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Monopoles of SU(15) grand unification

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In a recently analyzed grand unified model based on the gauge group SU(15), monopoles are automatically consistent with the cosmological mass density bound. The Parker bound of monopole flux puts some constraints on the model which can be easily satisfied.

The interesting idea of gauging symmetries like baryon number (B) and lepton number (L) was incorporated [1] in grand unification models based on the group SU(16). In these models, B and L are violated spontaneously. The resulting proton decay rate is much slower [1,2] compared to the prediction of more popular unification models based on SU(5) or SO(10).

Interest in such models has recently been revived by some discussion of a unification model [3, 4] based on SU(15). The difference of this model with those based on SU(16) is that in the fundamental representation containing the fermions there is no right-handed neutrino. Baryon number is still part of the gauge symmetry although lepton number is not. The spontaneous symmetry-breaking process breaks B [5, 6], giving rise to baryon-violating processes. It has been shown that, in a particular chain of symmetry breaking, the unification scale can be as low as 10^7 GeV. This prediction is very different from those of popular grand unification models where the unification scale is 10^{14} GeV or higher. However, a low unification scale in the context of SU(15) does not conflict with data on proton stability because proton decay is very suppressed [3, 7] in this model, just as it is in SU(16).

Here, we want to point out another aspect of the SU(15) model which has not been emphasized before. Because of the possibility of low unification scale, gauge monopoles in this model can be easily consistent with cosmological bounds. This characteristic, again, is very different from that of SU(5), where gauge monopoles violate cosmological energy density bounds by many orders of magnitude.

We start by outlining the derivation of the cosmological bound on the monopoles [8]. At the present era, the energy density of the monopoles is given by $\rho_M = m_M n_M^0$, where m_M is the mass of a monopole and n_M^0 is their present number density. This energy density must satisfy

$$m_M n_M^0 \lesssim 10^{-5} h^2 \,\mathrm{GeV/cm}^3 \,, \tag{1}$$

where h is the Hubble parameter in units of 100 $\mathrm{km\,s^{-1}Mpc^{-1}}$. We can parametrize the monopole mass by

$$m_M = \xi V_M/\alpha_M \,, \tag{2}$$

where $\xi \simeq 1$, and α_M is the fine-structure constant at the scale V_M of symmetry breaking which gives rise to the monopoles. To estimate the number density of the monopoles, we use Kibble's picture [9] in which monopoles are produced when domains of the broken-symmetry phase coalesce. This picture gives rise to about one monopole within the horizon at the time of the creation of the monopoles. Formally, one can write the monopole number density n_M at that era as

$$n_M = \beta/\ell_M^3 \,, \tag{3}$$

where ℓ_M is the horizon length at the time of monopole creation, and $\beta \simeq 1$. Using $\ell_M = 0.6 g_\star^{-1/2} M_P/T_M^2$ where $g_\star$ is the effective number of relativistic degrees of freedom and T_M is the temperature at that era, we obtain

$$n_M/n_\gamma \simeq 20\beta g_\star^{3/2} (T_M/M_P)^3 \,, \tag{4}$$

since the photon number density is related to temperature by $n_\gamma = 0.24\,T^3$. Since that era, the total number of monopoles has not changed noticeably since monopole-antimonopole annihilation cross sections are negligible [10]. Therefore, the number density of monopoles has changed only due to the expansion of the universe. The photon number density, on the other hand, has changed also due to annihilations of particle-antiparticle pairs. Denoting the increase of photon number density due to these reheatings by a factor f_{RH}, we obtain

$$n_M^0 = \frac{20\beta g_\star^{3/2}}{f_{\mathrm{RH}}} \left(\frac{T_M}{M_P}\right)^3 n_\gamma^0 \,, \tag{5}$$

where $n_\gamma^0 \approx 400\,\mathrm{cm}^{-3}$ is the present number density of photons. Using these estimates and introducing the parameter $a_{\mathrm{sc}} = T_M/V_M$, we obtain

$$\rho_M = 8 \times 10^{-54} \, \frac{\xi\beta g_\star^{3/2} a_{\mathrm{sc}}^3}{\alpha_M f_{\mathrm{RH}}} \left(\frac{V_M}{1\,\mathrm{GeV}}\right)^4 \mathrm{GeV/cm}^3. \tag{6}$$

The bound in Eq. (1) then translates to

$$V_M \lesssim \left(\frac{\alpha_M f_{\mathrm{RH}} h^2}{\xi\beta g_\star^{3/2} a_{\mathrm{sc}}^3}\right)^{1/4} \times 10^{12}\,\mathrm{GeV} \,. \tag{7}$$

Observational limits on the value of Hubble parameter imply $\frac{1}{2} \leq h \leq 1$. The parameters ξ and β are of order unity. The fine-structure constant is typically $\sim 10^{-2}$.

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The quantities $f_{\rm RH}$ and $g_\star$ depend on the number of degrees of freedom in the model, and are typically $\approx 10^2$. The factor $a_{\rm sc}$ denotes the supercooling needed below the symmetry-breaking scale before monopoles can appear. For second-order or weakly first-order phase transitions, one would naturally expect this not to be much smaller than unity. Then, Eq. (7) clearly shows why monopoles of SU(5) unification model were inconsistent with cosmology. In order to make SU(5) monopoles acceptable with the cosmological mass density bound, one therefore has to invoke either inflation, or a large supercooling (i.e., $a_{\rm sc} \ll 1$) as may be possible in the case of a strongly first-order transition.

For the recently discussed SU(15) model of grand unification, the situation is very different. The symmetry-breaking chain which gives a low unification scale is

$$\begin{aligned} \mathrm{SU}(15) &\xrightarrow{M_G} \mathrm{SU}(12)_q \times \mathrm{SU}(3)_l \\ &\xrightarrow{M_B} \mathrm{SU}(6)_L \times \mathrm{SU}(6)_R \times \mathrm{U}(1)_B \times \mathrm{SU}(3)_l \\ &\xrightarrow{M_A} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \\ &\xrightarrow{M_W} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q \,. \end{aligned} \tag{8}$$

At the scale M_B, a U(1) factor appears, which gives rise to monopoles. Thus, the monopole scale V_M in the earlier discussion should be identified with the scale M_B in this model. Renormalization-group analysis of the model shows that the scale M_B can easily satisfy Eq. (7), as we discuss below.

Introducing the notation $M_X = 10^{n_X}\,\mathrm{GeV}$ for any mass scale M_X, one gets the following relations connecting different mass scales [4, 5]:

$$\begin{aligned} 18n_G - 6n_B - 7n_A - 5n_W &= 3K\left[\tfrac{3}{8} - \sin^2\theta_W(M_W)\right] \,, \\ 18n_G - 10n_B - 5n_A - 3n_W &= K\left(\frac{3}{8} - \frac{\alpha(M_W)}{\alpha_{3c}(M_W)}\right) \,, \end{aligned} \tag{9}$$

where $K = 8\pi/[11\alpha(M_W)\ln 10]$. Using $\alpha^{-1}(M_W) = 128$, $\sin^2\theta_W(M_W) = 0.228$, $\alpha_{3c}^{-1}(m_W) = 9.35$ and putting $n_W = \log_{10} 81$, one can solve n_G and n_B in terms of n_A:

$$n_B = \tfrac{1}{2}n_A + 5.34\,, \quad n_G = \tfrac{5}{9}n_A + 5.42\,. \tag{10}$$

It was emphasized [5] that the above solution for n_B, together with the constraint $M_B \geq M_A$, implies $M_B \leq 5 \times 10^{10}\,\mathrm{GeV}$. Any value of M_B in this range is easily consistent with the bound in Eq. (7). If M_B happens to be close to the upper limit allowed by renormalization-group calculations, the monopoles can constitute the bulk of the energy density in the universe and therefore can solve the dark-matter problem.

We now discuss the Parker bound [11] on monopole fluxes in our Galaxy, which derives from the consideration that the galactic magnetic field $B_{\rm gal}$ is not destroyed by the acceleration of the monopoles. For monopoles with magnetic charge $\mathcal{G}$, the energy dissipation rate is of order $\mathcal{G}n_M^0 v_M B_{\rm gal}$, where v_M denotes their average velocity. Demanding that the energy density in the galactic magnetic field, $B_{\rm gal}^2/8\pi$, is not depleted in time $\tau \sim 10^8\,\mathrm{yr}$ which is the time needed to regenerate the field, one obtains

$$\frac{n_M^0 v_M}{4\pi} \lesssim \frac{B_{\rm gal}}{32\pi^2\mathcal{G}\tau} \approx 10^{-16}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}\,, \tag{11}$$

using $B_{\rm gal} \simeq 3 \times 10^{-6}\,\mathrm{G}$ and $\mathcal{G} \sim (2e)^{-1}$. The coherence length of this magnetic field within our Galaxy [11] is $\sim 10^{21}\,\mathrm{cm}$. For monopoles with masses in the range of $10^{11}\,\mathrm{GeV}$ or less, it is easy to see that a monopole crossing one coherence length can obtain velocities of order of the speed of light. Thus, using $v_M \sim 1$ in Eq. (11), we obtain

$$n_M^0 \lesssim 10^{-25}\,\mathrm{cm}^{-3}\,. \tag{12}$$

It should be noted that this bound on number densities is much stronger than that obtained for SU(5) monopoles. The reason is that the SU(5) monopoles are much heavier so that their average velocity is much smaller. However, though the bound on n_M^0 is stronger for SU(15), it can be satisfied easily since V_M can be much smaller in this model. To see this, we rewrite Eq. (12) by using the estimate of n_M^0 from Eq. (5), getting

$$V_M \lesssim \left(\frac{f_{\rm RH}}{\beta g_\star^{3/2} a_{\rm sc}^3}\right)^{1/3} \times 10^9\,\mathrm{GeV}\,. \tag{13}$$

This can put nontrivial constraints on the mass scales of the model. Through renormalization-group analysis and the values of the gauge coupling constants at the weak scale, the scales M_B and M_G can be expressed as functions of the scale M_A. For example, if we take the prefactor in Eq. (13) to be unity, we get $M_B \lesssim 10^9\,\mathrm{GeV}$, which implies $M_A \lesssim 2 \times 10^7\,\mathrm{GeV}$ and $M_G \lesssim 3 \times 10^9\,\mathrm{GeV}$ from Eq. (10). This is stronger than the bounds from any other consideration. If, within our Galaxy, the local density of monopoles is higher than the estimate of Eq. (5), the bound on V_M becomes stronger.

To summarize, we have shown that the monopoles appearing the course of symmetry breaking of SU(15) grand unified group are consistent with the cosmological energy density bound. In sharp contrast to monopoles arising, e.g., in SU(5) grand unified model, one does not have to invoke large supercooling to achieve this. The Parker bound on monopole flux in our Galaxy can put some constraints on the scales of the model. These constraints are consistent with renormalization-group calculations.

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