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Simple tests of the factorization assumption

Thomas Mannel
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
and Deutsches Elektronen-Synchrotron (DESY), Hamburg, Germany

Winston Roberts and Zbigniew Ryzak
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
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We suggest simple experimental tests to determine whether or not the factorization assumption is valid for the two-body, nonleptonic decays of heavy hadrons, when both daughter hadrons are also heavy.

I. INTRODUCTION

The nonleptonic weak decays of hadrons can drive the most optimistic and perservering theorist to distraction. For instance, consider the decay $\bar{B}^0 \to D^+\pi^-$, whose matrix element M is

$$M = \langle D^+(p_1)\pi^-(p_2)|\bar{d}L^\mu u \bar{c}L_\mu b|\bar{B}^0(p)\rangle . \tag{1}$$

The best that one can do is write down the most general form that can be constructed from the kinematic variables p_1, p_2, and p, and hide one's ignorance in various form factors. Without further input from some source, such as divine revelation or explicit model construction, this yields essentially no useful information.

In the limit when all of the hadrons involved are heavy, the recently developed heavy-quark effective theory (HQET) [1] lessens our ignorance somewhat, by allowing us to relate the matrix elements of some decays to others. Nevertheless, even in this case, the amount of information available is not overwhelming.

A popular assumption that has been used in treating nonleptonic decays is that the amplitude for the decay factorizes [2]. For our example of $B \to D\pi$, this means that one can write

$$\langle D^+(p_1)\pi^-(p_2)|\bar{d}L^\mu u \bar{c}L_\mu b|\bar{B}^0(p)\rangle \approx \langle D^+(p_1)|\bar{c}L^\mu b|\bar{B}^0(p)\rangle\langle \pi^-(p_2)|\bar{d}L_\mu u|0\rangle \tag{2}$$

so that the previously unknown form factors are simply related to the pion decay constant f_π, and to the form factors describing the semileptonic decay $B \to De\nu$. When coupled with HQET, this assumption allows a myriad predictions to be made for the decays of heavy hadrons.

Two questions that have to be addressed in all this are (1) "how good is the factorization assumption?" and (2) "when is it valid?" Recent work by Dugan and Grinstein [3] suggests that this assumption is a good one when the meson produced from the virtual W in the decay of a heavy hadron is very energetic, so that it can escape from the surrounding hadronic matter without undergoing significant strong interactions. This corresponds to production of a light meson, so that factorization is indeed valid for decays like $B \to D\pi$ and $\Lambda_b \to \Lambda_c\pi$.

In contrast with the formulation of [3], there is no comparable justification of factorization when the meson produced from the virtual W is a heavy one. Nevertheless, this assumption has been used to describe the decays $B \to DD_s^{(*)}$ and $\Lambda_b \to \Lambda_c D_s^{(*)}$ [4–8]. Because of HQET, a few predictions can be made for these decays, using the nonfactorized forms of the transition amplitudes [9,10]. With factorization, similar predictions can be made, but these can be quite different from the predictions made without factorization.

In what follows, we compare the two sets of predictions for these decays. In particular, we look for experimentally testable consequences that will enable us to determine whether or not factorization is a good assumption for the decays we consider. We point out here that the work of Refs. [7] and [8] have assumed factorization, have performed some simple tests of the assumption, and have made predictions based on this assumption. Our approach is somewhat different. Let us emphasize that we specifically compare the predictions of the factorized form of the decay amplitudes we consider with those of the nonfactorized form. We confine our attention to the two-body nonleptonic decays of heavy hadrons, in which both daughter hadrons are also heavy, since HQET allows us to write down reasonably simple forms for the nonfactorized matrix elements of such decays.

II. $B \to D^{(*)}D_s^{(*)}$

In a previous analysis [9] we described all of the decays $B \to D^{(*)}D_s^{(*)}$ using the tensor

$$\begin{aligned} T^{\mu\nu} &= \langle D(v_1)D_s(v_2)|\bar{c}\gamma^\mu(1+\gamma_5)b\bar{s}\gamma^\nu(1+\gamma_5)c|B(v)\rangle \\ &\equiv Mg^{\mu\nu} + N[v_1^\mu v_2^\nu + v_1^\nu v_2^\mu - \tfrac{1}{2}(v_1\cdot v_2)g^{\mu\nu}] \\ &\quad + S(v_1^\mu v_1^\nu - \tfrac{1}{4}g^{\mu\nu}) + Q(v_2^\mu v_2^\nu - \tfrac{1}{4}g^{\mu\nu}) \\ &\quad + P(v_1^\mu v_2^\nu - v_1^\nu v_2^\mu) + iL\epsilon^{\mu\nu\alpha\beta}v_{1\alpha}v_{2\beta} . \end{aligned} \tag{3}$$

We found that these decays split into three disjoint sets: the decay $B \to DD_s$ is described by the form factor M, the decays $B \to DD_s^*$ and $B \to D^*D_s$ are described by the form factors P and L, while the decay $B \to D^* D_s^*$ depends on N, S, and Q. Without factorization, all of these form factors are independent, so not much information is available. The only quantity that may give a clue to the question being addressed is the ratio of widths $\Gamma(B \to DD_s^*)/\Gamma(B \to D^* D_s)$, which, ignoring the less than 0.3% difference in phase space for the two decays, is equal to $(P-L)^2/(P+L)^2$.

In contrast with this, factorization of any sort tells us that $L=0$, so the ratio in question is unity. Without factorization, the ratio may have any value between 0 and ∞, the extreme values being realized when $P=\pm L$, respectively. Measurement of a value much different from unity for this ratio of widths would therefore be an indication that the factorization assumption is at best questionable for these decays. Note that conservation of angular momentum does not allow any transversely polarized vectors in the decays $B \to DD_s^*$ and $B \to D^* D_s$.

The decay $B \to D^* D_s^*$ potentially could give some information, since one could look at the ratio of widths for longitudinally and transversely polarized mesons. However, the fact that there are three arbitrary form factors describing this decay means that the prediction of the factorized form of the amplitude could somehow be mimicked. In addition, one may examine various other ratios of widths, such as $\Gamma(B \to DD_s)/\Gamma(B \to D^* D_s^*)$, to see if there are departures from the predictions of factorization.

III. $\Lambda_b \to \Lambda_c D_s^{(*)}$

One expects that more information may be available from these decays for two reasons. (1) Heavy Λ baryons are simpler to describe than mesons in HQET, since the spin of the light degrees of freedom is 0. This means that the significant Lorentz structure is simply that of the heavy quark. (2) There is a veritable plethora of polarization variables that can be measured, each of which may yield some information.

In the factorized form, and ignoring $1/m_c$ effects, these decays are described by a single form factor, and very specific predictions for the polarization variables can be made [6]. In addition, the ratios of widths Γ_L/Γ_T in $\Lambda_b \to \Lambda_c D_s^*$ and $\Gamma(\Lambda_b \to \Lambda_c D_s^*)/\Gamma(\Lambda_c \to \Lambda_c D_s)$ are easily evaluated, provided that one assumes that the form factor in question exhibits no pathological behavior between $q^2=m_{D_s}^2$ and $q^2=m_{D_s^*}^2$.

The work of Grinstein and Wise [10] shows that the nonfactorized form of the matrix element for the decays requires two independent form factors, A and B. In more detail,

$$
\begin{aligned}
\mathcal{A}(\Lambda_b(v) \to \Lambda_c(v')D_s(\bar v)) &= \bar u(v',s')\gamma^\mu(1-\gamma_5)\frac{\bar{\not v}-1}{2}\gamma_5 M\gamma_\mu u(v,s)\,,\\
\mathcal{A}(\Lambda_b(v) \to \Lambda_c(v')D_s^*(\bar v,\epsilon)) &= \bar u(v',s')\gamma^\mu(1-\gamma_5)\frac{\bar{\not v}-1}{2}\not\epsilon^* M\gamma_\mu u(v,s)\,,
\end{aligned}
\tag{4}
$$

with

$$
M=A(v\cdot v')I+B(v\cdot v')\not v' \tag{5}
$$

being the most general form possible for the matrix M, I is the identity matrix, and A and B are the same for both decays.

The various polarization variables α and γ in the decay $\Lambda_b \to \Lambda_c D_s$, and δ_1, δ_2, δ_3, α, α', γ, and γ' in the decay $\Lambda_b \to \Lambda_c D_s^*$, are given in terms of the decays rates as

$$
\begin{aligned}
\Gamma_{\Lambda_b\to\Lambda_c D_s} &= \Gamma^0_{\Lambda_b\to\Lambda_c D_s}[1+\alpha(\mathbf{S}_{\Lambda_c}+\mathbf{S}_{\Lambda_b})\cdot\hat{\mathbf{p}}+\gamma\mathbf{S}_{\Lambda_c}\cdot\mathbf{S}_{\Lambda_b}+(1-\gamma)\hat{\mathbf{p}}\cdot\mathbf{S}_{\Lambda_b}\hat{\mathbf{p}}\cdot\mathbf{S}_{\Lambda_c}]\,,\\
\Gamma_T &= \Gamma_T^0[1+\delta_1(\mathbf{S}_{\Lambda_b}-\mathbf{S}_{\Lambda_c})\cdot\hat{\mathbf{p}}-\mathbf{S}_{\Lambda_c}\cdot\hat{\mathbf{p}}\mathbf{S}_{\Lambda_b}\cdot\hat{\mathbf{p}}]\,,\\
\Gamma_L &= \Gamma_L^0[1+\delta_2(\mathbf{S}_{\Lambda_b}+\mathbf{S}_{\Lambda_c})\cdot\hat{\mathbf{p}}+\mathbf{S}_{\Lambda_c}\cdot\hat{\mathbf{p}}\mathbf{S}_{\Lambda_b}\cdot\hat{\mathbf{p}}+\delta_3\mathbf{S}_{\Lambda_c}\cdot\hat{\mathbf{p}}\times(\hat{\mathbf{p}}\times\mathbf{S}_{\Lambda_b})]\,,\\
\Gamma_{\Lambda_b\to\Lambda_c D_s^*} &= \Gamma^0_{\Lambda_b\to\Lambda_c D_s^*}[1+\alpha\mathbf{S}_{\Lambda_b}\cdot\hat{\mathbf{p}}+\alpha'\mathbf{S}_{\Lambda_c}\cdot\hat{\mathbf{p}}+\gamma\mathbf{S}_{\Lambda_c}\cdot\hat{\mathbf{p}}\times(\hat{\mathbf{p}}\times\mathbf{S}_{\Lambda_b})+\gamma'\mathbf{S}_{\Lambda_c}\cdot\hat{\mathbf{p}}\mathbf{S}_{\Lambda_b}\cdot\hat{\mathbf{p}}]\,,
\end{aligned}
\tag{6}
$$

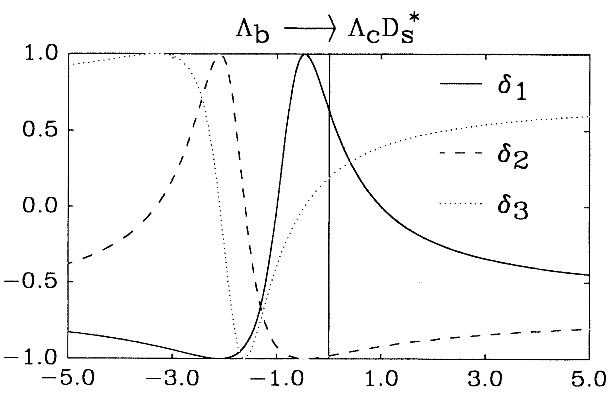

FIG. 1. The polarization variables δ_1, δ_2, and δ_3 for decay $\Lambda_b \to \Lambda_c D_s^*$, as functions of the ratio $r=B/A$.

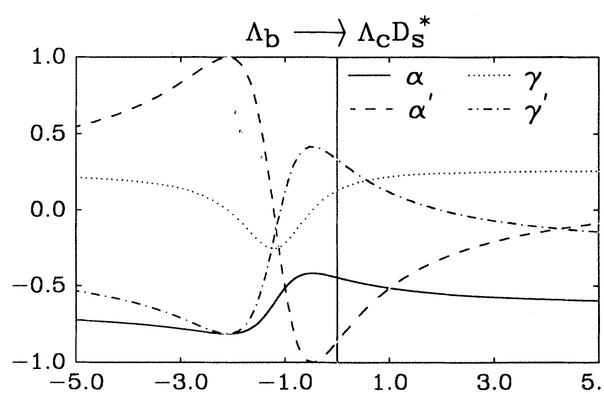

FIG. 2. The polarization variables α, α', γ, and γ' for the decay $\Lambda_b \to \Lambda_c D_s^*$, as functions of the ratio $r=B/A$.

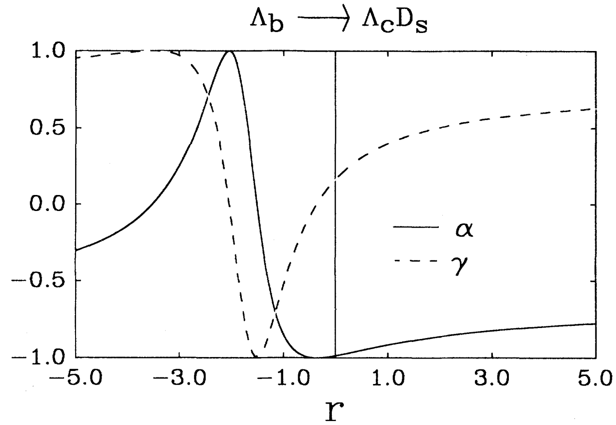

FIG. 3. The polarization variables α and γ for the decay $\Lambda_b \to \Lambda_c D_s$, as functions of the ratio $r = B/A$.

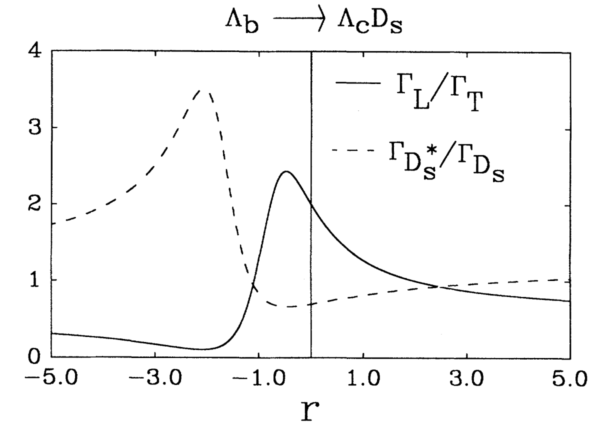

FIG. 4. The ratios Γ_L/Γ_T for the decay $\Lambda_b \to \Lambda_c D_s^*$, and $\Gamma(\Lambda_b \to \Lambda_c D_s^*)/\Gamma(\Lambda_b \to \Lambda_c D_s)$, as functions of the ratio $r = B/A$.

where the subscripts L and T refer to longitudinally and transversely polarized D_s^* mesons, respectively. Here, $\mathbf{S}_{\Lambda_c}$ and $\mathbf{S}_{\Lambda_b}$ are the polarizations of the Λ_c and Λ_b, respectively, and $\hat{\mathbf{p}}$ is a unit vector in the direction of motion of the Λ_c, in the rest frame of the Λ_b. $\Gamma^0_{\Lambda_b \to \Lambda_c D_s}$, Γ^0_L, Γ^0_T, and $\Gamma^0_{\Lambda_b \to \Lambda_c D_s^*}$ are the decay rates when the polarization of neither baryon is detected.

These variables may be evaluated as functions of $r = B/A$. We show these variables in Figs. 1–3, while in Fig. 4 we show the ratios of widths Γ_L/Γ_T in $\Lambda_b \to \Lambda_c D_s^*$ and $\Gamma(\Lambda_b \to \Lambda_c D_s^*)/\Gamma(\Lambda_b \to \Lambda_c D_s)$. In these figures, the values at $r = 0$ would be the predictions of the factorized form of the matrix element.

From these figures we see that there is indeed some hope of determining whether or not factorization is at work in these decays. In particular, if $r \approx -2$ ($B \approx -2A$), the predictions of the nonfactorized form are consistently and markedly different from those of the factorized form. Clearly, however, other variables of r would require precision measurements of these variables for any concrete conclusions to be made.

IV. CONCLUSION

It is clear from the above discussion that, barring some major contrivance on nature's part, it will take very precise measurements of polarization variables and ratios of widths for us to tell whether the factorization assumption is breaking down. For instance, for r in the range -0.5 to 0.5, (for the $\Lambda_b \to \Lambda_c D_s^{(*)}$ decays) deviations from the factorized predictions are typically of the order of $\approx 30\%$. Thus, while measurements to this precision could be considered consistent with factorization, we see that significant deviations from factorization would not be ruled out. Experimental measurements may therefore have to be precise to within $\approx 5\%$ (or less), if r is in this range, before any firm conclusions can be made.

Absolute rates may also offer some clue, since with factorization, and given assumptions about the behavior of the form factors, definite predictions can be made. In contrast, no such predictions can be made from nonfactorized matrix elements, since the normalizations of the form factors are unknown. This suggests that measurements of rates much different from those quoted in Refs. [4–6] would signal the demise of factorization.

Note, however, that we have omitted any discussion of terms suppressed by $1/m_c$. Inclusion of such terms for the nonfactorized matrix elements may introduce additional form factors. One can hope, however, that these new terms are sufficiently small that the leading-order analysis we have carried out is not completely invalidated.

Some indication of this may be gleaned from detailed analyses of decay rates for processes where factorization is expected to be valid, such as $\Lambda_b \to \Lambda_c \pi$ and $B \to D\pi$. Confirmation of the predictions of HQET and factorization for these decays will give us more confidence that we, in fact, understand the dynamics of the processes we are considering. Then we should be able to examine the data for decays such as $\Lambda_b \to \Lambda_c D_s$ and say whether or not factorization is valid. Of course, considering that the Λ_b has not yet been confirmed [11], it will be a while before our conjectures are seriously tested.

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