Investigations of dynamical models of $B \to \psi/J\, K^*_+$

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We investigate representative dynamical models of $B \to \psi/J\, K^*_+$ with an eye toward the suggestion of Kayser, Kuroda, Peccei, and Sanda and Lipkin that such modes may be used to reduce the required luminosity to observe *CP* violation in *B*-factory-type devices on the one hand and toward recent impending data from ARGUS which may be used to discriminate among such models, and to corroborate any conclusions from our analysis about such a reduction on the other hand. We find that all three of our relativistically invariant models give *CP*-even final-state dominance of this decay, in agreement with the initial reports from ARGUS. Further, the method of Lepage and Brodsky is consistent with the ARGUS observation of helicity 0 for the dominant *CP*-even final states.

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I. INTRODUCTION

There is currently considerable interest in exploring *CP* violation in *B* decay in *B*-factory-type scenarios, such as the SLAC Asymmetric *B*-factory device, which is currently under proposed development at SLAC and LBL, for example. Such machines extend the state of the art in e^+e^- colliding beam devices in several ways, most immediately in the luminosity parameter: For the *CP*-violation studies to proceed with a good range of parameter space accessible, in the gold plated $B \to \psi/J\, K_s$ mode, luminosities $\sim 3\times 10^{33}/\text{cm}^2\,\text{sec}$ are needed. The question of other possible modes is therefore extremely relevant. In this paper, following the work in Ref. [1], we consider several dynamical models of $B \to \psi/J\, K^*_+$ as a way to reduce the required luminosity for *CP*-violation studies in *B*-factory-type devices and as a way to test these models against preliminary data from ARGUS (and CLEO) [2]. Here, K^*_+ is the *CP*-even neutral K^*.

Indeed, in our initial study [3], we did not have access to the ARGUS data, so that our first motivation was preeminent. After the ARGUS result was announced [2], it provided confirmation of our initial study and it allowed us to discriminate among the various models which we had analyzed. In what follows, we will thus present the complete analysis which underlies our findings about $B \to \psi/J\, K^*_+$ and how the ARGUS (and CLEO) results relate to these findings. The implications of our work in Ref. [3] for the required luminosity for the *CP*-violation studies in *B*-factory-type devices (it implies a reduction by a factor of ~ 2.5) are then put on both a solid theoretical and experimental basis. Further, our study teaches us something fundamental about QCD effects in such bound-state transition amplitudes. Thus, we will illustrate a paradigm for another product of high-luminosity factory-type scenarios: the detailed measurements of angular and energy distributions in heavy-meson decays will in fact permit discrimination among the various models of such decays.

The key issue is whether the modes $B \to \psi/J\, K^*_+$ can be used to study *CP* violation in *B* decays in analogy with $B \to \psi/J\, K_s$. The problem is that, while the latter final state is a *CP* eigenstate, in the former final state a mixture of *CP* parities is present owing to the possibility of both odd and even orbital partial waves in the respective decay amplitudes: $CP=(-1)^L$, $L=0,1,2$, for the $\psi/J\, K^*_+$ final state if L is the total orbital angular momentum eigenvalue. As has been emphasized by Dorfan [4], if either *CP* parity dominates, the $\psi/J\, K^*_+$ mode can be used to reduce the required luminosity for *B*-factory-type devices such as the SLAC Asymmetric *B* Factory to observe *CP* violation in the *B* decays. Recent work by Dunietz *et al.* [5] has shown that the respective reduction factor is ~ 2.5, as we have noted above—this is, indeed, a significant reduction factor, so it is important to understand the dynamical mechanisms which determine the relative admixture of the two *CP* parities in the $B \to \psi/J\, K^*_+$ transition. Here, we do this by considering in detail several models of the dynamics with an eye toward isolating which aspect of it is relevant to the *CP* parity of our transition. Thus, it is important to consider an unbiased sample of the available models with the idea of understanding how the approximations inherent to each affect the ratio of *CP*-even to *CP*-odd final states in $B \to \psi/J\, K^*_+$. Accordingly, we consider four different representative models of the $B \to \psi/J\, K^*_+$ transition in our analysis. Such a detailed dynamical analysis of the interplay of the *CP* parity in $B \to \psi/J\, K^*_+$ with the specific approximations of representative models has not appeared elsewhere.

Our work is organized as follows. In the next section, we consider our first and notation setting model, that of Wirbel, Stech, and Bauer (WSB) in Ref. [6]. It is representative of relativistically invariant harmonic-oscillator-type potential models. In Sec. III, we treat the $B \to \psi/J\, K^*$ transition using the methods of Lepage and Brodsky in Ref. [7]. Here, the QCD corrections to the spectator approximation are computed in a manifestly Lorentz-covariant fashion. In Sec. IV, we illustrate, for completeness, what a nonrelativistic potential-type model would imply for our decay by considering it in the model of Isgur, Scora, Grinstein, and Wise (ISGW) [8]. Of course, since K^* is relativistic in our decay, we do not expect such a model to stand up to a good comparison with data. Nonetheless, it gives us a handle on the size of the relativistic effects by its degree of deviation from the relativistically invariant models. Finally, in Sec. V, we use recent heavy-quark-mass limit methods of Wise, Isgur, and Bjorken [9] (WIB theory) to discuss our decay. Again, we do not want to suggest that the strange-quark mass is heavy: even for its constituent value, we have $\Lambda_{\rm QCD}/m_s \sim 0.2$, so that 20% amplitude errors may easily occur in our WIB amplitudes. The implied $\sim 40\%$ rate errors, however, still leave an interesting limit for our work, as we shall see; for, in this limit, we believe the QCD corrections to be small, and a 40% check of our work will be useful for us. Section VI contains a comparison with available data. Section VII contains our summary remarks; the appendixes contain technical details.

II. MODEL OF WIRBEL, STECH, AND BAUER (WSB)

In this section, we analyze $B \to \psi/J\, K^*$ from the viewpoint of the WSB model. In the course of our analysis of this mode., we set our notational and kinematical conventions. We will use these conventions henceforward.

We should note that, while the WSB model has been applied extensively in the literature to various heavy-meson decays, the relative admixture of *CP*-even to *CP*-odd final states in $B \to \psi/J\, K^*_{+}$ has not been given for this model elsewhere. Thus, because of its familiarity, this model is a good straightforward one to use to set our notation and to represent the general class of relativistically invariant harmonic-oscillator-type potential models.

We illustrate the process of interest to us in Fig. 1. The WSB model, like all of the models we consider, exploits the smallness of m_b^2/M_W^2 compared to 1 to use the pointlike limit of the W propagator in Fig. 1. On taking the standard QCD corrections into account, one may identify, following the work in Ref. [10], the effective interaction Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}}[a_1 \bar{c}\gamma_\mu(1-\gamma_5)s'\bar{s}'\gamma^\mu(1-\gamma_5)c + a_2\bar{s}'\gamma_\mu(1-\gamma_5)s'\bar{c}\gamma^\mu(1-\gamma_5)c + \text{H.c.}] , \tag{1}$$

where a_i are determined by the standard QCD short-distance corrections, G_F is the Fermi constant, H.c. denotes Hermitian conjugation, and d', b', and s' are the usual Cabibbo-Kobayashi-Maskawa (CKM) rotated mixtures of the d, s, and b mass eigenstates. The values of a_i are well known if one takes them from QCD perturbation

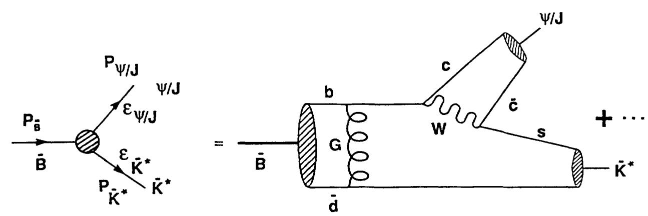

FIG. 1. The process $\bar{B} \to \psi/J\,\bar{K}^*$ to lowest order in the $SU(2)_L \times U(1)$ electroweak theory. P_A is the four-momentum of A, $A = \bar{B}, \psi/J, \bar{K}^*$, and $E_A = P_A^0$.

theory in conjunction with the partial differential equations of QCD. Here, we will leave a_i as parameters, for we will not need their values in what follows.

Let us now focus on the next step in the WSB approach to our decay. Specifically, the operator in the charge retention order in (1) suggests the use of the current field identity [11], which has had some success in the theory of the interactions of the lower-lying vector mesons such as ρ, ω, ϕ. Accordingly, with the general neglect of final-state interactions between ψ/J and K^*_{+}, WSB arrive at the amplitude

$$\mathcal{M}(\bar{B} \to \psi/JK^*_{+}) = \frac{iG_F}{\sqrt{2}} a_2 (2\pi)^4 \delta^4(P_{\bar{B}} - P_{\psi/J} - P_{K^*_{+}}) \times \langle K^*_{+} | J_\mu^{(s's')}(0) | \bar{B} \rangle \times \frac{\epsilon^{*\mu}_{\psi/J} F_{\psi/J} m_{\psi/J}}{[2E_{\psi/J}(2\pi)^3]^{1/2}} , \tag{2}$$

where we have introduced

$$\langle 0 | \bar{c}(0)\gamma_\mu(1-\gamma_5)c(0) | \psi/J \rangle = \epsilon_{\psi/J\mu} F_{\psi/J} m_{\psi/J} / [2E_{\psi/J}(2\pi)^3]^{1/2} \tag{3}$$

and

$$J_\mu^{(s's')} = \bar{s}'\gamma_\mu(1-\gamma_5)s' . \tag{4}$$

To arrive at the complete prediction for the process in Fig. 1 then requires a systematic analysis of the transition matrix element $\langle K^*_{+} | J_\mu^{(s's')}(0) | \bar{B} \rangle$. Here, our present state of understanding of the strong interaction necessitates that, eventually, a model of the dynamics in such a matrix element be used. From the standpoint of our work, it is at this point that the true distinctions between the different models in our analysis enter.

However, it is important to proceed as far as one can with the analysis of $\langle K^*_{+} | J_\mu^{(s's')}(0) | \bar{B} \rangle$ before invoking model-dependent results. Accordingly, we follow WSB and express the matrix element under study in a model-independent, Lorentz-covariant representation as follows:

$$\tilde{M}_\mu \equiv \langle K^*_{+} | J_\mu^{(s's')}(0) | \bar{B} \rangle = \frac{2}{m_B + m_{K^*_{+}}} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu}_{K^*_{+}} P_{\bar{B}}^{\rho} P_{K^*_{+}}^{\sigma} V(q^2) + i \left[\epsilon^*_{K^*_{+}\mu}(m_B + m_{K^*_{+}}) A_1(q^2) - \epsilon^*_{K^*_{+}} \cdot q (P_{\bar{B}} + P_{K^*_{+}})_\mu \frac{A_2(q^2)}{(m_{\bar{B}} + m_{K^*_{+}})} - \frac{\epsilon^*_{K^*_{+}} q}{q^2}(2m_{K^*_{+}}) q_\mu A_3(q^2) \right] + i \frac{\epsilon^*_{K^*_{+}} q}{q^2}(2m_{K^*_{+}}) q_\mu A_0(q^2) \tag{5}$$

for some invariant functions V and A_i of $q^2=m^2_{\psi/J}$ where $q=P_{\bar{B}}-P_{K^*_+}$. One may now make a pole approximation to the V and A_i such that they all have the form

$$P(q^2)=P_0/(1-q^2/m^2) \tag{6}$$

for some masses and residues (m,P_0). Note that the model details begin to enter in a pronounced way. In the WSB approach the (m,P_0) are determined from a relativistic harmonic-oscillator potential model with a Gaussian transverse-momentum distribution characterized by a scale $w\sim 0.35$ GeV. The complete specification of the (m,P_0) then allows one to determine the basic quantity of interest to us, namely,

$$r_{CP}\equiv\Gamma(CP=\text{odd})/[\Gamma(CP=\text{even})+\Gamma(CP=\text{odd})] \tag{7}$$

for our process in Fig. 1. Here, we define $\Gamma(CP=s)$ to be the total width of $\bar{B}\to\psi/J\,K^*_+$ final states with $CP=s, s=\text{even},\text{odd}$.

More precisely, in order to exploit the specific results of the WSB model for the V, A_i, we first observe that the respective $\psi/J\,K^*_+$ helicity amplitudes are readily seen to be

$$\mathcal{M}(00)=\frac{-G_F}{\sqrt{2}}\frac{a_2F_{\psi/J}m_{\psi/J}}{\sqrt{2E_{\psi/J}(2\pi)^3}}\left[\frac{\mathbf{P}^2+E_{\psi/J}E_{K^*_+}}{m_{K^*_+}m_{\psi/J}}(m_B+m_{K^*_+})A_1(q^2)-\frac{2m_B^2\mathbf{P}^2A_2(q^2)}{[m_{K^*_+}m_{\psi/J}(m_B+m_{K^*_+})]}\right], \tag{8}$$

$$\mathcal{M}\begin{bmatrix}+ & +\\ (-) & (-)\end{bmatrix}=-\frac{G_F}{\sqrt{2}}\frac{a_2F_{\psi/J}m_{\psi/J}}{\sqrt{2E_{\psi/J}(2\pi)^3}}\left[\begin{matrix}+\\(-)\end{matrix}\left[\frac{2m_B}{m_B+m_{K^*_+}}\right]|\mathbf{P}|V(q^2)+(m_B+m_{K^*_+})A_1(q^2)\right], \tag{9}$$

where $\mathbf{P}\equiv(\mathbf{P}_{K^*_+}-\mathbf{P}_{\psi/J})/2$ in the $\bar{B}$ rest frame. Entirely standard manipulations may now be used to express r_{CP} as

$$r_{CP}=2\frac{4m_B^2}{(m_b+m_{K^*_+})^2}\mathbf{P}^2|V(q^2)|^2\Bigg/\left[\left|\left[\frac{\mathbf{P}^2+E_{K^*_+}E_{\psi/J}}{m_{K^*_+}m_{\psi/J}}\right](m_B+m_{K^*_+})A_1(q^2)-\frac{2m_B^2\mathbf{P}^2A_2(q^2)}{[m_{K^*_+}m_{\psi/J}(m_B+m_{K^*_+})]}\right|^2\right.$$
$$\left.+2\frac{4m_B^2}{(m_B+m_{K^*_+})^2}\mathbf{P}^2|V(q^2)|^2+2(m_{K^*_+}+m_B)^2|A_1(q^2)|^2\right]. \tag{10}$$

This equation then expresses our desired ratio r_{CP} in terms of V and A_i explicitly so that we can use it as our basic expression for all model assessments of r_{CP}.

In particular, on substituting the values of A_i and V into (10) from the WSB relativistic harmonic-oscillator potential model [6], we get

$$r_{CP}\simeq 0.091\ . \tag{11}$$

This result is encouraging. We also may note that, in this WSB model, the ratio $\Gamma(00)/\Gamma(\text{all})\equiv r_{00}$ is given by

$$r_{00}\simeq 0.57\ ; \tag{12}$$

the latter result implies that the original idea of Kayser, Kuroda, Peccei, and Sanda (KKPS) [1] to focus on $\Gamma(00)$ in the K^* decay is not excluded completely. We shall return to this point shortly. We do understand [12], however, that such an isolation of $\Gamma(00)$ is a difficult one. Here, $\Gamma(00)$ is the rate associated with $\mathcal{M}(00)$: it can be given in terms of V and A_i, as we have done in Appendix A.

We need to stress at this point that the WSB framework is a model. The issue of model dependence is then apparent: how much does (11) depend on the details of the WSB model? We address this question here by considering other models. In this regard, we turn next to the methods of Lepage and Brodsky for an analysis of r_{CP}, r_{00}.

III. LEPAGE-BRODSKY THEORY OF $\bar{B}\to K^*_+\ \psi/J$

In this section, we will apply the methods of Lepage and Brodsky [7] to the process $\bar{B}\to K^*_+\ \psi/J$ in order to determine the respective prediction for r_{CP}. The virtues of these methods are that they are relativistically invariant and that they represent perturbative QCD corrections to the spectator model of the respective transition. More precisely, these corrections model the response of the would-be spectator to the decay of the b quark. Such corrections in the WSB model are implicit in the respective wave-function overlap integrals which determine, for example, the residues in A_i and V. Thus, the Lepage-Brodsky methods will allow us a different, more field-theoretic view of the $\bar{B}\to K^*_+$ transition in (5). We shall begin in a pedagogic way by recalling the elements of the Lepage-Brodsky theory relevant to our work.

We illustrate the method in Fig. 1 if we use, for example, their distribution amplitudes for the incoming and outgoing hadrons and follow their calculational methods. The transition will be handled precisely in analogy with our discussion of (2). This permits us to interpret the Lepage-Brodsky theory of our decay in terms of its predictions for A_i and V in (5). The first step in this interpretation is the computation of the relevant distribution amplitudes for B and K^*. On adapting our distribution amplitude results for the $D\to K^*$ transition in Ref. [13] to $B\to K^*$, we get the distribution amplitudes

$$\phi_{K^{0*}_{\parallel}}(y_1,y_2)=\sqrt{3}f_{K^*}y_2(-0.344y_1+2.69y_1^2)\ ,$$
$$\phi_{K^{0*}_{\perp}}(y_1,y_2)=\sqrt{3}f_{K^*}y_2(-0.609y_1+2.76y_1^2)\ ,$$
$$f_{K^*}\simeq 0.175\ \text{GeV}\ , \tag{13}$$
$$\phi_B(x_1,x_2)=(f_B/\sqrt{12})\delta(x_2-x_B)\ ,$$
$$x_B=\{m_d-[m_b/(m_b+m_d)]\times(m_b+m_d-m_B)\}/m_B\simeq 0.036\ ,$$

where f_B is the B-meson decay constant (its precise value is not important for our present purposes, but for definiteness, we note that potential model estimates suggest $f_B\simeq 0.136$ GeV, for example). In deducing x_B, some choice of masses for quarks has been made: $m_u=m_d\simeq 0.33$ GeV, $m_b=5.1$ GeV. We have taken $m_B=5.28$ GeV.

Using (13) we may then repeat our steps in our analysis of $D\to K^*$ in Ref. [13] (we do not use the results from these steps in Ref. [13]; we repeat each step anew for $B\to K^*$) to get the following predictions [in both (13) and (14), we should note that $\parallel$ and $\perp$ always denote the obvious: the K^* polarization]:

$$A_1^{\perp(\parallel)}=[m_B g_A/(m_B+m_{K^*_+})]F^{(1)}_{\perp(\parallel)}\ ,$$
$$A_2^{\parallel}=(m_B+m_{K^*_+})m_B g_A F^{(3)}_{\parallel}\ , \tag{14}$$
$$V^{\perp}=\tfrac{1}{2}m_B g_V(m_B+m_{K^*})F^{(2)}_{\perp}\ ,$$

where the functions $F_H^{(i)}$ are the generalization to $B\to K^*$ of the corresponding functions in $D\to K^*$ in Ref. [13]. They are recorded in Appendix B for definiteness and for completeness. Here $g_{A,V}$ are the usual axial-vector and vector form factors which we treat in the standard pole approximation from the WSB model, for example. From (14) we get the results

$$r_{CP}\simeq 0.083 \tag{15}$$

and

$$r_{00}\simeq 0.834\ . \tag{16}$$

Again, our result for r_{CP} is encouraging, and we have in addition the enhancement of $\Gamma(00)$ in the Lepage-Brodsky theory. Hence, in this approach the idea of KKPS looks theoretically more attractive. Nonetheless, it is still true that other variants of the transition amplitude in (5) might give yet another perspective. Indeed, two different aspects of (5) are hard to assess without exploring the respective limiting cases. These are the importance of the relativistic invariance of the WSB and Lepage-Brodsky approaches and the significance of the QCD corrections in the transition in (5). Here, we shall explore these aspects by considering, in turn, a nonrelativistic model of Isgur *et al.* for the A_i and V and the heavy-quark limit of Wise, Isgur, and Bjorken (WIB theory) for the transition in (5); for in the latter limit, the QCD corrections are expected to be small, so that, in comparing with (11) and (15), there will only enter the issue that m_s is not really all that heavy compared with Λ_{QCD}. We may emphasize, however, that the binding-related effects which we neglect in WIB theory are controlled by Λ_{QCD}/m_s, where m_s is the strange-quark mass relevant to its binding to the $\bar{d}$ in the $\bar{K}^*$: thus, in this view, $\Lambda_{\text{QCD}}/m_s\simeq 0.100/0.500=20\%$. Twenty-percent effects in our amplitudes correspond to $\sim 40\%$ effects in our rates, but in view of the size of (11) and (15), this kind of knowledge of our rates is still interesting, particularly when one realizes that (11) and (15) are certainly no more accurate than $\sim 20\%$ themselves. Thus, the WIB result can help us understand the role of QCD corrections in our results (11) and (15).

Similarly, the nonrelativistic calculations will help us isolate the role of relativistic corrections in (11) and (15). We turn now to the nonrelativistic limit.

IV. MODEL OF ISGUR, SCORA, GRINSTEIN, AND WISE (ISGW) [9]

For our exploration of the importance of the relativistic invariance of the WSB and Lepage-Brodsky methods for our results in (11), (12), (15), and (16), we consider the model of Isgur *et al.* This is a Cornell-type [14] potential model in which contact with the physical hadron matrix elements is made via the "mock-meson" method. Here, we use the procedures of Isgur *et al.* to compute the transition in (5). Since the K^*_+ is relativistic in (5), we do indeed expect to find that the relativistic effects are in fact important.

Indeed, following always the methods in Ref. [8], we may identify the form factors A_i and V as

$$A_1=-2\tilde{m}_B F_3/(m_B+m_{K^*_+})\ ,$$
$$A_2=\frac{m_B+m_{K^*_+}}{2\tilde{m}_{K^*_+}}\times F_3\left[1+\frac{m_d}{m_b}\left[\frac{\beta_B^2-\beta_{K^*_+}^2}{\beta_B^2+\beta_{K^*_+}^2}\right]-\frac{m_d^2}{4\mu_-\tilde{m}_B}\frac{\beta^4_{K^*_+}}{\beta^4_{K^*_+}}\right]\ , \tag{17}$$

and

$$V=-\frac{m_B+m_{K^*_+}}{2}F_3\left[\frac{1}{m_s}-\frac{1}{2\mu_-}\frac{m_d}{\tilde{m}_{K^*_+}}\frac{\beta_B^2}{\beta^2_{BK^*_+}}\right]\ ,$$

where

$$F_3(t)=\frac{\tilde{m}_{K^*_+}}{\tilde{m}_B}\left[\frac{\beta_B\beta_{K^*_+}}{\beta^2_{BK^*_+}}\right]^{1/2}\exp\left[-\frac{m_d^2}{4\tilde{m}_B\tilde{m}_{K^*_+}}\frac{t_m-t}{\kappa^2\beta^2_{BK^*_+}}\right], \quad (18)$$

with

$$\mu_-=(1/m_s-1/m_b)^{-1}, \quad \kappa=0.7,$$

$$t_m=(m_B-m_{K^*_+})^2, \quad \tilde{m}_B=m_b+m_d, \quad \tilde{m}_{K^*_+}=m_s+m_d, \quad (19)$$

$$\beta_B=0.41, \quad \beta_{K^*_+}=0.34, \quad \beta^2_{BK^*_+}=\tfrac{1}{2}(\beta_B^2+\beta^2_{K^*_+}),$$

$$m_b=5.12 \text{ GeV}, \quad m_s=0.55 \text{ GeV}, \quad m_d=0.33 \text{ GeV}.$$

Here, $t=m^2_{\psi/J}$. From (10), (A1), and (17) we then obtain

$$r_{CP}\simeq 0.516 \quad (20)$$

and

$$r_{00}\simeq 0.064. \quad (21)$$

Our initial expectations are thus borne out by the actual calculation: the nonrelativistic model disagrees with the methods of WSB and of Lepage and Brodsky on the ratio r_{CP} and on the ratio $\Gamma(00)/\Gamma(\text{all})$. The relativistic effects in these ratios are thus important. An intuitive argument for such importance is the following. In the *CP*-even modes, *S* and *D* waves are present; in the *CP*-odd modes only the *P* wave is present and the vector polarizations are $\epsilon^*_{(\pm)}$, which are invariant to boosts along the particles' directions. Hence, the relativistic effects associated with the boosts of the vector polarizations will tend to be smaller for the *P*-wave contribution than for the *S*- and *D*-wave contributions. While such intuitive arguments are useful as a guide, they cannot replace the actual calculations themselves. It is reassuring that, in this case, intuition and calculation are in agreement.

One may also view (20) more positively as saying that, even when we use a model which clearly suppresses the kind of large values of *t* as we have in (5), we still get that r_{CP} is only ~ 0.5. Hence, it is indeed plausible that the relativistic effects could further reduce r_{CP} to the values we have found in (11) and (15). The result (21), however, appears to reflect a clear inadequacy of the nonrelativistic methods for our decay scenario.

We will now look more deeply into the expectations for r_{CP} by discussing it from the standpoint of the recent methods of Wise, Isgur, and Bjorken. This we will do in the next section.

V. WIB THEORY

In this section, we wish to use the heavy-quark-mass limit methods of Ref. [9] to study our ratios r_{CP} and r_{00}. Thus, we must work at infinite mass for m_b and m_s at fixed velocities v_b and v_s. Clearly, we face immediately the question of how far m_s is from infinity.

This question can be discussed as follows. If we expand our amplitudes in the constituent quark masses,

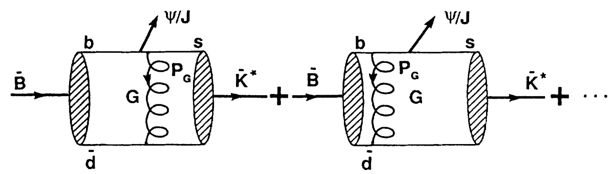

FIG. 2. Lepage-Brodsky approach to $\bar{B}\to\psi/J\,\bar{K}^*$. *G* is a QCD gluon of four-momentum P_G.

then dominant corrections of size Λ_{QCD}/m_s are expected in the amplitudes themselves. As we have noted, this would lead to 20% effects in the amplitudes and 40–50 % effects in rates. Since we only need to know that $r_{CP}\ll 1$ to have a useful result, if r_{CP} is indeed as small as we found in (11) and (15), a factor of 2 estimate of it which is believable is indeed useful. Hence, we will apply the WIB theory with this idea in mind: we may be off by a factor of order 2 but, for small r_{CP}, we will still get insight into whether $r_{CP}\ll 1$. If $r_{CP}\ll 1$, so are $\frac{1}{2}r_{CP}$ and $2r_{CP}$. Thus, we make a conservative use of the methods in Ref. [9] here as a general, qualitative check on our analyses in Secs. II and III. We will determine whether the differences between (11), (15), and our WIB predictions are consistent with 40% corrections to the rates that enter the latter predictions. This will give us some amount of assurance that our QCD "model" calculations in Secs. II and III have not made a gross error in the value of r_{CP}.

More precisely, we may call attention to the diagrams in Fig. 2 from the viewpoint of the methods of Lepage and Brodsky. In the desired WIB limit, so long as v_b and v_s are such that the gluon four-momentum transfer P_G in Fig. 2 satisfies

$$-P_G^2\lesssim 1/r^2_{\text{hadron}}, \quad (22)$$

where $r_{\text{hadron}}\sim 1$ fm is the typical hadron size, that particular gluon is already included in the Lepage-Brodsky distribution amplitudes due to the presence of the collinear projection operator $\mathcal{P}_Q$ which we illustrate in Fig. 3. Such small momentum-transfer glue is all collected by $\mathcal{P}_Q$ to form the respective Lepage-Brodsky distribution amplitude $\phi_{K^*}(x_1,x_2,Q)$ at a scale $Q\sim m_B$. Note that in this WIB limit, both *B* and K^*_+ have reduced mass m_d. Furthermore, when we neglect the intrinsic $P_\perp$ of the *B* and K^*_+, it means that their respective Bethe-Salpeter (BS) wave functions are effectively approximated by δ functions in the $P_\perp$ space. Such a BS wave function is clearly an eigenvector of $\mathcal{P}_Q$ in Fig. 3, and hence, the Lepage-Brodsky methods applied to Fig. 2 give just the result of Bjorken (and Georgi) [9] for $\bar{\mathcal{M}}_\mu$:

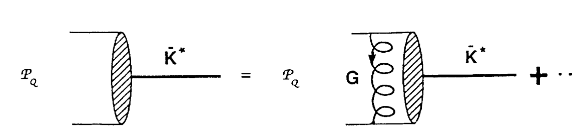

FIG. 3. Lepage-Brodsky equation for the $\bar{K}^*$ distribution amplitude. $\mathcal{P}_Q$ is the collinear projection operator for scale *Q*.

$$\tilde{\mathcal{M}}_\mu=\int d^2x\,d^2y\,\delta(1-x_1-x_2)\delta(1-y_1-y_2)\phi_{K^*}(y)\phi_{\bar B}(x)\mathrm{tr}\{[\not\epsilon^*_{K^*}(\not P_{K^*}+m_{K^*})/\sqrt{2}](\gamma_\mu g_V-\gamma_\mu\gamma_5 g_A)\gamma_5(\not P_{\bar B}-m_{\bar B})/\sqrt{2}\}\,, \tag{23}$$

where we get, by this argument, some QCD corrections in the form of $\phi_{\bar B,K^*}$.

Of course, the key issue is when will the value of $-P_G^2$ satisfy (22)? We note that, in the WIB limit, we get

$$-P_G^2\sim(0.36\text{ GeV})^2 \tag{24}$$

if we presume that 5.1 GeV is already close to $m_b=\infty$, so that we approximate the $\bar B$ and $K^*_{\mp}$ binding energies by ~ 0.15 GeV at $m_q=\infty$. Thus, in the WIB limit, we always have (22) essentially satisfied. In practice, the value of $-P_G^2$ is

$$-P_G^2\sim 0.69y_2\text{ GeV}^2\,, \tag{25}$$

and only $\sim 20\%$ of the support of the distribution amplitudes ϕ_{K^*} and $\phi_{\bar B}$ actually corresponds to (22). Hence, we expect sizable bound-state corrections to the WIB limit predictions in our $\bar B\to\psi/J\,K^*_{\mp}$.

Indeed, when we compute the values of the A_i and V which follow from (23), we get, suppressing trivial normalization factors,

$$V=(m_{K^*}+m_{\bar B})g_V/2\,,$$
$$A_1=-(E_{K^*}+m_{K^*})m_{\bar B}g_A/(m_{\bar B}+m_{K^*})\,, \tag{26}$$
$$A_2=-(m_{\bar B}+m_{K^*})g_A/2\,.$$

On introducing these results into (10) and (A1) we obtain

$$r_{CP}\simeq 0.196 \tag{27}$$

and

$$r_{00}\simeq 0.398\,. \tag{28}$$

The result for r_{CP} corroborates (11) and (15); that for r_{00} is consistent with (12) and differs by a factor ~ 2 from (16). Since we only believe (28) to a factor of 2, we can not draw any conclusion from it concerning the correctness of (12) over (16) or vice versa. Nonetheless, the result for r_{CP} in (27), taken together with (11) and (15), provides substantial theoretical argument for the smallness of r_{CP} compared to 1. This would then permit $\bar B\to\psi/J\,K^*_{\mp}$ to be used to supplement $\bar B\to\psi/J\,K_s$ in CP-violation studies at an asymmetric SLAC-LBL-type B-factory device, and hence, would reduce, in that way, the required luminosity for such a device for such studies by a factor of ~ 2.5 [5].

VI. COMPARISON WITH DATA

The study of r_{CP} and $\Gamma(00)/\Gamma(\text{all})$ in our three relativistically covariant models has left us with a good, encouraging result for the smallness of r_{CP} compared to 1 and in a quandry for $r_{00}=\Gamma(00)/\Gamma(\text{all})$, since our two quasirealistic complete models give values for the latter that differ by a factor of ~ 1.5, and our more model-independent but approximate (in our case) WIB method is only accurate to a factor ~ 2 for our work (due to the size of m_s) and gives a result for $\Gamma(00)/\Gamma(\text{all})$ which agrees with the WSB result and is just within a factor of ~ 2 of the Lepage-Brodsky prediction. Thus, we need further input to distinguish between the WSB and Lepage-Brodsky predictions for $\Gamma(00)/\Gamma(\text{all})$ insofar as which is closer to observation.

Recently [2], such input has become available. The ARGUS Collaboration has studied the final particle angular distributions in $B\to K^*\,\psi/J\to K\pi\,\psi/J$ and has found what amounts to a $\sin^2\theta$ dependence for the distribution of the K polar angle (relative to a transverse axis), so that they have found that the K^* is essentially 100% polarized in the 0 helicity state. This corresponds to a small value of r_{CP} and to $\Gamma(00)/\Gamma(\text{all})\sim 1$, exactly what we have found for the Lepage-Brodsky theory. More importantly, the ARGUS result corroborates our general result for r_{CP} and the conclusion that $\bar B\to\psi/J\,K^*_{\mp}$ can be used to reduce the required luminosity to study CP violation at an asymmetric B factory.

We should mention that, however, there is a very preliminary result from CLEO [2] which does not actually confirm the ARGUS result on r_{00}; we understand that this CLEO analysis is still in progress and we look forward to its final result.

VII. CONCLUSIONS

In this paper, we have investigated the size of the CP-odd part of the decay $\bar B\to\psi/J\,K^*_{\mp}$ with an eye toward the required luminosity for CP-violation studies at an asymmetric B factory of the kind that is under proposed development at SLAC and LBL. We have been encouraged by our findings, which are summarized in Table I.

Specifically, in three different relativistically covariant models, we found that the CP-odd part of the decay is small, so that the decay can be used in CP-violation studies at a B factory to supplement the studies in the gold plated $B\to\psi/J\,K_s$ mode. This would reduce the required B-factory luminosity by a factor ~ 2.5.

Further, we looked into the effect of the relativistic

TABLE I. Summary of the results in the text for r_{CP} and r_{00} for four different models. As we explain in detail in the text, the three relativistically invariant models all give that the CP-odd part of our $B\to\psi/J\,K^*_{\mp}$ decay is small.

Model	r_{CP}	r_{00}
WSB	0.091	0.57
Lepage-Brodsky	0.083	0.834
ISGW	0.516	0.064
WIB	0.196	0.398

corrections by considering the decay in the model of Isgur *et al.* We indeed found that these corrections are important as expected; the K_+^* is relativistic in the decay under study here.

We were pleased to discover that the WIB theory works at the level of a factor of 2 insofar as r_{CP} and r_{00} are concerned. There is now considerable debate about where the Wise-Isgur (WI) symmetry is useful. Here, for r_{00}, it is comparable to the WSB methods, and for r_{CP} it misses by a factor of ~ 2. Both of these results are consistent with amplitude errors $\sim \Lambda_{\rm QCD}/m_s \sim 20\%$. Looked at in another way, our WIB analysis is consistent with the conclusion that we are not making a gross error in our QCD models in the WSB and Lepage-Brodsky analyses. Of course, it does not actually prove this conclusion. It is a self-consistency check which increases our confidence in our WSB and Lepage-Brodsky predictions for r_{CP}.

It should be stressed that, from the standpoint of QCD, r_{00} and r_{CP} are very different quantities. Because QCD conserves parity and charge conjugation, it is impossible for pure QCD corrections to mix *CP*-odd and *CP*-even final states in a pure weak process. On the other hand, r_{00} is a helicity rate; since only the total angular momentum is conserved in general, the $|00\rangle$ helicity content of a pure weak-process final state can vary substantially from one QCD corrections model to the next. What we have calculated in the text bears this out explicitly.

Indeed, most importantly, we have found that the Lepage-Brodsky model not only predicts the smallness of the *CP*-odd contribution but agrees further with recent observations by ARGUS that the 0-helocity final state is the dominant one. While this has yet to be confirmed by CLEO, it is still true that we now have both theoretical and experimental evidence of the usefulness of $B \to \psi/J\, K_+^*$ as a vehicle for *CP*-violation studies at an asymmetric B factory. In view of the attendant factor of ~ 2.5 reduction in the required luminosity, we encourage the various B-factory proponents to pursue their proposals vigorously.

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APPENDIX A: $r_{00} \equiv \Gamma(00)/\Gamma(\text{all})$

In this brief appendix we give the expression for r_{00} as it is defined in the text. We have

$$r_{00} = |[(\mathbf{P}^2 + E_{K_+^*} E_{\psi/J})/(m_{K_+^*} m_{\psi/J})](m_{\bar B} + m_{K_+^*})A_1 - 2m_{\bar B}^2 \mathbf{P}^2 A_2/[m_{K_+^*} m_{\psi/J}(m_{\bar B} + m_{K_+^*})]|^2/D\ , \tag{A1}$$

where

$$D = |[(\mathbf{P}^2 + E_{K_+^*} E_{\psi/J})/(m_{K_+^*} m_{\psi/J})](m_{\bar B} + m_{K_+^*})A_1 - 2m_{\bar B}^2 \mathbf{P}^2 A_2/[m_{K_+^*} m_{\psi/J}(m_{\bar B} + m_{K_+^*})]|^2 + 2(m_{K_+^*} + m_{\bar B})^2 |A_1|^2 + 2[4m_{\bar B}^2/(m_{\bar B} + m_{K_+^*})^2]\mathbf{P}^2 |V|^2 \tag{A2}$$

for V and A_i as defined in (5).

APPENDIX B: $B \to K^*$ LEPAGE-BRODSKY FORM FACTORS

In this appendix we wish to record the generalization to $B \to K^*$ of the $D \to K^*$ transition form factors in Ref. [13] in the context of the methods of Lepage and Brodsky in Ref. [7]. This will then render the discussion in the text above entirely self-contained.

Specifically, following the procedures illustrated in Ref. [13] and invented in Ref. [7], we get the following expressions for the form factors $F^{(i)}$ in (5):

$$\begin{aligned} F_{\perp(\|)}^{(1)} = {} & -4ig_s^2 C_F \int_0^1 dx_2 \int_0^1 dx_1\, \delta(1-x_1-x_2) \int_0^1 dy_1 \int_0^1 dy_2\, \delta(1-y_1-y_2) \\ & \times \frac{\phi_{\bar B}(x_1,x_2)\phi_{K^*_{\perp(\|)}}(y_1,y_2)}{x_2 y_2 (m_{\bar B}^2 + m_{K_+^*}^2 - m_{\psi/J}^2)} \left\{ \frac{[(x_2 - \frac{1}{2})m_{\bar B}^2 + \frac{1}{2}(m_{\psi/J}^2 - m_{K_+^*}^2)]m_{K_+^*}}{\bar D_2(x_1,x_2)} \right. \\ & + \{ -m_{K_+^*}[(1+y_1)m_B^2/2 - (1-y_1)m_{\psi/J}^2/2 + (1-y_1)m_{K_+^*}^2/2 - 2m_B m_b] \\ & \left. + m_b(m_B^2 + m_{K_+^*}^2 - m_{\psi/J}^2)/2 + m_B[(1-2y_1)m_{K_+^*}^2 - m_B^2 + m_{\psi/J}^2]\}/D_2(y_1,y_2) \right|, \end{aligned} \tag{B1}$$

$$F_{\perp}^{(2)}=-4ig_s^2C_F\int_0^1 dx_2\int_0^1 dx_1\,\delta(1-x_1-x_2)\int_0^1 dy_1\int_0^1 dy_2\,\delta(1-y_1-y_2)\times\frac{\phi_{\bar B}(x_1,x_2)\phi_{K^*_\perp}(y_1,y_2)}{x_2y_2(m_B^2+m_{K^*_+}^2-m_{\psi/J}^2)}\left[\frac{-m_{K^*_+}}{\bar D_2(x_1,x_2)}+\frac{(1-y_1)m_{K^*_+}-m_b+2m_B}{D_2(y_1,y_2)}\right] \tag{B2}$$

$$F_{\perp(\parallel)}^{(3)}=-4ig_s^2C_F\int_0^1 dx_1\int_0^1 dx_2\,\delta(1-x_1-x_2)\int_0^1 dy_1\int_0^1 dy_2\delta(1-y_1-y_2)\times\frac{\phi_{\bar B}(x_1,x_2)\phi_{K^*_{\perp(\parallel)}}(y_1,y_2)}{x_2y_2(m_B^2+m_{K^*_+}^2-m_{\psi/J}^2)}\left[\frac{-m_{K^*_+}(\frac12-x_2)}{\bar D_2(x_1,x_2)}+\frac{(1-y_1)m_{K^*_+}/2+m_b/2-m_B}{D_2(y_1,y_2)}\right], \tag{B3}$$

where we have introduced

$$\bar D_2(x_1,x_2)=x_1x_2m_B^2+x_2m_{\psi/J}^2+x_1m_{K^*_+}^2\ ,$$
$$D_2(y_1,y_2)=(1-y_1)m_{\psi/J}^2+y_1m_B^2-y_1m_{K^*_+}^2+y_1^2m_{K^*_+}^2-m_b^2+i\epsilon\ , \tag{B4}$$

and the distribution amplitudes are given in (13). Here $g_s(m_B^2)$ is the QCD coupling constant at the scale m_B and $C_F=\frac43$ is the quadratic Casimir invariant of the quark color representation in QCD. The results (B1)–(B3) are the desired representation of A_1, A_2, and V in our work through the relations (14).

The integrations in (B1)–(B3) may now be done, with the consequent results,

$$F_{\perp(\parallel)}^{(j)}=\frac{(-4ig_s^2C_F/x_2)\mathcal{F}_{\perp(\parallel)}^{(j)}}{(m_B^2+m_{K^*}^2-m_{\psi/J}^2)}\ , \tag{B5}$$

where

$$\mathcal{F}_{\perp(\parallel)}^{(i)}=\frac{f_B}{\sqrt{12}}\sqrt3 f_{K^*}\left[a_{\perp(\parallel)}^{(i)}+\sum_{k=1}^{3}b_{\perp(\parallel)k}^{(i)}\bar f_k\right]. \tag{B6}$$

The constants $a_{\perp(\parallel)}^{(i)},b_{\perp(\parallel)}^{(i)}$ in (B6) are given by

$$a_{\perp(\parallel)}^{(1)}=i_{\perp(\parallel)}^0\frac{[(x_B-\frac12)m_B^2+\frac12(m_{\psi/J}^2-m_{K^*_+}^2)]m_{K^*_+}}{\bar D_2(1-x_B,x_B)}\ ,$$
$$a_{\perp(\parallel)}^{(2)}=i_{\perp(\parallel)}^0\frac{-m_{K^*_+}}{\bar D_2(1-x_B,x_B)}\ , \tag{B7}$$
$$a_{\perp(\parallel)}^{(3)}=i_{\perp(\parallel)}^0\frac{[-m_{K^*_+}(\frac12-x_B)]}{\bar D_2(1-x_B,x_B)}\ ,$$

with

$$i_\perp^0=0.616\ ,\quad i_\parallel^0=0.725\ ,$$

and

$$b_{\parallel(\perp)1}^{(1)}=\eta_{\parallel(\perp)1}[-m_{K^*_+}(\tfrac12m_B^2-\tfrac12m_{\psi/J}^2+\tfrac12m_{K^*_+}^2-2m_Bm_b)+\tfrac12m_b(m_B^2+m_{K^*_+}^2-m_{\psi/J}^2)+m_B(m_{K^*_+}^2-m_B^2+m_{\psi/J}^2)]\ ,$$
$$b_{\parallel(\perp)2}^{(1)}=\eta_{\parallel(\perp)2}[-m_{K^*_+}(\tfrac12m_B^2+\tfrac12m_{\psi/J}^2-\tfrac12m_{K^*_+}^2)-2m_Bm_{K^*_+}^2]+(\eta_{\parallel(\perp)2}/\eta_{\parallel(\perp)1})b_{\parallel(\perp)1}^{(1)}\ ,$$
$$b_{\parallel(\perp)3}^{(1)}=\frac{\eta_{\parallel(\perp)2}}{\eta_{\parallel(\perp)1}}\left[b_{\parallel(\perp)2}^{(1)}-\frac{\eta_{\parallel(\perp)2}}{\eta_{\parallel(\perp)1}}b_{\parallel(\perp)1}^{(1)}\right],$$
$$b_{\parallel(\perp)1}^{(2)}=\eta_{\parallel(\perp)1}(m_{K^*_+}-m_b+2m_B)\ ,$$
$$b_{\parallel(\perp)2}^{(2)}=-\eta_{\parallel(\perp)1}m_{K^*_+}+\eta_{\parallel(\perp)2}(m_{K^*_+}-m_b+2m_B)\ ,$$
$$b_{\parallel(\perp)3}^{(2)}=-\eta_{\parallel(\perp)2}m_{K^*_+}\ , \tag{B8}$$
$$b_{\parallel(\perp)1}^{(3)}=\eta_{\parallel(\perp)1}(m_{K^*_+}/2+m_b/2-m_B)\ ,$$
$$b_{\parallel(\perp)2}^{(3)}=\eta_{\parallel(\perp)1}(-m_{K^*_+}/2)+\eta_{\parallel(\perp)2}(m_{K^*_+}/2+m_b/2-m_B)\ ,$$
$$b_{\parallel(\perp)3}^{(3)}=\eta_{\parallel(\perp)2}(-m_{K^*_+}/2)\ .$$

The $\bar f_k$ are given by

$$\bar f_k=\int_0^1 dy\,y^k/D_2(y,1-y)\ , \tag{B9}$$

so that

$$f_1\simeq-\frac{i\pi y_0}{m_B^2-m_{\psi/J}^2}+\frac{1}{m_B^2-m_{\psi/J}^2}\left[1+y_0\ln\left|\frac{1-y_0}{y_0}\right|\right], \tag{B10}$$

$$f_2 \simeq -\frac{i\pi y_0^2}{m_B^2-m_{\psi/J}^2} + \frac{1}{m_B^2-m_{\psi/J}^2}\left[\frac{1}{2}+y_0+y_0^2\ln\left|\frac{1-y_0}{y_0}\right|\right], \tag{B11}$$

$$f_3 \simeq -\frac{i\pi y_0^3}{m_B^2-m_{\psi/J}^2} + \frac{1}{m_B^2-m_{\psi/J}^2}\left[\frac{1}{3}+\frac{1}{2}y_0+y_0^2+y_0^3\ln\left|\frac{1-y_0}{y_0}\right|\right] \tag{B12}$$

for

$$y_0=(m_b^2-m_{\psi/J}^2)/(m_B^2-m_{\psi/J}^2) , \tag{B13}$$

with $m_b \simeq 4.5$ GeV. Here, the η_{hi} are given by -0.344, 2.69, $i=1,2$, respectively, for $h=\parallel$ and by -0.609, 2.76, $i=1,2$, respectively, for $h=\perp$.

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