and $\Gamma_e^2\neq 0$, then their various linear combinations are generally not diagonalized by the same matrices U_R^e and U_L^e. In general the matrices A_R^e, B_1^e, and B_2^e are not diagonal. The lepton numbers L_e, L_μ, and L_τ are not conserved separately. The charged $H^\pm$ and the neutral H_i^0 Higgs interactions "remember" the off-diagonal elements of the lepton mass matrix M^l. Now, from the $U(1)_Y$ invariance of the Yukawa interaction (3.2), we have new relations among the lepton hypercharges,

$$-Y_{l_i}+1+Y_{e_j}=0;\quad i,j=1,2,3; \tag{3.21}$$

similarly, for the quark sector, one has

$$Y_{l_1}=Y_{l_2}=Y_{l_3}\equiv Y_l,\quad Y_{e_1}=Y_{e_2}=Y_{e_3}\equiv Y_e,\quad Y_e=Y_l-1. \tag{3.22}$$

The fermion hypercharges are fixed by the anomaly constraints (3.12) in the same way as in the one-generation case, and the charge is quantized.

Unfortunately, in models with more than one Higgs doublet lepton-number nonconservation is connected with flavor-changing neutral currents (FCNC) (matrices B_1^e and B_2^3 and similar matrices in the quark sector are not diagonal). As we well know FCNC are highly suppressed, relative to charged-current processes, so it would be desirable to eliminate them at the tree level. This can be done by imposing the discrete symmetry [27]

$$D:\ \begin{cases}\phi_2\to-\phi_2,\\ d_R\to-d_R,\end{cases} \tag{3.23}$$

which means in practice that only one Higgs doublet couples with each generation ($\Gamma_e^2=\Gamma_u^2=0$, $\Gamma_d^1=0$). In this case the lepton mass matrix M_e in (3.16) and the four matrices A_R^e, B_i^e $(i=1,2,3)$ are diagonalized by the same transformation [for quarks their mass matrices M^q and the neutral quark transition matrices B_i^q $(i=1,2,3)$ are commonly diagonalized]. This means, however, that we "lose" information about nondiagonal Γ_e^q matrix elements, and the charge dequantizes. To get this quantization of quark and lepton charges we have to assume the existence of small FCNC at the tree level.

The second above-mentioned possibility for getting quantization of the electric charge consists of introducing Majorana neutrinos. There are two simplest methods: (i) we can add a Higgs triplet, or (ii) there is only a Higgs doublet but we add at least one right-handed-neutrino singlet. The second possibility seems to be more attractive; the see-saw mechanism and radiative corrections explain why the neutrino masses are so small [28]. It is a known fact that Majorana neutrinos "save" the quantization of electric charges [9,12]. Here we would like to show explicitly that one right-handed-neutrino singlet with Majorana-mass term is able to recover charge quantization for three fermion generations. Having the right-handed-neutrino singlet $\nu_R\sim(1,1,Y_\nu)$, we can add the Dirac mass term

$$\mathcal{L}_D=-\bar{L}_L\tilde{\phi}\Gamma_\nu\nu_R+\text{H.c.}, \tag{3.24}$$

where Γ_ν is a 3×1 matrix, and the Majorana-mass term

$$\mathcal{L}_M=\tfrac{1}{2}\nu_R^T C^{-1}M_R\nu_R+\text{H.c.} \tag{3.25}$$

to the SM Lagrangian. After spontaneous symmetry breaking the neutrino mass Lagrangian is given by

$$\mathcal{L}_{\text{mass}}^\nu=-\bar{\nu}_L M_D^\nu\nu_R+\tfrac{1}{2}\nu_R^T C^{-1}M_R\nu_R+\text{H.c.}=\tfrac{1}{2}\omega_R^T C^{-1}M^\nu\omega_R+\text{H.c.}, \tag{3.26}$$

where

$$\omega_R=\begin{bmatrix} C\bar{\nu}_L^T \\ \nu_R \end{bmatrix},\quad M^\nu=\begin{bmatrix} 0 & M_D^\nu \\ M_D^{\nu T} & M_R \end{bmatrix},$$

and $M_D^\nu=\langle\phi\rangle^*\Gamma_\nu$. The symmetric 4×4 mass matrix M^ν can be diagonalized by the transformation

$$\omega_R=U^*\hat{\omega}_R,\quad \omega_R^T=\hat{\omega}_R^T U^\dagger , \tag{3.27}$$

and then

$$\mathcal{L}_{\text{mass}}^\nu=-\tfrac{1}{2}\hat{\omega}_R^T C^{-1}(U^\dagger M^\nu U^*)\hat{\omega}_R+\text{H.c.} \\ =-\tfrac{1}{2}\sum_{i=1}^{4} m_i\bar{N}_i N_i , \tag{3.28}$$

where $N_i=\hat{\omega}_{R_i}+C\bar{\hat{\omega}}_{R_i}^T$ is the Majorana-neutrino field with mass m_i. To find the lepton charged-current interaction let us parametrize the matrix U in the following way:

$$U=\begin{bmatrix} U_L \\ U_R^* \end{bmatrix} . \tag{3.29}$$

Then $\nu_L=U_L\hat{\omega}_{RC}=U_L P_L N$, and we have

$$\mathcal{L}_{\text{cc}}^\nu=-\frac{g}{\sqrt{2}}\bar{\nu}_L\gamma^\nu e_L W_\mu^+ +\text{H.c.} \\ =-\frac{g}{\sqrt{2}}\bar{N}P_R\gamma^\mu(U_L^\dagger U_L^e)\hat{e}_L W_\mu^+ +\text{H.c.}, \tag{3.30}$$

where $U_L^\dagger U_L^e$ is the 4×3 lepton mixing matrix. From (3.30) we see that, in the lepton charged-current interaction, there is mixing among mass eigenfields $(N,\hat{e}_L)$. This means that the off-diagonal elements of the charged-lepton mass matrix in (3.2) are "measurable" quantities, and the $U(1)_Y$ invariance gives the equality (3.22) among lepton hypercharges. As there is one more right-handed particle in the theory (ν_R), the anomaly-cancellation equation (i) in (3.12) will change:

$$\text{(i)}\rightarrow\text{(i}''\text{)}:\ \sum_i(6Y_{q_i}^3+2Y_{l_i}^2-3Y_{u_i}^3-3Y_{d_i}^3-Y_{e_i}^3)-Y_\nu^3=0 . \tag{3.31}$$

The $U(1)_Y$ invariance of the Majorana-mass Lagrangian (3.25) gives $Y_\nu=0$. Then, from (3.24), one has $Y_l=-1$, and only (iii) in Eq. (3.12) gives values (2.11) of all hypercharges. It is obvious that the models with more Higgs doublets and additional Majorana right-handed neutrinos also predict charge quantization. What will happen if the Majorana-mass term (3.25) is absent [29]? This question is interesting in light of the recent discovery of a heavy neutrino state of mass about 17 keV [30]. The experiments on double beta decay require this neutrino to be a Dirac one. We can use the formalism of this section with $M_R=0$ in the mass matrix M^ν in (3.26). After diagonalization of M^ν, we end up with three Dirac neutrinos (two massless and one massive). We have a nontrivial mixing, given by the 3×3 lepton mixing matrix, in the charged-current interaction (3.30). The lepton numbers L_e, L_μ, and L_τ are not conserved. The $U(1)_Y$ invariance of the Yukawa interaction (3.24) leads to

$$-Y_{l_i}-1+Y_\nu=0;\quad i=1,2,3 . \tag{3.32}$$

This set of equations, together with (3.8), requires the generation independence of lepton hypercharges, Eq. (3.22). Then Eq. (3.31) takes the form

$$3(6Y_q^3+2Y_l^3-3Y_u^3-3Y_d^3-3Y_e^3-3Y_\nu^3)+2Y_\nu^3=0 . \tag{3.33}$$

As in the one-generation case, the first part of (3.33) is equal to zero [cf. Eq. (2.19)], and we get

$$2Y_\nu^3=0\Longrightarrow Y_\nu=0 . \tag{3.34}$$

Generally, if we have several right-handed neutrinos in the theory and their number differs from the number of left-handed neutrinos, then the theory predicts charge quantization. If there are equal numbers of left- and right-handed neutrinos then the theory possesses an additional gaugeable U(1) symmetry $(B-L)^3$ and charge is dequantized.

IV. CONCLUSIONS

In a dynamical way, using the properties of a given Lagrangian, we discuss in full detail what it means for a theory to predict fermion charge quantization. The various approaches are reviewed. We also show why charge is dequantized in the three-generation SM. On these grounds we consider the simplest generalization of the SM which predicts charge quantization. Models with more than one Higgs doublet and/or at least one right-handed neutrino with Majorana-mass term possess the nice feature of charge quantization. Models with Dirac neutrinos with different numbers of left- and right-handed Weyl states also predict charge quantization. The question, *which simple generalization of the standard model is realized in nature?* arises. The answer should be obtained in future experiments.

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