Electromagnetic mass differences of the octet and decuplet baryons

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The general parametrization method [Phys. Rev. D **40**, 2997 (1989)], an exact consequence of any QCD-like relativistic field theory, is used to parametrize the electromagnetic mass differences of the baryons. First we show that the Coleman-Glashow relationship, derived neglecting flavor breaking, continues to hold if all flavor-breaking terms are kept, except the three-quark ones; this may explain why the formula agrees so well with the data. In addition, neglecting only three-quark terms, we reproduce some equalities between electromagnetic mass differences of **8** and **10** baryons derived a long time ago [A. Gal and F. Scheck, Nucl. Phys. **B2**, 110 (1967)] by the nonrelativistic quark model (NRQM). Also these equalities, now testable due to the improved knowledge of decuplet masses, are well satisfied; they now appear as a general consequence of a relativistic QCD-like field theory, not just of the NRQM.

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I. INTRODUCTION: THE PARAMETRIZATION METHOD

In this paper we apply the general parametrization method (a consequence of any QCD-like field theory) to the electromagnetic mass differences of the lowest octet and decuplet baryons. The main results, listed in the abstract, will be discussed also in the Conclusion (Sec. VI). The parametrization method was developed [1(a)]—and applied to a variety of cases (Refs. [1(a)–1(d); 2])—to understand why the nonrelativistic naive quark model [3] (NRQM) is often so successful quantitatively. For completeness and to introduce the notation we summarize in this section the basic idea of the method [1(a)]. Suppose that one has to calculate the expectation value $\Omega_{\rm av}$ of some operator Ω in the exact state $|\Psi\rangle$ of some hadron; here Ω is some operator constructed in terms of the quark and gluon fields and $|\Psi\rangle$ is the exact state. For instance, if we adopt QCD as the basic theory, $|\Psi\rangle$ is the exact eigenstate of the full QCD Hamiltonian H ($H|\Psi\rangle=E|\Psi\rangle$) for the hadron under consideration; of course $|\Psi\rangle$ has to be thought of as an infinite superposition of Fock states of quarks, antiquarks, and gluons. In terms of $|\Psi\rangle$, the expectation value of the operator Ω is $\Omega_{\rm av}=\langle\Psi|\Omega|\Psi\rangle$. We now write the exact state $|\Psi\rangle$ as $V|\phi\rangle$ so that

$$\Omega_{\rm av}=\langle\Psi|\Omega|\Psi\rangle=\langle\phi|V^{\dagger}\Omega V|\phi\rangle \ . \tag{1}$$

Here $|\phi\rangle$ (which we will call "the model state") is chosen as a very simple state endowed with all the quantum numbers of the exact state. It is defined in the Fock subspace of just three quarks for the (nonexotic) baryons or of one quark and one antiquark for the (nonexotic) mesons. For baryons $|\phi\rangle$ is chosen as

$$|\phi\rangle=\sum_{\substack{\mathbf{p},\mathbf{p}',\mathbf{p}''\\ r,s,t}} f_{rst}(\mathbf{p},\mathbf{p}',\mathbf{p}'')a^{\dagger}_{\mathbf{p}r}a^{\dagger}_{\mathbf{p}'s}a^{\dagger}_{\mathbf{p}''t}|0\rangle \ , \tag{2}$$

where $|0\rangle$ is the vacuum of the Fock space of quarks and gluons [4], and $a^{\dagger}_{\mathbf{p}r}$'s are creation operators of quarks in the state of momentum $\mathbf{p}$ and spin-flavor-color characterized by the index r, and $f_{rst}(\mathbf{p},\mathbf{p}',\mathbf{p}'')$ is the wave function of the model state in momentum space. For the lowest octet and decuplet baryons we choose for the wave function $f_{rst}(\mathbf{p},\mathbf{p}',\mathbf{p}'')$ (when written in coordinate space) the usual expression of the naive NRQM, that is,

$$\varphi_B=X_{L=0}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)W_B(1,2,3)S_c(1,2,3) \ , \tag{3}$$

where the subscript B reminds us of the specific baryon B under consideration and the symbol φ rather than ϕ makes it clear that (3) is just the wave function corresponding to the state $|\phi\rangle$. In Eq. (3) $X_{L=0}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ is the space factor of the wave function, with orbital angular momentum zero and symmetric; $W_B(1,2,3)$ is the spin-flavor factor, that, due to symmetry, is automatically endowed with the SU(6) structure; and $S_c(1,2,3)$ is the color-singlet factor that will be usually omitted in what follows to simplify the notation.

The unitary operator V in Eq. (1) transforms the model state $|\phi\rangle$ into the exact state $|\Psi\rangle$. V is of course a very complicated operator constructed in terms of the quark and gluon fields; it has many tasks at the same time.

(a) It transforms the naive pure three-quark state $|\phi\rangle$ into the exact state $|\Psi\rangle$ that, as stated, is a superposition of infinitely many Fock states; at the same time it introduces all kinds of configuration mixing.

(b) It transforms the two-component spinors in terms of which $|\phi\rangle$ is constructed into four-component Dirac spinors in terms of which the operator Ω is constructed and the relativistic field theory is formulated.

In Ref. [1(a)] it is shown how the operator V might be constructed by the Gell-Mann–Low procedure; V is related to the $U(-\infty,0)$ operator of Dyson, where the "perturbation" to be used in constructing the latter is essentially the difference between the exact Hamiltonian H and the "model" Hamiltonian (in the three-quark–no-gluon sector) of which the $|\phi\rangle$'s are eigenstates. But the procedure for constructing V is unimportant; the only points of importance are that V exists and that (being a function of the exact Hamiltonian H) has the same good quantum numbers as H.

The essential point of the parametrization method can now be stated: Consider on the right-hand side of Eq. (1) the projection of the operator $V^\dagger \Omega V$ on the three-quark sector:

$$\tilde{\Omega} = \sum_{3q} \sum_{3q'} |3q\rangle\langle 3q|V^\dagger \Omega V|3q'\rangle\langle 3q'| \ . \tag{4}$$

After normal ordering and contraction of all the creation and destruction operators present in $V^\dagger \Omega V$ as well as in $\langle \phi_B|$ and $|\phi_B\rangle$, the operator $\tilde{\Omega}$ becomes a function only of the spin-flavor-space variables of the three quarks in φ_B. Thus the problem reduces to that of constructing the most general three-quark operator (which we continue to call $\tilde{\Omega}$) of the spin $\boldsymbol{\sigma}_i$, flavor λ_i, and space variables $\mathbf{r}_i$'s of the three quarks ($i=1,2,3$) and taking its expectation value over the wave function φ_B. In other words, the calculation of the expectation in $|\phi_B\rangle$ of the field operator $V^\dagger \Omega V$ amounts to that of the expectation value on the wave function φ_B of some three-body operator $\tilde{\Omega}$ in ordinary nonrelativistic three-body quantum mechanics. Clearly $\tilde{\Omega}$ has the same rotation properties as Ω; if Ω is invariant with respect to rotations also $\tilde{\Omega}$ must be such; if Ω is a vector also $\tilde{\Omega}$ must be such and so on. To calculate the electromagnetic masses of the **8** and **10** baryons, as we shall do in the next section, one has to calculate a scalar Ω bilinear in the electromagnetic current $j_\mu(x)$. In a QCD-like theory the current is just that of the quarks; in terms of the quark field operators it is $j_\mu(x) = e\overline{\Psi}(x)Q\gamma_\mu\Psi(x)$ where Q (which was called P^q in Refs. [1(a)–1(c)]) stands for

$$Q = \tfrac{2}{3}P^{\mathcal{P}} - \tfrac{1}{3}P^{\mathcal{N}} - \tfrac{1}{3}P^{\lambda} \tag{5}$$

and we use instead of 1, λ_3, and λ_8, the projectors $P^{\mathcal{P}}, P^{\mathcal{N}}$, and P^{λ} defined as

$$\begin{aligned} P^{\mathcal{P}} &= \tfrac{1}{6}(2+3\lambda_3+\lambda_8) \ , \\ P^{\mathcal{N}} &= \tfrac{1}{6}(2-3\lambda_3+\lambda_8) \ , \\ P^{\lambda} &= \tfrac{1}{3}(1-\lambda_8) \ . \end{aligned} \tag{6}$$

Here $\mathcal{P}, \mathcal{N}, \lambda$ indicate the quark fields: $\mathcal{P}(x) \equiv u_R(x)$; $\mathcal{N}(x) \equiv d_R(x)$; $\lambda(x) \equiv s_R(x)$; the subscript R means "renormalized [4]."

At this point the properties of what we called a QCD-like field theory come into play and simplify considerably the form of the most general $\tilde{\Omega}$.

(1) The electromagnetic current is constructed only in terms of quark fields, as stated. The only charged fields in the Lagrangian are those of the quarks; in particular the Lagrangian does not contain, say, pion fields explicitly as extra degrees of freedom. Due to this, the expectation value of an operator (as, for, e.g., here the Ω representing the electromagnetic mass) bilinear in the current $j_\mu(x) = e\overline{\Psi}(x)Q\gamma_\mu\Psi(x)$ must necessarily be quadratic in Q, because all the flavor operators $P^{\mathcal{P}}, P^{\mathcal{N}}, P^{\lambda}$ that enter in its construction commute and because of property 2 below.

(2) In a QCD-like theory the only λ-flavor matrix appearing in the strong Lagrangian is λ_8 (or if one prefers P^{λ}), associated with the mass difference between strange and nonstrange quarks. Because the Lagrangian contains only λ_3 and λ_8, which commute, the Casimir operators of SU(3) (flavor) do not enter in the final results.

Using these properties and the fact that the model wave function φ_B in Eq. (3) is factorizable into a product of a symmetrical spin-flavor factor $W_B(1,2,3)$ times a space factor $X_{L=0}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ (with $L=0$), the most general structure of the three-quark operator $\tilde{\Omega}$ simplifies considerably. It becomes very similar to that emerging from a NRQM calculation. We discuss in the next section the three-quark operator $\tilde{\Omega}$ for the electromagnetic masses of baryons.

II. THE GENERAL PARAMETRIZATION OF THE ELECTROMAGNETIC MASS OPERATOR FOR THE LOWEST 8 AND 10 BARYONS

With the procedure sketched above, and described in full detail in Ref. [1(a)], the operator $\tilde{\Omega}$ acting in the space of the three quarks can be shown to have a structure of the form

$$\tilde{\Omega} = \sum_{\nu} G_\nu(\mathbf{r})\Gamma_\nu(s,f) \ , \tag{7}$$

where $G_\nu(\mathbf{r})$ is a factor depending on all the space coordinates (called cumulatively $\mathbf{r}$) and $\Gamma_\nu(s,f)$'s are the set of spin-flavor operators in the spin-flavor space of the three quarks to be constructed in accordance with the properties noted in Sec. I; the sum over ν in Eq. (7) extends to all the possible $\Gamma_\nu(s,f)$. The factorization property of the model wave function φ_B noted in Sec. I allows one to perform immediately the expectation value of $\tilde{\Omega}$ on the space part $X_{L=0}$ of the model wave function φ_B. Defining

$$t_\nu = \langle X_{L=0}|G_\nu(\mathbf{r})|X_{L=0}\rangle \tag{8}$$

the t_ν's are of course different from zero only if the $G_\nu(\mathbf{r})$'s are rotation-invariant functions of $\mathbf{r}$. Integrating (7) over the space coordinates, the general parametrization thus becomes

$$\tilde{\Omega} = \sum_{\nu} t_\nu \Gamma_\nu(s,f) \ , \tag{9}$$

where now the expectation value of $\tilde{\Omega}$ in Eq. (9) for the various baryons B must be taken just on their spin-flavor functions W_B. Thus, because $\tilde{\Omega}$ must be rotation invariant, the set of $\Gamma_\nu(s,f)$ in (9) must be scalars in the spin space constructed only with the Pauli spin operators of the three quarks. We proved in Ref. [1(a)] that, for three quarks, only the following scalars exist:

$$1; \ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k) \ (i \neq k) \ (i,k=1,2,3) \ . \tag{10}$$

The scalar $(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot\boldsymbol{\sigma}_3$ has vanishing expectation value on a spin-flavor state having a real wave function, as $W_B(1,2,3)$ in Eq. (3) is.

As to the flavor operators that appear in the $\Gamma_\nu(s,f)$'s when calculating the electromagnetic masses, they must all be bilinear in Q (as discussed at the end of Sec. I).

They will, moreover, contain a number of factors P_i^λ up to three, related to the order of flavor breaking up to which we push the calculation. The following list contains all the possible structures in flavor space; in forming the list we exploited the fact that

$$P_i^\lambda P_i^\lambda = P_i^\lambda \tag{11}$$

but we did not yet use the simplifications

$$Q_i P_i^\lambda = -\tfrac{1}{3} P_i^\lambda, \quad Q_i Q_i \equiv Q_i^2 = \tfrac{4}{9} P_i^{\mathcal{P}} + \tfrac{1}{9} P_i^{\mathcal{N}} + \tfrac{1}{9} P_i^\lambda , \tag{12}$$

to preserve the evidence that all our flavor structures in this problem must be bilinear in the Q_i's:

$$Q_i^2, \quad Q_i Q_k , \tag{13}$$

$$Q_i^2 P_i^\lambda, \quad Q_i^2 P_k^\lambda, \quad Q_i Q_k P_i^\lambda, \quad Q_i Q_k P_j^\lambda , \tag{14}$$

$$Q_i^2 P_i^\lambda P_k^\lambda, \quad Q_i^2 P_k^\lambda P_j^\lambda, \quad Q_i Q_k P_i^\lambda P_k^\lambda, \quad Q_i Q_k P_j^\lambda P_k^\lambda , \tag{15}$$

$$Q_i^2 P_1^\lambda P_2^\lambda P_3^\lambda, \quad Q_i Q_k P_1^\lambda P_2^\lambda P_3^\lambda . \tag{16}$$

Multiplying each of the flavor structures (13)–(16) by each spin structure (10) and symmetrizing over all the quark indices i,k,j we produce all the spin-flavor structures $\Gamma_\nu(s,f)$ that can appear in the parametrization (9); the symmetrization over the quark indices arises because the spin-flavor functions $W_B(1,2,3)$ in Eq. (3), on which one must then take the expectation value, are symmetrical in 1,2,3.

We display below the complete list of spin-flavor operators Γ_ν; the list is ordered according to the number of factors P^λ. The flavor-breaking term (proportional to P^λ) in the strong Hamiltonian is multiplied by a factor $\Delta m / m_\lambda$ ($\cong 0.34$); *but recall that, because of* $(P_i^\lambda)^n = P_i^\lambda$, *an infinite number of flavor-breaking terms (those affecting only one quark i at a time)* are taken *into account exactly in a calculation that, apparently, is of first order in* P^λ. Below, the symbols

$$\Sigma[i] = \sum_{i=1}^{3} , \quad \Sigma[i,k] = \frac{1}{2} \sum_{\substack{i,k=1 \\ i \neq k}}^{3} , \quad \Sigma[i,k,j] = \frac{1}{6} \sum_{\substack{i,k,j=1 \\ i \neq k \neq j}}^{3} ,$$

are symmetrizers

$$\Sigma[A_i] = A_1 + A_2 + A_3, \quad \Sigma[A_i B_k] = \tfrac{1}{2}(A_1 B_2 + A_1 B_3 + A_2 B_1 + A_2 B_3 + A_3 B_1 + A_3 B_2) ,$$

$$\text{if } A_i = B_i, \quad \Sigma[A_i A_k] = A_1 A_2 + A_1 A_3 + A_2 A_3, \text{ etc.}$$

(1) Γ's of *zero order* in flavor breaking:

$$\begin{aligned} &\Gamma_1 = \Sigma[Q_i^2], \quad \Gamma_2 = \Sigma[Q_i^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)], \quad \Gamma_3 = \Sigma[Q_i^2(\boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_j)] , \\ &\Gamma_4 = \Sigma[Q_i Q_k], \quad \Gamma_5 = \Sigma[Q_i Q_k(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)], \quad \Gamma_6 = \Sigma[Q_i Q_k(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_k) \cdot \boldsymbol{\sigma}_j] . \end{aligned} \tag{17}$$

(2) Γ's of *first order* in P^λ (acting in $\Lambda, \Sigma, \Sigma^*, \Xi, \Xi^*, \Omega^-$):

$$\begin{aligned} &\Gamma_7 = \Sigma[Q_i^2 P_i^\lambda], \quad \Gamma_8 = \Sigma[Q_i^2 P_i^\lambda(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)], \quad \Gamma_9 = \Sigma[Q_i^2 P_i^\lambda(\boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_j)] , \\ &\Gamma_{10} = \Sigma[Q_i^2 P_k^\lambda], \quad \Gamma_{11} = \Sigma[Q_i^2 P_k^\lambda(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)], \quad \Gamma_{12} = \Sigma[Q_i^2 P_k^\lambda(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_k) \cdot \boldsymbol{\sigma}_j] , \\ &\Gamma_{13} = \Sigma[Q_i Q_k P_i^\lambda], \quad \Gamma_{14} = \Sigma[Q_i Q_k P_i^\lambda(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)], \quad \Gamma_{15} = \Sigma[Q_i Q_k P_i^\lambda(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_k) \cdot \boldsymbol{\sigma}_j] , \\ &\Gamma_{16} = \Sigma[Q_i Q_k P_j^\lambda], \quad \Gamma_{17} = \Sigma[Q_i Q_k P_j^\lambda(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)], \quad \Gamma_{18} = \Sigma[Q_i Q_k P_j^\lambda(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_k) \cdot \boldsymbol{\sigma}_j] . \end{aligned} \tag{18}$$

(3) Γ's of *second order* in P^λ (acting only in Ξ, Ξ^*, Ω^-):

$$\begin{aligned} &\Gamma_{19} = \Sigma[Q_i^2 P_i^\lambda P_k^\lambda], \quad \Gamma_{20} = \Sigma[Q_i^2 P_i^\lambda P_k^\lambda(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)], \quad \Gamma_{21} = \Sigma[Q_i^2 P_i^\lambda P_k^\lambda(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_k) \cdot \boldsymbol{\sigma}_j] , \\ &\Gamma_{22} = \Sigma[Q_i^2 P_k^\lambda P_j^\lambda], \quad \Gamma_{23} = \Sigma[Q_i^2 P_k^\lambda P_j^\lambda(\boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_j)], \quad \Gamma_{24} = \Sigma[Q_i^2 P_k^\lambda P_j^\lambda(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_k) \cdot \boldsymbol{\sigma}_j] , \\ &\Gamma_{25} = \Sigma[Q_i Q_k P_i^\lambda P_k^\lambda], \quad \Gamma_{26} = \Sigma[Q_i Q_k P_i^\lambda P_k^\lambda(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k)], \quad \Gamma_{27} = \Sigma[Q_i Q_k P_i^\lambda P_k^\lambda(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_k) \cdot \boldsymbol{\sigma}_j] , \\ &\Gamma_{28} = \Sigma[Q_i Q_k P_i^\lambda P_j^\lambda], \quad \Gamma_{29} = \Sigma[Q_i Q_k P_i^\lambda P_j^\lambda(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)], \quad \Gamma_{30} = \Sigma[Q_i Q_k P_i^\lambda P_j^\lambda(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \cdot \boldsymbol{\sigma}_k] . \end{aligned} \tag{19}$$

(4) Γ's of *third order* in P^λ (acting only in Ω^-):

$$\Gamma_{31} = \Sigma[Q_i^2 P_i^\lambda P_k^\lambda P_j^\lambda], \quad \Gamma_{31} = \Sigma[Q_i^2 P_i^\lambda P_k^\lambda P_j^\lambda((\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k) + (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) + (\boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_j))] . \tag{20}$$

Note that in the above list we omitted (in the third-order terms) several Γ's that are immediately reducible to others using the properties (11); for instance, $Q_i^2 P_i^\lambda P_k^\lambda P_j^\lambda$ and $Q_i Q_k P_i^\lambda P_k^\lambda P_j^\lambda$ are both equal to $(1/9) P_i^\lambda P_k^\lambda P_j^\lambda$; it would have been a repetition to list them separately.

The following properties of the decuplet and octet states simplify the above expressions considerably. For any i and k $(i \neq k)$,

$$\langle 10 | (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k) | 10 \rangle = 1, \quad \langle 8 | (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k) | 8 \rangle = -(2\mathbf{J} \cdot \boldsymbol{\sigma}_j) = [-3\sigma_{jz}] \quad (i \neq k \neq j) . \tag{21}$$

Here **10** and **8** stand for $W_B(\mathbf{10})$ and $W_B(\mathbf{8})$ in Eq. (3). The expression in square brackets in the second equation above (that for the octet) refers to the total angular momentum **J** with z component up.

Using the simplifications (21) all decuplet terms of order zero reduce to $\Gamma_1=\Sigma[Q_i^2]$ and $\Gamma_4=\Sigma[Q_iQ_k]=\frac{1}{2}(Q^2-\Sigma[Q_i^2])$. The octet terms of order zero are also expressed in Eq. (22) below in terms of the four quantities $\Sigma[Q_i^2]$, Q^2, $\Sigma[Q_i^2\sigma_{iz}]$, and $Q\Sigma[Q_i\sigma_{iz}]$ where $Q\equiv\Sigma[Q_i]$ is the charge of the baryon:

$$\Gamma_1(8)=\Gamma_1(10)=\sum_{i=1}^{3} Q_i^2\equiv\Gamma_1\ , \tag{22a}$$

$$\Gamma_2(8)=-\tfrac{3}{2}\Gamma_1+\tfrac{3}{2}\sum_{i=1}^{3} Q_i^2\sigma_{iz},\quad \Gamma_2(10)=\sum_{i=1}^{3} Q_i^2\ , \tag{22b}$$

$$\Gamma_3(8)=-3\sum_{i=1}^{3} Q_i^2\sigma_{iz},\quad \Gamma_3(10)=\sum_{i=1}^{3} Q_i^2\ , \tag{22c}$$

$$\Gamma_4(8)=\Gamma_4(10)=\frac{1}{2}\left[Q^2-\sum_{i=1}^{3} Q_i^2\right]\equiv\Gamma_4\ , \tag{22d}$$

$$\Gamma_5(8)=-\frac{3}{2}\left[Q^2-\sum_{i=1}^{3} Q_i^2\right] +3\left[Q\sum_{i=1}^{3} Q_i\sigma_{iz}-\sum_{i=1}^{3} Q_i^2\sigma_{iz}\right],\quad \Gamma_5(10)=\Gamma_4\ , \tag{22e}$$

$$\Gamma_6(8)=-3\Gamma_5(8)-3\Gamma_4,\quad \Gamma_6(10)=2\Gamma_4\ . \tag{22f}$$

In the next section we rederive by the present technique the Coleman-Glashow formula for the electromagnetic masses at zero order in flavor breaking. In Sec. V we will examine the flavor corrections to that relationship.

III. THE COLEMAN-GLASHOW RELATIONSHIP AT ZERO ORDER IN FLAVOR BREAKING

The general expression $\delta_0 B$ of the electromagnetic mass of baryon B at zero order in flavor breaking is, according to Eq. (9):

$$\delta_0 B=a\Gamma_1+b\Gamma_2+c\Gamma_3+d\Gamma_4+e\Gamma_5+f\Gamma_6 \tag{23}$$

(where we used $a,\ldots,f$ instead of $t_1,\ldots,t_6$ and the index 0 in δ_0 recalls that we are working at zero order in flavor breaking). The values of the quantities $\Sigma[Q_i^2]$, Q^2, $\Sigma[Q_i^2\sigma_{iz}]$, $Q\Sigma[Q_i\sigma_{iz}]$ in terms of which all six Γ_i's of order zero are expressed [Eq. (22)] appear in Table I for each octet and decuplet baryon; all $\delta_0 B$'s of decuplet baryons can be expressed only in terms of $\Sigma[Q_i^2]$ and Q^2; therefore for them the quantities in the second and fourth columns of the table have not been calculated. It is easy to check that for each column of Table I one has

$$\delta_0 P-\delta_0 N=\delta_0\Sigma^+-\delta_0\Sigma^-+\delta_0\Xi^--\delta_0\Xi^0 \tag{24}$$
$$1.29 \qquad\qquad 1.67\pm0.6\ \text{MeV}$$

which is the Coleman-Glashow [5] relationship at zero order in flavor breaking; the figures under the left- and right-hand sides are the present experimental values [6].

TABLE I. The values of the quantities: ΣQ_i^2, $\Sigma Q_i^2\sigma_{iz}$, Q^2, $Q\Sigma Q_i\sigma_{iz}$ for all baryons. The Γ_i's ($i=1$–6) that contribute in the case of no flavor breaking can all be expressed in terms of such quantities.

	ΣQ_i^2	$\Sigma Q_i^2\sigma_{iz}$	Q^2	$Q\Sigma Q_i\sigma_{iz}$
P	1	$\frac{5}{9}$	1	1
N	$\frac{2}{3}$	0	0	0
Λ	$\frac{2}{3}$	$\frac{1}{9}$	0	0
Σ^+	1	$\frac{5}{9}$	1	1
Σ^0	$\frac{2}{3}$	$\frac{1}{3}$	0	0
Σ^-	$\frac{1}{3}$	$\frac{1}{9}$	1	$\frac{1}{3}$
Ξ^0	$\frac{2}{3}$	0	0	0
Ξ^-	$\frac{1}{3}$	$\frac{1}{9}$	1	$\frac{1}{3}$
Δ^{++}	$\frac{4}{3}$	—	4	—
Δ^+	1	—	1	—
Δ^0	$\frac{2}{3}$	—	0	—
Δ^-	$\frac{1}{3}$	—	1	—
Σ^{*+}	1	—	1	—
Σ^{*0}	$\frac{2}{3}$	—	0	—
Σ^{*-}	$\frac{1}{3}$	—	1	—
Ξ^{*0}	$\frac{2}{3}$	—	0	—
Ξ^{*-}	$\frac{1}{3}$	—	1	—
Ω^-	$\frac{1}{3}$	—	1	—

IV. THE GAL-SCHECK RELATIONSHIPS AT ZERO ORDER IN FLAVOR BREAKING

Using the nonrelativistic quark model (NRQM) Gal and Scheck [7] derived long ago a set of relationships between the electromagnetic masses both for mesons and for baryons. Whereas for mesons the assumptions in Ref. [7] are very restrictive, for baryons these assumptions amount mostly to the neglect of three-body electromagnetic interaction terms. With that neglect the NRQM leads to the following relationships that involve both the octet and the decuplet electromagnetic masses:

$$\delta\Delta^{++}-\delta\Delta^0=2(\delta P-\delta N)+(\delta\Sigma^+-\delta\Sigma^0)+(\delta\Sigma^--\delta\Sigma^0)\ , \tag{25}$$

$$\delta\Delta^{++}-\delta\Delta^-=3(\delta P-\delta N)\ , \tag{26}$$

$$\delta\Delta^+-\delta\Delta^0=\delta P-\delta N\ , \tag{27}$$

$$(\delta\Sigma^{*+}-\delta\Sigma^{*-})+(\delta\Xi^{*-}-\delta\Xi^{*0})=\delta P-\delta N\ , \tag{28}$$
$$-1.2\pm0.9 \qquad\qquad -1.29\ \text{MeV}$$

$$\tfrac{1}{2}(\delta\Sigma^{*+}+\delta\Sigma^{*-})-\delta\Sigma^{*0}=\tfrac{1}{2}(\delta\Sigma^++\delta\Sigma^-)-\delta\Sigma^0\ . \tag{29}$$
$$1.3\pm1.2 \qquad\qquad 0.85\pm0.12\ \text{MeV}$$

We have not given numbers for the relationships (25), (26), and (27) because the experimental uncertainties on the masses of Δ's are still large.

To check if the formulas above are generally true in a QCD-like theory (not just consequences of the nonrelativistic quark model) we must calculate the electromagnetic masses of the baryons that appear in (25)–(29) in terms of the six parameters a,b,c,d,e,f in (23). Of course Eq. (23) refers to zero order in flavor breaking; in this section we confine ourselves to this case, so that, in fact we are checking Eqs. (25)–(29) with δ replaced by δ_0; for Eqs. (26) and (27) terms proportional to P^{λ} do not contribute so that, for them, $\delta_0 \equiv \delta$.

It is straightforward to check from (22) and (23) that for the octet baryons we have

$$\begin{aligned}\delta_0 B[8] = &(\tfrac{1}{2}d - \tfrac{3}{2}e + \tfrac{9}{2}f)Q^2\\ &+(a - \tfrac{3}{2}b - \tfrac{1}{2}d + \tfrac{3}{2}e - \tfrac{9}{2}f)\Sigma Q_i^2\\ &+(\tfrac{3}{2}b - 3c - 3e + 9f)\Sigma Q_i^2\sigma_{iz}\\ &+(3e - 9f)Q\Sigma Q_i\sigma_{iz}\end{aligned} \tag{30}$$

and for the decuplet ones

TABLE II. The values of the quantities: ΣQ_i^2, $\Sigma Q_i^2\sigma_{iz}$, Q^2, $Q\Sigma Q_i\sigma_{iz}$, given in Table I, each multiplied by its coefficient [that appears in Eqs. (33) and (34)] for all baryons of interest.

	[Coeff]ΣQ_i^2	[Coeff]$\Sigma Q_i^2\sigma_{iz}$	[Coeff]Q^2	[Coeff]$Q\Sigma Q_i\sigma_{iz}$
P	$a-\frac{3}{2}b-\frac{d}{2}+\frac{3}{2}e$	$\frac{5}{9}(\frac{3}{2}b-3e)$	$\frac{d}{2}-\frac{3}{2}e$	$3e$
N	$\frac{2}{3}\left[a-\frac{3}{2}b-\frac{d}{2}+\frac{3}{2}e\right]$	0	0	0
Σ^+	$a-\frac{3}{2}b-\frac{d}{2}+\frac{3}{2}e$	$\frac{5}{9}(\frac{3}{2}b-3e)$	$\frac{d}{2}-\frac{3}{2}e$	$3e$
Σ^0	$\frac{2}{3}\left[a-\frac{3}{2}b-\frac{d}{2}+\frac{3}{2}e\right]$	$\frac{1}{3}(\frac{3}{2}b-3e)$	0	0
Σ^-	$\frac{1}{3}\left[a-\frac{3}{2}b-\frac{d}{2}+\frac{3}{2}e\right]$	$\frac{1}{9}(\frac{3}{2}b-3e)$	$\frac{d}{2}-\frac{3}{2}e$	e
Ξ^0	$\frac{2}{3}\left[a-\frac{3}{2}b-\frac{d}{2}+\frac{3}{2}e\right]$	0	0	0
Ξ^-	$\frac{1}{3}\left[a-\frac{3}{2}b-\frac{d}{2}+\frac{3}{2}e\right]$	$\frac{1}{9}(\frac{3}{2}b-3e)$	$\frac{d}{2}-\frac{3}{2}e$	e
Δ^{++}	$\frac{4}{3}\left[a+b-\frac{d}{2}-\frac{e}{2}\right]$	0	$4\left[\frac{d}{2}+\frac{e}{2}\right]$	0
Δ^+	$\left[a+b-\frac{d}{2}-\frac{e}{2}\right]$	0	$\frac{d}{2}+\frac{e}{2}$	0
Δ^0	$\frac{2}{3}\left[a+b-\frac{d}{2}-\frac{e}{2}\right]$	0	0	0
Δ^-	$\frac{1}{3}\left[a+b-\frac{d}{2}-\frac{e}{2}\right]$	0	$\frac{d}{2}+\frac{e}{2}$	0
Σ^{*+}	$a+b-\frac{d}{2}-\frac{e}{2}$	0	$\frac{d}{2}+\frac{e}{2}$	0
Σ^{*0}	$\frac{2}{3}\left[a+b-\frac{d}{2}-\frac{e}{2}\right]$	0	0	0
Σ^{*-}	$\frac{1}{3}\left[a+b-\frac{d}{2}-\frac{e}{2}\right]$	0	$\frac{d}{2}+\frac{e}{2}$	0
Ξ^{*0}	$\frac{2}{3}\left[a+b-\frac{d}{2}-\frac{e}{2}\right]$	0	0	0
Ξ^{*-}	$\frac{1}{3}\left[a+b-\frac{d}{2}-\frac{e}{2}\right]$	0	$\frac{d}{2}+\frac{e}{2}$	0

$$\delta_0 B[10]=(\tfrac{1}{2}d+\tfrac{1}{2}e+f)Q^2 +(a+b+c-\tfrac{1}{2}d-\tfrac{1}{2}e-f)\Sigma Q_i^2 \,. \tag{31}$$

The neglect of three-quark terms amounts to the omission of the terms Γ_3 and Γ_6 in (23) that is to

$$c=f=0 \,. \tag{32}$$

With this neglect the expressions (30) and (31) simplify to

$$\delta_0 B[8]=(\tfrac{1}{2}d-\tfrac{3}{2}e)Q^2 +(a-\tfrac{3}{2}b-\tfrac{1}{2}d+\tfrac{3}{2}e)\Sigma Q_i^2 +(\tfrac{3}{2}b-3e)\Sigma Q_i^2\sigma_{iz}+3eQ\Sigma Q_i\sigma_{iz} \,, \tag{33}$$

$$\delta_0 B[10]=(\tfrac{1}{2}d+\tfrac{1}{2}e)Q^2 +(a+b-\tfrac{1}{2}d-\tfrac{1}{2}e)\Sigma Q_i^2 \,. \tag{34}$$

Using Eqs. (33) and (34) it is easy to check that all Eqs. (25)–(29) are satisfied; it would not have been so if we had kept the three-quark terms. To simplify this check we display in Table II the values of ΣQ_i^2, $\Sigma Q_i^2\sigma_{iz}$, Q^2, and $Q\Sigma Q_i\sigma_{iz}$ times their coefficients for all the baryons involved [8]. In Table II [Coeff] means the coefficient that multiplies, respectively, ΣQ_i^2, $\Sigma Q_i^2\sigma_{iz}$, Q^2 and $Q\Sigma Q_i\sigma_{iz}$ in Eqs. (33) and (34) [9].

V. FLAVOR BREAKING DOES NOT MODIFY THE ELECTROMAGNETIC MASS RELATIONSHIPS IF THREE-QUARK TERMS ARE NEGLECTED

The derivation of the Coleman-Glashow formula for the octet (both the usual derivation and that given here in Sec. III) is general, except that it does not take into account terms P^λ or $P_i^\lambda P_k^\lambda$. As to the octet-decuplet relationships (25)–(29) of Sec. IV, their derivation excludes three-quark terms and also it does not take into account—for Eqs. (25), (28), and (29)—flavor-breaking terms. In this section we will show that, if we exclude three-quark terms, both the Coleman-Glashow and the octet-decuplet relationships are not modified by flavor breaking.

Before proving this, it is necessary to be precise in the definition of three-quark terms in our parametrization. In the list [(17),(18),(19)] they are all terms that depend on the indices of three different quarks, that is the terms Γ_{3n} with $n=1$ to 10 plus $\Gamma_{16},\Gamma_{17},\Gamma_{22},\Gamma_{23},\Gamma_{28},\Gamma_{29}$; note, however, that the use of (21) changes (and may reduce) the number of different quark indices. But Eqs. (21) introduce a collective variable of the quarks, the total angular momentum; we define three-quark terms as those that are such before introducing these collective variables; indeed this introduction simply conceals the three-quark nature of these terms [10].

In Table III below we give the values of all different non–three-quark terms with one and two P^λ's for the baryons of interest (P, N, and Δ's are not affected by P^λ terms, that is they would have all zeros in the tables; we have not listed them).

It is easy from Table III to check that each Γ_i does not contribute either to the Coleman-Glashow formula or to the formulas (25)–(29) thus showing that all these relationships are correct to any order in flavor breaking provided that three-quark flavor terms are omitted. For instance, consider the contribution from Γ_{11} to the

TABLE III. The values of all Γ_i's (except the three-quark ones) that enter in calculating the flavor contributions to each baryon; Γ_{25} and Γ_{26} are not listed because they are identical respectively to Γ_{19} and Γ_{20}.

	Γ_7	Γ_8	Γ_{10}	Γ_{11}	Γ_{13}	Γ_{14}	Γ_{19}	Γ_{20}
Λ	$\frac{1}{9}$	0	$\frac{5}{9}$	2	$-\frac{1}{9}$	$\frac{1}{3}$	0	0
Σ^+	$\frac{1}{9}$	$-\frac{4}{9}$	$\frac{8}{9}$	$\frac{11}{9}$	$-\frac{4}{9}$	$-\frac{1}{9}$	0	0
Σ^0	$\frac{1}{9}$	$-\frac{4}{9}$	$\frac{5}{9}$	$\frac{8}{9}$	$-\frac{1}{9}$	$\frac{2}{9}$	0	0
Σ^-	$\frac{1}{9}$	$-\frac{4}{9}$	$\frac{2}{9}$	$\frac{5}{9}$	$\frac{2}{9}$	$\frac{5}{9}$	0	0
Ξ^0	$\frac{2}{9}$	$-\frac{2}{9}$	$\frac{10}{9}$	$\frac{22}{9}$	$-\frac{2}{9}$	$\frac{10}{9}$	$-\frac{1}{3}$	$\frac{1}{9}$
Ξ^-	$\frac{2}{9}$	$-\frac{2}{9}$	$\frac{4}{9}$	$\frac{16}{9}$	$\frac{4}{9}$	$\frac{16}{9}$	$-\frac{1}{3}$	$\frac{1}{9}$
Σ^{*+}	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{8}{9}$	$\frac{8}{9}$	$-\frac{4}{9}$	$-\frac{4}{9}$	0	0
Σ^{*0}	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{5}{9}$	$\frac{5}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	0	0
Σ^{*-}	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	0	0
Ξ^{*0}	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{10}{9}$	$\frac{10}{9}$	$-\frac{2}{9}$	$-\frac{2}{9}$	$-\frac{1}{3}$	$\frac{1}{9}$
Ξ^{*-}	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{4}{9}$	$-\frac{1}{3}$	$\frac{1}{9}$
Ω^-	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	-1	$\frac{1}{3}$

Coleman-Glashow formula; from Table III we read $[\Sigma^{+}-\Sigma^{-}+\Xi^{-}-\Xi^{0}]=(11-5+16-22)/9=0$. The same is true for all Γ's and all relationships. Note again, finally, that the Coleman-Glashow relationship includes at zero order all terms, in particular three-quark terms, while the formulas (25)–(29) do not include three-quark terms already at zero order in flavor breaking.

VI. CONCLUSION

(1) That the Coleman-Glashow relationship is satisfied "too" well has been a fact known since the formula was written; now perhaps it can be understood. It would be of great interest to reduce the experimental error on the mass of Ξ^0, which is the reason for the error ± 0.6 MeV on the right-hand side of Eq. (24); the other masses there are known to ± 0.13 MeV or less.

(2) It is rewarding that the general parametrization method allows once more (compare Refs. [1(a)–1(d),2] to give a firm basis to the quantitative success of a NRQM calculation, as that of Gal and Scheck for the electromagnetic masses of the **10** and **8** baryons; once more the basis of this success is simply that the parametrization given by the NRQM is quite similar to the parametrization that would be obtained in a full calculation based on a QCD-like field theory. Note, again, that when we say QCD-like we mean that the theory must have in common with QCD *only* the following two features: (a) that the electromagnetic current is carried only by the quarks; (b) that the only flavor breaking is due to λ_8 terms. These features, in fact, simplify the parametrization eliminating all Casimir flavor operators from the final results.

(3) As to the quantitative agreement of the relations (28) and (29) with the data (1.2 ± 0.9 to be compared with 1.29; and 1.3 ± 1.2 to be compared with 0.85 ± 0.12), this is also a confirmation that three-quark terms are negligible. This confirmation adds to that provided in Ref. [2] by the modified Gell-Mann–Okubo mass formula, based on the same assumption, and satisfied, we recall, at the level of 1132.4 ± 0.8 compared to 1133.9 ± 0.1. That the above verification of the relations (28) and (29) is now possible is due to the decrease, during the years, of the experimental errors on the masses of the decuplet and suggests, if feasible, further efforts in this direction although here the difficulties, particularly for very broad resonances such as the Δ's, are evident.

Finally we note again [11] that the interpretation of the meaning of the NRQM brought by the general parametrization method makes more acute the need of understanding the meaning of the many descriptions of the internal dynamics of hadrons (relativistic quark models, bag models, quarks plus pions, Skyrmions, chiral models, etc.) and their compatibility with the underlying basic field theory; we hope to discuss all that soon.

[1] G. Morpurgo: (a) Phys. Rev. D **40**, 2997 (1989); (b) **40**, 3111 (1989); (c) **41**, 2865 (1990); (d) **42**, 1497 (1991). In (a) the method is formulated and applied to the magnetic moments, em transition matrix elements and masses of the baryons; in (b) it is applied to the semileptonic baryon decays, in (c) to the meson masses and in (d) to the $V\to P\gamma$ meson decays. Note the following misprints. In (a), Eq. (45) for G, read $\frac{1}{2}\Lambda$ instead of $\frac{1}{4}\Lambda$; the combination $\delta-\beta-2\gamma$ (not $\delta-\beta+2\gamma$) should appear in Eqs. (64) and (66). In (c), in the row after Eq. (26) insert a minus sign in front of η_1. We used the correct formulas in the calculations.

[2] G. Morpurgo, Phys. Rev. Lett. **68**, 139 (1992). In this paper we derive a modified Gell-Mann–Okubo formula for the baryon masses that takes into account all flavor-breaking terms except three-quark ones; it is satisfied at the level 1132.4 ± 0.8 versus 1133.9 ± 0.1 MeV.

[3] G. Morpurgo: (a) Physics **2**, 95 (1965) [also reproduced in J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969), p. 132]; (b) *Theory and Phenomenology in Particle Physics*, Proceedings of the International School of Physics "E. Majorana," Erice, Italy, 1968, edited by A. Zichichi (Academic, New York, 1969), pp. 83–217; (c) Annu. Rev. Nucl. Sci. **20**, 105 (1970).

[4] If QCD is taken as the underlying theory, the quark fields (of which the $a^{\dagger}_{pr}$ are the creation operators) are the fields that appear in the Lagrangian; we define them here, however, after the Lagrangian is written in terms of masses renormalized at a low value of q^2. The quark fields after this mass renormalization are identified with the so-called "constituent" quarks.

[5] S. Coleman and S. L. Glashow, Phys. Rev. Lett. **6**, 423 (1961).

[6] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[7] A. Gal and F. Scheck, Nucl. Phys. **B2**, 121 (1967); compare also Ref. [3(b)], p. 165.

[8] We used Table II to calculate the parametrized δB (now they should be called $\delta_0 B$) of the **8** baryons listed in footnote 9 of Ref. [2]. With the identifications $\mu=a-\frac{3}{2}b-d/2+\frac{3}{2}e$; $\nu=\frac{3}{2}b-3e$; $\eta=d/2-\frac{3}{2}e$; $\rho=3e$, one obtains from Table II the δB's given there.

[9] In Ref. [7] the NRQM was applied also to the meson electromagnetic masses, introducing however many assumptions. It is obvious that the general parametrization method does not lead very far when applied to the em masses of mesons; indeed there are four parameters and four mass differences; moreover one of these, that of the ρ, is essentially unknown. Note also that even if, at the expense of losing generality, one were to introduce some assumption, there would be no reason why the coefficients t_μ [Eq. (8)] and their analogues for mesons should be equal or similar; in Ref. [7] such a difference was considered as something difficult to understand in the NRQM.

[10] To clarify this point by a simple example consider, say, a three-quark term such as $P_1^\lambda P_2^\lambda P_3^\lambda$. Introducing the strangeness $S=-\Sigma_i P_i^\lambda$ we can rewrite $P_1^\lambda P_2^\lambda P_3^\lambda$ in the form $P_1^\lambda P_2^\lambda P_3^\lambda=\frac{1}{6}[(-S)^3+3S(\Sigma_i P_i^{\lambda 2})+2(\Sigma_i P_i^{\lambda 3})]$; clearly three-quark terms are concealed by the collective variable S; the same occurs (with **J** replacing S) when we use Eq. (21).

[11] On this point compare, e.g., the remark in Ref. [1(d)], p. 1506.