Feynman rules for Majorana-neutrino interactions

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Simple Feynman rules for Majorana neutrinos and Dirac fermions interacting with spin-1 or spin-0 bosons are presented. Several examples using these rules are given.

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I. INTRODUCTION

In perturbation field theory the Feynman-diagram technique is usually applied. This technique is very well known for neutral and charged spin-0 and spin-1 bosons and for charged spin-$\frac{1}{2}$ fermions while for neutral spin-$\frac{1}{2}$ fermions the diagrammatic methods are not fully developed. We know of only three papers which touch on the problem [1–3]. However, the presented Feynman methods are complicated and in many papers which describe Majorana-neutrino interactions the authors prefer to use the Wick theorem [4]. The Feynman rules for Majorana particles differ from those of Dirac fermions due to the presence of several different propagators, a related multiplicity of Feynman vertices, and the problem involved in the relative signatures of the various diagrams contributing to a given amplitude [5]. The self-conjugacy of Majorana fermions is responsible for all these differences. On the other hand, the Majorana neutrinos appear in a number of proposed extensions of the standard model so it becomes important to have simple diagrammatic rules to calculate cross sections. The number of necessary propagators and related Feynman vertices was reduced already in Ref. [3] but the applications of their Feynman rules are still complicated. First of all, the authors of Ref. [3] give the rule for an absolute signature of each Feynman diagram which depends on the order of fermion operators in the Green's function. In Refs. [1–3], the Majorana representation for the Dirac γ^μ matrices and spinors is used; therefore, it is not obvious which simplifications originate from the proposed method and which from the spin representation.

In this paper we propose a Feynman-diagram technique for Majorana neutrinos which is independent of the spinor representation and does not attempt to give a rule for the absolute sign of each diagram. We found a simple method which makes it possible to predict the relative signs between different diagrams, which are important in practical calculations. The proposed Feynman-diagram technique for neutral spin-$\frac{1}{2}$ fermions is very simple and we hope it will find applications in practice.

In the next section we describe the Majorana-neutrino interactions for which we have found Feynman rules. In Sec. III the Feynman technique is presented. At the beginning of this section we consider a very simple example which explains why it is possible to formulate explicit Feynman rules in spite of the multiplicity of vertices and propagators. In Sec. IV we give certain examples using our rules.

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II. MAJORANA-NEUTRINO INTERACTIONS

We will consider only that part of any gauge theory which describes the Majorana-neutrino interactions. The other parts of the gauge theory Lagrangian are not important for our purpose, and give Feynman rules which are well known. We take the interactions of Majorana neutrinos (N) with charged leptons (l) and charged ($W^\pm$) or neutral (Z^0) gauge bosons in the form

$$L_{NW^\pm}=\bar{N}\Gamma_l^\mu l W_\mu^+ + \bar{l}\,\bar{\Gamma}_l^\mu N W_\mu^- \;, \tag{2.1}$$

$$L_{NZ^0}=(\bar{N}\Gamma_N^\mu N+\bar{l}\Gamma_{lN}^\mu l)Z_\mu \;, \tag{2.2}$$

where

$$\Gamma_{(x)}^\mu=\gamma^\mu(P_L A_L^{(x)}+P_R A_R^{(x)}) \;, \quad x=l,N,lN \;,$$

and

$$\bar{\Gamma}_l^\mu\equiv\gamma_0\Gamma_l^{\mu\dagger}\gamma_0=\gamma^\mu(P_L A_L^{(l)*}+P_R A_R^{(l)*}) \;,$$

$$P_{L(R)}=\tfrac{1}{2}(1\mp\gamma_5) \;.$$

The parameters $A_{L,R}^{(x)}$ are real for $x=N,lN$ and generally complex for $x=l$ and depend on the theory under investigation. In the case of mixing between generations the $A_{L,R}^{(x)}$ quantities are matrices, but this does not complicate our approach to the Feynman rules.

In a similar way we consider the Majorana neutrino interaction (N) with the charged ($H^\pm$) and neutral (H^0) spin-0 Higgs particles:

$$L_{NH^\pm}=\bar{N}\Gamma_l l H^+ + \bar{l}\,\bar{\Gamma}_l N H^- \;, \tag{2.3}$$

$$L_{NH^0}=(\bar{N}\Gamma_N N+\bar{l}\Gamma_{lN} l)H^0 \;, \tag{2.4}$$

where

$$\Gamma_{(x)}=P_L B_L^{(x)}+P_R B_R^{(x)} \quad \text{for } x=l,N,lN \;,$$

$$\bar{\Gamma}_l\equiv\gamma_0\Gamma_l^\dagger\gamma_0\equiv P_R B_L^{(l)*}+P_L B_R^{(l)*} \;,$$

and

$$B_L^{(x)}=B_R^{(x)*} \quad \text{for } x=N,lN ,$$

and as previously $B_{L(R)}^{(x)}$ are theory-dependent numbers (or matrices). The Majorana fields are self-conjugate and have a plane-wave decomposition,

$$N_\alpha(x)=\sum_{\lambda=\pm 1/2}\int \frac{d^3k}{(2\pi)^3 2E}[u_\alpha(\mathbf{k},\lambda)a(\mathbf{k},\lambda)e^{-ikx} + v_\alpha(\mathbf{k},\lambda)a^\dagger(\mathbf{k},\lambda)e^{ikx}] , \tag{2.5}$$

$$N_\alpha^C(x)\equiv \mathbf{C}N_\alpha(x)\mathbf{C}^{-1}=C_{\alpha\beta}[\bar{N}^T(x)]_\beta=N_\alpha(x) ,$$

where $C_{\alpha\beta}$ is the 4×4 charge-conjugate matrix in Dirac space

$$C\gamma^\mu C^{-1}=-\gamma^{\mu^T}, \quad C^\dagger=C^{-1}, \quad C^T=-C . \tag{2.6}$$

The Feynman vertices will be described by the $\Gamma_{(x)}^\mu$ or $\Gamma_{(x)}$ matrices and their charge-conjugate quantities

$$\Gamma_{(x)C}^\mu\equiv C\Gamma_{(x)}^{\mu^T}C^{-1}=-\gamma^\mu(P_R A_L^{(x)}+P_L A_R^{(x)}) , \tag{2.7}$$

and

$$\Gamma_{(x)C}\equiv C\Gamma_{(x)}^T C^{-1}=\Gamma_{(x)} . \tag{2.8}$$

III. THE FEYNMAN RULES

We will present our Feynman rules for the Majorana fermion interactions given by (2.1)–(2.4). On Feynman diagrams the Dirac fermion will be depicted by a double continuous line (═════), the Majorana fermion by a single line (────) and other particles will be depicted traditionally.

The problems arising with the Feynman rules for Majorana neutrinos are connected with the fact that Majorana fields satisfy the self-conjugate condition (2.5). The basic propagator for Majorana neutrinos is the same as for Dirac fermions:

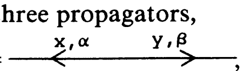
$$\langle 0|T[N_\alpha(x)\bar{N}_\beta(y)]|0\rangle \equiv iS_{\alpha\beta}(x-y)=$$ 

(3.1)

where

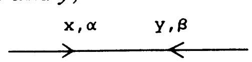
$$S(x-y)=i\int \frac{d^4x}{(2\pi)^4}e^{-i(x-y)k}\frac{\hat{k}+m}{k^2-m^2+i\epsilon} ,$$

and describe the fermion created at y and annihilated at x. Using the relations $N^T=\bar{N}C^T$ and $\bar{N}^T=C^{-1}N$ valid for Majorana particles (2.5) we can define the other three propagators,

$$\langle 0|T[N_\alpha(x)N_\beta(y)]|0\rangle \equiv -i[S(x-y)C]_{\alpha\beta}=$$ 

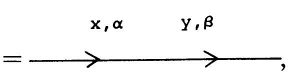
, (3.2)

describing the Majorana neutrino annihilated at x and y,

$$\langle 0|T[\bar{N}_\alpha(x)\bar{N}_\beta(y)]|0\rangle \equiv i[C^{-1}S(x-y)]_{\alpha\beta}=$$ 

, (3.3)

where the particle is created at x and y, and finally

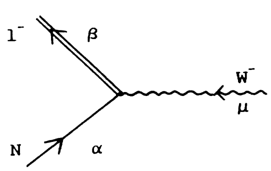
$$\langle 0|T[\bar{N}_\alpha(x)N_\beta(y)]|0\rangle \equiv -i[C^{-1}S(x-y)C]_{\alpha\beta}=$$ 

, (3.4)

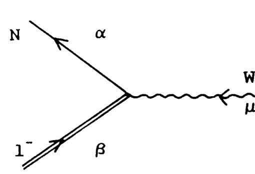
which describes the particle created at x and annihilated at y. For a given interaction between fermions and a gauge boson there are also four types of vertices. To show this, let us take the neutrino–charged-lepton interaction given by (2.1). Using the relations $N=C\bar{N}^T$ and $\bar{N}=-N^TC^{-1}$, the Lagrangian can be written in the form

$$L_{\rm CC}=\tfrac{1}{2}[\bar{l}_\beta\Gamma_{\beta\alpha}^\mu N_\alpha W_\mu^- +\bar{N}_\alpha\Gamma_{\alpha\beta}^\mu l_\beta W_\mu^+ +\bar{l}_\beta(\Gamma^\mu C)_{\beta\alpha}\bar{N}_\alpha^T W_\mu^- +N_\alpha^T(-C^{-1}\Gamma^\mu)_{\alpha\beta}l_\beta W_\mu^+] . \tag{3.5}$$

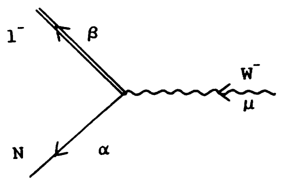
Hence four types of vertices which are connected successively with the terms of the Lagrangian (3.5) can be defined as



$=[i\Gamma^\mu]_{\beta\alpha}$, (3.6)

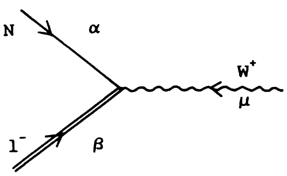


$=[i\Gamma^\mu]_{\alpha\beta}$, (3.7)



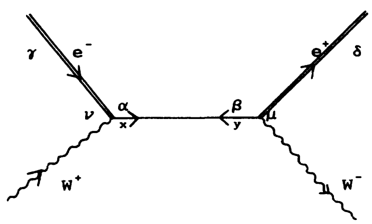
$$=i[\Gamma^\mu C]_{\beta\alpha}\ , \tag{3.8}$$

and the last term

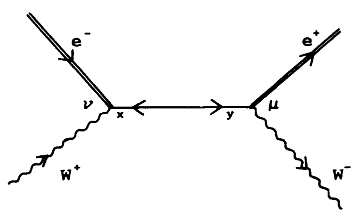


$$=i[-C^{-1}\Gamma^\mu]_{\alpha\beta}\ . \tag{3.9}$$

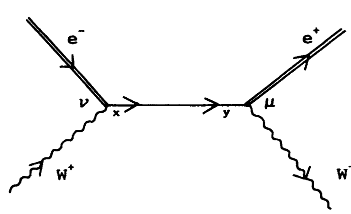
At first sight this appears to be complicated because it seems to be necessary to remember the particle-antiparticle direction of flow on the fermion lines. Fortunately, we shall see that the additional C matrices which appear in the vertices and propagator all cancel and hence we are able to formulate simple Feynman rules for Majorana neutrinos. To present the basic idea we will consider one simple example. The lepton-number-violating process $e^- W^+ \to e^+ W^-$ is described by one diagram which can be depicted in four different versions:



$$\begin{aligned}&=[i\Gamma^\nu]_{\alpha\gamma}[iC^{-1}S(x-y)]_{\alpha\beta}[i\Gamma^\mu]_{\beta\delta}\\&=[i\Gamma^\nu]^T[iC^{-1}S(x-y)][i\Gamma^\mu]\ ,\end{aligned} \tag{3.10}$$

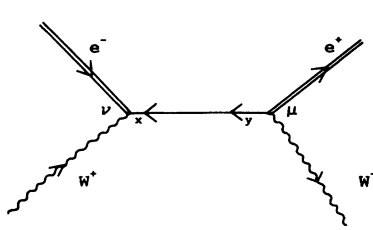


$$=[-iC^{-1}\Gamma^\nu]^T[-iS(x-y)C][-iC^{-1}\Gamma^\mu]\ , \tag{3.11}$$



$$=[i\Gamma^\nu]^T[-iC^{-1}S(x-y)C][-iC^{-1}\Gamma^\mu]\ , \tag{3.12}$$

and



$$=-[iC^{-1}\Gamma^\nu]^T[iS(x-y)][i\Gamma^\mu]\ . \tag{3.13}$$

We may see that in all four cases we end up with the same amplitude ($C^T=-C$):

$$T_{e^-W^+\to e^+W^-}\sim -iC^{-1}[\Gamma^\nu_C]S(x-y)[\Gamma^\mu]\ . \tag{3.14}$$

After appropriate attribution of spinors to external fermion lines the C matrix can be eliminated from the amplitude.

In the full amplitude the C matrix (3.14) is sandwiched with spinors, u^T for the incoming electron and v for the outgoing positron; then using the relation $-u^TC^{-1}=\bar{v}$ we get

$$T_{e^-W^+\to e^+W^-}=-iu^T(e^-)C^{-1}[\Gamma_C^\nu]S(k)[\Gamma^\mu]v(e^+)=i\bar{v}(e^-)[\Gamma_C^\nu]S(k)[\Gamma^\mu]v(e^+)\ . \tag{3.15}$$

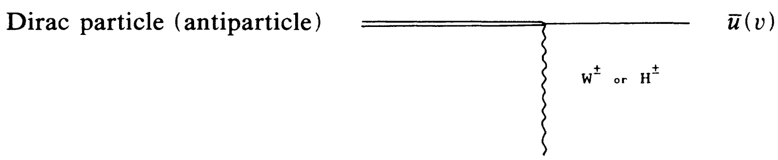
Hence, irrespective of which propagator (3.1)–(3.4) is used, with correct choice of vertices (3.6)–(3.9) and suitable attribution of spinors to the external lines, we obtained for the amplitude one simple expression (3.15). Now we will formulate the rules for Majorana-neutrino interactions (2.1)–(2.4) which can be used for a rapid calculation of the amplitude for any process.

At the beginning of our procedure we need to attribute spinors to fermion lines on the given Feynman diagram (not only to external lines). The propagators for the internal fermion will be built from spinors as described below.

Spinors

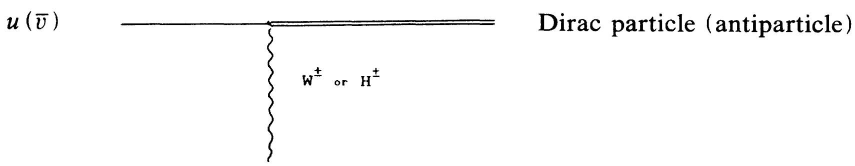
For the Majorana-Dirac coupling with charged bosons ($W^\pm,H^\pm$) the spinors' attribution to the Majorana line depends on the nature of the Dirac line.

(a) For the incoming Dirac particle (antiparticle) the outgoing Majorana fermion must also be treated as a particle (antiparticle):



(3.16)

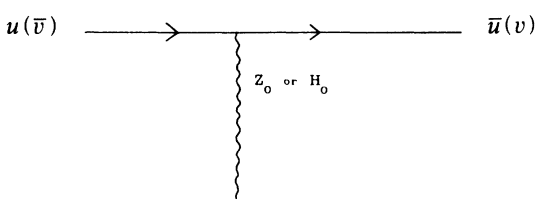
(b) For the outgoing Dirac fermion the fermion "nature" of the vertex is also conserved and for the outgoing Dirac particle (antiparticle) the incoming Majorana fermion must be treated as a particle (antiparticle):



(3.17)

We see that for the Dirac-Majorana transition the attribution of spinors to Majorana lines is definitive—the Dirac "particle nature" is remembered in the vertex. This is not so in the case of Majorana-Majorana coupling with the neutral bosons (Z^0,H^0). We can treat the Majorana fermion as a particle or an antiparticle, and the final result will be independent of this choice.

(c)



(3.18)

The situation sometimes arises where the "fermion flow" is opposite to the momentum flow on a line. In such a case we use the standard relation for spinors:

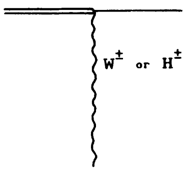
(d)

$$u(\pm k)=v(\mp k)\ . \tag{3.19}$$

Vertices

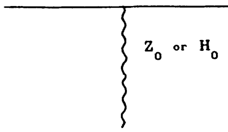
Contrary to previous approaches, our vertices are independent of the direction of the "fermion flow" on the line. We have only one vertex for the given Lagrangian as in the Dirac fermion case.

(a) For Dirac-Majorana fermion coupling with the bosons with spin 1 ($W^\pm$) or spin 0 ($H^\pm$) we have



$$\begin{cases} i\Gamma_l^{\mu} & \text{for outgoing } W^- \text{ or incoming } W^+ , \\ i\overline{\Gamma}_l^{\mu} & \text{for outgoing } W^+ \text{ or incoming } W^- , \\ i\Gamma_l & \text{for outgoing } H^- \text{ or incoming } H^+ , \\ i\overline{\Gamma}_l & \text{for outgoing } H^+ \text{or incoming } H^- . \end{cases} \tag{3.20}$$

(b) For two Majorana fermions' coupling with the neutral bosons Z^0 or H^0 we obtain



$$\begin{cases} i(\Gamma_N^{\mu}+\Gamma_{NC}^{\mu}) & \text{for } Z^0 , \\ i(\Gamma_N+\Gamma_{NC}) & \text{for } H^0 . \end{cases} \tag{3.21}$$

As we see, there is no "particle flow" indication on the vertices' lines. This means that the vertices are the same for particles and antiparticles, and are independent of the direction of the "particle flow" if they flow in the same direction. When the relation (3.19) is used, the vertices with opposite "particle flow" (e.g., —>—<— or —<—>—) are not needed.

Propagators

In the proposed Feynman technique we need only one propagator for internal Majorana particles, the same as for the Dirac fermion. To eliminate the problem connected with the sign ambiguity [3] the propagators taken will be made of spinors.

(a) For the internal Majorana line, we may choose any direction of momentum k. The final result will be independent of the direction of k (taking into account the sign convention).

(b) We use the relation of type

$$\overline{v}_2 O u_1 = -\overline{v}_1 O_C u_2 , \quad \overline{u}_2 O u_1 = -\overline{v}_1 O_C v_2 , \tag{3.22}$$

for $O=(\Gamma_x^{\mu},\Gamma_x,\ldots)$ and $O_C \equiv C O^T C^{-1}$.

Then, using relations (3.22) we transform the amplitude in order to get an expression of $u(k)\overline{u}(k)$ type and instead we put the normal Dirac propagator

$$u(k)\overline{u}(k) \to i \sum_{\text{spin}(\lambda)} \frac{u(\mathbf{k},\lambda)\overline{u}(\mathbf{k},\lambda)}{k^2-m^2+i\epsilon} = \frac{i(\hat{k}+m)}{k^2-m^2+i\epsilon} \equiv iS(k) . \tag{3.23}$$

(c) If we use relations (3.22) n times, in order to find the propagators (3.23) for Majorana or Dirac particles, we multiply the amplitudes by $(-)^n$, which is equivalent to disregarding the minus sign in formulas (3.22). If both spinors 1 and 2 in (3.22) describe the external particles, we have to take into account the minus sign.

Sign convention

Using the proposed rules for each Feynman diagram we obtain some analytical expression which contributes to the full amplitude with an unknown sign. To find the absolute sign we need to know the order of fermion operators inside the Green's function [3]. However, the amplitude is normally used to calculate a cross section; then we need only the relative sign between various Feynman diagrams. This relative sign problem can be resolved as follows.

(a) We choose any Feynman diagram which we call the reference diagram. In its amplitude, Dirac and Majorana fermions appear in the fermion chain in a given order.

(b) We compare all the other diagrams with the reference one. We permute the fermions in their "fermion chains" to obtain the same order as in the reference diagram.

(c) The signature of each diagram in the full amplitude depends on the fermion order. If parity of the permutation is even (odd), the sign of the diagram in the full amplitude remains unchanged (is changed). In this way we have resolved the relative signature problem.

The Lagrangians (2.1)–(2.4) are usually a part of some gauge-independent theory. The Feynman rules for bosons remain unmodified. For Dirac fermions the same rules as described above for the Majorana particles can be applied.

We now apply the presented Feynman technique in order to calculate amplitudes in a few simple examples.

IV. EXAMPLES

A. Electron-neutrino scattering $e^-N \to e^-N$

Three diagrams given in Fig. 1 describe the process on the tree level. If we compare the process with neutrino (v_e) or antineutrino ($\overline{v}_e$) scattering with electrons, in the former case the A_1 and A_2 diagrams contribute, and in the latter, only A_1 and A_3 are present. Our rules tell us that in the A_2 diagram the Majorana neutrino should be considered as a particle but in A_3 as an antiparticle and hence we have

$$A_1 = \overline{u}(3)[i\Gamma_{lN}^{\mu}]u(1) iD_{\mu\nu}^{Z^0}(k_1-k_3)\overline{u}(4) \times [i(\Gamma_N^{\nu}+\Gamma_{NC}^{\nu})]u(2) , \tag{4.1}$$

$$A_2 = \overline{u}(4)[i\Gamma_l^{\mu}]u(1) iD_{\mu\nu}^{W}(k_1-k_4)\overline{u}(3) \times [i\overline{\Gamma}_l^{\nu}]u(2) , \tag{4.2}$$

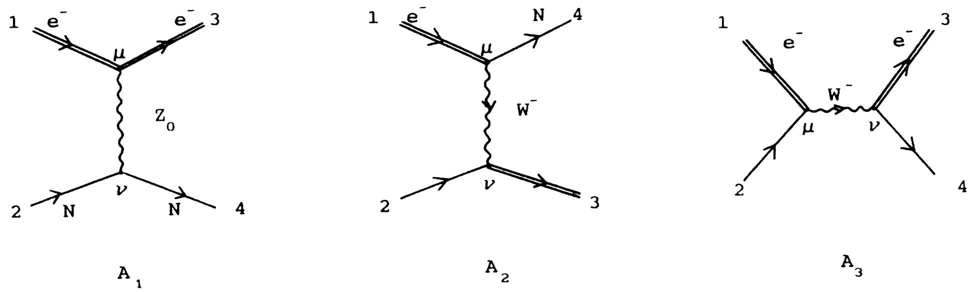


FIG. 1. Feynman diagrams for electron–Majorana-neutrino interaction $e^-N \rightarrow e^-N$.

$$A_3 = \bar{u}(-k_2)[i\Gamma_l^\mu]u(1) iD_{\mu\nu}^W(k_1+k_2)\bar{u}(3) \times [i\bar{\Gamma}_l^\nu]u(-k_4)$$
$$= \bar{v}(2)[i\Gamma_l^\mu]u(1) iD_{\mu\nu}^W(k_1+k_2)\bar{u}(3) \times [i\bar{\Gamma}_l^\nu]v(4) \ . \quad (4.3)$$

To find the relative sign between the amplitudes A_i $(i=1,2,3)$ let us analyze the order of the fermions in the "fermion chains." For diagram A_1 the order is (3,1,4,2), for $A_2 \sim (4,1,3,2)$ and for $A_3 \sim (2,1,3,4)$. Let us take diagram A_1 as the reference point. To find ordering as in A_1 we have to make an odd number of transpositions in A_2 and an even number in A_3, so that the full amplitude for the process is

$$M_{e^-N \rightarrow e^-N} = A_1 - A_2 + A_3 \ . \quad (4.4)$$

In diagram A_1 we treat the Majorana neutrinos as particles. Our rules tell us that these neutrinos can also be considered as antiparticles; then the neutral vertex will be described by

$$\bar{v}(2)[i(\Gamma_N^\nu + \Gamma_{NC}^\nu)]v(4)$$
$$= v^T(4)C^{-1}C[i(\Gamma_N^{\nu T} + \Gamma_{NC}^{\nu T})]C^{-1}C\bar{v}^T(2)$$
$$= -\bar{u}(4)[i(\Gamma_N^\nu + \Gamma_{NC}^\nu)]u(2) \ . \quad (4.5)$$

If we consider the Majorana neutrino as an antiparticle then the amplitude for the A_1 diagram will have the opposite sign. Taking the same A_1 diagram as a reference we obtain

$$M'_{e^-N \rightarrow e^-N} = -A_1 + A_2 - A_3 \ , \quad (4.6)$$

which differs in sign from the previous one and has no meaning for practical calculations.

B. The process $e^+e^+ \rightarrow W^+W^+$

Only two diagrams describe the process in the tree approximation (Fig. 2). Let us choose the internal neutrino momentum from the positron $2 \rightarrow 1$, as in Fig. 2. According to our rules we have

$$B_1 \Longrightarrow \bar{v}(2)[i\bar{\Gamma}_l^\mu]v(k)\bar{v}(1)[i\bar{\Gamma}_l^\nu]v(-k)$$
$$\Longrightarrow \bar{v}(1)[i\bar{\Gamma}_l^\nu]u(k)\bar{u}(k)[i\bar{\Gamma}_{lC}^\mu]u(2) \ ,$$

and hence

$$B_2 = \bar{v}(1)[i\bar{\Gamma}_l^\mu]iS^N(k_2-k_3)[i\bar{\Gamma}_{lC}^\nu]u(2)\epsilon_\mu^*(4)\epsilon_\nu^*(3) \ . \quad (4.7)$$

And for B_2 we have

$$B_2 = \bar{v}(1)[i\bar{\Gamma}_l^\mu]iS^N(k_2-k_3)[i\bar{\Gamma}_{lC}^\nu]u(2)\epsilon_\mu^*(4)\epsilon_\nu^*(3) \ .$$

As in both amplitudes, the external fermions ordering is identical and we have

$$M_{e^+e^+ \rightarrow W^+W^+} = B_1 + B_2 \ . \quad (4.8)$$

As previously stressed, our method is independent of the direction of internal Majorana fermion momentum. Let us choose the momentum in the B_2 diagram in the opposite direction (Fig. 3). Then we have

$$B'_2 \Longrightarrow \epsilon_\mu^*(4)\epsilon_\nu^*(3)\bar{v}(1)[i\bar{\Gamma}_l^\mu]v(k)\bar{v}(2)[i\bar{\Gamma}_l^\nu]v(-k) \ .$$

and hence

$$B'_2 = \epsilon_\mu^*(4)\epsilon_\nu^*(3)\bar{v}(2)[i\bar{\Gamma}_l^\nu]iS^N(k_1-k_4)[i\bar{\Gamma}_{lC}^\mu]u(1)$$
$$= \epsilon_\mu^*(4)\epsilon_\nu^*(3)u^T(1)C^{-1}C[i\bar{\Gamma}_C^{\mu T}]C^{-1}CiS^{N^T}(k_1-k_4) \times C^{-1}C[i\bar{\Gamma}^{\nu T}]C^{-1}C\bar{v}^T(2) = -B_2 \ . \quad (4.9)$$

But also the fermion order in $B'_2(1,2)$ is opposite to that in $B_2(2,1)$; therefore it is in agreement with our sign convention

$$M_{e^+e^+ \rightarrow W^+W^+} = B_1 - B'_2 = B_1 + B_2 \ ,$$

and the final amplitude is independent of the direction of momentum.

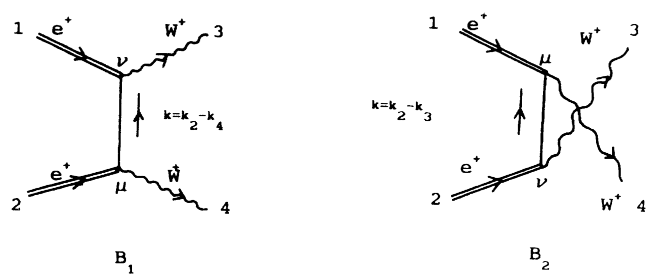
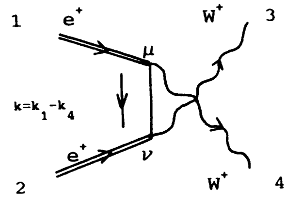


FIG. 2. Feynman diagrams for two-W^+ production process $e^+e^+ \rightarrow W^+W^+$. We changed the neutrino momentum (from the positron 2 to the positron 1).



B'2

FIG. 3. Diagram B_2 from previous figure where we change the direction of the neutrino momentum ($k=k_1-k_4$).

C. Process $e^-e^-\to e^-H^-N$

Let us consider a more complicated process which, in the framework of our Lagrangian (2.1)–(2.4), is described by six diagrams given in Fig. 4. Using our rules we get

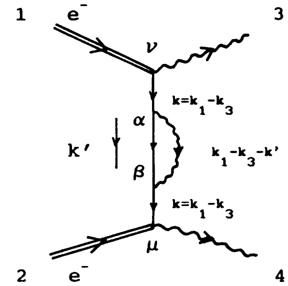
$$C_1=\bar{v}(1)[i\Gamma_{lC}]iS^N(k_4-k_1)[i\Gamma_l]u(2)iD^H(k_3+k_5)\times\bar{u}(3)[i\bar{\Gamma}_l]v(5)\ , \quad (4.10)$$

$$C_2=\bar{u}(3)[i\bar{\Gamma}_l]iS^N(k_1-k_4)[i\Gamma_l]u(1)iD^H(k_2-k_5)\times\bar{u}(5)[i\Gamma_l]u(2)\ , \quad (4.11)$$

$$C_3=\bar{u}(3)[i\bar{\Gamma}_l]iS^N(k_2-k_4)[i\Gamma_l]u(2)iD^H(k_1-k_5)\times\bar{u}(5)[i\Gamma_l]u(1) \quad (4.12)$$

$$C_4=\bar{v}(1)[i\Gamma_{lC}]iS^N(k_2-k_4)[i\Gamma_l]u(2)iD^H(k_3+k_5)\times\bar{u}(3)[i\bar{\Gamma}_l]v(5)\ , \quad (4.13)$$

$$C_5^{Z^0,\gamma}=\bar{u}(5)[i\Gamma_l]iS^e(k_5+k_4)[i\Gamma^{\mu}_{lN}]u(2)iD^{Z^0,\gamma}_{\mu\nu}(k_3-k_1)\times\bar{u}(3)[i\Gamma^{\nu}_{lN}]u(1)\ , \quad (4.14)$$



D1

FIG. 5. Diagram describing the virtual one-loop correction to the Majorana neutrino propagators in the process $e^-e^-\to W^-W^-$.

$$C_6^{Z^0,\gamma}=\bar{u}(5)[i\Gamma_l]iS^e(k_5+k_4)[i\Gamma^{\mu}_{lN}]u(1)iD^{Z^0,\gamma}_{\mu\nu}(k_2-k_3)\times\bar{u}(3)[i\Gamma^{\nu}_{lN}]u(2)\ . \quad (4.15)$$

Taking into account the order of fermions $C_1(1,2,3,5)$, $C_2(3,1,5,2)$, $C_3(3,2,5,1)$, $C_4(1,2,3,5)$, $C_5(5,2,3,1)$, and $C_6(5,1,3,2)$ and C_1 as the reference diagram, we have

$$M_{e^-e^-\to e^-H^-N}=C_1-C_2+C_3+C_4-C_5^{Z^0}-C_5^{\gamma}+C_6^{Z^0}+C_6^{\gamma}\ , \quad (4.16)$$

in agreement with the Wick theorem. To reach agreement it is important to use relation (3.22) without the minus sign in the amplitudes C_1 and C_4.

D. Virtual corrections

Our rules can be applied in any order of perturbation theory. As an example, let us calculate only one diagram (Fig. 5) which presents the one-loop correction to the neutrino propagator in the process $e^-e^-\to W^-W^-$:

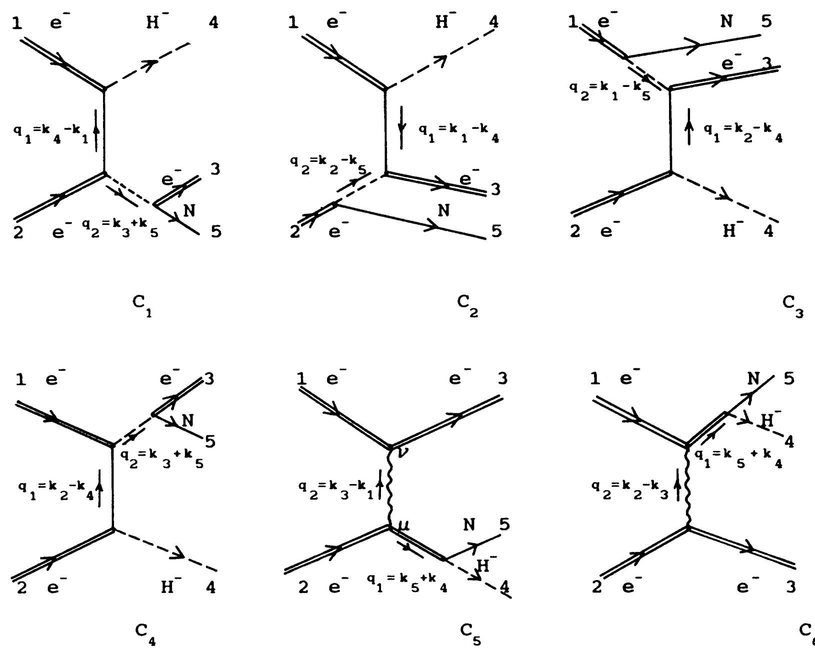


FIG. 4. Six Feynman diagrams which describe the three-body process $e^-e^-\to e^-H^-N$. All momenta inside the propagators are specified.

$$D_1=\epsilon_\mu^*(4)\epsilon_\nu^*(3)\int \frac{dk'}{(2\pi)^4}\bar{v}(2)[i\Gamma_{lC}^\mu]iS^N(k_4-k_2)[i(\Gamma_N^\beta+\Gamma_{NC}^\beta)]iS^N(k')[i(\Gamma_N^\alpha+\Gamma_{NC}^\alpha)]iS^N(k_1-k_3)$$
$$\times[i\Gamma_1^\gamma]u(1)D_{\alpha\beta}^{Z^0}(k_1-k_3-k') \ . \qquad (4.17)$$

The same convention as for charged leptons, known from QED, is applied now for the Majorana fermion; e.g., a factor of $-\frac{1}{2}$ must be associated with each closed Majorana fermion loop.

V. CONCLUSIONS

We present here very simple Feynman rules for Majorana fermions. We have used only one vertex for each Majorana fermion interaction, independent of the particle or antiparticle flow. We have also used one Majorana fermion propagator, identical to the Dirac one. Our rules are so simple that it appears to be advantageous to use them instead of the Wick theorem.

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