Heavy-quark symmetry and $D_1(2420) \to D^* \pi$ decay

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Heavy-quark symmetry relates the D-wave amplitude for $D_1(2420) \to D^* \pi$ to the amplitudes for the decays $D_2^*(2460) \to D^* \pi$ and $D_2^*(2460) \to D \pi$. We discuss the extraction of the S and D partial wave amplitudes for $D_1 \to D^* \pi$ and their significance as tests of the applicability of heavy-quark symmetry for charm quarks.

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The physics of hadrons containing a single heavy quark Q (by "heavy," we mean $m_Q \gg \Lambda_{\rm QCD}$) is greatly simplified by going over to an effective theory where the mass of the heavy quark goes to infinity with its four-velocity fixed [1–12]. For N heavy quarks, this effective theory has an SU($2N$) spin-flavor symmetry group that acts on heavy quarks with the same four-velocity [1,2]. The heavy-quark symmetry can be used to predict many nonperturbative properties of hadrons containing a heavy quark. The most important of these predictions are for exclusive semileptonic B-meson decays. They are expected to play an important role in determining the magnitudes of the elements V_{ub} and V_{cb} of the Cabibbo-Kobayashi-Maskawa matrix.

The weakest feature of this approach to determining $|V_{ub}|$ and $|V_{cb}|$ is treating the charm quark as heavy. In this paper we examine a system which bears on the issue of whether the charm quark mass is sufficiently large for the limit $m_c \to \infty$ to be a good approximation.

In addition to predicting weak matrix elements, heavy-quark symmetry relates the masses and strong decay amplitudes of excited hadrons containing a single heavy quark Q (Ref. [13]). In the limit $m_Q \to \infty$ the spin $\mathbf{S}_Q$ of the heavy quark and the spin $\mathbf{S}_l \equiv \mathbf{S} - \mathbf{S}_Q$ of the light degrees of freedom are separately conserved by the strong interactions; hadrons containing a single heavy quark can therefore be simultaneously assigned the quantum numbers s_Q, m_Q, π_Q, s_l, m_l, and π_l, where $\pi_Q = +1$ and π_l are the parities of the heavy quark and light degrees of freedom, respectively. Since the dynamics of the light degrees of freedom are independent of the mass and spin of the heavy quark it is convenient, in the limit $m_Q \to \infty$, to classify states by s_l and π_l. Then associated with each such state of the light degrees of freedom will be a degenerate doublet of hadrons with total spin parity

$$s_\pm^\pi = (s_l \pm \tfrac{1}{2})^{\pi_l} , \tag{1}$$

unless $s_l = 0$ in which case a single spin-$\frac{1}{2}$ state is obtained. Since the state of the light degrees of freedom is independent of the heavy-quark mass, the spectrum is identical for each heavy-quark flavor, up to an overall shift associated with the mass of the heavy quark.

Since the heavy-quark flavor symmetry applies to the complete set of n-point functions of the theory, not only mass splittings but also all strong decay amplitudes arising from emission of light states such as $\pi, \eta, \rho, \pi\pi$, etc., are independent of the heavy-quark flavor [13]. For a given flavor, because $\mathbf{S}_Q$ generates a symmetry, the two states with spin $s_\pm$ must have the same total widths. This equality between total widths typically arises in a nontrivial way: the two states of a given multiplet can decay to both states of every available multiplet with distinct partial widths whose sum must be identical. The spin symmetry predicts the ratios of these partial widths.

At the present time, only for $Q = c$ have excited hadrons containing a single heavy quark been observed. They are the charmed mesons $D_2^*(2460)$ and $D_1(2420)$. For mesons containing a single heavy quark both the constituent quark model and experiment tell us that the ground states have $s_l^{\pi_l} = \frac{1}{2}^-$ giving $s_-^\pi = 0^-$ and $s_+^\pi = 1^-$ states. In the case $Q = c$ these are the D and D^* mesons. The constituent quark model suggests that the lowest-lying excited states are likely to be those that correspond to the spin-$\frac{1}{2}$ constituent antiquark having a unit of orbital angular momentum, giving $s_l^{\pi_l} = \frac{1}{2}^+$ and $\frac{3}{2}^+$ multiplets.

Heavy-quark spin symmetry predicts [13,14] that the $s_+^\pi = 2^+$ state of the $s_l^{\pi_l} = \frac{3}{2}^+$ charm-meson multiplet has decay amplitudes in the proportion $\sqrt{2/5}$: $\sqrt{3/5}$ to the states $[D\pi]_D$ and $[D^*\pi]_D$, respectively, while its multiplet partner with $s_-^\pi = 1^+$ decays at the same total rate exclusively to $[D^*\pi]_D$. Note that in the heavy-quark limit $m_c \to \infty$, this latter state does not decay to $[D^*\pi]_S$ even though this is an allowed channel. (Here the subscript S or D denotes the partial wave of the pion). For the $s_l^{\pi_l} = \frac{1}{2}^+$ multiplet of charmed mesons heavy-quark symmetry predicts [13,14] that the $s_+^\pi = 1^+$ state decays exclusively to $[D^*\pi]_S$ (even though $[D^*\pi]_D$ is an allowed channel) and that its $s_-^\pi = 0^+$ partner decays to $[D\pi]_S$ with the same amplitude.

These predictions are not inconsistent with existing experimental information [15]. If one interprets the two

confirmed states $D_2^*(2460)$ and $D_1(2420)$ as members of the $s_l^{\pi_l}=\frac{3}{2}^+$ multiplet, then their mass difference is consistent with being a $\Lambda_{\rm QCD}/m_c$ correction to the limiting theory. Moreover, the $D_2^*\to D\pi$ and $D_2^*\to D^*\pi$ decays (which are *necessarily* D waves) have amplitudes that are in the ratio 0.8 ± 0.1, which is close to the prediction $\sqrt{2/3}$. [Here, and in what follows, we quote amplitudes with a phase space and "typical" barrier-penetration factor of $[p^{2L+1}\exp(-p^2/(1\ \text{GeV}^2))]^{1/2}$ removed. This amounts to including the effects of those $\Lambda_{\rm QCD}/m_c$ corrections which are enhanced in importance because p, the magnitude of the pion three-momentum, is not very large compared with the D^*-D mass splitting.]. The ratio of the $D_1(2420)\to D^*\pi$ and $D_2^*(2460)\to D^*\pi$ decay amplitudes is 2.3 ± 0.6. Given the substantial error, it is not clear whether this result is in disagreement with the $m_c\to\infty$ prediction of $\sqrt{5/3}$. The examination of the $D_1\to D^*\pi$ decay in more detail is the main purpose of this paper.

While heavy-quark symmetry does not relate S-wave to D-wave decay amplitudes, the constituent quark model suggests that the S-wave decay amplitudes are very strong [16]. This may explain the difficulty so far in seeing the members of the $s_l^{\pi_l}=\frac{1}{2}^+$ multiplet which consequently are expected to have large widths. This observation suggests that the decay of $D_1(2420)$ may be contaminated via $\Lambda_{\rm QCD}/m_c$ corrections by an S-wave component. (For example, such corrections could mix the $s_l^{\pi_l}=\frac{3}{2}^+$ and $s_l^{\pi_l}=\frac{1}{2}^+ D_1$ states. Since the quark model predicts that the S-wave widths are an order of magnitude larger than the D-wave widths, even a weak mixing could produce a substantial effect.) To rigorously test the predictions of heavy-quark symmetry, it will therefore be necessary to separately determine the D- and S-wave amplitudes of the decay $D_1(2420)\to D^*\pi$. Heavy-quark symmetry must then pass two tests: the D-wave amplitude must be correctly related to those of the $D_2^*(2460)$, and the S-wave amplitude must be small compared to other "typical" S-wave amplitudes, e.g., the decay amplitudes for the members of the $s_l^{\pi_l}=\frac{1}{2}^+$ multiplet. (Note, however, that it need not be small compared with the $D_1\to[D^*\pi]_D$ decay amplitude: a small grapefruit can be larger than a typical apple.) It already seems likely that the second test will be met. The total width of the $D_1(2420)$ is only about 20 MeV, and this is much smaller than typical S-wave widths (e.g., $a_1\to[\rho\pi]_S$, which is the light-quark analogue of the D_1 decays, has a width of about 350 MeV).

The $D_1\to D^*\pi$ decay matrix element can be written, in the rest frame of the D_1, as

$$M(D_1\to D^*\pi)=[Se^{i\phi_{SD}}-(\tfrac{1}{2})^{1/2}D]\epsilon^*\cdot\epsilon_1+\frac{3}{\sqrt{2}}D(\epsilon^*\cdot\hat{\mathbf{p}})(\epsilon_1\cdot\hat{\mathbf{p}})\ . \tag{2}$$

In Eq. (2), S and $D\equiv\tilde{D}(p/\beta)^2$ are real constants corresponding to the S- and D-wave amplitudes, $\phi_{\rm SD}$ is their relative phase, $\hat{\mathbf{p}}$ is a unit vector in the direction of the pion's three-momentum $\mathbf{p}$, β is a momentum scale characteristic of the decay, ϵ is the polarization vector for the D^* state, and ϵ_1 is the polarization vector for the D_1 state. A factor of $3/\sqrt{2}$ and the centrifugal barrier factor $(p/\beta)^2$ have been inserted into the definition of D for convenience [see, e.g., Eq. (5)]. Since the polarization of the D_1 depends on its production mechanism (i.e., in general it will be a mixed state), we use the density-matrix formalism to describe it. We assume that the density matrix depends only on the direction of the D_1, which we call $\hat{\mathbf{z}}$ (Ref. [17]). Then

$$\rho^{ij}=\left[\frac{1-f_L}{2}\right]\delta^{ij}+\left[\frac{3f_L-1}{2}\right]\hat{z}^i\hat{z}^j-\frac{i}{2}P\epsilon^{ijk}\hat{z}^k\ , \tag{3}$$

where f_L is the probability that the D_1 is in a state with zero helicity ($0\le f_L\le1$) and $-1+f_L\le P\le1-f_L$ is the net polarization of the D_1 with respect to the direction $\hat{z}$. Combining Eqs. (2) and (3) yields

$$\begin{aligned}\frac{d\Gamma}{d\Omega}=&\left[S^2+\frac{1}{2}D^2-\sqrt{2}\,SD\cos\phi_{SD}\right]p\left[\left[\frac{1-f_L}{2}\right]+\left[\frac{3f_L-1}{2}\right]|\epsilon\cdot\hat{\mathbf{z}}|^2-\frac{iP}{2}\epsilon^*\cdot(\epsilon\times\hat{z})\right]\\&+3\sqrt{2}\,\mathrm{Re}\left\{D[Se^{i\phi_{SD}}-(\tfrac{1}{2})^{1/2}D]p\left[\left[\frac{1-f_L}{2}\right]|\epsilon\cdot\hat{\mathbf{p}}|^2+\left[\frac{3f_L-1}{2}\right](\epsilon^*\cdot\hat{\mathbf{z}})(\epsilon\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\hat{\mathbf{z}})-\frac{iP}{2}\epsilon^*\cdot(\hat{\mathbf{p}}\times\hat{\mathbf{z}})(\epsilon\cdot\hat{\mathbf{p}})\right]\right\}\\&+\frac{9}{2}D^2p\left[\left[\frac{1-f_L}{2}\right]|\epsilon\cdot\hat{\mathbf{p}}|^2+\left[\frac{3f_L-1}{2}\right]|\epsilon\cdot\hat{\mathbf{p}}|^2|\hat{\mathbf{p}}\cdot\hat{\mathbf{z}}|^2\right]\ .\end{aligned} \tag{4}$$

From Eq. (4) we can derive [18] all of the results we need to determine D and S. Several distinct types of measurements are required because in addition to the magnitudes S and D, which are of immediate interest, measurements typically will depend on the relative phase ϕ_{SD} as well as the parameters f_L and P describing the density matrix. We can eliminate the dependence on P, thereby reducing the required number of measurements, by concentrating on unpolarized rates and polarization effects involving helicity-zero D^*'s (or ones longitudinally polarized with respect to $\hat{\mathbf{z}}$). See Eq. (4) with $\epsilon=-\hat{\mathbf{p}}$ (or $\epsilon=\hat{\mathbf{z}}$).

We first sum over the possible polarizations of the D^* (using $\sum_\epsilon\epsilon^{*i}\epsilon^j=\delta^{ij}$). Equation (4) then yields

$$\Gamma=4\pi p(D^2+S^2) \tag{5}$$

and

$$\frac{4\pi}{\Gamma}\frac{d\Gamma}{d\Omega}=1+\left[\frac{3f_L-1}{2}\right]\left[\frac{D^2+2\sqrt{2}\,DS\cos\phi_{SD}}{D^2+S^2}\right]\left[\frac{3\cos^2\theta-1}{2}\right]. \tag{6}$$

Here θ is the angle between the direction of the D_1 and the direction of the pion (in the D_1 rest frame). Note that if $f_L=\frac{1}{3}$, the distribution is isotropic, while if $f_L=0$ the angular distribution for a pure D wave is proportional to $5-3\cos^2\theta$. Instead of averaging over the D^* spins, we can also easily calculate from Eq. (4) the rate for D^*'s with zero helicity, i.e., with $\epsilon=-\hat{\mathbf{p}}$. This gives

$$\frac{4\pi}{\Gamma}\frac{d\Gamma_L}{d\Omega}=\left[1+\left[\frac{D^2+2\sqrt{2}\,DS\cos\phi_{SD}}{D^2+S^2}\right]\right]\left[\left[\frac{1-f_L}{2}\right]+\left[\frac{3f_L-1}{2}\right]\cos^2\theta\right]. \tag{7a}$$

Note that for $f_L=\frac{1}{3}$ the angular distribution is once again isotropic, while now for $f_L=0$ it is proportional to $\sin^2\theta$, independent of the D/S ratio. The total rate for D_1 decay to zero-helicity D^*'s is independent of f_L:

$$\frac{\Gamma_L}{\Gamma}=\frac{1}{3}\left[1+\left[\frac{D^2+2\sqrt{2}\,DS\cos\phi_{SD}}{D^2+S^2}\right]\right]. \tag{7b}$$

Since Eqs. (6) and (7) all depend on the same combination

$$\left[\frac{D^2+2\sqrt{2}\,DS\cos\phi_{SD}}{D^2+S^2}\right],$$

additional constraints are required. We therefore turn to the (P-independent) decay rate to D^*'s polarized longitudinally along $\hat{\mathbf{z}}$, which is

$$\frac{4\pi}{\Gamma}\frac{d\Gamma_{\hat{z}}}{d\Omega}=f_L+\frac{2\sqrt{2}\,f_L SD\cos\phi_{SD}}{S^2+D^2}\left[\frac{3\cos^2\theta-1}{2}\right]-\frac{D^2}{2(S^2+D^2)}\left[f_L+3\left[\frac{7f_L-3}{2}\right]\cos^2\theta-9\left[\frac{3f_L-1}{2}\right]\cos^4\theta\right]. \tag{8a}$$

The total decay rate to D^*'s polarized longitudinally along $\hat{\mathbf{z}}$ is

$$\frac{\Gamma_{\hat{z}}}{\Gamma}=f_L-\frac{3D^2}{5(D^2+S^2)}\left[\frac{3f_L-1}{2}\right]. \tag{8b}$$

The measurements (5)–(8) overdetermine D, S, ϕ_{SD}, and f_L, so that many strategies can be devised for extracting the interesting amplitudes D and S from them. As an extreme example, we note that a fit to the single angular distribution in (8a) can, in principle, simultaneously determine f_L, S/D, and ϕ_{SD}, so that it and the total rate determine all four parameters. A more realistic strategy might be to use the angular distribution in (7a) to determine f_L, (8b) to determine S/D, and then (5) to determine D and S separately. This strategy avoids ϕ_{SD}.

It should be noted that no strategy based on these measurements will work if the D_1's are produced with $f_L=\frac{1}{3}$: in this case Eqs. (6) and (8b) are independent of S/D, while Eqs. (7a), (7b), and (8a) depend only on

$$\left[\frac{D^2+2\sqrt{2}\,DS\cos\phi_{SD}}{(D^2+S^2)}\right].$$

This limitation is not, however, a serious one since there are specific production mechanisms for which the D_1's are aligned. For example, a D_1 produced in the decay $\bar{B}\to D_1\pi$ is in a zero-helicity state, $f_L=1$, to conserve angular momentum. Also, assuming factorization [19] of the weak nonleptonic decay amplitude, in $\bar{B}\to D_1\rho^-$ decay the D_1 is mostly emitted with zero helicity. Thus, if necessary, a selection of D_1's can be made which will have $f_L\neq\frac{1}{3}$. Of course in practice the optimal strategy will depend on experimental conditions and so must be devised by those doing these measurements.

In this paper we have seen how, in the decay $D_1\to D_1^*\pi$, such measurements as the angular distribution of the pion with respect to the D_1 direction, the longitudinal helicity of the D^*, and the longitudinal polarization of the D^* along the direction of the D_1, allow a determination of the fraction of the D_1 width that arises from the $D_1\to[D^*\pi]_D$ amplitude. This can be used to test a prediction based on heavy-quark symmetry that relates the $D_1\to[D^*\pi]_D$ decay amplitude to the D-wave $D_2^*\to D\pi$ and $D_2^*\to D^*\pi$ decay amplitudes. Such tests are important for defining the extent to which a charm quark may be sufficiently heavy for the predictions of heavy-quark symmetry to be applicable not only in hadron spectroscopy, but also in the prediction of weak matrix elements.

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$$\left[\frac{1}{4\pi}\right]\int d\Omega\,\hat{p}^{\,i}\hat{p}^{\,j}=\frac{1}{3}\delta^{ij}\,,$$

$$\left[\frac{1}{4\pi}\right]\int d\Omega\,\hat{p}^{\,i}\hat{p}^{\,j}\hat{p}^{\,k}\hat{p}^{\,l}=\frac{1}{15}(\delta^{ij}\delta^{kl}+\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk})\,.$$

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