# Analysis of $\eta, K_L \rightarrow \pi^+\pi^-\gamma$ using chiral models

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The decay $\eta \rightarrow \pi^+\pi^-\gamma$ is analyzed using two different approaches that incorporate vector mesons in the chiral Lagrangian, one which treats vector mesons as massive Yang-Mills bosons and one which treats them as dynamical gauge bosons of a hidden symmetry. From these approaches a common way of adding vector mesons to that decay emerges. A rate and photon spectrum are generated which compare reasonably to the experimental data. The procedure is then adapted into a simple pole model and used to calculate the more complicated decay $K_L \rightarrow \pi^+\pi^-\gamma$. Notwithstanding some uncertainties in the model, a rate that matches the experimental one is obtained with reasonable values of SU(3)-breaking parameters.

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## I. INTRODUCTION

In recent years there has been much work on effective chiral Lagrangians that include the Wess-Zumino non-Abelian anomaly. In particular, different models have attempted to incorporate vector mesons in an effort to provide a more detailed description of "intrinsic parity" odd processes, such as $\pi^0 \rightarrow \gamma\gamma$ and $\omega \rightarrow 3\pi$. Among the important questions has been the extent to which vector-meson dominance should be assumed for the photon coupling to hadrons. These efforts have been part of the larger endeavor to describe low-energy hadron physics in terms of effective chiral Lagrangians. A picture has emerged whereby low-energy QCD can be adequately described in terms of mesonic degrees of freedom instead of quarks and gluons when one incorporates correctly the underlying symmetries and anomalies of QCD.

There have been two fundamental approaches in developing these models, one based on treating the spin-one particles as massive Yang-Mills bosons and the other based on hidden symmetries. In the former approach, as developed by Schechter and collaborators [1–3], the chiral $U(3)_L \times U(3)_R$ symmetry is gauged by replacing the derivatives in the spin-zero Lagrangian according to the prescription

$$\partial_\mu U \rightarrow \partial_u U - igA_{\mu L}U + igUA_{\mu R} , \tag{1}$$

where $g$ is a gaugelike coupling constant. Early efforts [1] attempted to determine the parameters in the anomalous action by introducing the vector mesons as external gauge fields and gauging the strong Lagrangian. This procedure led to good predictions for the processes $\omega \rightarrow 3\pi$ and $\omega \rightarrow \pi^0\gamma$. However, the model did not conserve chiral symmetry without the introduction of additional terms [2]. Subsequent refinements led to a version which correctly satisfies the anomaly low-energy constraints, and the latest effort [3] has been the investigation of the most general $\pi\rho\omega$ Lagrangian of that type, which includes three parity- and chiral-invariant terms for the anomalous action. Combinations of the unknown constants were determined from the decays $\omega \rightarrow 3\pi$, $\phi \rightarrow 3\pi$, and $\phi \rightarrow \rho\pi$. Although there have been other developments along similar lines [4], here we will deal specifically with the model described in Ref. [3].

In the second approach, the vector mesons are identified as the dynamical gauge bosons of a hidden local symmetry in the nonlinear chiral Lagrangian. Bando *et al.* [5] first suggested the $\rho$ meson, and later Meissner and co-workers [6,7] expanded $H_{\text{local}}$ to include the $\omega$ meson. A form of the Wess-Zumino action in the presence of vector mesons was developed by Fujiwara *et al.* [8]. Here there are various ways in which gauge invariants can be included in the Lagrangian. A particular choice was made in Ref. [8] which yields the proper rate for $\omega \rightarrow 3\pi$, and other processes such as $\pi^0 \rightarrow 2\gamma$ and $\omega \rightarrow \pi^0\gamma$ are then correctly predicted.

The values of the parameters in these models determine the relative importance of the contact and pole terms, and also the extent to which vector-meson dominance is present in radiative processes. The symmetry principles limit the number of parameters quite drastically in both approaches. Moreover, aside from the numerical values of the parameters, the equivalence of the general structures of both schemes has been demonstrated [3,9]; there really seems to be little arbitrariness in the introduction of vector mesons in the effective theory. Indeed, in our case we find that radiative processes of the type $\eta \rightarrow \pi\pi\gamma$ can be described in a unique way by either method: the necessary requirements turn out to be that the anomaly constraint be met and that the combination of terms be such that the experimental rate for $\omega \rightarrow 3\pi$ be matched, and both requirements are met in either approach.

In this paper we first provide a brief outline of the models just mentioned. We then analyze the decay $\eta \rightarrow \pi^+\pi^-\gamma$ as a test of these models. This decay is particularly suited for that purpose because it does not involve the complications of weak interactions and the analysis is clean and straightforward, yet is not properly described by earlier vector-meson-dominance models [10,11]. We then calculate the rate for the decay

 

$K_L \to \pi^+\pi^-\gamma$. This process had been previously calculated [12] using the earliest version of the massive Yang-Mills model [1] but, as already mentioned, that version did not properly satisfy the anomaly constraints and therefore is not reliable for this decay. Indeed, the $\eta \to \pi^+\pi^-\gamma$ amplitude, which is itself an integral part of the $K_L \to \pi^+\pi^-\gamma$ calculation, was not adequately described [13]. In contrast, we find that both present approaches correctly predict the rate and spectrum for $\eta \to \pi^+\pi^-\gamma$ and also yield a result which matches the experimental rate for $K_L \to \pi^+\pi^-\gamma$ with reasonable values of SU(3)-breaking parameters.

## II. CHIRAL LAGRANGIANS

For completeness we give a brief outline of the models involved, emphasizing the results needed for our calculations. For a full description one may refer to the original papers or the excellent review by Meissner [9].

### A. Massive Yang-Mills

Following Ref. [3], the scalar nonet is denoted by $\phi$ and the linearly transforming matrix is

$$U = \exp\left[\frac{2i\phi}{F_\pi}\right], \tag{2}$$

where $F_\pi = 132$ MeV. The vector-meson nonet $\rho_\mu$ is related to the gauge fields by

$$\begin{aligned} A_\mu^L &= \xi\rho\xi^\dagger + \frac{i}{g}\xi\,\partial_\mu\xi^\dagger , \\ A_\mu^R &= \xi^\dagger\rho\xi + \frac{i}{g}\xi^\dagger\,\partial_\mu\xi , \end{aligned} \tag{3}$$

where $\xi = U^{1/2}$. The action is written down as

$$\Gamma = \int (\mathcal{L}_1 + \mathcal{L}_2)d^4x + \Gamma_3 . \tag{4}$$

$\mathcal{L}_1$ is a gauge-invariant kinetic term for the vectors, and $\mathcal{L}_2$ is a generalized mass term which breaks the strong gauge invariance. The third term $\Gamma_3$ contains terms proportional to the antisymmetric symbol $\epsilon_{\mu\alpha\beta\gamma}$, and it is primarily this one that concerns us here. One may write a general action which contains the Wess-Zumino (WZ) term plus all the possible chiral-invariant combinations. If one eliminates the axial-vector meson [2,3,9] and considers only the chiral-invariant terms which conserve $C$ and $P$, what is left is

$$\begin{aligned} \Gamma_3 = \Gamma_{WZ}(U) &+ \int \mathrm{Tr}\Big[ic_1(A_L\alpha^3) \\ &+ c_2(dA_L\alpha A_L - A_L\alpha A_L + A_L\alpha A_L\alpha) \\ &+ c_3\Big(-3iA_L^3\alpha + \frac{i}{g}A_L\alpha A_L\alpha\Big)\Big] , \end{aligned} \tag{5}$$

where $\Gamma_{WZ}$ is the Wess-Zumino term, and $\alpha = (\partial_\mu U)U^{-1}dx_\mu$. The constants $c_1, c_2, c_3$ are determined from the strong decays. This can be done by extracting from Eq. (5) the terms that describe the vector[2]-pseudoscalar and vector-pseudoscalar[3] couplings. Using Eqs. (2) and (3), we can expand $\Gamma_3$ as follows:

$$\begin{aligned} \Gamma_3 = \epsilon_{\mu\nu\alpha\beta}\int d^4x[&-ig_{VV\phi}\mathrm{Tr}(\partial_\mu\rho_\nu\,\partial_\alpha\rho_\beta) \\ &- h\,\mathrm{Tr}(\rho_\nu\,\partial_\nu\phi\,\partial_\alpha\phi\,\partial_\beta\phi + \cdots)] , \end{aligned} \tag{6}$$

where

$$g_{VV\phi} = \frac{4ic_2}{F_\pi}, \quad h = \frac{-4i}{F_\pi^3}\left[2c_1 - \frac{2c_2}{g} - \frac{c_3}{g^2}\right] . \tag{7}$$

Electromagnetic interactions are included as follows: under an infinitesimal local U(3)×U(3) transformation, a nonstrong gauge field $B_{L,R}$ with coupling constant $k$ changes by

$$\delta B_{L,R} = -[B_{L,R}, E_{L,R}] - \frac{i}{k}dE_{L,R} . \tag{8}$$

To include electromagnetic interactions we let $B_{L,R} \to eQA_\mu$, where $A_\mu$ is the photon field and $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$. $\Gamma_{WZ}(U)$ is gauged and becomes $\Gamma_{WZ}(U, B_L, B_R)$ [3]. Then the other terms in $\Gamma_3$ are required to be gauge invariant. This is done by constructing gauge-invariant terms out of the field-strength tensor and covariant derivatives of the field $U$. Four terms conserve $P$ and $C$ and are locally gauge invariant. They are

$$\begin{aligned} &\mathrm{Tr}(\alpha_1\alpha_2\alpha_1\alpha_2) = \mathrm{Tr}(\beta_1\beta_2\beta_1\beta_2) , \\ &\mathrm{Tr}(\alpha_1^3\alpha_2 - \alpha_2^3\alpha_1) = \mathrm{Tr}(\beta_1^3\beta_2 - \beta_2^3\beta_1) , \\ &\mathrm{Tr}\{F(A_L)[\alpha_1, \alpha_2]\} = \mathrm{Tr}\{F(A_R)[\beta_1, \beta_2]\} , \\ &\mathrm{Tr}\{F(B_L)[\alpha_1, \alpha_2] + F(B_R)[\beta_1, \beta_2]\} , \end{aligned} \tag{9}$$

where

$$\begin{aligned} F(A_L) &= dA_L - igA_L^2 , \\ \alpha_1 &= \alpha - igA_L + ikUB_RU^\dagger , \\ \alpha_2 &= \alpha + igA_L - ikB_L , \\ \beta_{1,2} &= U^\dagger\alpha_{1,2}U . \end{aligned} \tag{10}$$

To see how these terms in Eq. (9) participate in the action, set $B$ to zero to find the linearly independent combination of the $c_1, c_2, c_3$ terms in Eq. (5) that they correspond to. The fourth term vanishes. The proper action is then found to be

$$\begin{aligned} \Gamma_{WZ}(U, B_L, B_R) &+ \int \mathrm{Tr}\left[-\frac{c_1}{g}(\alpha_1^3\alpha_2 - \alpha_2^3\alpha_1)\right] - \frac{c_2}{ig}\{F(A_L)[\alpha_1, \alpha_2]\} \\ &+ \left[-\frac{c_1}{g} + \frac{c_2}{g^2} + \frac{c_3}{g^3}\right]\mathrm{Tr}(\alpha_1\alpha_2\alpha_1\alpha_2) + d_1\{F(B_L)[\alpha_1, \alpha_2] + F(B_R)[\beta_1, \beta_2]\} , \end{aligned} \tag{11}$$

where $d_1$ is a new constant.

The anomaly constraint requires that, in the low-momentum limit, all the terms in Eq. (9) vanish except for the contribution coming from $\Gamma_{WZ}(U,B_L,B_R)$. The values of $c_1, c_2, c_3$ are determined from strong-interaction experiments. In particular, the authors of Ref. [3] used the processes $\omega \to 3\pi$, $\phi \to 3\pi$, and $\phi \to \rho\pi$. In our case, in order to calculate the rate for $\eta \to \pi^+\pi^-\gamma$, the individual values of these parameters are not needed but certain conditions must be met, as will be elaborated later. Incidentally, while the low-energy expression for the amplitude is totally given by $\Gamma^{WZ}(U,B_L,B_R)$ [14], the corrections from the other parts of Eq. (11) are substantial at the momentum values involved in $\eta \to \pi^+\pi^-\gamma$; in fact, $\Gamma_{WZ}(U,B_L,B_R)$ alone [13] does not come close to matching the experimental rate.

### B. Hidden symmetry

We follow the work of Fujiwara *et al.* [8], whereby the scalar and pseudoscalar nonets are represented by $3\times 3$ matrix fields and the vector nonet by $V_\mu$. They transform accordingly under $[\mathrm{U}(3)_L \times \mathrm{U}(3)_R]_{\mathrm{global}} \times [\mathrm{U}(3)_V]_{\mathrm{local}}$. $U$ is written in terms of the Nambu-Goldstone pion fields as $U=\exp(2i\pi/f_\pi)$, where $f_\pi=93$ MeV is the pion decay constant. The matrix variables $\xi_L$ and $\xi_R$ are defined such that $U=\xi_L^\dagger \xi_R$. The full group is gauged with external fields $A_{L\mu}$ and $A_{R\mu}$ including, in particular, the electromagnetic field $B_\mu$, in which case alone we would have $A_{L\mu}=A_{R\mu}=eB_\mu Q$, with $Q$ the quark charge matrix. By expanding the Lagrangian and picking off individual terms, it is possible to relate the gauge coupling $g$ and $f_\pi$ to $m_V$, the vector-photon coupling $g_V$, and $g_{V\pi\pi}$. In particular,

$$\frac{g_V}{m_V^2}=\frac{1}{g}$$

and (12)

$$\frac{g_{V\pi\pi}}{m_V^2}=\frac{1}{2f_\pi^2 g} .$$

To deal with the anomaly including vector mesons, one defines the building blocks

$$\begin{aligned} &\alpha_{L,R}=d\xi_{L,R}\xi_{L,R}^\dagger, \quad \hat{A}_{L,R}=\xi_{L,R}A_{L,R}\xi_{L,R}^\dagger , \\ &\hat{\alpha}_{L,R}=\alpha_{L,R}-igV+i\hat{A}_{L,R} , \\ &F_V=dV-igV^2, \quad \hat{F}_{L,R}=\xi_{L,R}F_{L,R}\xi_{L,R}^\dagger , \end{aligned} \tag{13}$$

where $\alpha=(\partial_\mu U)U^{-1}dx^\mu$. Then the general form for the anomalous action is

$$\Gamma=\Gamma_{WZ}+\sum_i c_i\mathcal{L}_i , \tag{14}$$

where $\Gamma_{WZ}$ is the original Wess-Zumino term and the $\mathcal{L}_i$ are gauge-covariant terms which are homogeneous solutions of the original Wess-Zumino action. There are four terms which can be made $C$ and $P$ invariant, and they are

$$\begin{aligned} &\mathcal{L}_1=\mathrm{Tr}(\hat{\alpha}_L^3\hat{\alpha}_R-\hat{\alpha}_R^3\alpha_L) , \\ &\mathcal{L}_2=\mathrm{Tr}(\hat{\alpha}_L\hat{\alpha}_R\hat{\alpha}_L\hat{\alpha}_R) , \\ &\mathcal{L}_4=i\,\mathrm{Tr}[F_V(\hat{\alpha}_L\hat{\alpha}_R-\hat{\alpha}_L\hat{\alpha}_R)] , \\ &\mathcal{L}_6=i\,\mathrm{Tr}(\hat{F}_L\hat{\alpha}_L\hat{\alpha}_R-\hat{F}_R\hat{\alpha}_R\hat{\alpha}_L) . \end{aligned} \tag{15}$$

As shown in Ref. [3], when the last term is made $C$ invariant these four terms correspond exactly to those given in Eq. (9). Thus, all the $\mathcal{L}_i$ vanish in the low-energy limit, and $\Gamma$ converges to the old $\Gamma_{WZ}$ as required. Different values of $c_i$ will then determine the extent of vector-meson dominance and the balance between contact terms and those which contain intermediate vector mesons. The constants $c_i$ can be fitted to the data, and the authors in Ref. [8] chose $c_1-c_2=-1$, $c_4=c_6=1$. One may see the effect of this choice by expanding the terms in Eq. (4) and picking off the contributions to a particular coupling. Then, in the notation of Ref. [8], this particular choice of constants yields

$$\Gamma=[3(VV\pi)-2(\gamma\pi^3)+\cdots] . \tag{16}$$

For comparison, $\Gamma_{WZ}$ alone would generate terms of the form

$$\Gamma_{WZ}=[4(\gamma\pi^3)+\cdots] , \tag{17}$$

while a different choice of constants that would correspond to "complete vector dominance" would yield

$$[3(VV\pi)-2(V\pi^3)+\cdots] . \tag{18}$$

The particular combination in Eq. (16) chosen by these authors makes $\pi^0 \to 2\gamma$ proceed entirely via $\pi^0 \to \rho+\omega$ followed by $\rho \to \gamma$, $\omega \to \gamma$, and also makes the contact term in the decay $\omega \to 3\pi$ vanish. With this choice they calculated $\Gamma(\omega \to 3\pi) \simeq 9.1$ MeV compared to the experimental value $8.9\pm 0.3$ MeV. On the other hand, Eq. (18) leads to the "bad" prediction $\Gamma(\omega \to 3\pi) \simeq 6.1$ MeV. By contrast, Jain *et al.* [3] determined the combination of terms in Eq. (5) by doing a simultaneous fit of $\omega \to 3\pi$, $\phi \to 3\pi$, and $\phi \to \rho\pi$. That way they determined their $c_2$ and the linear combination of their $c_1, c_2, c_3$ called $h$. A vanishing $h$ would correspond to no contact term in $\omega \to 3\pi$. Instead, these authors found

$$\tilde{g}_{VV\phi}=g_{VV\phi}F_\pi=\pm 1.9, \quad \tilde{h}=hF_\pi^3=\pm 0.4 . \tag{19}$$

In other words, they used a broader set of inputs and found a combination of terms that satisfies the $\omega \to 3\pi$ rate which is somewhat different from the combination used by Fujiwara *et al.* [8]. The rate for $\pi^0 \to 2\gamma$ is guaranteed in any case by the anomaly constraint when the intermediate vector mass is zero.

### III. THE DECAY $\eta \to \pi^+\pi^-\gamma$

The two models described above have been found to be fundamentally equivalent [3,9]. Furthermore, the difference in the linear combinations used in the two approaches does not affect the rate for $\eta \to \pi^+\pi^-\gamma$. The reason for this can be seen by looking at the three contributions to that process, shown in Fig. 1. Figures 1(b) and

1(c) are intimately related to the decay $\omega \to 3\pi$; by erasing the photon lines one gets essentially the two graphs that can contribute to that process (with external $\omega, \pi$ instead of $\rho, \eta$). Therefore, any combination of the two that will match the $\omega \to 3\pi$ rate will give the same result for $\eta \to \pi^+\pi^-\gamma$. The balance required to satisfy the anomaly at zero momentum is supplied by the graph in Fig. 1(a). The correct contributions are ensured by the relations given in Eq. (12). The particular combination in Eq. (16) makes the contribution from Fig. 1(b) vanish.

To calculate $\eta \to \pi^+\pi^-\gamma$, we use the mixing prescription

$$|\eta\rangle = \cos\theta|\eta^8\rangle - \sin\theta|\eta^0\rangle ,$$
$$|\eta'\rangle = \sin\theta|\eta^8\rangle + \cos\theta|\eta^0\rangle , \quad (20)$$

with $\theta = -20.6°$. The amplitude is then given by

$$A(\eta \to \pi^+\pi^-\gamma) = \frac{ie}{4\pi^2 f_\pi^2}\epsilon^{\mu\nu\alpha\beta}p_\mu^+ p_\nu^- k_\alpha \epsilon_\beta \times \left[\frac{1}{2} - \frac{3}{2}\frac{m_\rho^2}{m_\rho^2 - p_\rho^2}\right] \times \left[\frac{1}{\sqrt{3} f_8}\cos\theta - \left[\frac{2}{3}\right]^{1/2}\frac{1}{f_0}\sin\theta\right] , \quad (21)$$

where $\epsilon_\beta$ is the photon polarization. We parametrize a source of uncertainty in SU(3) breaking by using the physical values $f_8 = 1.25 f_\pi$ and $f_0 = 1.04 f_\pi$, taken from Donoghue *et al.* [15]. The contact term interferes significantly with that generated by the pole diagrams, and thus the resulting rate is quite different from those of earlier attempts [10,11] to use simple vector-meson dominance to describe this decay. A numerical integration over phase space yields the decay width

$$\Gamma(\eta \to \pi^+\pi^-\gamma) = 62 \text{ eV} \quad (22)$$

compared to the experimental result [16] $\Gamma(\text{expt}) = 64 \pm 6$ eV. In stressing the effect of the propagator, the authors of Ref. [16] demonstrated that the rate obtained with the simplest gauge-invariant amplitude (without the $\rho$ propagator) does not match the experimental photon spectrum. Figure 2 shows our result (with the detector efficiency folded in) compared to the experimental spectrum of Gromley *et al.* [16].

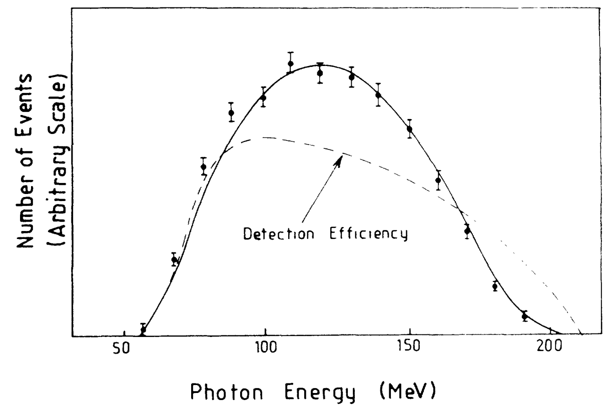


FIG. 2. Prediction for $\eta \to \pi^+\pi^-\gamma$ compared to the experimental spectrum of Gromley *et al.* [16].

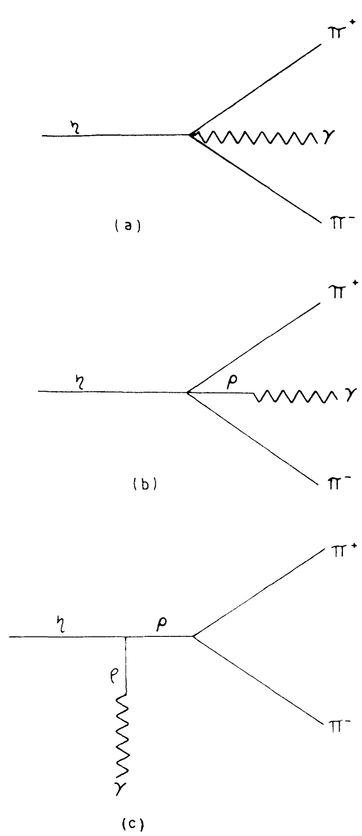


FIG. 1. Graphs for $\eta \to \pi^+\pi^-\gamma$.

## IV. THE DECAY $K_L \to \pi^+\pi^-\gamma$

The rate of *CP*-violating inner bremsstrahlung was calculated in Ref. [12] in a straightforward fashion. These authors studied the direct emission term as well, using

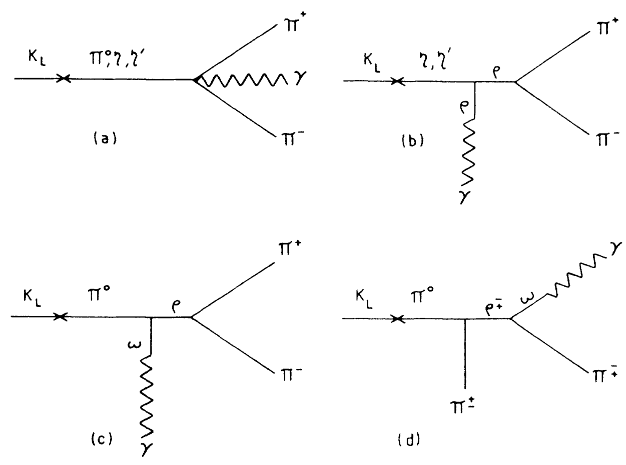


FIG. 3. Graphs for $K_L \to \pi^+\pi^-\gamma$ direction emission.

the model of Kaymakcalan *et al.* [1], with the hope of finding a substantial *CP*-violating contribution which might afford an opportunity to study direct *CP* violation. However, for reasons mentioned above, that approach was unsuitable [13]. We now calculate the direct emission term using the models described here. The relevant diagrams corresponding to the scheme in Eq. (16) are shown in Fig. 3. The graphs in Fig. 3(d) were not included in the earlier analysis [12], but they are important in ensuring consistency with the predictions of the anomaly at low energies, and they affect the rate and spectrum significantly. We assume that the vertices $K_L \to \pi^0$, $K_L \to \eta$, $K_L \to \eta'$ can be related by nonet symmetry. Following Ref. [15], we describe SU(3) breaking and the inclusion of the singlet via the parameters $\xi$ and $\rho$. Then we have

$$\frac{\langle \eta^8|L|K_L\rangle}{\langle \pi^0|L|K_L\rangle} = \sqrt{1/3}(1+\xi) , \qquad \frac{\langle \eta^0|L|K_L\rangle}{\langle \pi^0|L|K_L\rangle} = -2\sqrt{2/3}\rho . \tag{23}$$

This approach, although not uncommon, may introduce some additional uncertainty in our treatment of $\eta'$. The amplitude generated by the graphs in Fig. 3 then becomes

$$A(K_L\to\pi^+\pi^-\gamma) = -\frac{\langle \pi^0|L|K_L\rangle}{m_K^2-m_\pi^2}\frac{e}{8\pi^2 f_\pi^3} \times \epsilon^{\mu\nu\alpha\beta} p_\mu^+ p_\nu^- k_\alpha \epsilon_\beta (A_\pi + A_\eta + A_{\eta'}) ,$$

$$A_\pi = 1 - m_\rho^2 \left[ \frac{1}{m_\rho^2-(p^++p^-)^2} + \frac{1}{m_\rho^2-(p^++k)^2} + \frac{1}{m_\rho^2-(p^-+k)^2} \right] ,$$

$$A_\eta = \left[ 1 - \frac{3m_\rho^2}{m_\rho^2-(p^++p^-)^2} \right] \frac{m_K^2-m_\pi^2}{m_K^2-m_\eta^2} \times \left[ \frac{1+\xi}{\sqrt{3}}\cos\theta + 2\left[\frac{2}{3}\right]^{1/2} \rho\sin\theta \right] \times \left[ \frac{f_\pi}{\sqrt{3}f_8}\cos\theta - \left[\frac{2}{3}\right]^{1/2} \frac{f_\pi}{f_0}\sin\theta \right] ,$$

$$A_{\eta'} = \left[ 1 - \frac{3m_\rho^2}{m_\rho^2-(p^++p^-)^2} \right] \frac{m_K^2-m_\pi^2}{m_K^2-m_{\eta'}^2} \times \left[ \frac{1+\xi}{\sqrt{3}}\sin\theta - 2\left[\frac{2}{3}\right]^{1/2} \rho\cos\theta \right] \times \left[ \frac{f_\pi}{\sqrt{3}f_8}\sin\theta + \left[\frac{2}{3}\right]^{1/2} \frac{f_\pi}{f_0}\cos\theta \right] .$$

To be consistent with the choice of parameters [15], we use $\langle \pi^0|L|K_L\rangle = -0.035\ \mathrm{MeV}^2$. The branching ratio is obtained by a numerical integration over phase space.

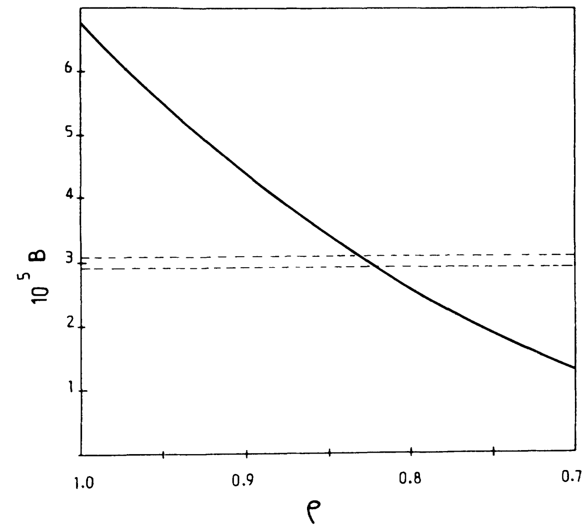


FIG. 4. Branching ratio $B(K_L\to\pi^+\pi^-\gamma)$ for direct emission as a function of the singlet parameter $\rho$. The dotted lines represent the experimental result [17].

The parameter $\xi$ was calculated [15] at one loop in chiral perturbation theory, obtaining $\xi=0.17$. $\rho$ cannot be similarly calculated, but the authors of Ref. [15] found that the rate for $K_L\to\gamma\gamma$ is consistent with a value for $\rho$ close to but noticeably smaller than 1. In Fig. 4 we show the branching ratio $B(K_L\to\pi^+\pi^-\gamma)$ for $\xi=0.17$ for different values of $\rho$. The latest experimental result [17] $B(\mathrm{expt})=(2.98\pm0.08)\times10^{-5}$ can be easily matched with a value $\rho\simeq0.83$. One should note that for $K_L\to\pi^+\pi^-\gamma$, as was the case [15] for $K_L\to\gamma\gamma$, the rate is sensitive to small changes in these parameters. It should also be pointed out that the calculation only includes weak transitions on the external kaon leg, as shown in Fig. 3, and does not include any direct weak amplitudes. Here, as in previous similar calculations of kaon decays, we are constrained by our incomplete understanding of weak interactions. There is evidence, for example [18] for the case $K_L\to\pi\gamma\gamma$, that direct amplitudes may interfere significantly with external weak transition amplitudes.

## V. CONCLUSION

Two recent chiral models that incorporate vector mesons in the Wess-Zumino anomaly have been reviewed with the aim of analyzing the decay $\eta\to\pi^+\pi^-\gamma$. $\eta$ decays are, in principle, easier to analyze in chiral theories because weak interactions are not present. We found that both types of approaches, one based on massive Yang-Mills bosons and one based on the hidden local symmetry of the nonlinear chiral Lagrangian, generate the same amplitude for this kind of decay and lead to a reasonable comparison with the experimental rate and photon spectrum.

We then similarly analyzed the decay $K_L\to\pi^+\pi^-\gamma$. The amplitude was obtained by assuming that the transition is mediated by $K_L\to\pi^0,\eta,\eta'$, with these particles decaying into the final state as in $\eta \to \pi^+ \pi^- \gamma$. Notwithstanding the uncertainties inherent in this approach, we found that we can obtain a rate which matches the experimental one with reasonable values of SU(3)-breaking parameters, consistent with those used [15] for the analysis of $K_L \to \gamma\gamma$.

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