Initial condition for the minimal isocurvature scenario

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We derive a proper expression for the power spectrum of baryon-number fluctuations arising from decay of a heavy Majorana lepton with space-dependent *CP* violation due to a spatially varying Majoron field. The spectrum is explicitly calculated in a power-law inflation model and found to be almost scale invariant on small scales and white noise on large scales. Under reasonable assumptions, we present a natural particle-physics model which provides an appropriate initial condition for the minimal isocurvature scenario of large-scale structure formation.

I. INTRODUCTION

One of the primary purposes of cosmology is to explain how the observed large-scale structures were formed in the course of cosmic evolution. Though several competing scenarios have been proposed so far, too-large observational uncertainties make it impossible for us to single out the right one. Indeed we do not know yet even what kind of matter dominates our Universe, nor do we know precise values of fundamental parameters such as the Hubble constant, density parameter, or cosmological constant. In this situation Peebles and Silk listed various observational constraints that a successful scenario should satisfy and made a cosmic book [1], in which they concluded that the canonical cold-dark-matter (CDM) scenario in the inflationary cosmology [2] and the minimal baryon isocurvature scenario in a low-density universe [3–6] surpass other candidates at present.

The cold-dark-matter scenario is attractive in that its initial condition is based on a rather definite prediction of inflationary cosmology, that is, adiabatic fluctuations with a scale-invariant (Harrison-Zel'dovich) spectrum [7] in a spatially flat universe. Unfortunately, however, quite a few serious difficulties have been revealed for the scenario recently [8]. In particular it seems incapable of explaining very large-scale structures on scales over 100 Mpc whose existence has been claimed by a number of recent observations [9]. Another uninteresting feature is that, in spite of intensive efforts by experimentalists, we have not seen any evidence for the existence of an elementary particle which may serve as cold dark matter.

On the other hand, the minimal isocurvature scenario, which has been proposed by Peebles [3], attempts to explain the large-scale structure formation in terms of a very different philosophy, or on a purely phenomenological basis. A nice feature of this scenario is that it assumes as the material ingredients only those that we know exist, namely, baryons and radiation (photons and massless neutrinos). However, as is well known, it is very difficult in general to account for galaxy formation only with baryonic matter without violating the observed isotropy of the cosmic-microwave-background (CMB) radiation [10]. Hence it is necessary to assume a very *ad hoc* type of primordial density fluctuations as the initial condition, namely, isocurvature fluctuations with a steep spectrum. Having less power on larger scales, one may avoid a generic difficulty of models with isocurvature fluctuations so that the scenario can be consistent with the large-angle isotropy of CMB. On the other hand, the large initial amplitude of fluctuations on smaller scales allows early star formation soon after the recombination. Stars then reionize the medium and baryons recouple with photons, erasing small-angle anisotropies of CMB through diffusion in the plasma. Thus once we admit this seemingly unnatural initial condition, the scenario does not contradict with the isotropy of CMB. In addition it has some attractive features which are absent in CDM scenarios: for example, galaxies may be formed much earlier and large-scale coherence can be obtained [3,11].

Recently Yokoyama and Suto [11] proposed a new mechanism to produce baryon isocurvature fluctuations which may provide an appropriate spectrum for the minimal isocurvature scenario. They considered a baryogenesis model with both hard and soft *CP* violation. The former is responsible for the homogeneous part of the baryon/entropy ratio, $n_b/s \simeq 10^{-10}$, and the latter for its spatial fluctuations. To be more specific, in their model the soft *CP* violation is induced by a spatially varying Majoron field associated with a heavy Majorana lepton field which decays into three quarks or three antiquarks and thereby violates baryon-number conservation. In the chaotic inflation model [12] they have shown that the resultant spectrum of baryon-number fluctuations has less power on larger scales, using an intuitive approach by Kofman and Linde [13], and that the amplitude may be large enough to provide the necessary initial condition

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for the minimal isocurvature scenario if the Majorana leptons are produced maximally in the reheating phase.

In the present paper we improve the previous intuitive calculation [13] to formulate a more proper expression for the power spectrum of baryon-number fluctuations and apply it to power-law inflation, keeping in mind that many viable inflationary universe models such as extended inflation [14] or soft inflation [15] predict power-law [16,17] rather than exponential [18] inflation. We find that the resultant spectrum is almost scale invariant on small scales and white noise on large scales and that it may be obtained from a much more natural particle-physics model than the one needed in the chaotic inflation model.

The rest of the paper is organized as follows. In Sec. II, we review particle-physics aspects of our model. In Sec. III we present a generic formula for the power spectrum of baryon-number fluctuations and explicitly evaluate it in a power-law inflation background. In Sec. IV we consider a specific class of inflation models which realize our purpose and discuss cosmological constraints on it. Astrophysical constraints on model parameters of particle physics are studied in Sec. V. Finally, Sec. VI is devoted to discussion and conclusion.

II. BARYON-NUMBER FLUCTUATION FROM SOFT CP VIOLATION

We consider baryon-number violation due to a heavy Majorana lepton N, which is supposed to generate tiny neutrino masses via the seesaw mechanism [19]. Baryogenesis through N decay has been considered by a number of authors [20], although their analyses were focused only on hard CP violation. The coupling between N and the Nambu-Goldstone boson (called Majoron, which arises when the lepton number is spontaneously broken), however, naturally leads to soft CP violation as well. Therefore an appreciable amount of baryon-number production is expected if a coherent Majoron field is spatially varying so that its complex coupling cannot be removed by a global transformation. Yoshimura [21] proposed a mechanism to generate isothermal baryon-number perturbations on the basis of the above soft CP violation but did not give the spectrum of resultant fluctuations. For a different mechanism of generation of baryon-number fluctuations, see [22].

Let us first describe the particle contents of our model using, for simplicity, the SU(5) representation. The Yukawa coupling between fermions and Higgs bosons is given as

$$\mathcal{L}_Y=[h_1\bar{\psi}^c_R(10)\psi_L(10)+h_2\bar{\psi}_L(10)\psi_R(5)+h_3\bar{\psi}_R(5)N^c_L(1)]S(5)+h_4N^T_R(1)\mathcal{C}N_R(1)S_0(1)+\text{H.c.}\ , \tag{2.1}$$

where h_1 through h_4 denote Yukawa coupling matrices whose generation indices are suppressed throughout the paper, $L,R=(1\mp\gamma_5)/2$ are chiral projections, and $\mathcal{C}$ is the charge-conjugation matrix. Here $\psi(5)$ and $\psi(10)$ are ordinary fermion fields in the **5**, and **10** representations of SU(5), and $S(5)$ denotes Higgs bosons in the **5** representation whose color-triplet components S_3 violate baryon-number conservation. $S_0(1)$ is a complex scalar field in the **1** representation which generates Majorana mass to N_R and spontaneously breaks lepton-number conservation when it acquires a vacuum expectation value $\langle S_0\rangle\equiv f$. Then a Nambu-Goldstone boson, called Majoron $A(x)$, appears and it interacts with N through

$$\tfrac{1}{2}M_N N^T_R(1)\mathcal{C}N_R(1)\exp\left[i\frac{A(x)}{f}\right]+\text{H.c.}$$

From this coupling CP violation arises even if the mass matrix M_N is real. The Majorana lepton N violates baryon number through three-body decays such as

$$N\rightarrow qqq,\ \overline{qqq}\ .$$

Through the above-mentioned CP violation, the space-dependent net baryon number

$$B(x)\equiv B_*\sin\left[\frac{A(x)}{f}\right] \tag{2.2}$$

is generated from decay of a pair of N through the diagram shown in Fig. 1. The coefficient B_* depends critically on the masses of decaying N, exchanged N, and S_3 as well as on coupling constants. In the case these masses are of the same order of magnitude, we find $B_*\simeq\alpha_2\alpha_3$, where $\alpha_i\equiv h_i^2/4\pi$ [20]. Note that the mass of S_3,M_S, should be larger than $\simeq 10^{11}$ GeV in order to meet the experimental bound on the proton lifetime [23]. Thus there may appear spatial fluctuations of the baryon-number associated with that of a coherent Majoron field.

On the other hand, there are a number of possible mechanisms to explain the homogeneous part of the baryon-to-entropy ratio, $n_b/s\simeq 10^{-10}$. For example, if appropriate hard CP violation is also present in Fig. 1, that decay process can account for it as well. Decays of

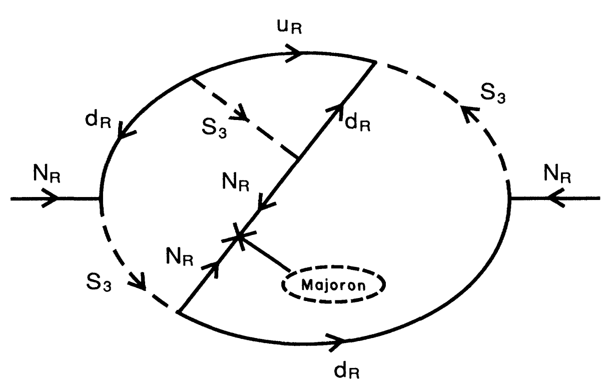

FIG. 1. Interference diagram of Majorana-lepton decay producing space-dependent baryon asymmetry.

S_3 may also generate the right magnitude of the observed asymmetry, if they are adequately produced in the reheating phase [24]. Since we are interested in its fluctuations, here we simply assume that one of these mechanisms is responsible for the homogeneous part.

III. POWER SPECTRUM OF BARYON-NUMBER FLUCTUATIONS

Here we consider the power spectrum of baryon-number fluctuations (2.2). We assume the Majoron field $A(x)$ can be regarded as a free minimally coupled massless scalar field in inflating spacetime. In reality, the scalar field S_0 suffers from fluctuations in the radial direction as well. In order that these radial fluctuations are negligible and $A(x)$ can be regarded as a free field, we assume that S_0 has a steep potential with a self-coupling of order of unity or larger.

First note that the two-point correlation function of $B(x)$ is expressed as

$$\langle B(x)B(y)\rangle = B_*^2\left\langle \sin\left[\frac{A(x)}{f}\right]\sin\left[\frac{A(y)}{f}\right]\right\rangle = \frac{B_*^2}{2}\mathrm{Re}\left\langle \exp\left[\frac{i}{f}[A(x)-A(y)]\right] - \exp\left[\frac{i}{f}[A(x)+A(y)]\right]\right\rangle . \tag{3.1}$$

Hence we need to evaluate $\langle e^{i[A(x)\pm A(y)]/f}\rangle$. In order to do so, let us consider the generating functional of the Green function for A:

$$Z(j) = \left\langle T\left[\exp\left[i\int d^4x\, A(x)j(x)\right]\right]\right\rangle = \left\langle \exp\left[\frac{i}{2}\int d^4x\, d^4y\, j(x)G_F(x,y)j(y)\right]\right\rangle , \tag{3.2}$$

where $G_F(x,y)$ is the Feynman propagator defined by $G_F(x,y)=i\langle TA(x)A(y)\rangle$ with T representing the time-ordered product. Since what we are interested in is the spatial autocorrelation of A, we may assume the points x and y are spatially separated. This means we have

$$-iG_F(x,y)=G(x,y)=G(y,x)\equiv\langle A(y)A(x)\rangle . \tag{3.3}$$

That is, we may neglect the presence of the time-ordered product and assume the symmetry in the arguments.

Then choosing the source density j as

$$j(z)=\frac{\delta^4(z-x)\pm\delta^4(z-y)}{f} ,$$

and inserting it into Eq. (3.2), one readily obtains

$$\left\langle \exp\left[\frac{i}{f}[A(x)\pm A(y)]\right]\right\rangle = \exp\left[-\frac{1}{f^2}[G(0,t)\pm G(r,t)]\right] , \tag{3.4}$$

where we have assumed the homogeneity and isotropy of the vacuum state and set $t=x^0=y^0$ and $r=|\mathbf{y}-\mathbf{x}|$. Thus the spatial autocorrelation function of B is given by

$$\langle B(x)B(y)\rangle = \frac{B_*^2}{2}\left[\exp\left[\frac{1}{f^2}[G(r,t)-G(0,t)]\right] - \exp\left[-\frac{1}{f^2}[G(r,t)+G(0,t)]\right]\right] . \tag{3.5}$$

Before we proceed, we note that the general n-point autocorrelation function can be straightforwardly calculated by setting j in Eq. (3.2) to be

$$j(z)=\sum_{i=1}^{n}(-1)^{\sigma_i}\delta^4(z-x_i) ,$$

for all possible choices of $\sigma_i=0,1$.

In the above, we tacitly assumed that $\theta_0\equiv\langle A(x)\rangle/f=0$. However, in a region of space which corresponds to our observable Universe it may well be nonvanishing since θ_0 may take an arbitrary value (modulo 2π) when the spontaneous breakdown occurs. The effect of a nonvanishing θ_0 can be easily seen by replacing $A(x)$ with $f\theta_0+A(x)$ in Eq. (3.1). Then instead of Eq. (3.5) we have

$$\langle B(x)B(y)\rangle = \frac{B_*^2}{2}\left[\exp\left[\frac{1}{f^2}[G(r,t)-G(0,t)]\right] - \cos 2\theta_0 \exp\left[-\frac{1}{f^2}[G(r,t)+G(0,t)]\right]\right] . \tag{3.6}$$

Thus the change brought about by a nonzero initial θ_0 is the appearance of a nontrivial coefficient in the second term. It will be shown below, however, that this θ_0 dependence turns out to be unimportant.

Let us now calculate the correlation function $G(r,t)$ in a power-law inflation background. The spacetime metric is assumed to be spatially flat:

$$ds^2=-dt^2+a^2(t)dx^2=a^2(\eta)(-d\eta^2+dx^2) , \tag{3.7}$$

with $a(t)$ being a power-law solution:

$$a(t)\propto t^{1+n},\quad n>0 . \tag{3.8}$$

In terms of the conformal time $\eta\propto -t^{-n}$ the scale factor $a(\eta)$ and the Hubble parameter $H(\eta)$ are expressed as

$$a(\eta)=\frac{1}{(-H_*\eta)^{1+1/n}} ,\qquad H(\eta)=\left[1+\frac{1}{n}\right]H_*(-H_*\eta)^{1/n} , \tag{3.9}$$

where H_* is a constant.

As usual, we decompose the scalar field as

$$A(x)=\int \frac{d^3k}{(2\pi)^{3/2}}\left[\hat{a}_{\mathbf{k}}A_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}+\hat{a}_{\mathbf{k}}^{\dagger}A_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\right], \tag{3.10}$$

where $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^{\dagger}$ are annihilation and creation operators, respectively, for a suitably chosen vacuum state, and $A_k(\eta)$ is the corresponding positive-frequency mode function. From the field equation and the canonical commutation relations, the mode function $A_k(\eta)$ satisfies the equation

$$\left[\frac{d^2}{d\eta^2}-\left[1+\frac{1}{n}\right]\frac{2}{\eta}\frac{d}{d\eta}+k^2\right]A_k(\eta)=0\ , \tag{3.11}$$

with the normalization

$$A_k\frac{d}{d\eta}A_k^*-A_k^*\frac{d}{d\eta}A_k=\frac{i}{a^2(\eta)}\ . \tag{3.12}$$

The solution is given by

$$A_k(\eta)=\left[\frac{\pi}{4}\right]^{1/2}H_*^{1+1/n}(-\eta)^{\nu}[c_{k1}H_\nu^{(1)}(-k\eta)+c_{k2}H_\nu^{(2)}(-k\eta)]\ ,$$

where $|c_{k1}|^2-|c_{k2}|^2=1, \nu=\frac{3}{2}+1/n$, and $H_\nu^{(j)}(z)$ is the Hankel function of the jth kind. We consider the case $c_{k1}=1$ and $c_{k2}=0$ since it corresponds to the usual Minkowski vacuum at $\eta\to-\infty$ and it is natural to assume that any k mode is in this vacuum when $-k\eta\simeq k/aH\gg 1$, i.e., when the effect of cosmic expansion can be neglected.

Then the symmetric two-point function $G^{(1)}(x,x')\equiv\langle A(x)A(x')+A(x')A(x)\rangle$ is given by

$$\begin{aligned}G^{(1)}(x,x')&=\int\frac{d^3k}{(2\pi)^3}[A_k(\eta)A_k^*(\eta')+A_k(\eta')A_k^*(\eta)]e^{i\mathbf{k}\cdot\mathbf{r}}\\&=(H_*^2\eta\eta')^{1/2}\frac{\pi}{4}H_*^2(\eta\eta')^{3/2}\int\frac{d^3k}{(2\pi)^3}[H_\nu^{(1)}(-k\eta)H_\nu^{(2)}(-k\eta')+\text{c.c.}]e^{i\mathbf{k}\cdot\mathbf{r}}\\&\equiv(H_*^2\eta\eta')^{1/n}G_{\mathrm{dS}}^{(1)}(x,x')\ ,\end{aligned}$$

where $\mathbf{r}=\mathbf{x}-\mathbf{x}'$. Here $G_{\mathrm{dS}}^{(1)}(x,x')$ is formally equivalent to the symmetric two-point function of a scalar field in de Sitter spacetime with the Hubble parameter H_*, whose mass squared m_*^2 is negative and given as

$$\nu=\frac{3}{2}+\frac{1}{n}=\left[\frac{9}{4}-\frac{m_*^2}{H_*^2}\right]^{1/2}\quad\text{or}\quad m_*^2=-\left[\frac{1}{n^2}+\frac{3}{n}\right]H_*^2\ . \tag{3.13}$$

This implies that a massless minimally coupled field in a power-law background is manifestly infrared unstable, while it is only marginally unstable in an exactly de Sitter background.

The regularization of this infrared instability has been explicitly done by Nambu and Sasaki [25] with a method appropriate to the inflationary universe. They find

$$G_{\mathrm{dS}}^{(1)}(x,x')=G_{\mathrm{dS0}}^{(1)}(Z)+\bar{G}_{\mathrm{dS}}^{(1)}(\eta\eta'),\quad Z\equiv\frac{\eta^2+\eta'^2-r^2}{2\eta\eta'}\ . \tag{3.14}$$

Here $G_{\mathrm{dS0}}^{(1)}(Z)$ is the symmetric two-point function for the Euclidean vacuum with m_*^2 analytically continued from a positive value to negative:

$$G_{\mathrm{dS0}}^{(1)}(Z)=\frac{H_*^2}{8\pi^2}\Gamma\left[3+\frac{1}{n}\right]\Gamma\left[-\frac{1}{n}\right]\times F\left[-\frac{1}{n},3+\frac{1}{n},2,\frac{1+Z}{2}\right]\ , \tag{3.15}$$

with F being the hypergeometric function. $\bar{G}_{\mathrm{dS}}^{(1)}(\eta\eta')$ is given by

$$\bar{G}_{\mathrm{dS}}^{(1)}(\eta\eta')\simeq\frac{nH_*^2}{4\pi^2}\left[\frac{r_0^2}{4\eta\eta'}\right]^{1/n}\ , \tag{3.16}$$

where r_0 is the comoving length of infrared cutoff which should be chosen much larger than that of the present horizon.

Since we are interested in the behavior of $G_{\mathrm{dS}}^{(1)}(x,x')$ with x and x' being spacelike separated much further than the Hubble length, we may expand $G_{\mathrm{dS0}}^{(1)}(Z)$ assuming $Z\ll-1$ to get

$$\begin{aligned}G_{\mathrm{dS0}}^{(1)}(Z)&\simeq\frac{H_*^2}{8\pi^2}\frac{\Gamma(3+2/n)\Gamma(-1/n)}{\Gamma(2+1/n)}\left[\frac{1-Z}{2}\right]^{1/n}\\&\simeq-\frac{nH_*^2}{4\pi^2}\left[\frac{r^2-(\eta-\eta')^2}{4\eta\eta'}\right]^{1/n}\ ,\end{aligned} \tag{3.17}$$

where the last approximation is good for $n\gg 1$ which we assume hereafter. Thus we finally obtain the following spatial correlation function:

$$\begin{aligned}G(r,\eta)&=\tfrac{1}{2}G^{(1)}(r,\eta)\\&\simeq-\frac{nH^2(\eta)}{8\pi^2}\left[\left[\frac{r^2}{\eta^2}\right]^{1/n}-\left[\frac{r_0^2}{\eta^2}\right]^{1/n}\right]\ .\end{aligned} \tag{3.18}$$

Note that scales of our interest are in the range $|\eta|\ll r\ll r_0$.

The bare $G(0,\eta)=\langle A(x)^2\rangle$ is also a divergent quantity and should be regularized. However, in this case it is a usual ultraviolet divergence and the renormalized $G(0,\eta)$ may be estimated by introducing an ultraviolet cutoff at the horizon scale $r=|\eta|$:

$$G(0,\eta)\simeq -\frac{nH^2(\eta)}{8\pi^2}\left[1-\left[\frac{r_0^2}{\eta^2}\right]^{1/n}\right], \tag{3.19}$$

which is in agreement with Sahni [26] and Pathinayake and Ford [27] at the large-n limit.

Now we turn to the evaluation of the spatial correlation of $B(x)$, Eq. (3.6). From Eqs. (3.18) and (3.19), we have

$$\langle B(\mathbf{x},\eta)B(\mathbf{x}+\mathbf{r},\eta)\rangle \simeq \frac{B_*^2}{2}\exp\left\{\frac{nH^2(\eta)}{8\pi^2 f^2}\left[1-\left[\frac{r^2}{\eta^2}\right]^{1/n}\right]\right\}\left[1-\cos2\theta_0\exp\left\{-2\frac{nH^2(\eta)}{8\pi^2 f^2}\left[\left[\frac{r_0^2}{\eta^2}\right]^{1/n}-\left[\frac{r^2}{\eta^2}\right]^{1/n}\right]\right\}\right]. \tag{3.20}$$

Since scales of our interest are in the range $|\eta| \ll r \ll r_0$, we see the term proportional to $\cos2\theta_0$ is negligible. Thus both the unwanted initial-value dependence and the cutoff dependence disappear as it is desired.

If we identify η_k with the epoch when the k mode leaves the horizon, the correlation function then reads

$$\langle B(0,\eta)B(\mathbf{r},\eta)\rangle = \frac{B_*^2}{2}\exp\{n\beta_k[(k\eta)^{2/n}-(kr)^{2/n}]\},$$
$$\beta_k \equiv \frac{H^2(\eta_k)}{8\pi^2 f^2}. \tag{3.21}$$

We define the power spectrum $P_B(k)$ of the baryon-number fluctuation as

$$P_B(k,\eta)=\int d^3r\langle B(0,\eta)B(\mathbf{r},\eta)\rangle e^{-i\mathbf{k}\cdot\mathbf{r}}. \tag{3.22}$$

From Eqs. (3.21) and (3.22), the power spectrum at the end of inflation, $\eta=\eta_f$, is expressed as

$$\begin{aligned}P_B(k,\eta_f)&=4\pi\int dr\, r^2\frac{\sin kr}{kr}\langle B(0,\eta_f)B(\mathbf{r},\eta_f)\rangle\\ &=\frac{\pi B_*^2}{k^3}\exp[n\beta_k(k\eta_f)^{2/n}]\\ &\quad\times\int_0^\infty ds\, s\sin s\exp(-n\beta_k s^{2/n})\\ &=\frac{\pi B_*^2}{k^3}e^{n\beta_f}J(n,k),\end{aligned} \tag{3.23}$$

where we have used the relation $\beta_k=\beta_f(k\eta_f)^{-2/n}$ and assumed $n\beta_f \lesssim 1$. We note that the lower bound of the integral in the second line of Eq. (3.23) is actually $k|\eta_f|$, corresponding to the cutoff at the horizon scale. The assumption $n\beta_f \lesssim 1$ enables us to put the lower bound zero. Of course one may consider the case $n\beta_f \gg 1$. In fact, the result for this case coincides with that for a pure de Sitter background ($n\to\infty$). However, it turns out to give either the amplitude too small or the spectrum too flat for cosmological interests [28].

The integral $J(n,k)$ may be analytically estimated in two extreme cases; for $n\beta_k \ll 1$ and $n\beta_k \gg 1$. In the former case, the leading-order contribution reads

$$\begin{aligned}J(n,k)&\simeq -n\beta_k\int_0^\infty ds\, s^{1+2/n}\sin s\\ &=n\beta_k\Gamma(2+2/n)\sin(\pi/n)\\ &\simeq \pi\beta_k=\pi\beta_f(k\eta_f)^{-2/n},\quad n\beta_k\ll 1.\end{aligned} \tag{3.24}$$

In the latter case, introducing a new variable $u \equiv n\beta_k s^{2/n}$, we find

$$\begin{aligned}J(n,k)&\simeq\frac{n}{2}(n\beta_k)^{-3n/2}\int_0^\infty u^{-1+3n/2}e^{-u}du\\ &=\frac{n}{2}(n\beta_k)^{-3n/2}\Gamma\left[\tfrac{3}{2}n\right]\\ &\simeq\left[\frac{n\pi}{3}\right]^{1/2}\left[\frac{2e\beta_f}{3}\right]^{-3n/2}k^3|\eta_f|^3,\quad n\beta_k\gg 1.\end{aligned} \tag{3.25}$$

The resultant power spectrum in the two asymptotic regions takes the form,

$$P_B(k)\simeq\begin{cases}\pi^2 B_*^2\dfrac{e^{n\beta_f}}{nk^3}\left[\dfrac{k}{k_c}\right]^{-2/n} & \text{for } k\gg k_c,\\[2ex] \dfrac{\pi B_*^2 e^{n\beta_f}}{k_c^3}\left[\dfrac{n\pi}{3}\right]^{1/2}\left[\dfrac{3n}{2e}\right]^{3n/2} & \text{for } k\ll k_c,\end{cases} \tag{3.26}$$

where k_c is the comoving wave number corresponding to $n\beta_{k_c}=1$ and given explicitly by

$$k_c=\frac{(n\beta_f)^{n/2}}{|\eta_f|}=(n\beta_f)^{n/2}H(\eta_f)a(\eta_f). \tag{3.27}$$

We can interpret the above spectral shape as follows. For $k\gg k_c$, the amplitude of fluctuation is so small that we can expand $B(x)$ as $B(x)=B_*\sin[A(x)/f]$

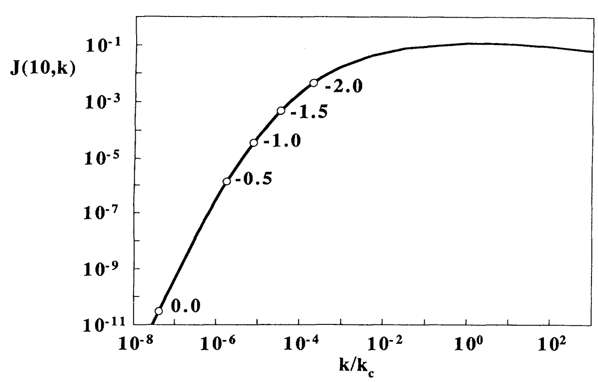

FIG. 2. Numerical values of $J(n,k)$ for $n=10$ as a function of k/k_c. The numbers indicated at several points on the line are the values of $p_{\rm eff}$ at respective points.

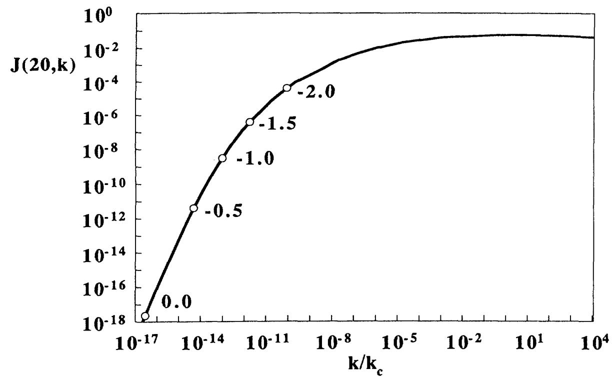

FIG. 3. The same as Fig. 2 but for $n=20$.

$\simeq B_* A(x)/f$. Hence fluctuation of $B(x)$ has the same spectral shape as that of $A(x)$, which is almost scale invariant. On the other hand, for $k \ll k_c$, fluctuation of $A(x)$ is larger than $2\pi f$, so that $\sin[A(x)/f]$ takes a random value between -1 and $+1$ on that scale, which implies that there is no correlation in $\sin[A(x)/f]$ on such a large scale. Thus the power spectrum becomes white noise on large scale.

Numerical values of $J(n,k)$ are shown in Figs. 2 and 3 for $n=10$ and 20, respectively. As can be seen from the figures, the span of intermediate scales between almost scale-invariant region and white-noise region becomes wider as n increases. Since astrophysical consideration is usually given to primordial fluctuation spectra of a power-law form, it is convenient to introduce an effective power-law index for the baryon-number fluctuation on scale k, defined as

$$p_{\rm eff}(k) \equiv \frac{d\ln P_B(k)}{d\ln k} = \frac{\partial \ln J(n,k)}{\partial \ln k} - 3 . \tag{3.28}$$

The values of $p_{\rm eff}(k)$ for several points of k are also shown in Figs. 2 and 3.

Thus provided that there exists a reasonable inflationary model which implements the desirable particle-physics contents and which yields an appropriate transition scale k_c^{-1} and amplitude of fluctuations, an adequate initial condition for the minimal isocurvature scenario may be realized very naturally. In the next two sections we consider these points.

IV. MODEL CONSTRUCTION AND COSMOLOGICAL CONSIDERATION

In order to compare predictions of the present model with observations, we must specify evolution of the Universe from inflationary era to present. To do this we should first specify a scenario of power-law inflation. There are two generic classes of models which predicts power-law inflation. One is those containing a scalar field with an effectively exponential potential in Einstein gravity theory [17]. The other is models with an ordinary inflation-driving (inflaton) field with a nearly flat potential in modified Einstein theories such as the Brans-Dicke model or induced gravity theory [14,15,29]. In fact, the latter class may be transformed to the former by a conformal transformation [30]. Hence in order to avoid unnecessary complications we consider the following simple model for the inflaton ϕ [31], in which generic features of more complicateed but viable models are maintained:

$$\mathcal{L}_{G\phi} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \right] ,$$
$$V(\phi) = V_0[\cosh(\lambda\kappa\phi) - 1] , \tag{4.1}$$

where $\kappa^2 = 8\pi G$ and V_0 and λ are constant. The potential has the asymptotic forms

$$V[\phi] \simeq \begin{cases} \frac{1}{2} V_0 \exp(\lambda\kappa\phi) & \text{for } \phi \gg \dfrac{1}{\lambda\kappa} , \\ \frac{1}{2} V_0 \lambda^2\kappa^2\phi^2 \equiv \frac{1}{2} M_\phi^2 \phi^2 & \text{for } |\phi| \ll \dfrac{1}{\lambda\kappa} . \end{cases} \tag{4.2}$$

If the Universe starts classical evolution with a large value of ϕ as in the chaotic inflation scenario, the system soon approaches the solution

$$a(t) = a_0 t^{1+n}, \quad n = \frac{2-\lambda^2}{\lambda^2} ,$$
$$\phi(t) = \frac{2}{\lambda\kappa} \ln \left[\left[\frac{6-\lambda^2}{\lambda^2 V_0} \right]^{1/2} \frac{2}{\lambda\kappa t} \right] . \tag{4.3}$$

Thus power-law inflation is realized for $\lambda < \sqrt{2}$ and sufficient inflation to solve the horizon and flatness problems may easily be obtained with a large initial value of ϕ [31]. Inflation ends when $\phi(t)$ decreases to $\phi \approx 1/(\lambda\kappa)$. We identify this epoch with the time $t_f(\eta_f)$. Just after $t=t_f$, the Universe is dominated by a coherent field oscillation of ϕ and cosmic expansion law is identical to that for a matter-dominated era. It is after ϕ decays into light particles when the Universe enters the radiation-dominated era.

Denoting by T_R the reheating temperature, we find the Hubble radius at the end of inflation, $H(\eta_f)^{-1} \equiv H_f^{-1}$, corresponds to the present length

$$l_{f0} \simeq 7 \times 10^{-13} H_{f10}^{-1/3} T_{R7}^{-1/3} \ \text{pc} , \tag{4.4}$$

where $H_{f10} \equiv H_f/10^{10}$ GeV and $T_{R7} \equiv T_R/10^7$ GeV. The time, t_g, when a length scale relevant to galaxy formation or $l_{0g} \sim 1-10^3$ Mpc left the horizon during inflation is related to t_f as

$$t_g = \left[\frac{l_{0f}}{l_{0g}} \right]^{1/n} t_f . \tag{4.5}$$

From Eqs. (4.3), (4.4), and (4.5), t_g is determined once we specify the value of n or λ. For definiteness of the discussion below we consider two specific values of n; $n=10$ and 20, hereafter.

The first constraint we consider is the amplitude of adiabatic fluctuations. It is given by

$$\left.\frac{\delta\rho}{\rho}\right|_k \simeq \left.\frac{H^2}{2\pi|\dot{\phi}|}\right|_{t=t_k} = \begin{cases} 4\kappa t_k^{-1} & \text{for } n=10 , \\ 11\kappa t_k^{-1} & \text{for } n=20 , \end{cases} \tag{4.6}$$

on comoving scale k^{-1}, where as usual the epoch $t=t_k$ is when the k mode left the horizon during inflation [17]. The amplitude is larger for a larger length scale. We demand

$$\left.\frac{\delta\rho}{\rho}\right|_k < 10^{-5} , \tag{4.7}$$

on the present horizon scale $\sim 3\times 10^3$ Mpc, which is translated to the following constraint on H_f:

$$H_{f10} \lesssim \begin{cases} 20 & \text{for } n=10 , \\ 80 & \text{for } n=20 . \end{cases} \tag{4.8}$$

Then adiabatic fluctuations would be both dynamically unimportant and harmless to the observed isotropy of CMB. From Eq. (4.8), we obtain the following constraint on the inflaton mass M_ϕ:

$$M_\phi \lesssim \begin{cases} 2\times 10^{11}\ \text{GeV} & \text{for } n=10 , \\ 8\times 10^{11}\ \text{GeV} & \text{for } n=20 . \end{cases} \tag{4.9}$$

Next we consider reheating processes. If one assumes ϕ is coupled to some other specific fields, then those particles may be dominantly produced from the coherent oscillation of ϕ with high efficiency [11,24]. Such an assumption, however, is artificial and *ad hoc*. In the present model, on the contrary, ϕ may be naturally related to gravity sector and interacts with all the other fields with gravitational strength, as in the case of soft inflation [15] or supergravity models [32]. A plausible form of the interaction is

$$\mathcal{L}_f = e^{-\lambda\kappa\phi}(M_F\bar{F}F + \tfrac{1}{2}M_B^2 B^2) , \tag{4.10}$$

for fermion F and boson B. Then ϕ will decay dominantly into heaviest particles that are lighter than $M_\phi/2$ with a quite small decay rate,

$$\Gamma_{\phi\to F\bar{F}} \simeq \lambda^2 \left[\frac{M_F}{M_{\text{Pl}}}\right]^2 M_\phi \quad \text{for fermionic two-body decay ,} \tag{4.11}$$

$$\Gamma_{\phi\to B\bar{B}} \simeq \lambda^2 \frac{M_B^4}{M_{\text{Pl}}^2 M_\phi} \quad \text{for bosonic two-body decay ,} \tag{4.12}$$

where M_{Pl} stands for the Planck mass. Hence it is likely that Majorana leptons N are dominantly produced in the reheating stage with decay rate (4.11), provided their mass is $M_N \sim 10^{10-11}$ GeV $< M_\phi/2$ [33]. We shall assume this is the case in the present model. We mention that the above value of Majorana mass is also appropriate to generate a tiny neutrino mass via the seesaw mechanism [19] which may solve the solar-neutrino problem [34].

The reheating takes place when the Hubble parameter decreases to the value $H(t) \sim \Gamma_\phi \simeq \Gamma_{\phi\to NN}$, where Γ_ϕ is the total decay rate of ϕ. Then the produced Majorana leptons decay into lighter particles almost instantaneously to reheat the Universe up to a temperature T_R. The Majorana lepton-number density n_N and the reheating temperature T_R are given from the relation

$$\Gamma^2_{\phi\to NN} \simeq \frac{\kappa^2}{3} M_N n_N \simeq \frac{\kappa^2}{3}\frac{\pi^2}{30} g_* T_R^4 , \tag{4.13}$$

as

$$\begin{aligned} n_N &\simeq \lambda^4 M_N^3 \left[\frac{M_\phi}{M_{\text{Pl}}}\right]^2 , \\ T_R &\simeq \lambda M_N \left[\frac{M_\phi}{M_{\text{Pl}}}\right]^{1/2} \simeq 10^7 M_{N11} M_{\phi 11}^{1/2}\ \text{GeV} , \end{aligned} \tag{4.14}$$

respectively, where $M_{N11} = M_N/10^{11}$ GeV, $M_{\phi 11} = M_\phi/10^{11}$ GeV, and $g_* \simeq 100$ is the effective number of massless degrees of freedom. Thus the magnitude of baryon/entropy fluctuations arising from the process depicted in Fig. 1 reads

$$\delta\left[\frac{n_b(x)}{s}\right] \simeq 10^{-5} B(x) M_{\phi 11}^{1/2} . \tag{4.15}$$

Since the amplitude of the entropy perturbation $S(x)$ is

$$S(x) \equiv \frac{\delta(n_b(x)/s)}{n_b/s} \approx 10^{10}\delta(n_b/s) ,$$

its power spectrum is given by

$$P_S(k) \simeq 10^{10} M_{\phi 11} P_B(k) \equiv Q^2 P_B(k) . \tag{4.16}$$

We require that the amplitude of the above primordial entropy fluctuations should not exceed unity on any scales so that the standard primordial nucleosynthesis scenario works [35]. Hence,

$$\begin{aligned} \max_k \langle S^2 \rangle &\simeq \max_k \frac{k^3}{(2\pi)^3} P_S(k) \\ &= \max_k \frac{(QB_*)^2}{8\pi^2} e^{n\beta_f} J(n,k) < 1 . \end{aligned} \tag{4.17}$$

One should probably keep in mind that the above requirement is not really a constraint but just imposed because we do not know precisely what would happen if the baryon-number perturbation was large during the nucleosynthesis. Using numerical values of $J(n,k)$, the condition (4.17) implies

$$(QB_*)^2 e^{n\beta_f} < \begin{cases} 7\times 10^2 & \text{for } n=10 , \\ 1\times 10^3 & \text{for } n=20 . \end{cases} \tag{4.18}$$

V. ASTROPHYSICAL CONSTRAINTS ON MODEL PARAMETERS

Starting with the initial condition (4.16) we can calculate the time evolution of the isocurvature fluctuations using linear theory [36], according to which the power spectrum of density fluctuations at recombination, $P(k,z_{\text{rec}})$, is related to $P_S(k)$ as

$$P(k,z_{\rm rec}) \approx \begin{cases} P_S(k), & k \gtrsim k_{\rm eq} \ , \\ P_S(k)(k/k_{\rm heq})^4, & k \lesssim k_{\rm eq} \ , \end{cases} \tag{5.1}$$

if $\Omega h^2 \lesssim 0.05$. Here $z_{\rm rec} \simeq 1300$ and $k_{\rm eq} \simeq 2\pi/[10(\Omega h^2)^{-1}$ Mpc] is the comoving wave number corresponding to the horizon scale at the matter-radiation equal time, where h is the present Hubble parameter normalized by 100 km s^{-1}Mpc^{-1}.

As is seen in the previous section, there are essentially two free parameters for each choice of n, namely, B_*, which determines the magnitude of density fluctuations, and k_c, which determines the spectral shape on scales relevant to galaxy formation. We are now in a position to determine them from astrophysical requirements.

Astrophysical aspects of the minimal isocurvature scenario is a flat Universe with a nonvanishing cosmological constant have been extensively discussed by Yokoyama and Suto [11]. Making use of their results, we specify values of model parameters. We take a model with $\Omega=\Omega_b=0.1$ and $h=0.5$ as an example, although various dynamical estimates typically gives a larger value [37] and standard nucleosynthesis predicts somewhat smaller value [38]. Note, however, that the latter critically depends on the abundance of primordial lithium. We also assume the existence of a positive cosmological constant to render the Universe spatially flat as predicted from inflation. For convenience, however, we leave the h dependence in formulas below.

In order to meet the constraints imposed by the large-scale isotropy of CMB [39], the effective power-law index should satisfy $p_{\rm eff} \gtrsim -1.5$ on a present scale $l_0 \simeq 25h^{-1}$ Mpc [11]. Although this constraint has been derived with a particular normalization scheme of density fluctuations, namely the so-called J_3 normalization; $J_3(r=25h^{-1}\ {\rm Mpc})=780h^{-3}\ {\rm Mpc}^3$ [40], a different normalization scheme by mass irregularity, $\langle(\delta M/M)^2\rangle \simeq 1$ on scale $r=8h^{-1}$ Mpc, changes the resultant amplitude of CMB anisotropies by no more than 30% [11]. Hence for simplicity we adopt this latter scheme to normalize the fluctuation amplitude, which reads

$$\left\langle \left[\frac{\delta M}{M}\right]^2 \right\rangle_{r=8h^{-1}\ {\rm Mpc}} \simeq \frac{k_{nl}^3}{(2\pi)^3} P(k_{nl},z_{\rm rec}) \left[\frac{D_0}{D_{\rm rec}}\right]^2 = 1 \ . \tag{5.2}$$

Here $2\pi k_{nl}^{-1}$ is the comoving length scale corresponding to $8h^{-1}$ Mpc today, which is smaller than $2\pi k_{\rm eq}^{-1}$, and $D_0/D_{\rm rec}$ is the linear growth factor from the recombination era up to present. This growth factor depends very much on the thermal history of the post-recombination era.

After recombination at $z_{\rm rec} \simeq 1300$, density fluctuations on scales larger than the Jeans mass scale $M_J \simeq 2\times 10^6\ h^{-1} M_\odot$, which corresponds to $2\pi k_J^{-1} \simeq 50 h^{-1}$ kpc today, start to grow in proportion to the scale factor. By the time the root mean square of fluctuations on the Jeans scale reaches $\frac{1}{2}$, typical peaks become nonlinear and star formation presumably takes place to reionize the medium again [6]. Hence the redshift at this epoch, $z_{\rm ion}$, is given from the equality

$$\frac{k_J^3}{(2\pi)^3} P(k_J,z_{\rm rec}) \left[\frac{1+z_{\rm rec}}{1+z_{\rm ion}}\right]^2 \simeq \frac{1}{4} \ . \tag{5.3}$$

Evolution of fluctuations is different depending on whether $z_{\rm ion} \gtrsim z_{cd} \equiv 130h^{2/3} \simeq 100$ or not, where $z=z_{cd}$ is the epoch after which the Compton drag force is no longer effective [41]. The linear growth factor from $z=z_{cd}$ to present has been calculated to be $\simeq 74h^{2/5} \simeq 56$ [11]. Thus we have

$$\frac{D_0}{D_{\rm rec}} \simeq 74h^{2/5} \frac{1+z_{\rm rec}}{1+z_{\rm ion}} \quad \text{for } z_{\rm ion} \gtrsim z_{cd} \ , \tag{5.4}$$

and

$$\frac{D_0}{D_{\rm rec}} \simeq 74h^{2/5} \frac{1+z_{\rm rec}}{1+z_{cd}} \simeq 7\times 10^2 \quad \text{for } z_{cd} \gtrsim z_{\rm ion} >> 1 \ . \tag{5.5}$$

Now let us show that values of model parameters specified in terms of the above normalization scheme lie in their plausible ranges from both astrophysics and particle-physics points of view.

(i) $n=10$. In this case, star formation may start fairly early and we may use Eq. (5.4) with large enough QB_*. From Eqs. (5.1), (5.2), and (5.4), we have $J(n,k_J)/J(n,k_{nl}) \simeq 56^2/4 \simeq 8\times 10^2$ with $k_J/k_{nl} \simeq 2\times 10^2$. Using these ratios we can specify the scales k_J and k_{nl} from Fig. 2 to find $k_J \simeq 1\times 10^{-3} k_c$ and $k_{nl} \simeq 6\times 10^{-6} k_c$ independent of QB_*. Then the magnitude of the fluctuation on the Jeans scale at recombination turns out to be about $0.15(QB_*/QB_*^{\max})$, where $QB_*^{\max}$ is the upper bound of QB_* imposed by Eq. (4.18). This in turn means $z_{\rm ion} \simeq 10^3(QB_*/QB_*^{\max})$ from Eq. (5.3). Thus in this case the Universe may be reionized soon after the recombination as in Peeble's original scenario [3]. Using the definition of k_c (3.27) and Eq. (4.4), we find

$$\beta_f \simeq 1\times 10^{-4} H_{f10}^{-1/15} T_{R7}^{-1/15} \ , \tag{5.6}$$

hence

$$f \simeq 1\times 10^{11} H_{f10}\ {\rm GeV} \lesssim 2\times 10^{12}\ {\rm GeV} \ , \tag{5.7}$$

from Eq. (4.8). Since $e^{n\beta_f}=1.0$, we have $QB_*^{\max} \simeq 30$, so $z_{\rm ion} \gtrsim z_{cd}$ for $QB_* \gtrsim 3$. The above argument together with Eq. (4.9) yields

$$B_* \simeq (3\sim 30)\times 10^{-4} M_{\phi 11}^{-1/2} \gtrsim 2\times 10^{-5} \ . \tag{5.8}$$

If we take smaller values of B_*, on the other hand, k_J and k_{nl} must be specified by a different procedure, which we demonstrate for $n=20$ below.

(ii) $n=20$. In this case, star formation becomes possible only after $z=z_{cd}$ even if we take the maximum value $QB_*^{\max}$. In fact if one tried to determine k_J and k_{nl} through the above procedure, the amplitude of $J(20,k_J)$ would be found too small compared with the maximum value of $J(20,k)$. This is because the rapid cosmic expansion makes the increase of $p_{\rm eff}$ so slow as a function of length scale that the amplitude of fluctuations are suppressed too much on astrophysically interesting scales where $p_{\rm eff}$ should be large enough. Thus Eq. (5.5) applies and from Eq. (5.2) and Fig. 3, one can specify k_{nl} as a function of QB_*. For example, if we adopt $QB_*^{\rm max}$, we find $k_{nl} \simeq 8\times10^{-13}k_c$ and $k_J \simeq 1\times10^{-10}k_c$. The magnitude of Jeans-scale fluctuations at recombination turns out to be about 0.03, which yields $z_{\rm ion} \simeq 80$. Following the same steps as before, we find

$$\beta_f \simeq 8\times10^{-3}H_{f10}^{-1/30}T_{R7}^{-1/30} ,$$
$$f \simeq 1\times10^{10}H_{f10}\ {\rm GeV} \lesssim 8\times10^{12}\ {\rm GeV} , \qquad (5.9)$$
$$B_* \simeq 1\times10^{-4}M_{\phi 11}^{-1/2} \gtrsim 2\times10^{-4} .$$

For either of the above two cases, we find the effective power index $p_{\rm eff}$ on $25h^{-1}$ Mpc is larger than -1.5, which makes the anisotropy of CMB well below the observational upper bound, while large-scale coherence is maintained as shown in [11]. If one adopted a different normalization scheme of density fluctuations, values of various parameters should be determined by a different procedure and they might change by a factor of 2 or so accordingly. However, we believe the physical significance of the present model would hardly be affected, since our model allows a wide parameter space as discussed above.

VI. DISCUSSION

In concluding the present paper, let us consider the naturalness of model parameters of particle physics obtained above. Though the value of $z_{\rm ion}$ turns out to be very different in the two cases considered, we have got similar constraints on model parameters from astrophysical consideration.

First the value $f \sim 10^{12}$ GeV is quite appropriate to generate a Majorana mass of $\sim 10^{10}-10^{11}$ GeV. Next $B_* \sim 10^{-4}-10^{-5}$ is possible with natural magnitudes of coupling constants provided the value of M_S lies near its lower bound [20]. However, this condition on the Higgs-boson mass may be removed if we consider a scenario such that lepton-number fluctuations are first generated from Majorana leptons through a lower-order process without exchanging any Higgs particle S_3 and that they are converted to baryon-number fluctuations at the electroweak phase transition due to anomaly [42].

In order for the fluctuation in the Majoron field to survive until baryogenesis (or leptogenesis, if one prefers the above scenario), the temperature should not exceed $\simeq f$ so that the symmetry of S_0 remains broken in the post-inflationary Universe. One should note that an adequate baryon asymmetry can be generated through gravitational interaction even with a low reheating temperature $T_R \ll M_N$.

Finally let us consider cases with n other than 10 and 20. If we decrease n, the upper bound of the inflaton mass is lowered to avoid too-large adiabatic fluctuations. Then M_N should also be smaller so that N's are adequately produced in the reheating phase through gravitational interaction. This may cause, however, too much decrease in B_*. On the other hand, if we increase n, the span of intermediate scales between the almost scale-invariant region and the white-noise region in the power spectrum becomes larger and star formation may not start until too late an epoch as discussed in Sec. VI. Thus we conclude neither $n \ll 10$ nor $n \gg 20$ is appropriate.

In summary, we have derived a proper expression for the power spectrum of baryon-number fluctuations arising from decay of a heavy Majorana lepton in which CP violation is space dependent due to a spatially varying Majoron field. We have constructed a natural particle-physics model, which is implemented in a power-law inflation model with the power-law index around $n \simeq 10 \sim 20$, to provide an appropriate initial condition for the minimal isocurvature scenario of large-scale structure formation.

Note added in proof. After submitting this manuscript, we became aware that the Majorana lepton N should also have an intrinsic mass term in order to guarantee CP violation discussed here. Although a certain kind of symmetry is necessary to avoid harmful domain walls resulting from quantum corrections, other conclusions of the present paper should remain unchanged. We thank C. Hill for pointing this out.

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