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Test of special relativity by a determination of the Lorentz limiting velocity: Does $E = mc^2$?

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We describe a method by which existing precision measurements may be used to provide a new laboratory test of the special theory of relativity. In this test the speed of light $c_{\rm em}$ is compared to the limiting velocity of massive particles, c_m. Although it is conventionally assumed that $c_{\rm em} = c_m \equiv c$, this need not be the case in phenomenological alternatives to special relativity. Our results impose limits on the quantity $(1 - c_m/c_{\rm em})$. Unlike null tests of special relativity, this limit does not depend on assumptions concerning the motion of the laboratory with respect to a preferred frame.

Recent experimental and theoretical advances have renewed interest in tests of special relativity and their interpretation. Experimentally, results from several measurements of the Hughes-Drever (HD) type have set stringent limits on possible spatial anisotropies that could signal a breakdown of Lorentz invariance, and hence of special relativity [1–7]. Theoretical work by Haugan and Will (HW) has led to a framework within which a breakdown of relativity is described in a self-consistent manner [8–11]. In the present paper we describe a new test of special relativity which complements existing tests derived from the HD and other experiments.

A central result of the synthesis represented by the special theory of relativity is the existence of a unique velocity equal to both the propagation velocity of electromagnetic radiation in a vacuum, and the limiting velocity for ponderable matter. The existence of a single characteristic velocity is central to special relativity and it can be argued that this uniqueness is in fact a fundamental *assumption* of special relativity. Such an argument follows from the observation (see, e.g., Ref. [12]) that the form of the Lorentz transformations can be derived from very limited assumptions concerning the transformation properties of purely mechanical physical observables. Such a derivation provides the exact mathematical form for transformations of mechanical properties between inertial frames, but requires the introduction of an arbitrary constant having the units of velocity. This velocity, which we shall call $c_{\rm m}$, corresponds to the limiting velocity to which an object of nonzero mass can be accelerated. To appreciate the generality of this approach, we note that Galilean relativity may be viewed as a special case with $c_{\rm m} = \infty$. Note that from such purely kinematic considerations alone we are unable to make a statement about the magnitude of $c_{\rm m}$.

A second fundamental assumption of special relativity arises from the consequences of *electrodynamic relativity.* Here it is assumed that the form of Maxwell's equations (or any equivalent formulation of classical electrodynamics) is invariant under transformations between inertial frames. This is the familiar assertion that the speed of light is a constant for all inertial frames. This assumption introduces a second characteristic velocity, $c_{\rm em}$, which is associated with electrodynamics.

In the above view, the special theory of relativity follows from the assertion that $c_{\rm m} = c_{\rm em}$. Note that the validity of this assertion is ultimately an experimental question. Indeed, as we shall discuss, measurements which are viewed as "tests of special relativity" may be characterized as tests of this fundamental assumption.

Although the preceding arguments make it clear that $c_{\rm m}$ and $c_{\rm em}$ may be viewed as distinct, they do not provide a theoretical basis for the interpretation of experimental tests of their equality. This requires a *dynamical* formulation which self-consistently incorporates the distinction between $c_{\rm m}$ and $c_{\rm em}$ into the interactions of test objects. The HW formalism provides such a framework for the electromagnetic interactions of charged particles, but requires additional assumptions about the nature of the coupling. Specifically HW assume that there is a coordinate system in which the action I is rotationally symmetric. They further assume that in this coordinate system all equations of motion are linear in the appropriate fields, even when $c_{\rm m} \neq c_{\rm em}$. Under these assumptions,

$$I = \int L\,dt = \int dt \sum_i \left[-m_i c_{\rm m}^2 \left(1 - \frac{v_i^2}{c_{\rm m}^2}\right)^{1/2} + \frac{e_i}{c_{\rm m}} A_\mu v_i^\mu \right] + \frac{1}{8\pi} \int d^3x\,dt \left[\mathbf{E}^2 - \left(\frac{c_{\rm em}}{c_{\rm m}}\right)^2 \mathbf{B}^2 \right] . \quad (1)$$

In Eq. (1) m_i, e_i are the rest mass and charge of particle i, and $v_i^\mu = dx_i^\mu/dt$ where $x_i^\mu(t)$ is the particle's world line with $x^0 = t$. The potential and fields are given by

$$A_\mu = (\Phi, -\mathbf{A}), \quad \mathbf{E} = -\nabla\Phi - \frac{1}{c_{\text{em}}}\frac{\partial \mathbf{A}}{\partial t},$$

and

$$\mathbf{B} = \nabla \times \mathbf{A}.$$

In Eq. (1), which employs the electromagnetic conventions of Ref. [13], c_{m} enters in the Lagrangian for the free electromagnetic field through a field redefinition, as explained in Refs. [10] and [11].

If $c_{\text{m}} = c_{\text{em}}$ the action in Eq. (1) is Lorentz invariant, and thus assumes the same form in any inertial frame. However, if $c_{\text{m}} \neq c_{\text{em}}$ there is only one preferred frame in which Eq. (1) holds. Thus in the HW formalism one manifestation of (and an experimental signal for) a breakdown of special relativity is the appearance of anomalous velocity-dependent effects which arise from motion of a test system with respect to the preferred frame. Experiments of the Michelson-Morley type [14, 15] constitute the most familiar of such measurements. HD experiments, as interpreted by HW, also fall in this category. Eq. (1) suggests another class of experimental tests of special relativity—those which directly check the equality of c_{m} and c_{em}. In Eq. (1) it is c_{m} which determines the energy of a particle of mass m; thus, $E = mc_{\text{m}}^2$. By contrast the propagation of light is determined by c_{em}, and the energy E of a photon of frequency ν is given by $E = h\nu = hc_{\text{em}}/\lambda$. A measurement of the wavelength λ of a photon emitted in a transition where a mass Δm is converted into electromagnetic radiation provides an experimental relation between c_{m} and c_{em} of the form

$$\Delta m c_{\text{m}}^2 = \frac{hc_{\text{em}}}{\lambda}. \tag{2}$$

Equation (2) provides the basis for our test of the assumption that $c_{\text{m}} = c_{\text{em}}$.

To formulate a precision test of the equality of c_{m} and c_{em} based on Eq. (2), it is convenient to define two distinct "fine-structure constants" $\alpha_{\text{em}} = e^2/4\pi\epsilon_0\hbar c_{\text{em}}$ and $\alpha_{\text{m}} = e^2/4\pi\epsilon_0\hbar c_{\text{m}}$. These quantities may be viewed as "electromagnetic" and "mechanical" fine-structure constants respectively [16]. Each is capable of being independently determined using existing precision measurements. Before proceeding, we note that some care is necessary in the interpretation of experimental results as the definition of the unit for length in the *Système International* (SI) is directly based on c_{em}. The meter is now defined as the distance traveled by electromagnetic radiation in a vacuum in 1/299 792 458 sec [17]. The meter is "realized" by measuring the frequency ν of visible (or infrared) radiation from a stabilized laser with respect to the cesium atomic clock which defines the second. The wavelength λ of this radiation is given by $\lambda = c_{\text{em}}/\nu$ and serves as a de facto length standard. We note several obvious, but important points. First, an appearance of c using the *defined* value is, in effect, an appearance of c_{em}. Second, the determination of the frequency of any electromagnetic radiation from its wavelength through the use of the relation $c = \lambda\nu$ implies the use of c_{em}. Finally, care must be exercised in the interpretation of any result involving a length measurement in SI units, since c_{em} may enter implicitly through the definition of the meter.

We first consider the "quantum Hall" determination of α via a measurement of the von Klitzing constant $R_K = h/e^2$. While one may directly express $\alpha = \mu_0 c/2R_K$, an experimental determination of α in this fashion is limited by the accuracy of SI electrical measurements. A more accurate procedure is to carry out the determination of R_K in "as maintained" electrical units, $R_{K\ \text{NIST}}$, and then to convert this "as maintained" resistance measurement to SI ohms [18]. Following this procedure one has

$$\alpha = \frac{\mu_0 c}{2\Omega_{\text{NIST}} R_{K\ \text{NIST}}}, \tag{3}$$

where Ω_{NIST} is the "as maintained" ohm expressed in SI units. The subscript "NIST" refers to "as maintained" electrical measurements carried out at the National Institute of Standards and Technology [18]. These currently provide the most accurate determination of the quantum Hall α.

The quantum Hall relation $R_K = h/e^2$ is derived from classical electromagnetism and nonrelativistic quantum mechanics for the electron. The determination of Ω_{NIST} is a purely classical electrical measurement involving the determination of the length of a calculable capacitor in SI units [19]. Finally, the "defined" value of c_{em} is employed on the right of Eq. (3). The quantities on the right of Eq. (3) involve neither the mechanical c_{m} nor the use of relativistic particle mechanics. We conclude that the α determined by Eq. (3) involves only c_{em}, and is therefore the "electromagnetic" fine-structure constant α_{em}. The experimental value for α_{em} is [18]

$$\alpha_{\text{em}}^{-1} = 137.035\,997\,9(32). \tag{4}$$

It has long been realized that determinations of the Rydberg constant R_∞, and the Compton wavelength of the electron λ_C, can be combined to provide a determination of α through the relation [20–23]

$$\alpha = (2R_\infty\lambda_C)^{1/2}. \tag{5}$$

In current R_∞ determinations, the wavelength of the radiation associated with a transition between two states of atomic hydrogen having different principal quantum numbers is measured with respect to a stabilized laser of known wavelength (see for example Ref. [24]). To lowest order the difference in the electronic energy between states having principal quantum numbers n_1 and n_2 is given by $E = (1/n_1^2 - 1/n_2^2)[m_e e^4/(8\epsilon_0^2 h^2)]$. After correcting for higher-order effects, this energy is equated to the energy of the transition photon $E = hc_{\text{em}}/\lambda$. From such considerations, an experimental value of $R_\infty = m_e e^4/8\epsilon_0^2 ch^3$ is obtained. The determination of R_∞ (to lowest order) involves only nonrelativistic dynamics and does not depend on c_{m}. The c which appears in this experimental R_∞ arises from the relation between the photon energy and wavelength as discussed above, and is identified as c_{em}.

We now consider a series of measurements which, taken together, yield a value for the SI wavelength of positron annihilation radiation. If we neglect corrections for binding energy in positronium, assume negligible positron and electron kinetic energies, and view the annihilation in the center-of-mass frame, then the energy of the annihilation photon will be given by the rest energy of the electron. (There are stringent experimental limits on the equality of the electron and positron masses from experiments of an essentially nonrelativistic character [25].) The rest energy of the electron is given by $E = m_e c_{\rm m}^2$, while the energy of the annihilation radiation, expressed in terms of its wavelength λ_C (the Compton wavelength), is given by $E = hc_{\rm em}/\lambda_C$. It follows that $\lambda_C = hc_{\rm em}/m_e c_{\rm m}^2$.

An accurate determination of λ_C requires several steps. First the lattice spacing of a perfect single crystal of silicon is determined by direct comparison with a primary optical wavelength standard [26] (the stabilized laser). In the second step, the lattice spacing of this particular crystal is compared with another perfect crystal suitable for γ-ray diffraction [27]. In the third step, the wavelength of a suitable γ reference line is determined by measuring its Bragg angle with the calibrated diffraction crystal. In the present discussion an appropriate reference γ ray is the so-called Au 411-keV line, which results from a radiative cascade in ^{198}Hg following neutron capture in ^{197}Au, and the subsequent β decay of ^{198}Au [28]. The fourth and final step involves the comparison of this Au 411-keV line with the 511-keV positron annihilation line using a curved crystal γ-ray spectrometer [29]. This chain involves only a series of length comparisons using essentially arbitrary "transfer" standards (the lattice spacing of silicon, the wavelength of the Au 411-keV line). Thus, this procedure may be viewed as providing a determination of λ_C *in terms of the SI definition of the meter.* It follows from the previous discussion that a determination of the fine structure constant via Eq. (5) provides a value for the "mechanical" fine structure constant $\alpha_{\rm m}$. Our analysis assumes that electromagnetic radiation suffers no dispersion in vacuum. Adequate experimental limits on such a dispersion have been set [30, 31].

The experimental value for the wavelength of the Au 411-keV line [28] λ_{411} must be corrected for a change in the best value for the silicon lattice spacing. Using the value for the silicon lattice spacing given in Ref. [32], $\lambda_{411} = 3.010\,773\,1(11) \times 10^{-12}\,$m. The wavelength ratio λ_{411}/λ_C is $\lambda_{411}/\lambda_C = 1.240\,884$ (12 ppm) [29]. Thus $\lambda_C = 2.426\,313 \times 10^{-12}\,$m (12 ppm). From Eq. (5) and the recommended value [24] for $R_\infty = 10\,973\,731.5709(18)\,{\rm m}^{-1}$ we obtain a value for the mechanical $\alpha_{\rm m}$ given by

$$\alpha_{\rm m}^{-1} = 137.0359 \text{ (12 ppm)} . \quad (6)$$

Finally from Eqs. (4) and (6) we find

$$1 - \frac{c_{\rm m}}{c_{\rm em}} = 1(12) \times 10^{-6} . \quad (7)$$

The error in the above result is dominated by the uncertainty in λ_C, arising from the width of the positron annihilation line [29].

As noted above, null tests of special relativity are based on the observation that when $c_{\rm m} \neq c_{\rm em}$, anomalous effects may arise which depend on the vector velocity $\mathbf{v}$ of the test system with respect to a preferred frame. For this reason such experiments set limits on expressions of the form $(1 - c_{\rm m}^2/c_{\rm em}^2)|\mathbf{v}|^2$, in contrast with the velocity-independent result in Eq. (7). Searches for preferred-frame effects generally utilize such direction-dependent quantities as $v^i v^j$ $(i,j = 1,2,3)$, or $\mathbf{v}\cdot\mathbf{V}$ where $\mathbf{v}$ is the velocity of the center of a rotating disk with respect to the preferred frame, and $\mathbf{V}$ is the (time-varying) velocity of the edge of a rotating disk. The anisotropic effects produced by such factors generate characteristic periodic signals that can be isolated and measured with high precision, and this is one reason for the great sensitivity of such experiments. HW [9] have analyzed a variety of searches for preferred-frame effects *under the assumption* that the velocity $\mathbf{v}$ relative to the preferred frame is the velocity of the Earth relative to the cosmic microwave background [33] ($|\mathbf{v}| \cong 380\,{\rm km\,s^{-1}}$). Their most stringent limit is

$$|1 - c_{\rm m}^2/c_{\rm em}^2| < 3 \times 10^{-22} , \quad (8)$$

utilizing the data of Ref. [4]. Although the constraint in Eq. (8) is far more stringent than our limit in Eq. (7), it depends on the observation of different physical phenomena, and on an assumption not required in our analysis: that *the preferred frame is that frame in which the microwave background is isotropic.*

HW note [9] that the result in Eq. (8) constrains the tensor contribution to any anomalous coupling which violates Lorentz invariance, but not the scalar contribution which does not lead to anisotropic effects. Since the scalar and tensor anomalies are not necessarily related [9, 34], separate limits on scalar (or rotationally invariant) anomalies are important, and Eq. (7) provides one such constraint. Another set of constraints on scalar couplings comes from the Eötvös experiment [9, 35, 36], particularly some of the recent high-precision versions of the experiment [37]. However, anomalies can arise in the Eötvös experiment by a violation not only of local Lorentz invariance, but also of local position invariance [10, 11]. The Eötvös experiments can nonetheless be used to test Lorentz invariance by making the plausible assumption that there is no fortuitous cancellation between these distinct anomalies. The significance of Eq. (7) is that, in contrast with the Eötvös results, it provides an unambiguous test of local Lorentz invariance which is independent of other dynamical assumptions.

In principle, it is also possible to set limits on the quantity $(1 - c_{\rm m}^2/c_{\rm em}^2)$ by simultaneously determining both the energy and the velocity of an electron (or other particle) in an accelerator (or storage ring). However it is likely to be difficult to set a limit at an interesting level of precision by such a procedure, since this would require highly accurate knowledge of particle energies in absolute units.

There are experimental approaches which could provide a significant reduction in the error on $\alpha_{\rm m}$ as determined above. It has been suggested [20] that a much more accurate determination of λ_C would be possible with a cold, thermalized positronium source. Doppler broadening in such a source is greatly reduced and an improvement in accuracy of 2 orders of magnitude in λ_C

might be obtainable.

An alternative procedure for the determination of α_m is to measure the change in atomic mass of a radiating system, along with the wavelength of the emitted photon. However, it should be noted [38] that such a transition process is more complicated than an annihilation because only part of the mass energy is converted into radiation. Hence a simple application of the mass-energy relation to the photon energy may require additional dynamical assumptions. Nonetheless, it has been previously noted [39–41] that it is possible to determine α from the independent determination of neutron binding energies in light elements in both atomic mass units and in wave numbers. It can be shown that such a determination gives a value for α_m. New mass spectroscopic techniques based on trapped ion technology [42,43] combined with very high accuracy γ-ray wavelength determinations [39–41,44] may ultimately provide a determination of α_m with an error at the level of $(1\text{–}2)\times10^{-7}$.

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[1] T. E. Chupp *et al.*, Phys. Rev. Lett. **63**, 1541 (1989).
[2] S. K. Lamoreaux, J. P. Jacobs, B. R. Heckel, F. J. Raab, and E. N. Fortson, Phys. Rev. A **39**, 1082 (1989).
[3] P. R. Phillips, Phys. Rev. Lett. **59**, 1784 (1986).
[4] S. K. Lamoreaux, J. P. Jacobs, B. R. Heckel, F. J. Raab, and E. N. Fortson, Phys. Rev. Lett. **57**, 3125 (1986).
[5] J. D. Prestage, J. J. Bollinger, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **54**, 2387 (1985).
[6] R. W. P. Drever, Philos. Mag. **6**, 683 (1961).
[7] V. W. Hughes, H. G. Robinson, and V. Beltran-Lopez, Phys. Rev. Lett. **4**, 342 (1960).
[8] M. D. Gabriel and M. P. Haugan, Phys. Rev. D **41**, 2943 (1990).
[9] M. P. Haugan and C. M. Will, Phys. Today **40**(5), 69 (1987).
[10] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1981), pp. 45–50.
[11] M. P. Haugan, Ann. Phys. (N.Y.) **118**, 156 (1979).
[12] J. Lévy-Leblond, Am. J. Phys. **44**, 271 (1976).
[13] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).
[14] A. Brillet and J. L. Hall, Phys. Rev. Lett. **42**, 549 (1979).
[15] A. A. Michelson and E. H. Morley, Am. J. Sci. **34**, 333 (1887).
[16] E. Fischbach, G. L. Greene, and R. J. Hughes, Phys. Rev. Lett. **66**, 256 (1991), consider the possibility that different determinations of α may depend on metrologically distinguishable Planck constants; for present purposes we assume e and $\hbar$ to be universal constants.
[17] *Comptes Rendus des Séances de la 17e Conférence Générale des Poids et Mesures, 1983* (Bureau International des Poids et Mesures, Sèvres, France, 1983).
[18] M. E. Cage *et al.*, IEEE Trans. Instrum. Meas. **38**, 284 (1989).
[19] A. M. Thompson and D. E. Lampard, Nature **177**, 888 (1956).
[20] W. C. Sauder, in *Precision Measurement and Fundamental Constants*, Natl. Bur. Stand. Special Publication No. 343 (U.S. GPO, Washington, D.C., 1971), p. 275.
[21] J. W. Knowles, in *Proceedings of the Second International Conference on Nuclidic Masses*, Vienna, 1963 (Springer-Verlag, Berlin, 1964), p. 113.
[22] J. W. Knowles, Can. J. Phys. **40**, 257 (1962).
[23] J. W. M. DuMond, D. A. Lind, and B. B. Watson, Phys. Rev. **75**, 1226 (1949).
[24] J. C. Garreau, M. Allegrini, L. Julien, and F. Biraben, J. Phys. (Paris) **51**, 2293 (1990).
[25] S. Chu, A. P. Mills, Jr., and J. L. Hall, Phys. Rev. Lett. **52**, 1689 (1984).
[26] R. D. Deslattes, E. G. Kessler, Jr., W. C. Sauder, and A. Henins, Ann. Phys. (N.Y.) **129**, 378 (1980).
[27] R. D. Deslattes, M. Tanaka, G. L. Greene, A. Henins, and E. G. Kessler, Jr., IEEE Trans. Instrum. **IM-36**, 166 (1987).
[28] E. G. Kessler, Jr., R. D. Deslattes, A. Henins, and W. C. Sauder, Phys. Rev. Lett. **40**, 171 (1978).
[29] P. H. M. Van Assche, H. Börner, W. F. Davidson, and H. R. Koch, *Atomic Masses and Fundamental Constants 5* (Plenum, New York, 1975), p. 37.
[30] B. C. Brown *et al.*, Phys. Rev. Lett. **30**, 763 (1973).
[31] Z. Bay and J. A. White, Phys. Rev. D **5**, 796 (1972).
[32] E. R. Cohen and B. N. Taylor, Rev. Mod. Phys. **59**, 1121 (1987).
[33] P. Lubin, T. Villela, G. Epstein, and G. Smoot, Astrophys. J. **298**, L1 (1985).
[34] E. Fischbach, M. P. Haugan, D. Tadić, and H. Y. Cheng, Phys. Rev. D **32**, 154 (1985).
[35] R. von Eötvös, D. Pekár, and E. Fekete, Ann. Phys. (Leipzig) **68**, 11 (1922).
[36] E. Fischbach, D. Sudarsky, A. Szafer, C. Talmadge, and S. H. Aronson, Ann. Phys. (N.Y.) **182**, 1 (1988).
[37] E. G. Adelberger *et al.*, Phys. Rev. D **42**, 3267 (1990), for a summary of recent experimental results.
[38] C. M. Will (private communication).
[39] M. S. Dewey *et al.*, Nucl. Instrum. Methods A **284**, 151 (1989).
[40] E. G. Kessler, Jr. *et al.*, J. Phys. G Suppl. **14**, S167 (1988); also in *Capture Gamma-ray Spectroscopy 1987*, edited by K. Abrahams and P. Van Assche, Institute of Physics Conference Series No. 88 (IOP, Bristol, 1988), pp. 167–174.
[41] R. D. Deslattes, G. L. Greene, and E. G. Kessler, Jr., J. Phys. (Paris) Colloq. **45**, C3 (1984).
[42] E. A. Cornell *et al.*, Phys. Rev. Lett. **63**, 1674 (1989).
[43] D. J. Wineland, J. J. Bollinger, and W. M. Itano, Phys. Rev. Lett. **50**, 628 (1983).
[44] E. G. Kessler, Jr., M. S. Dewey, G. L. Greene, R. D. Deslattes, and H. Börner, *Capture Gamma-Ray Spectroscopy*, Pacific Grove, California, 1990, edited by R. W. Hoff, AIP Conf. Proc. No. 238 (AIP, New York, 1991).