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$S_3 \times Z_3$ model of lepton mass matrices

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A recently proposed $S_3 \times Z_3$ model of quark mass matrices is extended in parallel to include leptons, assuming that neutrinos are in fact not massless. As an example, it is shown how a massive τ neutrino with a coupling of no more than 6% to the electron neutrino may naturally fit into such a scheme.

Recently a model was proposed [1] for quarks which obtained two successful empirical relationships, $|V_{us}| \simeq (m_d/m_s)^{1/2}$ and $m_s^2/m_b^2 \simeq -V_{ub}V_{cb}/V_{us}$, as the result of the discrete symmetry $S_3 \times Z_3$. Obviously one can apply this idea to leptons as well; but if neutrinos are all massless, mixing in the leptonic sector is theoretically undefined and experimentally unobservable. On the other hand, if neutrino masses are not zero and not degenerate, then this model does predict to a certain extent what the mixing angles should be.

In exact parallel with the assignment of the quarks, the usual 6 leptons are grouped into doublets and singlets under S_3. The 4 scalar SU(2) doublets already introduced for the quark sector are used in the same way. In addition, because neutrinos may also have self-conjugate (Majorana) masses, 3 neutral scalar singlets are assumed to be present. All are listed in Table I together with their $S_3 \times Z_3$ and lepton-number (L) assignments. As a result, certain Yukawa couplings are forbidden and the contributions to the charged-lepton mass matrix from the neutral scalar fields imply

$$M_l = \begin{pmatrix} 0 & h_1\langle\phi_2^0\rangle & h_2\langle\eta_1^0\rangle \\ h_1\langle\phi_2^0\rangle & 0 & h_2\langle\eta_2^0\rangle \\ h_3\langle\eta_1^0\rangle & h_3\langle\eta_2^0\rangle & h_4\langle\phi_1^0\rangle \end{pmatrix} \tag{1}$$

in exact analogy to M_d in Ref. [1]. Since

$$\frac{\langle\eta_1^0\rangle}{\langle\eta_2^0\rangle} = \frac{m_u}{m_c} (\simeq 4\times 10^{-3}) \tag{2}$$

from Ref. [1], M_l can be rewritten as

$$M_l = \begin{pmatrix} 0 & a & \xi b \\ a & 0 & b \\ \xi c & c & d \end{pmatrix}, \tag{3}$$

where a,b,c,d are real and ξ is complex with $|\xi| = m_u/m_c$. In the neutrino sector, the 3×3 mass matrix linking $(\overline{\nu_e, \nu_\mu, \nu_\tau})_L$ to $(\nu_1, \nu_2, \nu_3)_R$ is given by

$$M_\nu = \begin{pmatrix} h_5\langle\bar\eta_1^0\rangle & 0 & 0 \\ 0 & h_5\langle\bar\eta_2^0\rangle & 0 \\ 0 & 0 & h_6\langle\bar\phi_1^0\rangle \end{pmatrix}, \tag{4}$$

in exact analogy to M_u in Ref. [1]. However, there is also the Majorana mass matrix linking $(\nu_1, \nu_2, \nu_3)_R$ to itself:

$$M_\nu' = \begin{pmatrix} h_7\langle\chi_1^0\rangle & h_8\langle\chi_3^0\rangle & 0 \\ h_8\langle\chi_3^0\rangle & h_7\langle\chi_2^0\rangle & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{5}$$

This structure guarantees that ν_τ is a Dirac particle and that ν_e and ν_μ are Majorana particles with masses given by the seesaw mechanism. It is also a specific realization of the general form for the 6×6 neutrino mass matrix advocated by Glashow in a recent paper [2].

The motivation for all this comes from two recent experimental observations [3,4] of a possible 17-keV component of the electron neutrino with a mixing parameter measured by Hime and Jelley [4] to be

$$\sin^2\theta = 0.0085 \pm 0.0006 \pm 0.0005 . \tag{6}$$

The relatively large mixing excludes the possibility that this massive component is ν_μ because $\nu_\mu \rightarrow \nu_e$ oscillation

TABLE I. Assignment of leptons and scalars under $S_3 \times Z_3$ and L. The elements of Z_3 are 1, ω, and ω^2, with $\omega^3 = 1$.

	S_3	Z_3	L
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$, l_{3R}, ν_{3R}	1	1	1
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$, $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	2	ω	1
(l_{2R}, l_{1R}), (ν_{2R}, ν_{1R})	2	ω^2	1
$\begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}$	1	1	0
$\begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$	1	ω^2	0
$\begin{pmatrix} \eta_1^+ \\ \eta_1^0 \end{pmatrix}$, $\begin{pmatrix} \eta_2^+ \\ \eta_2^0 \end{pmatrix}$	2	ω	0
χ_3^0	1	ω^2	-2
(χ_2^0, χ_1^0)	2	ω^2	-2

data require $\sin^2(2\theta) < 0.0034$, but allows it to be ν_τ with only the limit $\sin^2(2\theta) < 0.07$ from $\nu_e \nrightarrow \nu_e$ data. However, if ν_τ is a Majorana particle, then its effective contribution to neutrinoless double β decay is (0.0085) (17 keV) = 145 eV which is 2 orders of magnitude larger than the present experimental bound [5]. Hence the 17-keV neutrino, if indeed it exists, is likely to be a Dirac particle. It should be emphasized at this point that a 17-keV neutrino was first reported by Simpson several years ago [6] but with a much larger $\sin^2\theta$ of about 3%. Subsequent other experimentation [5] did not confirm his result and has placed upper limits on $\sin^2\theta$ lower than the present value. Obviously this issue will only be resolved with further more precise data.

Consider now Eq. (3). If we assume that $d^2 \ll c^2$ (which will be justified *a posteriori*), then the diagonalization of M_l according to

$$M_l M_l^\dagger = V \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} V^\dagger \tag{7}$$

results in $m_\tau \simeq c$, $m_\mu \simeq (a^2+b^2)^{1/2}$, $m_e = |a^2 d - 2\xi abc| / c(a^2+b^2)^{1/2}$, $V_{e\tau} \simeq a/c$, $V_{\mu\tau} \simeq bd/c^2 + \xi^* a/c$, and $V_{e\mu} \simeq -[abd/c + \xi(a^2-b^2)]/(a^2+b^2)$. Since ν_τ does not mix with ν_e or ν_μ in the mass matrices of Eqs. (4) and (5), $V_{e\tau}$ does represent the observed mixing of ν_e and ν_τ. However, $a < m_\mu$ is required in this model, so $V_{e\tau} \simeq a/c < m_\mu/m_\tau \simeq 0.06$, which is below the recently reported value [4] of $(0.0085)^{1/2} \simeq 0.092$. If it is indeed confirmed in the future that $V_{e\tau}$ is greater than 0.06, then this model can be ruled out. Using $m_\tau = 1784$ MeV, $m_\mu = 105.66$ MeV, $m_e = 0.511$ MeV, and assuming that $b^2 \ll a^2$, so that $V_{e\tau} \simeq 0.06$, we then find $d \simeq m_e m_\tau / m_\mu \simeq 8.6$ MeV, yielding the predictions $|V_{e\mu}| \simeq 3.8\times 10^{-3}$ and $|V_{\mu\tau}| \simeq 2.3 \times 10^{-4}$. Whereas the former will be changed by the ν_e-ν_μ mixing in the neutrino mass matrix to be discussed, the latter will not. It is thus interesting to note that it is well below the existing limit of $\sin^2(2\theta) < 4\times 10^{-3}$ from $\nu_\mu \rightarrow \nu_\tau$ oscillation data [5]. In other words, this model offers a natural explanation for the smallness of $V_{\mu\tau}$. Indeed,

$$|V_{\mu\tau}| / |V_{e\tau}| \simeq m_u / m_c \, . \tag{8}$$

Consider now the 4×4 neutrino mass matrix spanned by $\bar\nu_e$, $\bar\nu_\mu$, ν_{1R}, and ν_{2R}:

$$M = \begin{pmatrix} 0 & 0 & A & 0 \\ 0 & 0 & 0 & B \\ A & 0 & C & E \\ 0 & B & E & D \end{pmatrix} , \tag{9}$$

where $A/B = m_u/m_c$ as in Eq. (2). Assume $B \ll C$, $D < E$, then the eigenvalues of M are $\pm E + (C+D)/2$, $B^2 C/E^2$, and $-A^2/C$. The last two are clearly seesaw Majorana masses for ν_μ and ν_e, respectively, with $m(\nu_\mu) \gg m(\nu_e)$. Mixing between ν_e and ν_μ is easily calculated to be AE/BC. If we choose, for example, $A \simeq 200$ eV, $B \simeq 50$ keV, $C \simeq 4$ GeV, and $E \simeq 100$ GeV, then $m(\nu_\mu) \simeq 10^{-3}$ eV, $m(\nu_e) \simeq 10^{-5}$ eV, and for ν_e-ν_μ mixing, $\sin\theta \simeq 0.1$. We may thus ignore the contribution of $|V_{e\mu}| \simeq 3.8\times 10^{-3}$ from the charged-lepton sector. We have obtained the particular values of $\Delta m^2 \simeq 10^{-6}$ eV^2 and $\sin^2(2\theta) \simeq 4\times 10^{-2}$ to show that this model can easily accommodate the necessary neutrino parameters [7] for an explanation of the solar-neutrino problem in terms of the Mikheyev-Smirnov-Wolfenstein (MSW) effect [8].

So far it has been shown that the $S_3 \times Z_3$ model proposed earlier for quark mass matrices is equally applicable to the lepton sector, resulting in a natural description of neutrino masses and mixing which is consistent with the MSW explanation of the solar-neutrino problem and possibly the newly reported 17-keV neutrino [3,4]. However, the existence of a stable 17-keV neutrino is against cosmological expectations [9]. This means that the ν_τ of this model must decay. The mechanism of choice is the emission of a Majoron, which is a massless Goldstone scalar boson arising from the spontaneous breaking of the lepton-number global symmetry L. In this model, the Majoron is a linear combination of $\chi^0_{1,2,3}$ and $\bar\chi^0_{1,2,3}$ which are SU(2) singlets [10]; hence, they do not contribute to the invisible width of the Z boson and are thus consistent with the observation at the CERN e^+e^- collider LEP that the effective number of light neutrinos is just three [11]. On the other hand, ν_τ does not mix with ν_e or ν_μ in the mass matrices of Eqs. (4) and (5), so it has no tree-level coupling to the Majoron (which will be called χ in the following, for simplicity).

The dominant one-loop diagrams for ν_τ decay are depicted in Fig. 1. The effective coupling is approximately given by

$$F \simeq \frac{V_{e\tau}}{16\pi^2} \frac{m(\nu_\tau) m_\tau^2 \langle\chi\rangle}{\langle\phi_1^0\rangle\langle\phi_2^0\rangle\langle m_\phi^2\rangle} \simeq 3\times 10^{-15} , \tag{10}$$

where the vacuum expectation values are estimated to be $\langle\phi_1^0\rangle = \langle\phi_2^0\rangle = 100$ GeV, $\langle\chi\rangle = 1$ TeV, and the effective m_ϕ^2 is taken to be 0.6 TeV^2. The decay lifetime $\tau(\nu_\tau)$ is then roughly 3×10^{11} s or about 10^4 yr. In particular

$$[m(\nu_\tau)]^2 \tau(\nu_\tau) \simeq 10^{20} \text{ eV}^2\text{s} \tag{11}$$

which is just below the upper limit [12] of 2×10^{20} eV^{2}s

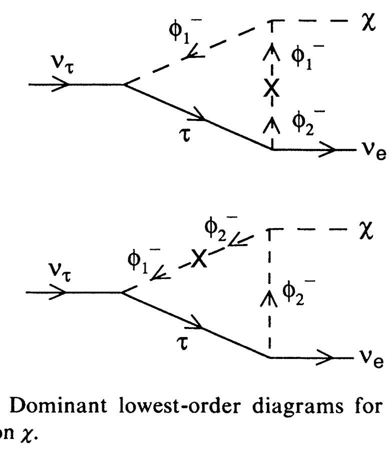

FIG. 1. Dominant lowest-order diagrams for ν_τ decay into the Majoron χ.

allowed by cosmology. However, $\langle\chi\rangle$ is not constrained by electroweak symmetry breaking and may well be much greater than 1 TeV, so $\tau(\nu_\tau)$ may in fact be much shorter. Note also that χ_i^0 interacts with the other scalars always in the combination $\chi_i^0\bar{\chi}_i^0$, which means that if there were only one χ^0, single Majoron emission off a scalar line would be impossible and $\tau(\nu_\tau)$ would be very much longer than is allowed. As it is, there are off-diagonal mass terms $\chi_i^0\bar{\chi}_j^0$ in this model which break the S_3 symmetry softly; hence, single Majoron emission is not a problem. Furthermore, if χ is replaced by a photon, the requirement of gauge invariance implies that the amplitude is reduced by a factor of the order $m(\nu_\tau)/\langle\chi\rangle$; hence, $\nu_\tau \rightarrow \nu_e\gamma$ is entirely negligible here.

In this model, ν_τ is a Dirac neutrino at the tree level. However, since lepton number is not conserved, there are induced radiative mass terms linking it with the other neutrinos of order $\delta \simeq 10^{-3}$ eV. Therefore, ν_τ splits up into a pair of nearly degenerate Majorana particles with a mass difference given by $\Delta m \simeq \delta^2 C/E^2 \simeq 4\times 10^{-19}$ eV. This is too small to have any observable consequences.

In conclusion, the $S_3 \times Z_3$ model proposed earlier [1] for quark mass matrices is remarkably suited also for understanding a plethora of neutrino phenomena. Whereas there are undoubtedly many theoretical models [13] which can accommodate a 17-keV neutrino, this one at least does so in the context of a unified description involving the quarks as well.

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