Superconducting cosmic string: Equation of state for spacelike and timelike current in the neutral limit

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The equation of state relating the tension T and the energy per unit length U of a cosmic string is investigated in the simplest nontrivial case, namely, that of a field theory with $\mathrm{U}(1)^{\mathrm{local}}\times\mathrm{U}(1)^{\mathrm{global}}$ invariance, in four dimensions, which is interpretable as the zero-charge-coupling-constant limit of the more general superconducting string models that have been previously investigated. This limit has the advantage of giving vacuum vortex defects that are strictly local so that the quantities such as U and T that are relevant for the macroscopic description can be computed without ambiguity. In the case of "electric" states (with timelike current) for which no comparable previous calculations exist, it is shown there is a critical frequency w_c beyond which the vortex becomes unstable due to "charge" carrier emission. In the case of "magnetic" states (with spacelike current), the present analysis provides more precise results than those of previous investigations, whose predictions are broadly confirmed for typical moderate models in which the tension T remains comparable to the energy density U though not for extreme models, in which serious discrepancies are revealed.

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INTRODUCTION

The purpose of this work is to derive the macroscopic quantities characterizing the dynamics of a current-carrying cosmic-string model of a charge-uncoupled or neutral kind whose complexity is intermediate between that of the original nonconducting kind proposed by Kibble [1] and that of the charge-coupled current-carrying kind proposed by Witten [2]. The most significantly new feature of this work is a first quantitative investigation of "electric" states, meaning those for which the current is timelike whereas previous numerical results (confirmed in moderate situations by the more precise numerical method used here) were restricted to "magnetic" states, meaning those for which the current is spacelike. One of the reasons why only a very few authors [3, 4] have paid much attention to "electric"-type states was the consideration that the corresponding electromagnetic fields would tend to be screened. However, more recent discussions [5] have made the point that the most important effects of currents in cosmic strings are mechanical rather than electromagnetic and as such can be studied even in the neutral (uncoupled) limit to which the present work is restricted.

Early discussions of the potential astrophysical significance of strings of the latter kind concentrated on spectacular effects (such as the formation of cosmological voids [6]) that might have been expected as a consequence of electromagnetic radiation resulting from the charge coupling of the currents. However it later became apparent that the most important effects of currents in cosmic strings are of a purely mechanical nature, and can be analyzed as a first approximation in terms of cosmic-string models of the simpler kind considered here, which are to be interpreted as corresponding to the limit in which the electromagnetic charge coupling constant e is considered to be sufficiently small to be neglected.

The dominant importance of purely mechanical rather than electromagnetic effects of currents in cosmic strings seems to have been first noticed by Davis and Shellard [5]. They drew attention to the fact that unlike nonconducting cosmic-string loops of the Kibble variety which must ultimately lose all their energy by gravitational radiation (and thus finally decay away to nothing), in current-carrying loops (whether of neutral or charge-coupled kind) gravitational (and when relevant electromagnetic) radiation losses will cease as the loop approaches a finite-energy centrifugally supported ground state in which it can survive indefinitely, the simplest configuration for such a stationary state having the form of a ring [5, 7]. This means that for massive [grand-unified-theory (GUT) energy scale] strings the currents whose existence was postulated in earlier discussions [2] can be excluded in advance on the grounds that the resulting relic distribution of cosmic-string loops would have been so dense as to have brought about closure and premature collapse of the universe. According to a tentative quantitative estimate by Carter [8] the avoidance of such a catastrophe requires that the characteristic energy scale for any (neutral or charged) conducting string distribution in the universe should not greatly exceed that of the electroweak unification as predicted by the diverse variants (see, e.g., Fayet [9]) of the original Glashow-Salam-Weinberg model.

A particularly convenient feature of the kind of uncoupled or neutral cosmic-string model to be considered here is that, as for a cosmic-string model of the simple nonconducting type originally discussed by Kibble [1], it is appropriately describable from a macroscopic point of view as a "string" in the strict sense. This means that its effective cross section is negligible compared with its length (its macroscopic motion being governed by a two-dimensional action which in the Kibble model is just that of Goto and Nambu), because its underlying microscopic structure is that of a vortex defect of the vacuum that is strictly local: the relevant fields are effectively confined to the immediate vicinity of the vortex center (at least in the approximation in which the extremely weak effect of gravitation is neglected). This is to be contrasted with the situation in a typical so-called superconducting string model of the type originally discussed by Witten [2] (and also of course in the gravitationally coupled version of the original Kibble model) in which the vacuum vortex defect is local in only a rather loose approximate sense, having an associated field distribution that is effectively unbounded at least in the idealized limit of an isolated state, the cross-sectional integral of physical quantities such as the energy density being divergent, due to the contribution of the accompanying electromagnetic field that arises from the presence of a source current along the vortex. The description of such a nonlocal vortex as a "string" at macroscopic level is only justifiable in the limit of applications for which, relative to their values in the core, the density of quantities such as the energy is sufficiently small to be neglected in the outer part, whose extent will in practice usually be limited by an effective cutoff at a distance beyond which effects of the external environment become relevant.

The weakness of the electromagnetic coupling constant e (and the even greater weakness that is typical of gravitational coupling) ensures that the appropriately truncated versions of what (in an idealized isolated state) would have been divergent integrals will in practice very often be negligible in comparison with the core contributions, so that macroscopic description as a narrowly confined "string" will after all be appropriate. It is only in such a limit (which might also result from the effect of screening in the charge-coupled case) that it will be justifiable to apply a formalism of the kind developed by Carter [10–12] whereby the macroscopic motion is calculable simply from knowledge of the effective equation of state of the string model.

In order to set up the macroscopic string formalism it suffices to specify the appropriate two-dimensional action (generalizing the simple Goto-Nambu action that governs the nonconducting Kibble model). Specification of the action is equivalent to the specification of two distinct equations of state relating the tension T of the string to the corresponding rest mass energy per unit length U say, one of these equations applying to the "magnetic" regime in which the current is spacelike, and the other applying to the "electric" regime in which the current is timelike [10, 12].

The most qualitatively new physical insight provided by the present work concerns the "electric" regime, which was largely ignored in previous studies except for what is implicit in the pionnering work of Davis and Shellard [5]. This regime has also been studied microscopically by Aryal, Vilenkin, and Vachaspati [3] as well as by Spergel, Piran, and Goodman [4] who were especially interested in electromagnetic effects (pair creation and radiation). Also we have to mention studies concerned with contexts such as that of the Kaluza-Klein mechanism [13, 14, 12, 15] or the noise mechanism [16, 17, 15] that are of a different nature from the ordinary (four-dimensional) conducting vortex mechanism under consideration here. For the "electric" (i.e., timelike current) regime, the effect of allowance for charge coupling can be expected to be more important than it has been found to be in the "magnetic" case, but considerations of this aspect will be postponed for future work [18]. An important new feature that is brought to light in the present study of the uncoupled limit is the existence of a phase frequency threshold at which the vortex ceases to be strictly local so that (as inevitable in the charge-coupled case) a macroscopic "string" description ceases to be exactly valid in the strictest sense, and beyond which it rapidly comes to break down altogether even as an approximation, a phenomenon that is interpretable as due to creation and ejection of current-carrying particles.

As far as magnetic states are concerned, the results of the present work can be summarized as confirming the general picture provided by previous studies [19, 20] in typical moderate cases for which the parameters of the underlying field theory are such that the tension remains comparable with the energy density U (one new detail emerges however, namely, that contrary to what occurs in the simplified model that is most commonly used [4, 12, 15] for which longitudinal perturbations propagate at the speed of light and therefore faster than transverse perturbations, in the more accurate study carried out here, it is found that longitudinal perturbations always propagate more slowly than transverse ones, a result that may have important implications for the problem of stability of stationary loop states [21]). However, in the more extreme cases for which a "cosmic-spring" limit of vanishing tension had been claimed to exist [20, 22], the accurate numerical method used in the present work leads to conclusions that are significantly different, so as to raise serious doubt about the reality of such states (whose physical relevance had already been called into question in view of stability considerations [11, 12]). Of course, the previous work differed from ours by the inclusion of charge coupling which violates the strictly local character of the vortex, but work to be described in a following article suggests that allowance for this effect is not sufficient to account for the discrepancy. It would appear from preliminary investigations presently in progress [23] that the results of previous studies of the more extreme cases in question were seriously misleading due to poor convergence of the numerical methods that were used. It can be conjectured that the so-called "cosmic-spring" states may merely have been an illusory artifact of an unsatisfactory approximation scheme with no underlying mathematical (still less physical) reality.

I. EQUATIONS OF MOTION

The field-theoretical model used by Kibble [1] is a special case within the class first discussed by Englert and Brout [24], in which spontaneous symmetry breaking occurs in the manner required by the Higgs mechanism [25]. In this case, the symmetry is that of the simplest Abelian gauge Lie group, namely $U(1)^{local}$: the Lagrangian describes a complex scalar Higgs field Φ coupled with a gauge vector B^μ by a charge-coupling constant, q say. This is the simplest field theory in which cosmic strings can form. Witten generalized this Lagrangian so as to allow the existence of what he referred to as "superconducting cosmic strings," by introducing a second scalar field Σ, which we shall refer to as the *current carrier.* Witten took the carrier scalar to be coupled by a second charge-coupling constant, e say, to a second gauge vector A_μ, which he assumed to be the ordinary electromagnetic field. There is however a simpler extension of the Kibble model that still allows the existence of a current in the string but which is characterized by a global, rather than local $U(1)^{global}$ symmetry; that is, its Lagrangian is only invariant under the addition of a constant in the phase of the *current carrier* field Σ. In this model, whose examination is the subject of the present work, the conserved current is effectively neutral, having no coupling to any associated gauge vector (this being the feature that allows the corresponding vortex to be strictly local as in the simple Kibble model but unlike what occurs in the general Witten model). Explicitly, this "neutral carrier" model is given by the (renormalizable) Lagrangian

$$\mathcal{L} = -\frac{1}{2}(D_\mu\Phi)(D^\mu\Phi)^\star - \frac{1}{2}(\nabla_\mu\Sigma)(\nabla^\mu\Sigma)^\star - \frac{1}{16\pi}H_{\mu\nu}H^{\mu\nu} - V(\Phi,\Sigma),$$

$$V(\Phi,\Sigma) = \frac{\lambda_\phi}{8}(|\Phi|^2-\eta^2)^2 + f(|\Phi|^2-\eta^2)|\Sigma|^2 + \frac{\lambda_\sigma}{4}|\Sigma|^4 + \frac{m_\sigma^2}{2}|\Sigma|^2, \tag{1}$$

$$D_\mu\Phi \equiv (\nabla_\mu + iqB_\mu)\Phi,$$
$$H_{\mu\nu} \equiv \nabla_\mu B_\nu - \nabla_\nu B_\mu,$$

where the signature of the metric is +2. This model can be considered either as a lowest-order approximation to the charged-coupled model of Witten or as a description of a conceivable physical situation in which the relevant current might be of exactly neutral type. In this Lagrangian, each parameter is supposed real and positive, so that, contrary to Witten, m_σ is the actual mass of the *current carrier* Σ. Note also that the positivity of the coupling constants and of the squared mass is a consequence of the requirement that the vacuum has to be a global minimum of the potential with respect to the bosonic Σ field, i.e., of the requirement

$$\frac{\delta^2 V}{\delta\Sigma^2}(|\Sigma|=0,|\Phi|=\eta) > 0, \tag{2}$$

which is needed for the theory to be physically meaningful.

We are concerned with vortex strings solution in which the $U(1)^{global}$ symmetry is broken by the Kibble-Witten mechanism which requires that the potential

$$V(|\Phi|=0) = \frac{\lambda_\phi}{8}\eta^4 + \frac{1}{2}(m_\sigma^2 - 2f\eta^2)|\Sigma|^2 + \frac{\lambda_\sigma}{4}|\Sigma|^4 \tag{3}$$

should be minimized with a nonzero vacuum expectation value of the field Σ. It is necessary for this that the combination

$$m_W^2 \equiv 2f\eta^2 - m_\sigma^2 \tag{4}$$

(where m_W^2 is what Witten [2] denoted simply by m^2) should be positive:

$$m_W^2 > 0. \tag{5}$$

We now set

$$\Phi = \varphi e^{i\alpha} \quad \text{and} \quad \Sigma = \sigma e^{i\psi}, \tag{6}$$

where the amplitudes φ and σ and the phase variables α and ψ are real. Then the independent variables are these functions, B_μ and their derivatives. This yields the following equations of motion (note that these would be completely different if the fields were complex):

$$\nabla_\mu[\varphi^2(\nabla^\mu\alpha + qB^\mu)] = 0, \tag{7}$$

$$\nabla_\mu(\sigma^2\nabla^\mu\psi) = 0, \tag{8}$$

$$\nabla_\mu\nabla^\mu\varphi = \varphi(\nabla_\mu\alpha + qB_\mu)(\nabla^\mu\alpha + qB^\mu) + \frac{\lambda_\phi}{2}\varphi(\varphi^2-\eta^2) + 2f\varphi\sigma^2, \tag{9}$$

$$\nabla_\mu\nabla^\mu\sigma = \sigma\nabla_\mu\psi\nabla^\mu\psi + 2f\varphi^2\sigma + \lambda_\sigma\sigma^3 + (m_\sigma^2 - 2f\eta^2)\sigma, \tag{10}$$

$$\nabla_\mu H^{\mu\nu} = 4\pi q\varphi^2(\nabla^\nu\alpha + qB^\nu). \tag{11}$$

The current conservation laws (7) and (8) are interpretable as the Nœther identities expressing invariance with respect to changes in the phases of Φ and Σ. The conserved current associated with the latter (global) invariance is given by

$$\mathcal{J}^\mu \equiv \sigma^2\nabla^\mu\psi. \tag{12}$$

We now restrict our attention to a straight stationary string configuration which we take to be aligned along the z axis. We postulate that the solution should be cylindrically symmetric modulo phase transformations, which means that the amplitudes φ and σ can depend only on a radial variable r say. As in the case of a nonconducting cosmic-string model of the type considered by Kibble [1], we suppose that the phase α of the Higgs field Φ depends only on the cylindrical angle variable θ in the form

$$\alpha = 2\pi n\theta, \tag{13}$$

where n is necessarily an integral winding number while as in the more general conducting string models of the type introduced by Witten [2] the phase of the carrier field Σ is postulated to be independant both of θ and of r, varying only as a function of the longitudinal variable

z and the time t in the form

$$\psi = \omega t - kz, \tag{14}$$

where k and ω are constants. Then, setting

$$w \equiv k^2 - \omega^2, \tag{15}$$

and introducing the abbreviation $Q(r) = n + qB_\theta$, we obtain the cylindrical equations of motion [because of (13) and (8), combined with the fact that σ is a function of r only, B_θ is the only nonzero component of the gauge field B_μ]

$$\begin{aligned} \frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} &= \frac{1}{r^2}\varphi Q^2 + \frac{1}{2}\lambda_\phi \varphi(\varphi^2 - \eta^2) + 2f\varphi\sigma^2, \\ \frac{d^2\sigma}{dr^2} + \frac{1}{r}\frac{d\sigma}{dr} &= w\sigma + 2f\varphi^2\sigma + \lambda_\sigma \sigma^3 \\ &\quad + (m_\sigma^2 - 2f\eta^2)\sigma, \\ \frac{d^2Q}{dr^2} - \frac{1}{r}\frac{dQ}{dr} &= 4\pi q^2 \varphi^2 Q, \end{aligned} \tag{16}$$

where

$$w \begin{cases} > 0 & \text{in the magnetic case,} \\ < 0 & \text{in the electric case,} \\ = 0 & \text{in the null case,} \end{cases}$$

according to the classification of Carter [10, 11]. We may now impose the physical boundary conditions

$$\begin{aligned} &\varphi(0) = 0, \quad \varphi(\infty) = \eta, \\ &\frac{d\sigma}{dr}(0) = 0, \quad \sigma(\infty) = 0, \\ &Q(0) = n, \quad Q(\infty) = 0, \end{aligned} \tag{17}$$

and scale the variables and fields by setting

$$\varphi = \eta X, \quad \sigma = \frac{m_\sigma}{\sqrt{\lambda_\sigma}} Y, \quad r = \frac{\rho}{\sqrt{\lambda_\phi}\eta}, \tag{18}$$

$$w = \frac{\lambda_\phi \lambda_\sigma}{m_\sigma^2}\eta^4 \tilde{w}, \quad q^2 = \lambda_\phi \tilde{q}^2. \tag{19}$$

We can now define the free parameters of the theory as

$$\alpha_1 = \frac{m_\sigma^2}{\lambda_\sigma \eta^2}, \quad \alpha_2 = \frac{f m_\sigma^2}{\lambda_\phi \lambda_\sigma \eta^2}, \quad \alpha_3 = \frac{m_\sigma^4}{\lambda_\phi \lambda_\sigma \eta^4}, \tag{20}$$

all of them being positive as a restatement of Eq. (2). Note that this scaling and the definition of the parameters seem identical to that of Babul, Piran, and Spergel [20], but here, as emphasized before, m_σ is the actual mass of the *current carrier*. Therefore, knowing the mass of the Higgs particle to be $m_\varphi = \sqrt{\lambda_\phi}\eta$, the physical meaning of these parameters is clear:

$$\alpha_1 = \frac{\lambda_\phi}{\lambda_\sigma}\left(\frac{m_\sigma}{m_\varphi}\right)^2, \quad \alpha_2 = \frac{f}{\lambda_\sigma}\left(\frac{m_\sigma}{m_\varphi}\right)^2, \quad \alpha_3 = \frac{\lambda_\phi}{\lambda_\sigma}\left(\frac{m_\sigma}{m_\varphi}\right)^4. \tag{21}$$

The relationship of the parameters defined here to those used by Babul, Piran, and Spergel [20], which we shall distinguish by a dagger, is given by

$$\begin{aligned} \alpha_1 &= 2\frac{\alpha_1^\dagger \alpha_2^\dagger}{\alpha_3^\dagger} - \alpha_1^\dagger, \\ \alpha_2 &= 2\frac{(\alpha_2^\dagger)^2}{\alpha_3^\dagger} - \alpha_2^\dagger, \\ \alpha_3 &= \frac{(\alpha_3^\dagger - 2\alpha_2^\dagger)^2}{\alpha_3^\dagger}, \end{aligned} \tag{22}$$

with the "renormalization"

$$Y = \frac{Y^\dagger}{\sqrt{2\frac{\alpha_2^\dagger}{\alpha_3^\dagger} - 1}} \quad \text{and} \quad \tilde{w} = \left(2\frac{\alpha_2^\dagger}{\alpha_3^\dagger} - 1\right)\tilde{w}^\dagger \tag{23}$$

for the *current carrier* field. However, there exists another set of parameters, namely,

$$\beta_1 = \frac{\alpha_3}{\alpha_1^2} = \frac{\lambda_\sigma}{\lambda_\phi}, \quad \beta_2 = \frac{\alpha_2}{\alpha_1} = \frac{f}{\lambda_\phi}, \quad \beta_3 = \sqrt{\frac{\alpha_3}{\alpha_1}} = \frac{m_\sigma}{m_\varphi}, \tag{24}$$

whose immediate physical significance makes them useful for the interpretation of the results obtainable in exploring the α_i space. In a physically realistic model where the Higgs-boson-mass scale is at least that of the electroweak unification scale and in which the relevant particle trapped in the string is the lightest charged particle, i.e., the electron, one expects these free parameters to be very small quantities that could be used in a perturbative way, noting that α_3 will certainly be a few orders of magnitude smaller than α_1 and α_2 in this case. Moreover, there is another restriction in the fact that we require the vacuum to be "uncharged" and "nonconducting" [regarding the $U(1)^{\text{global}}$ charge], i.e.,

$$V(|\Sigma| = 0, |\Phi| = \eta) < V(|\Sigma| \neq 0, |\Phi| = 0), \tag{25}$$

which implies

$$(\alpha_3 - 2\alpha_2)^2 < \frac{\alpha_3}{2}, \tag{26}$$

i.e., we require the ratio of the masses not to be less than a fixed value, namely,

$$\frac{m_\sigma^2}{m_\varphi^2} > 2\frac{f}{\lambda_\phi} - \sqrt{\frac{\lambda_\sigma}{2\lambda_\phi}}, \tag{27}$$

or, stated differently, the ratios between f and λ_ϕ and between λ_σ and λ_ϕ must be small enough (or close enough in case of a fine-tuned problem) so that (27) holds. We can now write down the dimensionless equations of motion, in terms of the variable ρ, as

$$X'' + \frac{1}{\rho}X' = \frac{1}{\rho^2}XQ^2 + \frac{1}{2}X(X^2-1) + 2\alpha_2 XY^2,$$

$$Y'' + \frac{1}{\rho}Y' = \frac{\tilde{w} + 2\alpha_2(X^2-1)}{\alpha_1}Y + \frac{\alpha_3}{\alpha_1}Y(Y^2+1), \tag{28}$$

$$Q'' - \frac{1}{\rho}Q' = 4\pi\tilde{q}^2 X^2 Q.$$

We shall return to these equations after having defined all the quantities of physical interest such as the energy, the tension, and the action.

II. EQUATION OF STATE

In order to obtain the equation of state, for which we take it for granted that only the case of unit winding number $n = 1$ is relevant, we need to compute the energy-momentum tensor from the variations of the Lagrangian with respect to the metric by means of

$$T^{\mu}_{\ \nu} = -2g^{\mu\alpha}\frac{\delta \mathcal{L}}{\delta g^{\alpha\nu}} + \delta^{\mu}{}_{\nu}\mathcal{L}, \tag{29}$$

which, with the Lagrangian (1), gives

$$T^{tt} = \omega^2\sigma^2 - \mathcal{L},$$
$$T^{zz} = k^2\sigma^2 + \mathcal{L}. \tag{30}$$

The quantities U and T are then understood to be

$$U = 2\pi \int r\,dr\,T^{tt}, \tag{31}$$

and

$$T = -2\pi \int r\,dr\,T^{zz}. \tag{32}$$

Also we use the current density (12) to define the magnitude $\mathcal{C}$ of a line density current in the string with internal string coordinate

$$\mathcal{C}^a = 2\pi \int r\,dr\,\mathcal{J}^{\,a}, \quad a = t \text{ or } z, \tag{33}$$

as

$$\mathcal{C} = \sqrt{|\mathcal{C}_t^2 - \mathcal{C}_z^2|}, \tag{34}$$

which we will denote as "charge number density" throughout the rest of this work. Now we can write down our actual definitions of U, T, and $\mathcal{C}$ as integrals over the microscopic fields in terms of cylindrical variables, but since we are going to concentrate on a particular example, we are obliged to define a referential in which ψ is a function of t or z only, in which case the phase of the *current carrier* field Σ will be ωt or $-kz$, where ω or k are constants. In the null case, with $w = 0$, we find, as in an ordinary relativistic string,

$$T^{tt} + T^{zz} = 0, \text{i.e., } U = T, \tag{35}$$

and obviously $\mathcal{C} = 0$. In the magnetic sector, we have

$$U = \pi \int r\,dr \left(\varphi'^2 + \sigma'^2 + \frac{Q'^2}{4\pi q^2 r^2} + \frac{\varphi^2 Q^2}{r^2} + w\sigma^2 + 2V\right) = -L, \tag{36}$$

$$T = \pi \int r\,dr \left(\varphi'^2 + \sigma'^2 + \frac{Q'^2}{4\pi q^2 r^2} + \frac{\varphi^2 Q^2}{r^2} - w\sigma^2 + 2V\right) = -\tilde{L}, \tag{37}$$

and, in the electric sector,

$$U = \pi \int r\,dr \left(\varphi'^2 + \sigma'^2 + \frac{Q'^2}{4\pi q^2 r^2} + \frac{\varphi^2 Q^2}{r^2} - w\sigma^2 + 2V\right) = -\tilde{L}, \tag{38}$$

$$T = \pi \int r\,dr \left(\varphi'^2 + \sigma'^2 + \frac{Q'^2}{4\pi q^2 r^2} + \frac{\varphi^2 Q^2}{r^2} + w\sigma^2 + 2V\right) = -L, \tag{39}$$

while in both cases the charge number density will be given by

$$\mathcal{C} = 2\pi\sqrt{|w|}\int r\,dr\,\sigma^2. \tag{40}$$

Now, if we set $\nu = w/\sqrt{|w|} = \text{sgn}\,(w)\sqrt{|w|}$, we have

$$\mathcal{C} = 2\pi|\nu|\int r\,dr\,\sigma^2, \tag{41}$$

and Eqs. (36)–(39) yield an analytical equation of state,

$$U - T = |\nu|\mathcal{C}, \tag{42}$$

of the form originally obtained by Carter [11], and with the help of which we can already deduce some conclusions because $\mathcal{C}$ has been defined positive. When $|w| \longmapsto \infty$, then either $U - T$ also diverges, or $\mathcal{C}$ tends to zero. We will see numerically that both cases occur. Also, as U (for $\nu \geq 0$) or T (for $\nu \leq 0$) does represent the action from which the equations of motion are derived, we have

$$\left.\frac{dU}{d\nu}\right|_{\nu\geq 0} = \mathcal{C} \quad\text{and}\quad \left.\frac{dT}{d\nu}\right|_{\nu\leq 0} = \mathcal{C}, \tag{43}$$

and, therefore, differentiating Eq. (42) with respect to ν leads to

$$\left.\frac{dT}{d\nu}\right|_{\nu\geq 0} = -\nu\left.\frac{d\mathcal{C}}{d\nu}\right|_{\nu\geq 0} \quad\text{and}\quad \left.\frac{dU}{d\nu}\right|_{\nu\leq 0} = -\nu\left.\frac{d\mathcal{C}}{d\nu}\right|_{\nu\leq 0}. \tag{44}$$

Moreover, we know that $\mathcal{C}(\nu = 0) = 0$ and $\mathcal{C} \geq 0$, so that $d\mathcal{C}/d\nu \geq 0$ for $\nu \geq 0$ and $d\mathcal{C}/d\nu \leq 0$ for $\nu \leq 0$.

Therefore, Eq. (43) permits us to show that the condition (Carter [11]) necessary for the string to be stable with respect to longitudinal perturbations $dT/dU \leq 0$ is satisfied at least in a small neighborhood around $\nu = 0$. Indeed, let us define $\mathcal{V}^+$ as the set $\{\nu \geq 0 \; ; \; d\mathcal{C}/d\nu \geq 0\}$ and $\mathcal{V}^- = \{\nu \leq 0 \; ; \; d\mathcal{C}/d\nu \leq 0\}$. Then, for $\nu \in \mathcal{V}^+$, Eq. (42) shows that $dU/d\nu \geq 0$, which implies, because of the first of Eqs. (43), $dT/d\nu \leq 0$; and for $\nu \in \mathcal{V}^-$, one has $dT/d\nu \geq 0$ and $dU/d\nu \leq 0$. Thus one finds that $dT/dU \leq 0$, for all $\nu \in \mathcal{V}^+ \cup \mathcal{V}^-$, so that the string is stable with respect to longitudinal perturbations either in magnetic and in electric regimes if this condition is a sufficient one.

Next we use U and T to define a reduced action per unit length as an integral over the microscopic fields which may be used as a multiplicative factor of the Goto-Nambu action in a macroscopic description of the string. As we must start from the Lagrangian (1) and integrate over all space-time, we see that, up to a sign, in the magnetic regime, this action is just the energy U, and in the electric regime, it is the tension T. This explains the notation L in (36) and (39), and the $\tilde{L}$ in (37) and (38) comes from the duality relation which exists between energy and tension, as developed by Carter [10]. Explicitly, for this action per unit length expressed in terms of dimensionless variables, we have

$$\frac{L}{\eta^2} = -\pi \int \rho\, d\rho \Big(X'^2 + \alpha_1 Y'^2 + \frac{Q'^2}{4\pi\tilde{q}^2\rho^2} + \frac{X^2Q^2}{\rho^2} + \tilde{w}Y^2 + \frac{1}{4}(X^2-1)^2 + 2\alpha_2 Y^2(X^2-1) + \alpha_3 Y^2(\tfrac{1}{2}Y^2+1)\Big). \quad (45)$$

It is convenient to convert this to a finite-range integral by means of $\rho = \tan\xi$, because the density of points will then be very large at the origin and small at infinity, so that we can compute with great accuracy the core structure while taking into account the boundary conditions. We obtain

$$\frac{L}{\eta^2} = -\pi \int_0^{\pi/2} d\xi \tan\xi \Bigg\{ \cos^2\xi \left[\left(\frac{dX}{d\xi}\right)^2 + \alpha_1 \left(\frac{dY}{d\xi}\right)^2 + \frac{\tan^{-2}\xi}{4\pi\tilde{q}^2}\left(\frac{dQ}{d\xi}\right)^2 \right] + \frac{1}{\cos^2\xi}\left(\frac{X^2Q^2}{\tan^2\xi} + \tilde{w}Y^2 + \frac{1}{4}(X^2-1)^2 + 2\alpha_2 Y^2(X^2-1) + \alpha_3 Y^2(\tfrac{1}{2}Y^2+1)\right)\Bigg\}. \quad (46)$$

In order to compute numerical solutions, we write this as a finite sum over a grid, vary this sum with respect to the values of the fields at each point of the grid and solve the resulting algebraic equations by means of a successively overrelaxed (SOR) method (cf., e.g., Adler and Piran [26]). The necessary integrals (36)–(42) are computed by means of an open alternative extended Simpson's rule, which gives the best accuracy. As a precision check of the numerical results obtained, we made use of Eq. (43), that is we computed separately $\mathcal{C}$ and $dU/d\nu$ or $dT/d\nu$ and compared both results; we obtained the same numerical values with an accuracy better than 10^{-8} in this way. A typical result for the Higgs field X and the *current carrier* field Y is shown on Fig. 1, in a stable electric situation.

Finally, as it can be useful for the macroscopic formalism (Carter [12, 15]), we computed the function K, given by

$$K(\nu) = -2\frac{dL}{dw} = 2\frac{dU}{dw}\Big|_{\nu\geq 0} = 2\frac{dT}{dw}\Big|_{\nu\leq 0} = 2\frac{\mathcal{C}}{|\nu|}, \quad (47)$$

and which is shown in Figs. 2 and 3 as a function of $\tilde{\nu}$ and $\tilde{w}$, respectively, as a dashed line. We see on these curves that the first and second derivatives of K with respect to ν are zero for $\nu = 0$, so that an expansion as

$$-L = T_0 + \frac{1}{2}K_0 w + O(w^2) \quad (48)$$

will be a very good approximation in the neighborhood of $\nu = 0$.

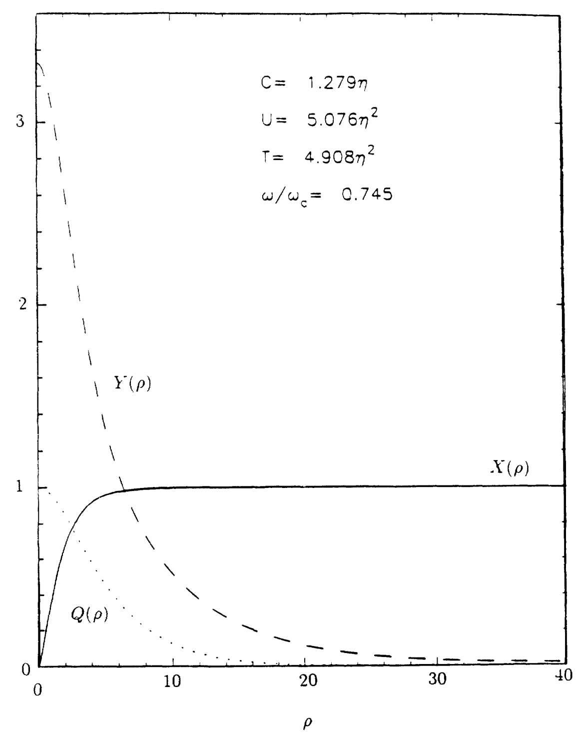

FIG. 1. The solutions of the field equations for X, Y, and Q. This represents a stable "charged" configuration with $\tilde{w} < 0$.

In order to compare our results with the clearest and most precise previous results, namely those of Babul, Piran, and Spergel [20], with $\mathrm{U}(1)^{\mathrm{local}}$ coupling constant given by

$$4\pi\tilde{q}^2 = 0.1, \tag{49}$$

and a nonzero charge coupling given by

$$(m_\sigma^2 e^2/m_\varphi^2) = 0.1 \tag{50}$$

(with e the usual electromagnetic charge coupling constant), we have chosen two sets of parameter values that were used by them, the first being a moderate situation (differing little from the zero current case) with

$$\alpha_1 = 1.68\times10^{-2},\quad \alpha_2 = 5.26\times10^{-3},\quad \alpha_3 = 5.26\times10^{-4}, \tag{51}$$

which corresponds to

$$\alpha_1^\dagger = 0.32,\quad \alpha_2^\dagger = 0.1,\quad \alpha_3^\dagger = 0.19, \tag{52}$$

while the second, with

$$\alpha_1 = 2\times10^{-3},\quad \alpha_2 = 5\times10^{-4},\quad \alpha_3 = 2\times10^{-6}, \tag{53}$$

i.e.,

$$\alpha_1^\dagger = 1\ ,\quad \alpha_2^\dagger = 0.25\ ,\quad \alpha_3^\dagger = 0.499 \tag{54}$$

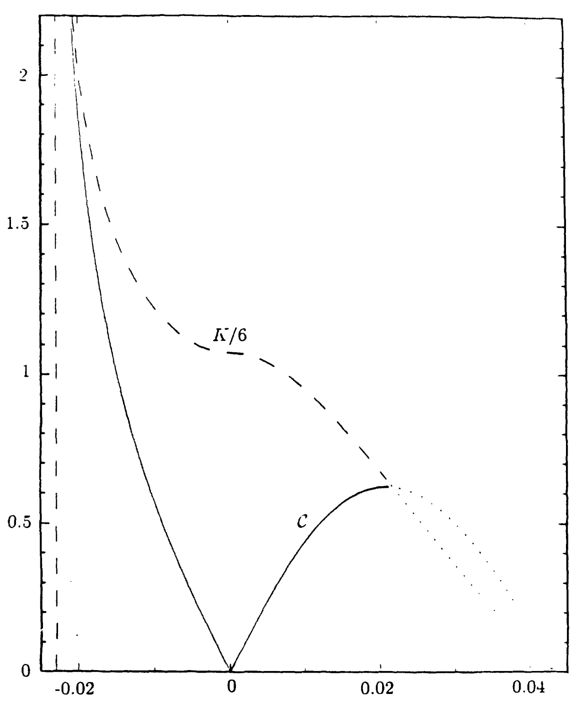

FIG. 2. The integrated charge (solid line) per unit length or current given by expression (40) as a function of $\tilde{\nu}$, i.e., $\mathcal{C} = (\eta/\sqrt{\lambda_\phi})2\pi\sqrt{\alpha_1}\tilde{\nu}\int\rho\, d\rho\, Y^2$ and the dimensionless K function (dashed line) given by $K = 4\pi(\alpha_1/\lambda_\phi)\int\rho\, d\rho\, Y^2$.

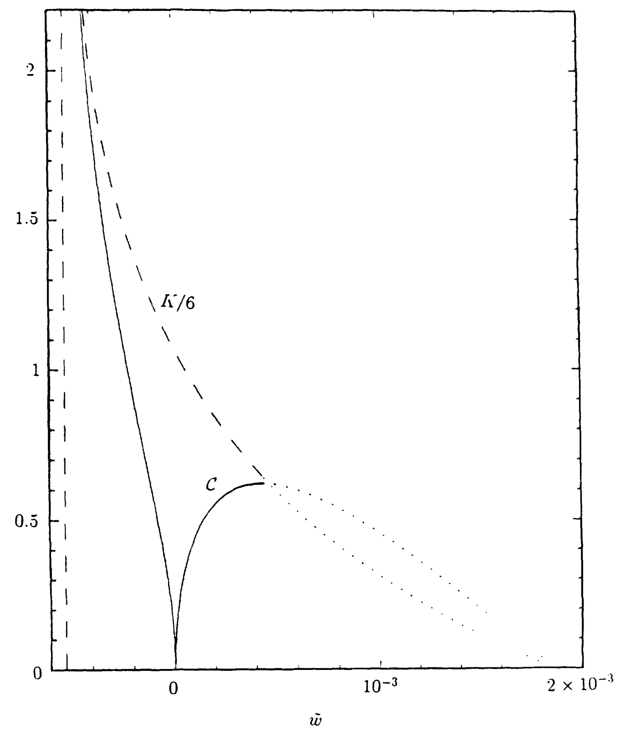

FIG. 3. Same as Fig. 2 but as a function of $\tilde{w}$.

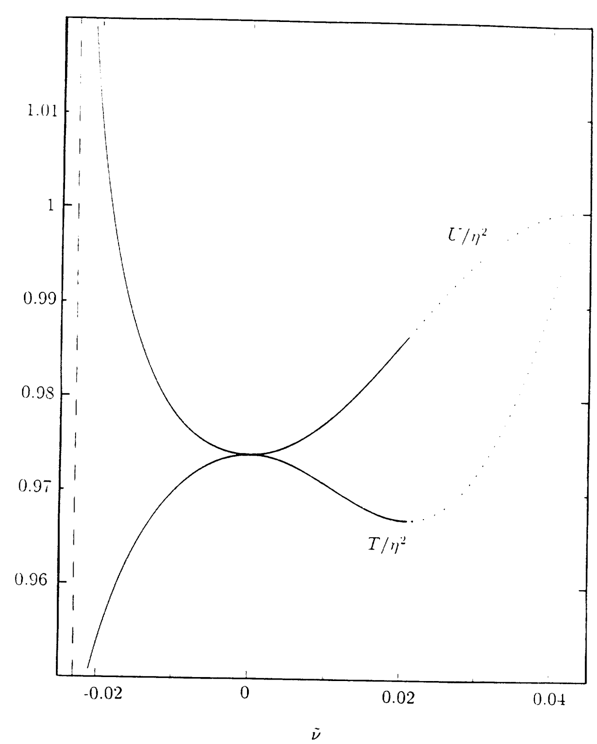

FIG. 4. The energy and tension integrated with (36)–(39) as a function of $\tilde{\nu}$ normalized to unity when the string has no current. The dashed line is the limit $\tilde{\nu} = \tilde{\nu}_c$ and the curves are drawn in dotted lines when the configuration is unstable against longitudinal perturbations [11].

corresponds to a much more extreme situation in which the original calculation leads to the prediction of a tension T decreasing to zero at a "cosmic-spring" limit.

Let us concentrate first on the moderate case for which we obtain a result agreeing closely with that of Babul, Piran, and Spergel in the magnetic sector to which their investigation was restricted. It is found that every quantity of physical interest is continuous while passing through $w = 0$. This is evident in Figs. 2–5 where U, T, and $\mathcal{C}$ are shown as functions of $\tilde{\nu}$ and $\tilde{w}$, with the energy and the tension normalized to their values in the Kibble case. It is apparent on these curves that when w becomes large and positive, the *current carrier* field decouples itself from the Higgs field, leaving just an ordinary Kibble-type cosmic string, which is a now well-known phenomenon, but before attaining this situation, there is a regime (for the chosen parameter values, it is for $\tilde{\nu}$ greater than approximately 0.021) in which the string is unstable under longitudinal perturbations ($dU/dT > 0$). This corresponds in Fig. 2 to the region where $\mathcal{C}$ is a decreasing function of ν and represented as a dotted curve.

Therefore, there is a maximum current contained in the string because, as the phase gradient of the *current carrier* field increases, it becomes energetically favored for the "charged" bosons to jump out of the core [2], but also because the string itself may decay for any longitudinal perturbation. However, we computed numerically that, as Witten emphasized, this phenomenon occurs when the impulsion k along the string exceeds the mass of the boson in vacuum, and we find that the maximum value for the current is obtained (up to numerical errors) for $w = m_\sigma$.

A direct plot of T versus U is given in Fig. 6, which shows explicitly the stable regimes attainable by the string, namely those for which $dT/dU \leq 0$. We can see on this plot that the stability is only possible in a finite (and in this case small) part of the w space in the magnetic case, whereas in the electric situation (as will be shown in next section) there is a critical value of w given by

$$\tilde{w}_c = -\alpha_3, \tag{55}$$

beyond which no stationary solution exists at all. The energy, the tension, and the charge number density [see Eq. (71) for the latter] all diverge as this value is approached as can be seen in Figs. 5 and 3. In accordance with Eq. (43) we see that the tension decreases in the electric regime as $\tilde{w}$ goes to $\tilde{w}_c$, so that the divergence of T should finally lead to negative values. This is actually what we find numerically, but for the parameters used here, this occurs only for values of $\tilde{\nu}$ very close to $\tilde{\nu}_c$ so that the only effective requirement here is $w > w_c$. More precisely, the stability requirement $T > 0$ against transverse perturbation [11] is verified as long as ν is less than ν_T say, where we define ν_T by means of $U = \nu_T \mathcal{C}$ [Eq. (42) with $T = 0$], and we find $\nu_T \gtrsim \nu_c$. This behavior is qualitatively similar to what has been observed [15] for the nondispersive self-dual equation of state that

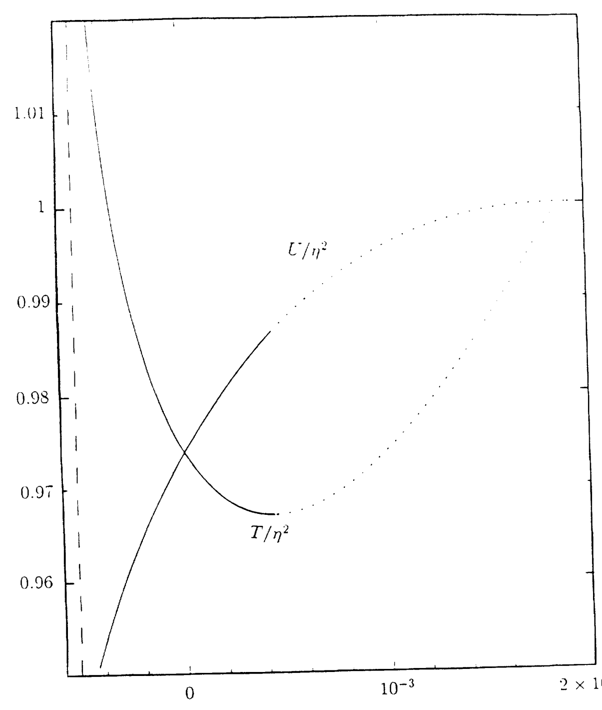

FIG. 5. Same as Fig. 4 but as a function of $\tilde{w}$.

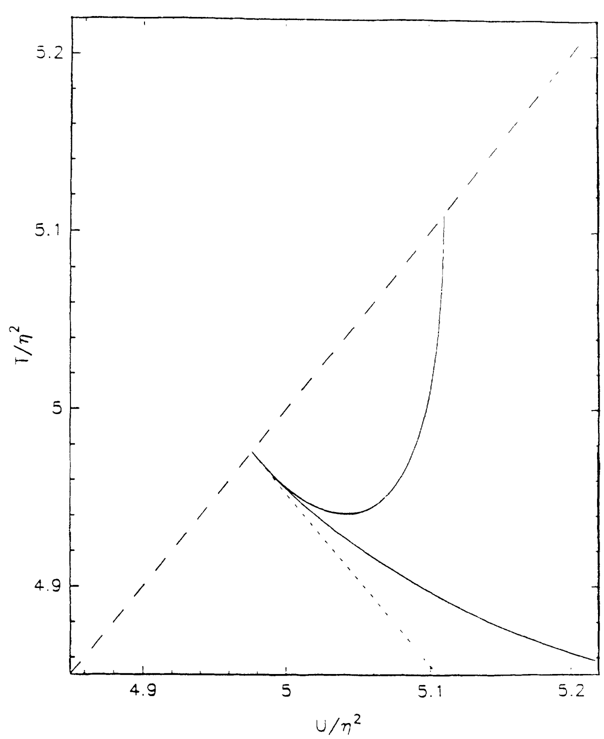

FIG. 6. The tension versus the energy per unit length. The "magnetic equation of state" is above the "electric equation of state." It can be observed that there is approximate self-duality [12] (in the sense that the two coincide with considerable precision) in the vicinity of the null limit. The limiting Goto-Nambu case is on the dashed line $U = T$ while the dotted curve represents the integrable equation of state $UT = U_0^2$.

results from the very different physical mechanism of microscopic noise excitation [16, 17].

The most striking feature that may be seen on the curves described above is that they look much the same as those obtained for the weakly charge-coupled model used by Babul, Piran, and Spergel: apart for small corrections whose significance will be analyzed in a future work [18], we find the energy per unit length, the tension, and the current to have nearly the same numerical values whether or not the current carrier is charge coupled. We can therefore conclude that most of the effects due to the string superconductivity in the magnetic regime (e.g., the saturation phenomenon) are qualitatively mechanical, their existence being in no way related to the charge coupling of the theory.

While the qualitative picture obtained by previous work is confirmed in moderate cases exemplified by the parameter values (51) and (52), the nontriviality of this conclusion is shown by the analysis of the more extreme cases exemplified by the parameter values (53) and (54) for which our result differs rather drastically from those obtained before (see Fig. 7). In particular, we do not find evidence that the tension tends to a zero "cosmic-spring" limit for any finite current value. This significant deviation from the previously accepted picture might be conceived to be due to the simplification constituted by our neglect of charge coupling in the present work, but an investigation to be described in a following article [23] indicates that, as in the more moderate cases described above, the effect of the comparatively weak charge-coupling value as given by Eq. (50) is still unimportant even in the extreme case characterized by (53) and (54) so the source of the discrepancy must be sought elsewhere. A preliminary investigation suggests that for the extreme case in question, the numerical methods used in previous work gave seriously misleading results due to bad convergence properties. Further work will be needed to clarify this question, but it can already be conjectured that the tension has been systematically underestimated and that it will in fact always remain positive.

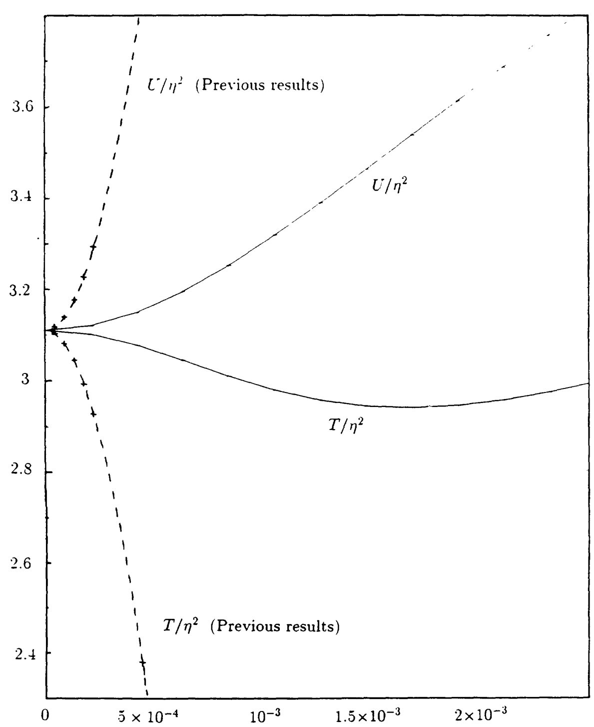

FIG. 7. Energy per unit length and tension as functions of $\tilde{\nu}$ in the extreme case. Solid curves represents our solutions and dashed curves are obtained from previous work [20, 22]. Previous results lead to zero or even negative values for the effective tension already for the relatively small values of $\tilde{\nu}$ greater than 10^{-3}.

Another new feature that arises from our solutions concerns the perturbations' propagation speeds as given by

$$c_T^2 = \frac{T}{U}, \tag{56}$$

for the transverse ones and

$$c_L^2 = -\frac{dT}{dU}, \tag{57}$$

for the longitudinal ones [12]. Contrary to previous simplified equations of state [10, 11, 15] such as the linear one $U + T = \mathrm{C^{ste}}$, we find that the longitudinal propagation speed is in fact smaller than the transverse perturbation velocity. This is shown in Fig. 8 with c_L as a dashed line and c_T being represented by the solid line in the moderate situation with parameter values given by (51). This result, even if obtained here in the neutral limit, may have important implications on stability

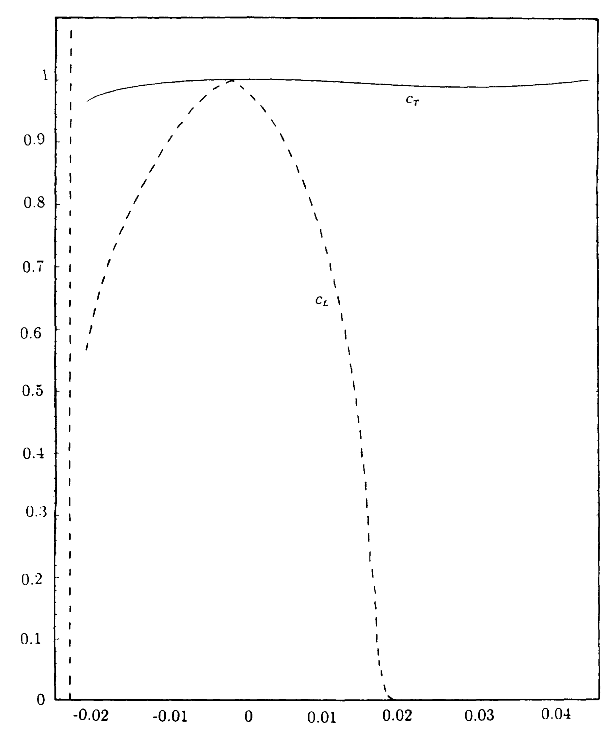

FIG. 8. Transverse (solid line) and longitudinal (dashed line) perturbation speeds as functions of ν.

requirements for loop configurations [12, 21] if verified in the charge-coupled cases [23].

III. LOCAL ANALYSIS

In order to understand the numerical results described above, we turn back to the system (28) which we may solve for $\rho \ll 1$ by means of a Taylor expansion around the origin as

$$X(\rho) = \sum_{i=0}^{\infty} x_i \rho^i, \quad Y(\rho) = \sum_{j=0}^{\infty} y_j \rho^j, \quad Q(\rho) = \sum_{i=0}^{\infty} q_k \rho^k. \tag{58}$$

Remembering the boundary conditions (17), one obtains the leading behavior for the Higgs field,

$$X \sim x_{|n|}\rho^{|n|} + x_{|n|+2}\rho^{|n|+2} + \cdots, \tag{59}$$

for a string with a net winding number n, which in turn implies that Q is an even function of ρ :

$$Q \sim n + q_2\rho^2 + \sum_{k=2}^{\infty} q_{2k}\rho^{2k}, \tag{60}$$

with $4k(k+1)q_{2(k+1)} = 4\pi\tilde{q}^2 x_{|n|}^2 q_{2(k-|n|)}$, so that one still has an unknown (namely, q_2) that is ultimately computed with a matching with the asymptotic solution. As for Y, we find

$$Y \sim y_0 \left(1 + \left[\tilde{w} - 2\alpha_2 + \alpha_3(y_0^2 + 1)\right] \frac{\rho^2}{4\alpha_1} + O(\rho^4)\right), \tag{61}$$

which gives us a constraint on y_0 since we require Y to be a decreasing function of ρ,

$$y_0^2 \leq \frac{2\alpha_2 - \alpha_3 - \tilde{w}}{\alpha_3}, \tag{62}$$

and, as we saw previously that σ, and therefore Y, is real, this leads to $\tilde{w} < 2\alpha_2 - \alpha_3$, or, stated in dimensionful variables, $w < m_W^2$ in which m_W is the mass as defined by (4). This restriction is merely a restatement of condition (5) with the potential (3) replaced by an effective potential $\tilde{V}$ given by

$$\tilde{V} = \frac{\lambda_\phi}{8}\eta^4 + \frac{1}{2}(m_\sigma^2 - 2f\eta^2 + w)\sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4, \tag{63}$$

which must also be minimizable by a nonzero vacuum expectation value for σ, now considered as a real self-interacting Klein-Gordon uncoupled field in the core of the string.

The values of $x_{|n|}$ (dotted) and q_2 (dashed) are displayed in Fig. 9 as function of $\tilde{\nu}$ for the special (but presumably the only stable) case $n = 1$, while the quantity y_0 as a function of $\tilde{\nu}$ is shown in the same figure as a solid line for the particular set of parameters (51). This provides a mathematical explanation for the shape of $\mathcal{C}(\nu)$: as ν is increased, the condensate σ becomes more coupled with itself than with the Higgs boson; in that sense, the X^2Y term in Eq. (28) is subdominant and the only solution satisfying the boundary conditions tends to the vacuum expectation value of the free self-interacting bosonic field, which is zero. An interesting feature concerning these curves and which seems to be independent of any special values of the parameters α_i is the abruptness at which they join the Kibble situation. This may also be seen in Fig. 10 where we defined a "string radius" ρ_c as $X(\rho_c) - 1 < \varepsilon$, with ε a small numerical constant. We observe on these curves the now well-known phenomenon of the increase of radius when there is some current (or charge) in the string, and we notice that as the current goes to zero, this radius falls to $\rho_c = \rho_c^{\text{Kibble}}$, but one may never get $\rho_c < \rho_c^{\text{Kibble}}$ since this would lead to an energy per unit length greater than that obtained in the Kibble case, and therefore an unstable configuration.

The case $\rho \gg 1$ yields

$$X \sim 1 - c_1\frac{e^{-\rho}}{\sqrt{\rho}}, \quad Q \sim c_2 e^{-2\sqrt{\pi}\tilde{q}\rho}, \tag{64}$$

for some constants c_1 and c_2, while in examining the asymptotic equation for Y, we see that Eq. (55) gives the phase frequency threshold w_c in terms of which we obtain

$$Y'' + \frac{1}{\rho}Y' \sim \frac{\tilde{w} - \tilde{w}_c}{\alpha_1}Y + \frac{\alpha_3}{\alpha_1}Y^3, \tag{65}$$

in which, if $\tilde{w} \neq \tilde{w}_c$ we can neglect the cubic term and get ordinary Bessel equations, which gives

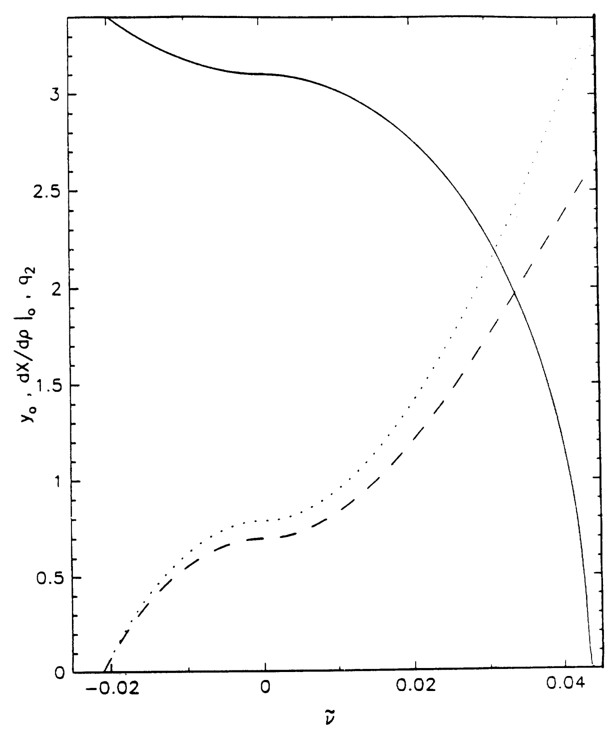

FIG. 9. The functions y_0, x_1, and q_2 defined by Eqs. (61), (59), and (60) versus $\tilde{\nu}$, and rescaled by means of $x_1 \rightarrow 50(x_1 - 0.404)$ and $q_2 \rightarrow 480(q_2 - 4.44 \times 10^{-2})$. The solid line is y_0, the dashed line is q_2 and the dotted line is x_1.

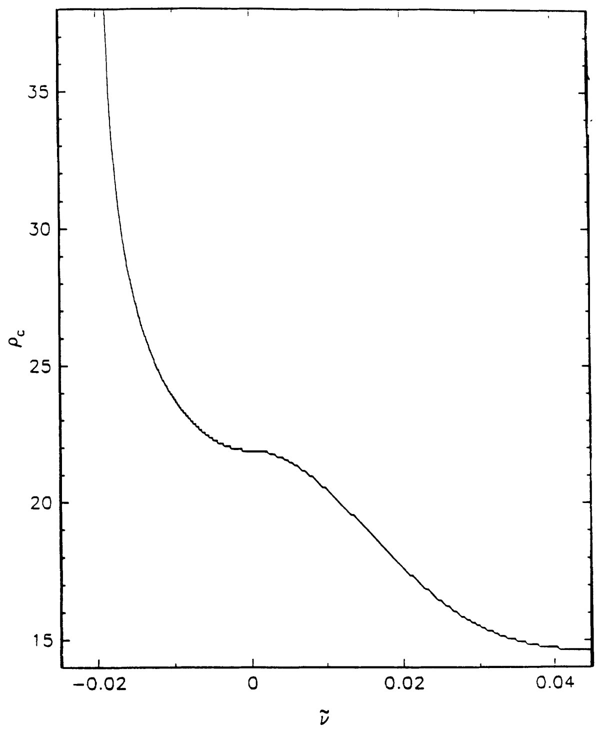

FIG. 10. The "effective radius" ρ_c of the vortex (see text) versus $\tilde{\nu}$.

$$\text{if } \tilde{w} > \tilde{w}_c, \quad Y \sim A\frac{e^{-\kappa\rho}}{\sqrt{\kappa\rho}}, \tag{66}$$

$$\text{if } \tilde{w} < \tilde{w}_c, \quad Y \sim \frac{A\cos(\kappa\rho + \delta)}{\sqrt{\kappa\rho}}, \tag{67}$$

while in the last case

$$\text{if } \tilde{w} = \tilde{w}_c, \quad Y \sim \sqrt{\frac{\alpha_1}{\alpha_3}}\rho^{-1}, \tag{68}$$

where $\kappa^2 = |(\tilde{w} - \tilde{w}_c)/\alpha_1|$ and the amplitude A and the phase displacement δ are constants that should ultimately be computed by a matching with the solution at the origin. Yet, we can only see in Eq. (28) that, as $X(0) = 0$, the equation at the origin for Y does not depend on κ except in the trivial case where $\alpha_2 = 0$, which is when Φ and Σ are not coupled. Thus, A and δ are functions of $\tilde{w}$, α_1, α_2, and α_3, but not of the special combination κ. The presence of this latter constant in Eq. (66) seems to us to be the source of the tension underestimation problem outlined at the end of the previous section. It turns out that for the α parameters used by previous authors [20, 22] such as those introduced earlier as the extreme case (53), one has $\kappa \ll 1$, so that Y is a very slowly decreasing function of ρ. This implies that the cutoff needed for numerical reasons must be somehow related to the value of κ. We shall return to that point in the framework of the fully coupled Witten model in a future article [23].

In the unstable regime $\tilde{w} < \tilde{w}_c$, we can interpret the result as cylindrical waves around the string, because the effective field Σ also has a phase by virtue of Eq. (14) with $k = 0$ for $w < 0$ as we choose a particular reference frame ($\Sigma = \sigma e^{i\omega t}$ with $\omega = \sqrt{-w}$), and as we supposed the string was in a stationary configuration, we can rewrite the solution as

$$Y \sim \frac{A}{\sqrt{\kappa\rho}}(e^{-i\kappa\rho - i\delta} + e^{i\kappa\rho + i\delta});$$

i.e., the number of incoming waves has to be the same as the number of outgoing waves. This means that in order to have a stationary solution, one has to impose that there exists a bosonic source at infinity to take into account the incoming waves. Moreover, we note that the pulsation ω also has a critical value (for which the critical solution for the fields is shown in Fig. 11) $\omega_c = m_\sigma$ which is exactly the mass of the charged boson in the real vacuum, so we can interpret the solution in saying that, as soon as the classical bosonic field oscillates topologically with enough energy, a particle is created at a quantum level and is ejected of the string, this phenomenon being exponentially reduced (tunnel effect) when the oscillations remain in a classical regime (frequency less than the mass). In the former regime, it may then be argued that stationary solutions do not exist since, in order to avoid an extra source at infinity, the current carrier's excitation has to be produced with radial momentum κ directed outward.

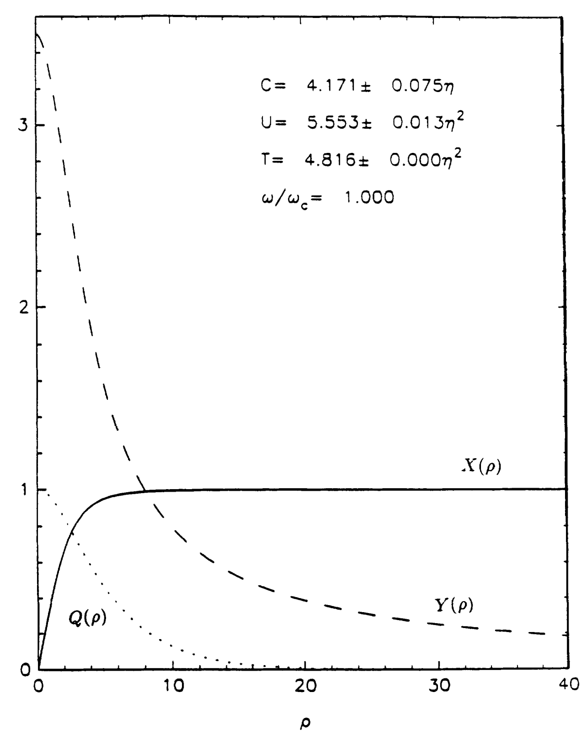

FIG. 11. The solutions of the field equations for X, Y, and Q. This represents the critical configuration with $\tilde{w} = \tilde{w}_c$.

Now the solution (64) for X is found to be subdominant when $w \leq w_c$ because of the coupling between the Higgs boson and the *current carrier* field. Inserting solution (67) into (28), and setting $X = 1 - \varepsilon(\rho)$, one obtains the linearized equation

$$\frac{d^2(\rho\varepsilon)}{d\rho^2} - \varepsilon' = \rho\varepsilon - 2\alpha_2 \frac{A^2}{\kappa} \cos^2(\kappa\rho + \delta), \qquad (69)$$

and we solve it in the hypothesis that ε' is negligible, which is verified afterwards. We find

$$X \sim 1 - \frac{2\alpha_2 A^2}{\kappa\rho}\left(1 + \frac{\cos\left[2(\kappa\rho + \delta)\right]}{1 + 4\kappa^2}\right) - c_1 \frac{e^{-\rho}}{\rho}, \qquad (70)$$

for some c_1. This result shows that there is no emission of Higgs particles because the dominant contribution is $\propto 1/\rho$ and not $1/\sqrt{\rho}$ and the current associated with Eq. (7) is damped, so that all the energy lost by the string is ejected in the flux of "charge."

Actually, we note that the only divergent terms in Eqs. (38)–(40) with $w \leq w_c$ are those which contain σ^2 alone, and if the integration is carried over all the surrounding space, we see that when $\tilde{w}$ approaches $\tilde{w}_c$, the total charge density behaves like

$$\mathcal{C} \sim \frac{\omega}{\omega^2 - m_\sigma^2} \propto (\tilde{w} - \tilde{w}_c)^{-1} \qquad (71)$$

if we assume that A is independent of $\tilde{w} - \tilde{w}_c$, and it becomes divergent when $\tilde{w} \leq \tilde{w}_c$. This kind of divergence corresponds to what is obtained in the integrable self-dual case $UT = U_0^2$ and in the Kaluza-Klein situations [15–17], so it may eventually be understood as a general result in the electric regime.

CONCLUSIONS

Let us summarize by stating the following new results that are actually obtained in this investigation of the neutral limit.

(i) It is shown in moderate cases that the previous results concerning the "magnetic" side of the equation of state (current saturation, for instance) are, as guessed beforehand by several other authors [5], of purely mechanical origin and are *not* due to electromagnetic coupling effect.

(ii) More specifically, it is shown in such cases that the speed of longitudinal perturbations is systematically lower than that of transverse perturbations.

(iii) It is shown that in more extreme cases, the tension remains significantly positive and higher than predicted by previous calculations.

(iv) The equation of state for the "electric regime" (whose existence was postulated earlier [10, 11]) is explored for the first time.

(v) It is found numerically that the equation of state is approximately "self–dual" [12] in some neighborhood of $\nu = 0$. It is found that stability requirements with respect to longitudinal perturbations are satisfied in the whole "electric" side.

(vi) It is shown that there exists a phase frequency threshold beyond which the string emits particles. This frequency corresponds exactly to the mass of the *current carrier* and, as might have been guessed for topological reasons, there is no Higgs particle emission. This, again, is not due to electromagnetic coupling effects.

(vii) Finally, the behavior of the amplitude of the current turns out to be that predicted by independent macroscopic models [15–17] when w approaches its critical value. It may be conjectured that this can be quite general so long as the electromagnetic coupling constant remains negligible.

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