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Ansatz for the quark mass matrices allowing for a high top-quark mass

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We present a new *Ansatz* for the quark mass matrices, in which the correct Cabibbo-Kobayashi-Maskawa matrix is obtained for an arbitrarily large value of the top-quark mass. Our *Ansatz* predicts $|V_{ub}/V_{cb}|^2 \approx m_u/m_c$ and $|V_{td}/V_{ts}|^2 \approx m_d/m_s$. The first prediction is close to the accepted lower bound; the second one may be tested in the near future.

In the standard model of the electroweak interactions the family structure of Yukawa couplings is not constrained by gauge invariance, and as a result the values of the quark masses [1] and of the mixing parameters [2] are completely arbitrary. They are not related among themselves and are not related to the value of any other parameter of the model. Following the original idea of Weinberg [3], several authors have tried to partially eliminate this shortcoming by devising *Ansätze* for the quark mass matrices [4]. The number of independent measurable physical quantities that are explained by the quark mass matrices is ten: six quark masses, and four parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix V [5]. On the other hand, the number of parameters in any particular *Ansatz* is smaller than ten, and therefore each of the *Ansätze* predicts some relationships between the quark masses and mixings. The most popular *Ansätze* are the ones of Fritzsch [6] and of Stech [7]; most of the other *Ansätze* [8–10] are indeed only variations of one of these. Although these *Ansätze* are usually not justified by a complete model, it is hoped that they might provide clues to some unknown physics underlying the origin of the fermion mass matrices.

Unfortunately, all these *Ansätze* have been either excluded or severely impaired by the recent experimental lower bounds on the top-quark mass [11]. Indeed, the *Ansätze* of Stech and of Gronau, Johnson, and Schechter [9] (GJS) have been completely excluded, since they predicted the top-quark mass to be smaller than 50 GeV. The *Ansatz* of Fritzsch and other related *Ansätze* [8, 10] cannot accommodate a top quark heavier than 120 GeV, and are at present in only marginal agreement with experiment [12]. One may be led to the conclusion that the interesting idea of obtaining relationships between the quark masses and mixing parameters through simple *Ansätze* has to be put aside because of the high value of the top-quark mass.

In this Rapid Communication we will show that this is not so. We suggest here a new *Ansatz* for the quark mass matrices, which has a different standpoint, completely distinct both from the one of Fritzsch and from the one of Stech. The main relevance of our *Ansatz* lies in the fact that its predictions are essentially independent of the values of both the top and the bottom masses. This has as a consequence that this new *Ansatz* is totally immune to any new experimental finding concerning the top-quark mass [13].

Our *Ansatz* predicts the ratios $|V_{ub}/V_{cb}|$ and $|V_{ts}/V_{td}|$ to be equal to some well-defined functions of the quark masses. The prediction for the first of these ratios is rather low, but it is within the experimentally allowed range of values. Furthermore, it should be noted that the determination of $|V_{ub}/V_{cb}|$ from the available experimental data has a strong input of quite uncertain theory. On the other hand, the prediction for the second ratio leads to a prediction for the amount of B_s-$\overline{B_s}$ mixing.

In order to illustrate the idea of our *Ansatz*, we start with a simple two-generation version of it, i.e., supposing that the top and bottom quarks did not exist. We denote by M_U and M_D the mass matrices of, respectively, the "up-type" (charge $\frac{2}{3}$) and "down-type" (charge $-\frac{1}{3}$) quarks. They are in this case 2×2 matrices. We assume that they are Hermitian and may be written as

$$\begin{aligned} M_U &= \mathrm{diag}(0, u_0) + U_U \mathrm{diag}(0, u_0) U_U^\dagger, \\ M_D &= \mathrm{diag}(0, d_0) + U_D \mathrm{diag}(0, d_0) U_D^\dagger, \end{aligned} \tag{1}$$

where u_0 and d_0 are real parameters with the dimension of mass, while U_U and U_D are unitary matrices. In general, we denote by $\mathrm{diag}(a, b, ...)$ a diagonal matrix of the appropriate dimension, having diagonal matrix elements a, b, and so on.

We parametrize the second columns of U_U and U_D in the following way:

$$\begin{aligned} (U_U)_{i2} &= \begin{pmatrix} \sin\theta_U \exp(i\theta_1) \\ \cos\theta_U \exp(i\theta_2) \end{pmatrix}, \\ (U_D)_{i2} &= \begin{pmatrix} \sin\theta_D \exp(i\theta_3) \\ \cos\theta_D \exp(i\theta_4) \end{pmatrix}, \end{aligned} \tag{2}$$

θ_U and θ_D being angles of the first quadrant. We define $\chi \equiv \theta_1-\theta_2-\theta_3+\theta_4$. The eigenvalues of the mass matrices are the quark masses

$$m_c = u_0(1+\cos\theta_U), \quad m_u = u_0(1-\cos\theta_U),$$
$$m_s = d_0(1+\cos\theta_D), \quad m_d = d_0(1-\cos\theta_D). \tag{3}$$

Notice that m_c and m_u have the same sign, just as m_s and m_d have the same sign. We now easily find that the Cabibbo angle is given as a function of the quark masses and of χ by

$$\sin^2\theta_C = \frac{m_c m_d + m_u m_s - 2(m_c m_d m_u m_s)^{1/2}\cos\chi}{(m_c+m_u)\,(m_s+m_d)}. \tag{4}$$

Taking into account that $m_c \gg m_s \gg m_d > m_u$, we obtain the well-known relationship [3] between the Cabibbo angle and the down-type-quark masses: $\sin^2\theta_C \approx m_d/m_s$. The fact that we could recover this nontrivial relationship shows that our two-generation *Ansatz* is successful.

The result for $\sin^2\theta_C$ of this two-generation *Ansatz* is equal to the corresponding result of the two-generation version of the Fritzsch *Ansatz* [3, 6]. However, the Hermitian quark mass matrices of the two *Ansätze* are not equivalent, as can be seen by noting that, while in our *Ansatz* the signs of their eigenvalues are equal, in the Fritzsch *Ansatz* they are opposite.

It is amusing to note that there are only two parameters with the dimension of mass in Eq. (1), corresponding however to a total of four distinct quark masses. The splittings between m_c and m_u in one charge sector, and m_s and m_d in the other, are, as seen in Eq. (3), provided by the angles in the unitary matrices U_U and U_D.

Encouraged by the success of this two-generation *Ansatz*, we tried to extend it to the three-generation case. There are, of course, many possible extensions, most of them incompatible with the known values of the quark masses and mixings. We propose the following three-generation *Ansatz*:

$$M_U = \mathrm{diag}(0,u_2,u_2) + W_U \mathrm{diag}(0,0,u_1)\, W_U^\dagger,$$
$$M_D = \mathrm{diag}(0,d_2,d_2) + W_D \mathrm{diag}(0,0,d_1) W_D^\dagger. \tag{5}$$

M_U and M_D are now 3×3 matrices. As before, u_1, u_2, d_1, and d_2 are real mass parameters, and W_U and W_D are unitary matrices.

Making use of the peculiar form of the first contribution in Eqs. (5) to each of the matrices M_U and M_D, we may, without loss of generality, parametrize the third columns of W_U and W_D in the following way:

$$(W_D)_{i3} = \begin{pmatrix} \sin\phi_D \\ \cos\phi_D \\ 0 \end{pmatrix},$$
$$(W_U)_{i3} = \begin{pmatrix} \sin\phi_U \exp(i\psi_1) \\ \cos\phi_U \cos\gamma \exp(i\psi_2) \\ \cos\phi_U \sin\gamma \end{pmatrix}. \tag{6}$$

Here, ϕ_D, ϕ_U, and γ are by definition angles of the first quadrant. From now on we denote $\cos\gamma$ by c_γ and $\sin\gamma$ by s_γ.

From Eqs. (5) and (6) we find

$$M_D = M_D', \quad M_U = K^\dagger M_U' K, \tag{7}$$

where

$$M_D' = \begin{pmatrix} d_1\sin^2\phi_D & d_1\sin\phi_D\cos\phi_D & 0 \\ d_1\sin\phi_D\cos\phi_D & d_2 + d_1\cos^2\phi_D & 0 \\ 0 & 0 & d_2 \end{pmatrix}, \tag{8}$$

while $M_U' = M_D'(d_1 \to u_1, d_2 \to u_2, \phi_D \to \phi_U)$; and K is the following unitary matrix:

$$K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{pmatrix} \mathrm{diag}(e^{i\sigma}, 1, e^{i\psi_2}). \tag{9}$$

We have defined $\sigma \equiv \psi_2 - \psi_1$. Therefore, if we define the orthogonal matrices R_D and R_U to be such that

$$R_D^T M_D' R_D = \mathrm{diag}(m_d, m_s, m_b),$$
$$R_U^T M_U' R_U = \mathrm{diag}(m_u, m_c, m_t), \tag{10}$$

we obtain for the CKM matrix V the result

$$V = R_U^T K R_D. \tag{11}$$

At this point, we should emphasize that m_d, ..., m_t, in Eqs. (10), are the real eigenvalues of the real symmetric matrices M_D' and M_U', and coincide with the current quark masses only in their absolute value. Their signs are however, for the moment, arbitrary.

We now proceed to calculate R_D. This is particularly simple because, as is seen in Eq. (8), d_2 is one of the eigenvalues of M_D'. Therefore, one has indeed to diagonalize only a 2×2 matrix. It is readily seen that d_2 can indeed only be either m_s or m_b, but not m_d. Furthermore, if $d_2 = m_s$ the signs of m_s and m_d have to be equal, and if $d_2 = m_b$ those signs are opposite. It turns out that the relevant case (the one in which the resulting CKM matrix fits well the experimental data) is the one in which $d_2 = m_b$. We then have $d_1 = -m_b + m_s + m_d$ and $\sin^2\phi_D = m_s m_d/[m_b(-m_b+m_s+m_d)]$. The signs of m_s and m_d must be opposite in order for $\sin^2\phi_D$ to be positive. Without loss of generality, we take m_d as positive and m_s as negative, a twofold ambiguity in the sign of m_b remaining. The matrix R_D is in this case given by

$$R_D = \begin{pmatrix} \sqrt{\dfrac{m_s(m_b-m_d)}{m_b(m_s-m_d)}} & \sqrt{\dfrac{m_d(m_s-m_b)}{m_b(m_s-m_d)}} & 0 \\ -\sqrt{\dfrac{m_d(m_s-m_b)}{m_b(m_s-m_d)}} & \sqrt{\dfrac{m_s(m_b-m_d)}{m_b(m_s-m_d)}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

Similarly, for the "up-type" sector we take m_u as positive, m_c as negative, $u_2 = m_t$, $u_1 = -m_t + m_c + m_u$, and $\sin^2\phi_U = m_c m_u/[m_t(-m_t+m_c+m_u)]$. An ambiguity in the sign of m_t remains, and $R_U = R_D(m_d \to m_u, m_s \to$

$m_c, m_b \rightarrow m_t$).

Finally, we are in a position to study the consequences of our *Ansatz*. Using Eqs. (9), (11), and (12), we first observe that $|V_{tb}| = c_\gamma$. This fixes the angle γ. It is very small, because $|V_{tb}|$ is very close to 1. We then see that $|V_{ub}|^2 = [m_u(m_c - m_t)/m_t(m_c - m_u)]s_\gamma^2$, while $|V_{cb}|^2 = [m_c(m_t - m_u)/m_t(m_c - m_u)]s_\gamma^2$. We thus obtain the first prediction of the *Ansatz*:

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 = \frac{m_u(m_c - m_t)}{m_c(m_t - m_u)} \approx -\frac{m_u}{m_c}. \tag{13}$$

Similarly, by comparing V_{td} and V_{ts} we find the second prediction of the *Ansatz*:

$$\left|\frac{V_{td}}{V_{ts}}\right|^2 = \frac{m_d(m_s - m_b)}{m_s(m_b - m_d)} \approx -\frac{m_d}{m_s}. \tag{14}$$

Finally, we obtain for V_{us} the result

$$V_{us} = -c_\gamma \sqrt{\frac{m_u(m_c - m_t)}{m_t(m_c - m_u)}} \sqrt{\frac{m_s(m_b - m_d)}{m_b(m_s - m_d)}} + e^{i\sigma} \sqrt{\frac{m_c(m_t - m_u)}{m_t(m_c - m_u)}} \sqrt{\frac{m_d(m_s - m_b)}{m_b(m_s - m_d)}} \tag{15}$$

$$\approx e^{i\sigma}\sqrt{-\frac{m_d}{m_s}} - \sqrt{-\frac{m_u}{m_c}}. \tag{16}$$

As the three-generation CKM matrix is completely parametrized by the moduli of four of its matrix elements [14], Eqs. (13)–(15) contain, together with $|V_{tb}| = c_\gamma$, all the relevant information.

Equation (16) also holds in the *Ansatz* of Fritzsch [12]; but the exact equation for V_{us}, Eq. (15), is much simpler than the analogous equation in the *Ansatz* of Fritzsch. At this point, it is worth emphasizing the extreme mathematical simplicity of our *Ansatz*. It is an eight-parameter *Ansatz*, and it yields two linear relationships between the squared moduli of the CKM matrix elements, Eqs. (13) and (14). The Stech *Ansatz* has seven parameters and it yields three linear relationships among the squared moduli [15], while the Fritzsch *Ansatz* is an eight-parameter *Ansatz*, leading to two relationships among the moduli, of which only one is linear in the $|V_{ij}|^2$ [16]. However, the linear relationships between the squared moduli are, in the *Ansätze* of Fritzsch and of Stech, rather complicated, and the best method to obtain them is through the use of mass-matrix invariants [14, 16, 15]. On the other hand, in our *Ansatz* those equations are obtained by a simple glance at the explicit form of the matrix elements. Of course, equations between the mass-matrix invariants — for instance, $\det(d_2 M_U - u_2 M_D) = 0$ — can also be derived in this *Ansatz*, but they are here of no practical usefulness.

At this point, it is worth analyzing why our *Ansatz*, unlike the ones of Fritzsch and of Stech, does not encounter any difficulty in accommodating a heavy top quark. The main problem in the *Ansatz* of Fritzsch is that, while obtaining Eq. (16) for V_{us}, and therefore the correct Weinberg relationship $|V_{us}| \approx (m_d/m_s)^{1/2}$, it is driven to a similar equation for V_{cb}, i.e., $V_{cb} \approx (m_s/m_b)^{1/2} + e^{i\rho}(m_c/m_t)^{1/2}$. As experimentally $m_s/m_b \gg |V_{cb}|^2$, a very delicate cancellation has to be assumed between the two contributions to V_{cb}. This leads to $m_t \sim m_c m_b/m_s$, a rather low value for m_t. The great advantage of our *Ansatz* is that, while reproducing the successful Eq. (16) of the Fritzsch scheme, it leaves $|V_{cb}|$ completely free. Indeed, in our *Ansatz* $|V_{cb}| \approx s_\gamma$ does not depend on the quark masses. Moreover, the predictions of our *Ansatz*, Eqs. (13) and (14), become, in the limit of a large hierarchy of the quark masses, independent of m_t and m_b; and, in particular, of their signs. Thus, in this *Ansatz* the exact value of the top-quark mass is essentially irrelevant, provided only that it is much larger than the charm mass. By the same mechanism, the fourfold ambiguity of the signs of m_b and m_t present in this *Ansatz* is of negligible practical consequences for its predictions. We recall that similar ambiguities in the signs of the quark masses, present in the *Ansatz* of Stech, had very bad consequences for its predictive power, as was emphasized in Ref. [15].

The prediction embodied in Eq. (13) gives, taking into account the "standard" [1] values of the current quark masses, and their error bars, $|V_{ub}/V_{cb}| = 0.061 \pm 0.010$. The recent experimental results [17] lead to one estimate $|V_{ub}/V_{cb}|$ to be 0.011 ± 0.002 (see however Ref. [2]). However, there are theoretical uncertainties in extracting $|V_{ub}/V_{cb}|$ from the experimental data, and in particular the error bar of 0.002 given above is grossly underestimated [18]. We find that the prediction of our *Ansatz* for $|V_{ub}/V_{cb}|$ is somewhat low, but a more precise experimental determination of that quantity is required in order to decide on the fate of our *Ansatz* [19]. It should also be pointed out that the *Ansatz* of Fritzsch does not perform better than ours in this respect: as was recently pointed out [12], for top-quark masses higher than the present lower bound, the *Ansatz* of Fritzsch also predicts $|V_{ub}/V_{cb}|$ to be around 0.06. On the other hand, there is a variation of the Fritzsch *Ansatz* [10] which allows $|V_{ub}/V_{cb}|$ to have a very large value.

We consider now the other prediction of our *Ansatz*, contained in Eq. (14). It gives $|V_{ts}/V_{td}|^2 = 19.6 \pm 1.6$ [1]. This is a nontrivial prediction: even after fixing $|V_{ub}/V_{cb}|$ to be 0.06, $|V_{ts}/V_{td}|^2$ is allowed by unitarity of the CKM matrix to vary between 11 and 39, approximately. If $|V_{ub}/V_{cb}|$ has a larger value, the domain allowed by unitarity for $|V_{ts}/V_{td}|^2$ is even much larger: for instance, for $|V_{ub}/V_{cb}| = 0.1$ we get $8 < |V_{ts}/V_{td}|^2 < 70$. The prediction for $|V_{ts}/V_{td}|^2$ in Eq. (14) has important consequences since, in the limit of exact SU(3) symmetry, the ratio of the mixing in the B_s-$\overline{B_s}$ system to the one in the B_d-$\overline{B_d}$ system is precisely equal to $|V_{ts}/V_{td}|^2$. The observation and measurement of B_s-$\overline{B_s}$ mixing is at present one of the great challenges of experimental particle physics; a mixing of the order of magnitude suggested in this Rapid Communication might be observed, for instance, in a high-luminosity collider working at the Z^0 pole [20].

We may confront the two predictions for the CKM matrix of our *Ansatz*, Eqs. (13) and (14), with the standard-model calculations of the parameters ϵ of CP violation and x_d of B_d-$\overline{B_d}$ mixing. The results of such a confrontation must however be taken with great reserve, because the values of the above parameters, in particular of ϵ [21], are very sensitive to any physics beyond the standard model. The result of the confrontation essentially tells us that our *Ansatz* requires a top-quark mass larger than about 130 GeV in order to fit x_d. The minimum value for m_t quoted above is higher than the one obtained from the fitting of x_d in the standard model not constrained by our *Ansatz*; this was to be expected, since Eq. (14) requires $|V_{td}|^2$ to be about three times smaller than what it could be from the sole condition of unitarity of the CKM matrix, and therefore m_t must be about twice as large in order to obtain the same x_d. On the other hand, the calculation of ϵ in our *Ansatz* does not lead to any new insight; that parameter is easily fitted whatever the value that we take for the relevant matrix element, i.e., for any B_K between 0.4 and 1.

We might in principle try to obtain further information, from the confrontation of our *Ansatz* with the experimental results on ϵ'/ϵ. This is in practice rather difficult, due to the still badly determined experimental value of that parameter [22], and also because of its unclear theoretical status, in particular in what concerns the values of the relevant matrix elements [23]. Moreover, ϵ'/ϵ is extremely sensitive to any physics beyond the standard model [24].

An interesting point about the predictions of our *Ansatz* is that they are almost invariant under the renormalization-group evolution of the quark masses and mixings. If our *Ansatz* is valid at some very-high-energy (say, grand-unification) scale, its predictions will not only be exact at that high scale, but also approximately valid at the Fermi scale. This is because, due to the large hierarchy between the top-quark mass and the other up-type-quark masses, and between the bottom mass and the other down-type-quark masses, the values of $|V_{us}|$, $|V_{ub}/V_{cb}|$, $|V_{td}/V_{ts}|$, m_d/m_s, and m_u/m_c are only very little changed by their renormalization-group evolution [25].

We have not attempted to construct a model giving rise in a natural way to the quark mass matrices proposed in our *Ansatz*. The form of our *Ansatz* suggests that each mass matrix can be naturally separated into two parts: one a rank-1 matrix and the other a rank-2 matrix. A popular idea which might give rise to such a structure is the suggestion that the fermion mass matrices are generated radiatively [26]. Each mass matrix is then usually the sum of three rank-1 matrices, the eigenvalues of which have a hierarchy. However, this idea certainly does not apply to our *Ansatz* since in it, as we have seen, the two mass parameters in each mass matrix are of the same order of magnitude, their signs only being different.

In conclusion, we have suggested a simple *Ansatz* for the quark mass matrices which is in agreement with the present experimental knowledge on the CKM matrix, and in particular fits correctly both $|V_{us}|$ and $|V_{cb}|$, while leaving the top-quark mass completely free. Our *Ansatz* gives clear-cut predictions both for $|V_{ub}/V_{cb}|$ and for $|V_{ts}/V_{td}|$ as functions of the quark masses. Both these predictions might be tested in the near future.

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