BRIEF REPORTS

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Cosmic no-hair theorem in power-law inflation

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We prove a cosmic no-hair theorem for Bianchi models in power-law inflation. Provided that the potential of an inflaton ϕ is $\exp(-\lambda\kappa\phi)$ with $0 \le \lambda < \sqrt{2/3}$, we find that the isotropic power-law solution is the unique attractor for any initially expanding Bianchi-type models except type IX. For Bianchi type IX, this conclusion is also true if the initial ratio of the vacuum energy to the maximum three-curvature is larger than one half.

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Inflation is definitely a great idea in modern cosmology [1], but unfortunately, so far there has been no natural model based on any fundamental high-energy physics. In order to look for such a model, a new approach has been proposed [2–4] by modifying the Einstein gravity. One attempt in this approach is the so-called extended inflation originally proposed by La and Steinhardt [2]. They adopted Jordan-Brans-Dicke (JBD) theory and reanalyzed the old inflationary scenario, finding that the phase transition can complete, in contrast with old inflation in the Einstein theory. It was, however, found that the bubbles lead to unacceptable distortions of the microwave background radiation. So Steinhardt and Accetta proposed an improved version, hyperextended inflation [3], in which they took into consideration higher-order couplings of the JBD scalar field to the scalar curvature, and showed that their model gives an almost model-independent bubble distribution. Another method is soft inflation, proposed by Berkin, Maeda, and Yokoyama [4]. They considered new and chaotic inflationary scenarios in generalized Einstein theories (GET's), which include induced gravity, R^2 theory, and Kaluza-Klein theories as well as JBD theory. Taking advantage of the equivalence between GET's and the Einstein gravity via a conformal transformation [5], inflationary models were systematically investigated in the conformal frame. They found that new inflation as well as chaotic inflaton with a massive inflaton allow not only natural initial data but also natural coupling constants. Such approaches may hence give us a natural inflationary model.

Apart from the naturalness of inflationary models, we have another question about naturalness in inflation, namely whether an inflationary solution is the unique attractor in the most general spacetime. In the discussion about inflation, we usually assume Friedmann-Robertson-Walker spacetimes. Since we have not so far learned anything about the initial state of the Universe, the ansatz of isotropy and homogeneity of the Universe should be also justified by inflation. We have to investigate whether or not inflation really occurs even in anisotropic and/or inhomogeneous spacetimes and whether the spacetime is isotropized and homogenized during inflation. This problem is related to the so-called cosmic no-hair conjecture.

In the most general inhomogeneous case, the proof is very difficult, and hence a numerical approach may be the only way to show whether or not it is true [6], except for perturbation analysis [7]. However, in an anisotropic but homogeneous case, we have the cosmic no-hair theorem proved by Wald [8], which tells us that all initially expanding Bianchi models except type IX approach de Sitter spacetime in one *e*-folding time if a positive cosmological constant exists. In old or new inflationary models, the vacuum energy is almost a constant, and thus behaves as a cosmological constant. This theorem is, hence, applicable. This theorem was also extended to chaotic inflationary models [9]. For type IX, however, since some spacetimes may recollapse, more study is necessary [10,11].

A natural question may arise, namely, whether or not this theorem is extendable to a power-law inflationary

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model [12]. This problem becomes more important in the above new approach, because such an inflation in GET's usually shows a power-law expansion (including hyperextended inflation, in which the cosmic expansion is power-law at the initial stage of interest). In fact, using a conformal transformation, we can show that such a model is equivalent to the Einstein theory with a scalar field whose potential is an exponential type, and thus leads to a power-law inflation [4]. Hence, if we believe either (hyper)extended or soft inflation, we have to worry about the cosmic no-hair conjecture in these scenarios, because only a small set of initial data may lead to an isotropic inflationary solution.

So far, there are a few previous works on this problem, both numerically and analytically [13]. In this Brief Report, we prove a cosmic no-hair theorem for Bianchi models. This theorem is just a simple extension of Wald's theorem [8] into the power-law inflationary case.

Our model Lagrangian is

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}(\nabla\phi)^2-V(\phi)+L_{\text{matter}}\right], \tag{1}$$

where $\kappa^2=8\pi G$, ϕ is an inflaton field and L_{matter} is the matter Lagrangian aside from ϕ. The potential of the inflaton is assumed to be an exponential type,

$$V(\phi)=V_0\exp(-\lambda\kappa\phi) \tag{2}$$

where V_0 and λ are positive constants. In order for power-law inflation to occur, λ must be smaller than $\sqrt{2}$. However, here, we restrict our discussion to

$$0\le\lambda<\sqrt{\tfrac{2}{3}} \tag{3}$$

for a mathematical reason. This restriction corresponds to $\omega>\frac{9}{2}$ for JBD theory [4], which is consistent with observations ($\omega>500$). We will comment on this point later.

Assuming a Bianchi-type homogeneous spacetime, we investigate whether or not a power-law inflationary solution is the unique attractor and such an anisotropic spacetime is really isotropized in finite time. The Einstein equations are written as

$$G_{ab}=\kappa^2(T_{ab}^{(\phi)}+T_{ab}), \tag{4}$$

where $T_{ab}^{(\phi)}$ and T_{ab} are the energy-momentum tensors of the inflaton and of matter. Here we assume that the matter fluid satisfies the strong and dominant energy conditions [14]. The latter condition implies the weak energy condition. The strong and weak energy conditions are described as

$$T_s(t)\equiv(T_{ab}-\tfrac{1}{2}Tg_{ab})t^at^b\ge 0 \quad\text{and}\quad T_w(t)\equiv T_{ab}t^at^b\ge 0 \tag{5}$$

for all timelike vectors t^a. Following Wald [8], we consider two components of the Einstein equations: one is the Hamiltonian constraint,

$$0=G_{ab}n^an^b-\kappa^2\left[\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2+V(\phi)+T_w(\boldsymbol{n})\right], \tag{6}$$

and the other is the Raychaudhuri equation,

$$0=R_{ab}n^an^b-\kappa^2\left[\left(\frac{d\phi}{dt}\right)^2-V(\phi)+T_s(\boldsymbol{n})\right], \tag{7}$$

where n^a is the unit vector orthogonal to the homogeneous hypersurfaces and t is the proper cosmic time.

For our purpose, it is convenient to use a new time coordinate τ, which is defined by

$$d\tau=\exp(-\lambda\kappa\phi/2)dt, \tag{8}$$

rather than the cosmic time t. In an isotropic and homogeneous spacetime, the attractor (a power-law inflationary solution) becomes a time-independent fixed point with this time coordinate [15]. Hereafter, an overdot denotes differentiation with respect to τ.

The above two equations and the scalar-field equation are then rewritten as

$$\tilde{K}^2=3\kappa^2(\tfrac{1}{2}\dot{\phi}^2+V_0)+\tfrac{3}{2}\tilde{\sigma}_{ab}\tilde{\sigma}^{ab}-\tfrac{3}{2}{}^{(3)}\tilde{R}+3\kappa^2\tilde{T}_w(\boldsymbol{n}), \tag{9}$$

$$\dot{\tilde{K}}=\kappa^2(-\dot{\phi}^2+V_0)+(\lambda\kappa/2)\tilde{K}\dot{\phi}-\tfrac{1}{3}\tilde{K}^2-\tilde{\sigma}_{ab}\tilde{\sigma}^{ab}-\kappa^2\tilde{T}_s(\boldsymbol{n}), \tag{10}$$

$$\ddot{\phi}=(\lambda\kappa/2)\dot{\phi}^2-\tilde{K}\dot{\phi}+\lambda\kappa V_0, \tag{11}$$

where we introduced a new variable $\tilde{K}$ defined by $\tilde{K}=K\exp(\lambda\kappa\phi/2)$, where K is the trace of the usual extrinsic curvature with respect to time t. The other variables with tilde relations are defined similarly, e.g., shear $\tilde{\sigma}_{ab}\tilde{\sigma}^{ab}=\sigma_{ab}\sigma^{ab}e^{\lambda\kappa\phi}$ and three-curvature ${}^{(3)}\tilde{R}={}^{(3)}Re^{\lambda\kappa\phi}$. Our proof is always reduced to Wald's by setting $\lambda=0$ and $\phi=0$. The terms with $\tilde{K}\dot{\phi}$ in (10) and with $\dot{\phi}^2$ in (11) appear due to the new time coordinate and new variables. We assume that the Universe is initially expanding, i.e., $K>0\Longrightarrow\tilde{K}>0$ at an initial time.

First we consider all Bianchi models except type IX. In this case, spacetime has nonpositive scalar curvature: ${}^{(3)}R\le 0$ [8]. From the constraint equation (9) and the weak energy condition, we get $\tilde{K}^2>0$ and then find $\tilde{K}>0$ forever if the spacetime is initially expanding. Now we shall define a function $\tilde{S}$, which plays the same role as $(K^2-3\Lambda)$ in Wald's paper [8] and $\frac{3}{2}S$ in Moss and Sahni's [9], as

$$\begin{aligned}\tilde{S}&\equiv\tilde{K}^2-3\kappa^2(\tfrac{1}{2}\dot{\phi}^2+V_0)\\&=\tfrac{3}{2}\tilde{\sigma}_{ab}\tilde{\sigma}^{ab}-\tfrac{3}{2}{}^{(3)}\tilde{R}+3\kappa^2\tilde{T}_w.\end{aligned} \tag{12}$$

This equation is nothing but the constraint equation (9). Because of ${}^{(3)}R\le 0$ and the weak energy condition, $S\ge 0$. The time derivative of S is derived from Eqs. (10) and (11), and then is estimated by $\tilde{K}>0$ and the strong energy condition as follows:

$$\begin{aligned}\dot{\tilde{S}}&=-\tfrac{1}{3}(2\tilde{K}-3\lambda\kappa\dot{\phi})\tilde{S}-2\tilde{K}(\tilde{\sigma}_{ab}\tilde{\sigma}^{ab}+\kappa^2\tilde{T}_s)\\&\le-\tfrac{1}{3}(2\tilde{K}-3\lambda\kappa\dot{\phi})\tilde{S}.\end{aligned} \tag{13}$$

From Eq. (12) with $\tilde{S}\geq 0$,

$$\tilde{K}^2-\tfrac{3}{2}\kappa^2\dot{\phi}^2\geq 3\kappa^2 V_0 \ , \tag{14}$$

and then from this inequality with $\tilde{K}>0$ and the ansatz $0\leq\lambda<\sqrt{\tfrac{2}{3}}$, we find

$$2\tilde{K}-3\lambda\kappa\dot{\phi}\geq\frac{3}{\tau_{\rm iso}}, \tag{15}$$

where

$$\tau_{\rm iso}\equiv\left[\frac{4}{3}\left[1-\frac{3\lambda^2}{2}\right]\kappa^2 V_0\right]^{-1/2} \tag{16}$$

is a given positive constant. Inequality (13) is then

$$\dot{\tilde{S}}\leq-\frac{1}{\tau_{\rm iso}}\tilde{S}\leq 0 \ . \tag{17}$$

Integrating this inequality, we find

$$0\leq\tilde{S}\leq\tilde{S}_0\exp\left[-\frac{(\tau-\tau_0)}{\tau_{\rm iso}}\right] , \tag{18}$$

where τ_0 is an initial time and $\tilde{S}_0=\tilde{S}(\tau_0)$. $\tilde{S}$ decays to zero exponentially with respect to time τ. Within one e-folding time $\tau_{\rm iso}$, the expansion rate of the Universe is dominated just by the inflaton energy density [16].

As for the shear, the three-curvature and the energy density of matter, we see

$$0\leq\frac{1}{2}\tilde{\sigma}_{ab}\tilde{\sigma}^{ab}, \quad -\frac{1}{2}{}^{(3)}\tilde{R} \ , \quad \kappa^2\tilde{T}_w(\boldsymbol{n})\leq\frac{\tilde{S}}{3}\leq\frac{\tilde{S}_0}{3}\exp\left[-\frac{\tau-\tau_0}{\tau_{\rm iso}}\right] \tag{19}$$

from Eq. (12). Thus the shear and the three-curvature of the homogeneous hypersurfaces and the matter energy density rapidly vanish, and hence all components of the energy-momentum tensor of matter fluid vanish too, because of the dominant energy condition, just as in Wald's case. In particular, the decay of anisotropy leads to an isotropic and homogeneous spacetime. We can thus show that a power-law inflationary solution is the unique attractor in spacetimes with non-negative curvature [15]. Thus, for $(\tau-\tau_0)>>\tau_{\rm iso}$, anisotropic spacetimes except for type IX are isotropized, and a power-law inflationary solution is realized.

One may want to know the time scale of isotropization in terms of the cosmic time t rather than τ. Since τ depends on ϕ, the isotropization time scale also depends on ϕ, for which we have to solve the equation of motion. However, since an isotropic power-law inflationary solution is realized rapidly in the τ time coordinate, we may probably estimate it using the isotropic attractor solution. The attractor solution is given by

$$a=a_0(t/t_0)^{2/\lambda^2} \ , \tag{20}$$

$$\kappa\phi=\kappa\phi_0+(2/\lambda)\ln(t/\kappa) \ , \tag{21}$$

where t_0 is some constant and $\kappa\phi_0 \equiv(1/\lambda)\ln[\lambda^4\kappa^4V_0/2(6-\lambda^2)]$ [15]. Using this solution, we can obtain the relation between t and τ to find

$$\exp\left[-\frac{\tau}{\tau_{\rm iso}}\right]\propto\left[\frac{t}{t_0}\right]^{-p_{\rm iso}} \tag{22}$$

where $p_{\rm iso}\equiv 4[(1-3\lambda^2/2)(1-\lambda^2/6)]^{1/2}/\lambda^2$. This gives us the following results in terms of t [17];

$$S,\ \sigma_{ab}\sigma^{ab},\ {}^{(3)}R, \text{ and } T_w(\boldsymbol{n})\propto(t/t_0)^{-(2+p_{\rm iso})} \ , \tag{23}$$

$$K^2 \text{ and } \rho_\phi\propto(t/t_0)^{-2} \ , \tag{24}$$

where $S\equiv\tilde{S}e^{-\lambda\kappa\phi}$ and $\rho_\phi=(d\phi/dt)^2+V(\phi)$. Thus, the additional power $p_{\rm iso}$ in (23) guarantees the anisotropy to vanish and allows us to ignore contributions from matter and the three-curvature. The remaining isotropic contribution from the inflaton field ϕ provides a power-law inflation.

Next we turn to the Bianchi-type IX case. First we show the upper bound of $\tilde{S}$. Although $\tilde{S}$ is not positive definite, it might be non-negative initially, i.e., $\tilde{S}_0=\tilde{S}(\tau_0)\geq 0$. In that case, as long as $\tilde{S}(\tau)\geq 0$, the inequality (17) holds as in the non–type-IX case, $\tilde{S}$ is monotonically decreasing and hence $\tilde{S}$ is bounded from above by Eq. (18). Thus, for general initial conditions, we find the upper bound as $\tilde{S}\leq\max\{0,\tilde{S}_0\exp[-(\tau-\tau_0)/\tau_{\rm iso}]\}$.

As opposed to the non–type-IX case, however, $\tilde{S}$ is not bounded from below. We hence have to estimate the lower bound as well. In the present model, we can give a vanishing lower bound as follows. In this closed model, fixing the proper volume, the three-curvature has its maximum positive value when the spacetime is isotropic [8]

$${}^{(3)}R\leq{}^{(3)}R_{\max}\propto\exp(-2\alpha) \ , \tag{25}$$

where $\exp(3\alpha)\equiv[\det(h_{ab})]^{1/2}$ is the volume element of the spatial metric $h_{ab}=g_{ab}+n_a n_b$. We require the additional condition that

$$\frac{\Lambda_{\rm eff}}{{}^{(3)}R_{\max}}=\frac{\tilde{\Lambda}_{\rm eff}}{{}^{(3)}\tilde{R}_{\max}}>\frac{1}{2} \tag{26}$$

is initially satisfied as in Wald's theorem, where $\Lambda_{\rm eff}\equiv\kappa^2V(\phi)=\kappa^2V_0e^{-\lambda\kappa\phi}$ is an effective cosmological constant, and $\tilde{\Lambda}_{\rm eff}\equiv\Lambda_{\rm eff}e^{\lambda\kappa\phi}=\kappa^2V_0$. Setting

$$\frac{\Lambda_{\rm eff}}{{}^{(3)}R_{\max}}\equiv\frac{1}{2}(1+\delta)\propto\exp(2\alpha-\lambda\kappa\phi) \ , \tag{27}$$

the ansatz (26) reads $\delta_0\equiv\delta(\tau_0)>0$.

From the Hamiltonian constraint (9) with $0\leq\lambda<\sqrt{\tfrac{2}{3}}$, if $\tilde{K}>0$ and $\delta>0$,

$$2\tilde{K}-3\lambda\kappa\dot{\phi}\geq 2\tilde{K}-\sqrt{6}\kappa|\dot{\phi}|>0 \ , \tag{28}$$

while

$$\frac{d}{d\tau}\ln(1+\delta)=\frac{d}{d\tau}\ln\left[\frac{\Lambda_{\rm eff}}{{}^{(3)}R_{\max}}\right] =-\frac{d}{d\tau}\ln{}^{(3)}\tilde{R}_{\max}=\frac{1}{3}(2\tilde{K}-3\lambda\kappa\dot{\phi}) \ , \tag{29}$$

where $\tilde{K}=3\dot{\alpha}$. Hence, if $\tilde{K}>0$ and $\delta>0$ initially, we can prove that the following two inequalities are satisfied for all $\tau\geq\tau_0$:

$$\delta \geq \delta_0 \text{ and then } \frac{\Lambda_{\text{eff}}}{{}^{(3)}R_{\max}} > \frac{1}{2} , \tag{30}$$

$$2\tilde{K} - 3\lambda\kappa\dot{\phi} \geq \frac{3}{\tau_{\text{iso,IX}}} , \tag{31}$$

where

$$\tau_{\text{iso,IX}} \equiv \tau_{\text{iso}} \left[1 + \frac{1}{\delta_0} \right]^{1/2} \tag{32}$$

with τ_{iso} defined previously. Inequality (31) is obtained in a similar way to the non-type-IX case.

From Eq. (12) with the weak energy condition, we find

$$\tilde{S} \geq -\frac{3}{2} {}^{(3)}\tilde{R}_{\max} \tag{33}$$

while ${}^{(3)}\tilde{R}_{\max}$ vanishes faster than $\exp[-(\tau-\tau_0)/\tau_{\text{iso,IX}}]$ from Eq. (29) with (31).

Finally, we get

$$-\frac{3}{2} {}^{(3)}\tilde{R}_{\max}(\tau_0) \exp\left[-\frac{(\tau-\tau_0)}{\tau_{\text{iso,IX}}} \right] \leq \tilde{S} \leq \max\left[0, \tilde{S}_0 \exp\left[-\frac{(\tau-\tau_0)}{\tau_{\text{iso}}} \right] \right] \tag{34}$$

and can verify S vanishes [17]. From Eq. (12), $-2\tilde{S}/3 \leq {}^{(3)}\tilde{R} \leq {}^{(3)}\tilde{R}_{\max}$, and then ${}^{(3)}R$ damps to zero just as S and ${}^{(3)}R_{\max}$.

The initially expanding Bianchi type-IX universe is, hence, also isotropized within one $\tau_{\text{iso,IX}}$, if $(\Lambda_{\text{eff}}/{}^{(3)}R_{\max}) > \frac{1}{2}$ initially. Because $\tau_{\text{iso,IX}}$ is larger than τ_{iso}, the convergence time in type IX is longer by the factor $\sqrt{1+1/\delta_0}$ than that in other types. As for the time scale in terms of the cosmic time t, p_{iso} should be replaced with $p_{\text{iso,IX}} \equiv p_{\text{iso}}\sqrt{\delta_0/(1+\delta_0)}$. The physical variables change with time t as Eqs. (23) and (24) and

$$\frac{{}^{(3)}R}{\Lambda_{\text{eff}}} \leq \frac{{}^{(3)}R_{\max}}{\Lambda_{\text{eff}}} \propto \left[\frac{t}{t_0} \right]^{-p_{\text{iso,IX}}} . \tag{35}$$

Note that in type IX, the initial time scale of convergence depends on the initial data $\delta_0 = 2(\Lambda_{\text{eff}}/{}^{(3)}R_{\max})(\tau_0) - 1$ (>0). The nearer δ_0 approaches to zero, the longer the convergence time becomes, but the speed of convergence grows to the same value near the attractor as that for other Bianchi types.

The cosmic no-hair theorem in power-law inflation presented in this paper is just an extended version of Wald's theorem. We can generalize the initial condition in type IX in the present cosmic no-hair theorem to a wider class both in conventional inflation and in power-law inflation, and also find the similar cosmic no-hair theorem for $\sqrt{\frac{2}{3}} \leq \lambda < \sqrt{2}$, although the time scale of convergence now depends on initial conditions for all Bianchi types. This will be presented elsewhere.

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[16] In our definition, our e-folding time τ_{iso} with $\lambda=0$ is exactly one-half of Wald's e-folding time α.

[17] Original variables without tildes decay faster than tilde variables because ϕ increases in the inflationary phase.