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CP violation in the Z_4 model with four generations

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It is pointed out that the CP-violating phases in the Z_4 model with four generations are much more restricted than those in the standard Kobayashi-Maskawa model with four generations, which involve a large number of arbitrary parameters in the CP-violating sector.

I. INTRODUCTION

In the Glashow-Salam-Weinberg model of unified electroweak interactions the quark masses and the Kobayashi-Maskawa (KM) angles are essentially free parameters, which makes this model somewhat unaesthetic. If one imposes additional discrete symmetries, the quark mass matrices become more restrictive, leading to relations among the quark masses and and KM angles. One hopes that the origin of such additional symmetries in terms of some deep underlying physics may be discovered in the future.

The same state of affairs holds in the left-right-symmetric (LRS) extension of the standard model.[1] Z_4 symmetry has been imposed on this model by many authors to investigate the relationship between the masses and mixing angles.[2,3] In an earlier paper[4] some aspects of the Z_4 model with four generations of quarks and leptons were examined. The CP properties of this model, however, were not explored. We wish to point out that an attractive feature of this model is the economical description of CP violation since all CP phases in this model arise from only one parameter; the relative phase between the vacuum expectation values (VEV's) of two Higgs-boson fields, which can be fixed by fitting only one CP-violating observable (say, ϵ). In contrast, the usual four-generation KM model has three independent CP-violating phases.

As an illustration of the highly constrained nature of this model we compute both the left- and the right-handed KM matrices (K_L and K_R) by using a small number of phenomenological inputs and the popular small-phase approximation.[5] We have also updated the analysis of Ref. 1 in the following ways.

(i) In Ref. 1 the top-quark mass (m_t) was assumed to be in the range 30–50 GeV, following the then UA1 limits.[6] Since these limits were subsequently withdrawn, we relax the above restriction. However, we keep in mind the current lower bounds (in GeV) $m_t \geq 67$ (UA1[7]), 69 (UA2[7]), 76,[8(a)] and 90,[8(b)] [Collider Detector at Fermilab (CDF) assuming standard-model decays of t].

(ii) In Ref. 1 the limit[9] $m_{t'} \leq 300$ GeV was used, where t', b' are quarks belonging to the fourth generation. We have considered the updated limit $m_{t'} - m_{b'} \leq 200$ GeV.[10]

(iii) Recent analyses reveal that the limits on $m_{b'}$ depend on the details of the decay scenario although the dependence is rather mild.[11] A conservative lower limit from e^+e^- annihilation is $m_{b'} \geq 35$ GeV,[11] since, at Fermilab Tevatron energies, heavy-flavor production via QCD processes dominates and, hence, the above lower limit for m_t is likely to be applicable to $m_{b'}$ as well. It should, however, be borne in mind that these $\bar{p}p$ collider limits involve some theoretical uncertainties[12] and are not as clean as the corresponding limits from e^+e^- annihilation.

It turns out from our analysis that the parameter space of the model is drastically reduced due to the tighter bounds in (i) and (ii). In fact if one takes the CDF lower bound on m_t (and $m_{b'}$) very seriously, then the model is strongly disfavored, since a lower bound on $m_{b'}$ in this restrictive model leads to a lower theoretical bound on m_t which violates the lower bound quoted in (ii). A possible way out may be to give up the small-phase approximation, which in turn may drastically alter the relations among the quark masses. CP violation, however, will still be very economical, involving just one free parameter. The analysis in this scenario, however, becomes very complicated and we have not attempted it. If on the other hand the CDF lower bound is relaxed by 10–15 %, which may not be very unrealistic in view of the theoretical uncertainties mentioned above, then the model with the small phase approximation is still allowed. But it strongly predicts a b' quark in the mass range 75–80 GeV and a heavier t quark.

We plan the paper as follows. In Sec. II we present the formulas. The numerical results for K_L and K_R calculated in terms of a relatively small number of input parameters is presented in Sec. III. An order-of-magnitude estimate for ϵ'/ϵ and the electric dipole moment of the neutron (EDMN), which are in principle predictions of this model, are also presented in this section. Our conclusions are summarized in the last section.

II. THE BASIC FORMULAS

The quark mass matrices in this model are given by (in units $k \approx 173$ GeV, where k is the VEV which gives mass to the W boson):

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$$\hat{M}_u = \hat{g} + r\cos\alpha\hat{h} - ir\sin\alpha\,\hat{h}\ , \tag{1}$$

$$\hat{M}_d = \hat{h} + r\cos\alpha\,\hat{g} + ir\sin\alpha\,\hat{g}\ , \tag{2}$$

where $\hat{g}$ is a 4×4 diagonal matrix with real elements, $\hat{h}$ is a real symmetric matrix with zero diagonal elements, and $h_{13}=h_{24}=0$. $r=|k'/k|$ is the magnitude of ratio of the VEV's of the two neutral-Higgs-boson fields in this model and α is the relative phase between these VEV's.[4]

In the small phase approximation the problem is solved in two steps. First, the quark masses and the elements of the KM matrix (without *CP* phases) are obtained by neglecting the $r\sin\alpha$ terms in Eqs. (1) and (2). Using seven quark masses ($m_u - m_t$, $m_{b'}$) and the experimental informations on K_{ub} and K_{cb} as the inputs, parameters $\hat{g}$, $\hat{h}$, and $r\cos\alpha$, are determined. The remaining elements of the quark mixing matrix (including the Cabibbo angle) and $m_{t'}$ are predictions of this model. To make our calculations simple we treat the $r\cos\alpha$ term in $\hat{M}_u$ as perturbations and obtain the eigenvectors and eigenvalues of $\hat{M}_u$ by applying straightforward perturbation theory. In $\hat{M}_d$, however, the term $r\cos\alpha\,\hat{g}$ cannot be treated as a perturbation since it involves the heavy masses m_t and $m_{t'}$. Instead we determine the eigenvectors and eigenvalues by treating h_{23} and h_{24}, the parameters which control the mixing between heavy and light generations as perturbations. The relevant equations for the masses are (in units of k)

$$m_{i0}^u = g_{ii} + O(r^2\cos^2\alpha)\ , \tag{3}$$

where $i=u,c,t,t'$,

$$m_{d0}(m_{s0}) = g'_{22}/2 \mp \tfrac{1}{2}(g'^2_{22} + 4h_{12}^2)^{1/2}\ , \tag{4}$$

$$m_{b0}(m_{b'0}) = (g'_{33}+g'_{44})/2 \mp \tfrac{1}{2}[(g'_{44}-g'_{33})^2 + 4h_{34}^2]^{1/2}\ , \tag{5}$$

$$\sum_{i=1}^{4} g_{ii} = m_u + m_c + m_t + m_{t'} = (m_{d0} + m_{s0} + m_{b0} + m_{b'0})/r\cos\alpha \tag{6}$$

where $g'_{ii} = g_{ii} r\cos\alpha$. The subscript 0 indicates that these are not physical masses. We choose m_{d0} to be negative. A suitable chiral rotation of the d field is then called for. For m_{b0} we leave both the options $m_{b0}=\eta m_b$ with $\eta=\pm 1$ open. m_{s0} and $m_{b'0}$ are identified with the physical masses. Using seven quark masses m_u to m_t and m'_b as inputs the parameters g_{ii} ($i=1$ to 4), $r\cos\alpha, h_{12}$, and h_{34} can be determined. One obtains in particular

$$r\cos\alpha = (m_s - m_d)/m_c\ , \tag{7a}$$

$$h_{12} = (m_s m_d)^{1/2}/k\ , \tag{7b}$$

and

$$h_{34}^2 = (r^2\cos^2\alpha m_t m_{t'} - m_{b0} m_{b'0})/k^2\ , \tag{7c}$$

which justifies the treatment of the $r\cos\alpha\hat{h}$ term in Eq. (1) as perturbation. The other important relation that follows from Eqs. (3) and (7) is

$$[(m_c - m_s + m_d)/(m_s - m_d)](\eta m_b + m'_b) - (m_t - \eta m_b) = (m'_t - m'_b) \le 200\ \text{GeV} \tag{8}$$

where we have used the bound $m_{t'} - m_{b'}$ from Ref. 10. It follows from Eq. (8) that for each m'_b there is a corresponding lower bound on m_t. This bound is given in Table I. It is clear from Eq. (8) that, for $\eta=+1$, $(m_t)_{\min}$ is even larger, and we do not consider this possibility any further. In this model, therefore, the b' quark is predicted to be lighter than the t quark. It is also found that if the b'-quark mass is close to the CDF lower bound[8] ($m'_b \ge 90$ GeV) then $(m_t)_{\min}$ turns to be too large to be compatible with the bound in Ref. 10. However, if, in view of the uncertainties (mainly theoretical) in the collider bounds, $(m'_b)_{\min}$ is relaxed by 10–15 %, a reasonable result may still be obtained. This tight result, however, could be an artifact of the small phase approximation and perturbation theory. An alternative approach would be to give up these approximations in analyzing the mass matrices, which does not affect the economical description of *CP* violation. In practice, however, such an analysis will be extremely complicated and we do not attempt it here. For an illustration of the highly constrained nature of the *CP* violation in this model we continue to work with the small phase approximation and use m'_b reasonably close to its current lower bound.

The KM matrix without the *CP* phases is given by

$$(K_0)_{ij} = x_i^{uT} x_j^d\ , \tag{9}$$

where $i=u,c,t,t'$ and $j=d,s,b,b'$. x_i^u and x_j^d are the eigenvectors of $\hat{M}_u$ and $\hat{M}_d$ if the $r\sin\alpha$ terms in (1) and (2) are neglected.

The eigenvectors arising from $M_u^0 = \hat{g} + r\cos\alpha\,\hat{h}$ are given by

$$|x_i^u\rangle = |x_{i0}^u\rangle + \sum_{j\neq i} \frac{\langle x_{j0}^u|\hat{h}|x_{i0}^u\rangle}{m_{i0}^u - m_{j0}^u}|x_{j0}^u\rangle \tag{10}$$

where $(x_{i0}^u,)_j = \delta_{ij}$, $i=1,2,3,4, j$ being the row index of the column vector x_{i0}^u and we have absorbed multiplicative factors such as $r\cos\alpha$ and k into the definition of $\hat{h}$.

The eigenvectors of $m_d^0 = A + r\cos\alpha B$ are given by (multiplicative factors are absorbed in the definition of B)

$$|x_i^d\rangle = |x_{i0}^d\rangle + \sum_{i\neq j} \frac{\langle x_{j0}^d|B|x_{i0}^d\rangle}{m_{i0}^d - m_{j0}^d}|x_{j0}^d\rangle \tag{11a}$$

where

$$B = r\cos\alpha\, k \begin{bmatrix} g_{11} & 0 & 0 & h'_{14} \\ 0 & 0 & h'_{23} & 0 \\ 0 & h'_{23} & 0 & 0 \\ h'_{14} & 0 & 0 & 0 \end{bmatrix} \tag{11b}$$

and

TABLE I. The lower limit on the top-quark mass $(m_t)_{\min}$ as a function of other parameters of the model (see text for details). We use $m_d=0.01$ and $m_s=0.2$. (All masses are in GeV.)

$\eta=-1$		$\eta=+1$	
$m_{b'}$	$(m_t)_{\min}$	$m_{b'}$	$(m_t)_{\min}$
85	199.00	85	264.68
80	172.42	80	238.11
75	145.84	75	211.53

$$A=k\begin{bmatrix}0 & h_{12} & 0 & 0\\ h_{12} & g'_{22} & 0 & 0\\ 0 & 0 & g'_{33} & h_{34}\\ 0 & 0 & h_{34} & g'_{44}\end{bmatrix} \tag{11c}$$

and the unperturbed eigenvectors x^d_{i0} can be obtained in terms of already known parameters from the matrix A in a straightforward way. The parameters h_{14} and h_{23} can be determined from K_{cb} and K_{ub} using the equations

$$h'_{14}=\frac{b_1(K_0)_{cb}-b_2(K_0)_{ub}}{a_1b_2+a_2b_1}\ , \tag{12a}$$

$$h'_{23}=\frac{a_1(K_0)_{cb}+a_2(K_0)_{ub}}{a_1b_2+a_2b_1}\ , \tag{12b}$$

$$a_{1,2}=\mp r\cos\alpha\, k\left[\frac{(x^d_{d0})_1(x^d_{d0})_{1,2}}{m_{b0}-m_{d0}}+\frac{(x^d_{s0})_1(x^d_{s0})_{1,2}}{m_{b0}-m_{s0}}\right](x^d_{b0})_4\ , \tag{12c}$$

$$b_{1,2}=r\cos\alpha\, k\left[\frac{(x^d_{d0})_2(x^d_{d0})_{1,2}}{m_{b0}-m_{d0}}+\frac{(x^d_{s0})_2(x^d_{s0})_{1,2}}{m_{b0}-m_{s0}}\right](x^d_{b0})_3\ , \tag{12d}$$

Equations (3), (7a)–(7c), (12a), and (12b) complete the phenomenological determination of all the unknown parameters in the mass matrices except $r\sin\alpha$. For numerical computations we use $m_u=0.004$, $m_d=0.01$, $m_s=0.2$, $m_c=1.2$, $m_b=5.2$ (all in GeV). $|K_{ub}|=0.006$, $|K_{cb}|=0.05$, $m_{b'}$ and m_t have been varied (see Tables I–III). If the $r\sin\alpha$ terms are retained, the matrices in (1) and (2) are diagonalized by biunitary transformations U_L, U_R and D_L, D_R, respectively, with $U_L=U_R^*$ and $D_L=D_R^*$ (since M_u and M_d are symmetric). The eigenvalue condition with $\lambda=r\sin\alpha\neq 0$ is given by

$$M_{ij}(Y_k^*)_j=m_k(Y_k)_i\ , \tag{13}$$

where the orthonormal vectors Y_k, $k=1,2,3,4$ span the columns of the the unitary matrix $U_L=U_R^*$. One can show that the physical masses change negligibly due to a nonzero λ. The vectors Y_k for both the up and down sectors are determined by straightforward perturbation theory. We obtain, up to first order in λ,

$$Y_i(\lambda)=\eta_i x_i+\lambda\sum_j C_i^j x_j\ , \tag{14}$$

where x_i's, $i=1,2,3,4$, are the eigenvectors [already used in Eqs. (10) and (11a)] in the limit $\lambda=0$ and C_i^j's are calculable coefficients. The phase factors η_i reflect the arbitrariness in the choice of the phase of the unperturbed

TABLE II. The predicted Kobayashi-Maskawa matrix for three different choices of the model (see the text for details). (All masses are in GeV.)

(a) $m_b=-5.2$, $m_{b'}=85$, $m_t=200$, $m_{t'}=304$, $\lambda=-0.0088$

	d	s	b	b'
u	0.977, 0.0	0.213, 0.0	0.63×10^{-2}, 0.0	0.51×10^{-3}, 0.0
c	−0.213, 0.0	0.977, -0.15×10^{-4}	0.056, 0.49×10^{-2}	-0.28×10^{-2}, -0.17×10^{-3}
t	0.41×10^{-2}, 0.0	−0.044, -0.24×10^{-2}	0.81, 0.10	0.59, −0.20
t'	-0.43×10^{-2}, 0.0	0.035, 0.46×10^{-2}	−0.59, −0.14	0.81, −0.19

(b) $m_b=-5.2$, $m_{b'}=80.01$, $m_t=173$, $m_{t'}=299.484$, $\lambda=-0.0103$

	d	s	b	b'
u	0.977, 0.0	0.213, 0.0	0.63×10^{-2}, 0.0	0.58×10^{-3}, 0.0
c	−0.213, 0.0	0.977, -0.83×10^{-5}	0.057, 0.53×10^{-2}	-0.28×10^{-2}, -0.18×10^{-3}
t	0.41×10^{-2}, 0.0	−0.045, -0.26×10^{-2}	0.82, 0.11	0.58, −0.21
t'	-0.43×10^{-2}, 0.0	0.035, 0.50×10^{-2}	−0.58, −0.15	0.82, −0.19

(c) $m_b=-5.2$, $m_{b'}=75$, $m_t=146$, $m_{t'}=294.842$ $\lambda=-0.0125$

	d	s	b	b'
u	0.977, 0.0	0.213, 0.0	0.63×10^{-2}, 0.0	0.65×10^{-3}, 0.0
c	−0.213, 0.0	0.977, 0.33×10^{-5}	0.057, 0.58×10^{-2}	-0.27×10^{-2}, -0.19×10^{-3}
t	0.42×10^{-2}, 0.0	−0.045, -0.29×10^{-2}	0.83, 0.12	0.56, −0.22
t'	-0.43×10^{-2}, 0.0	0.034, 0.55×10^{-2}	−0.56, −0.17	0.83, −0.21

TABLE III. The predicted right-handed Kobayashi-Maskawa matrix for three different choices of the parameters of the model (see text for details). (All masses are in GeV.)

(a)
$m_b=-5.2$, $m_{b'}=85$, $m_t=200$, $m_{t'}=304$, $\lambda=-0.23\times10^{-3}$

	d	s	b	b'
u	-0.977, 0.18×10^{-2}	0.213, -0.31×10^{-3}	-0.63×10^{-2}, -0.77×10^{-4}	0.51×10^{-3}, -0.51×10^{-6}
c	0.213, 0.45×10^{-3}	0.977, 0.24×10^{-2}	-0.057, -0.78×10^{-3}	-0.28×10^{-2}, -0.38×10^{-5}
t	-0.41×10^{-2}, 0.77×10^{-4}	-0.044, 0.87×10^{-3}	-0.81, 0.63×10^{-2}	0.59, -0.51×10^{-2}
t'	0.43×10^{-2}, -0.51×10^{-4}	0.035, -0.53×10^{-3}	0.59, -0.24×10^{-2}	0.81, -0.41×10^{-2}

(b)
$m_b=-5.2$, $m_{b'}=80.01$, $m_t=173$, $m_{t'}=299.484$, $\lambda=-0.257\times10^{-3}$

	d	s	b	b'
u	-0.977, 0.20×10^{-2}	0.213, -0.35×10^{-3}	-0.63×10^{-2}, -0.77×10^{-4}	0.58×10^{-3}, -0.68×10^{-6}
c	0.213, 0.49×10^{-3}	0.977, 0.26×10^{-2}	-0.057, -0.80×10^{-3}	-0.28×10^{-2}, -0.43×10^{-5}
t	-0.41×10^{-2}, 0.78×10^{-4}	-0.045, 0.89×10^{-3}	-0.82, 0.66×10^{-2}	0.58, -0.52×10^{-2}
t'	0.43×10^{-2}, -0.50×10^{-4}	0.035, -0.51×10^{-3}	0.58, -0.22×10^{-2}	0.82, -0.39×10^{-2}

(c)
$m_b=-5.2$, $m_{b'}=75$, $m_t=146$, $m_{t'}=294.842$, $\lambda=-0.288\times10^{-3}$

	d	s	b	b'
u	-0.977, 0.22×10^{-2}	0.213, -0.40×10^{-3}	-0.63×10^{-2}, -0.76×10^{-4}	0.65×10^{-3}, -0.91×10^{-6}
c	0.213, 0.53×10^{-3}	0.977, 0.28×10^{-2}	-0.057, -0.82×10^{-3}	-0.28×10^{-2}, -0.48×10^{-5}
t	-0.42×10^{-2}, 0.79×10^{-4}	-0.046, 0.91×10^{-3}	-0.83, 0.67×10^{-2}	0.56, -0.50×10^{-2}
t'	0.43×10^{-2}, -0.47×10^{-4}	0.034, -0.49×10^{-3}	0.56, -0.20×10^{-2}	0.83, -0.36×10^{-2}

eigenvectors. It can be checked that the choice $\eta_i^{*2}=-1$ is needed to generate a physical mass if m_{i0} turns out to be negative. The KM matrix to first order in λ is given by

$$(K_L)_{ij}=(K_0)_{ij}+i(K_1)_{ij} , \tag{15}$$

where K_0 is given in Eq. (9) and the matrix $(K_1)_{ij}$ is given by perturbation theory:

$$(K_1)_{ij}=\lambda\left[\sum_{l=1}^{4}\frac{\langle x_l^u|\hat{h}|x_i^u\rangle}{m_{i0}^u+m_{l0}^u}(K_0)_{lj}+\sum_{l=1}^{4}\frac{\langle x_l^d|\hat{g}|x_j^d\rangle}{m_{l0}^d+m_{j0}^d}(K_0)_{il}\right] . \tag{16}$$

In terms of the phenomenological inputs K_0 and K_1 can be now determined. The resulting analytical expressions for K_0 and K_1 are, however, very complicated. We have obtained K_L numerically from Eqs. (9), (15), and (16) by using Eqs. (3)–(7) and (10)–(14). The small phase approximation allows one to approximate Eq. (15) as

$$(K_L)_{ij}\approx(K_0)_{ij}e^{i\beta_{ij}} , \tag{17}$$

where $\beta_{ij}=(K_1)_{ij}/(K_0)_{ij}$. We now fix our phase convention by redefining the phases of the left-handed u- and d-quark fields once more to reduce Eq. (17) to the form used in the original standard form given by Kobayashi-Maskawa with the first row and the first column containing real elements only. The corresponding transformations must be given on the right-handed quark fields to keep the masses real. The right-handed KM matrix in this phase convention is then given by

$$K_R=P^2D_u^2K_L^*D_d^2P'^2 , \tag{18}$$

where D_u^2 (D_d^2) are diagonal matrices with diagonal elements equal to $+1$ (±1). A negative element in D_d results if the mass eigenvalue for the corresponding quark [m_{d0} or m_{b0} in Eqs. (4) and (5)] turns out to be negative. P^2 and P'^2 are diagonal phase matrices given by

$$(P^2)_{jk}=\delta_{jk}\exp(-2i\beta_{jd}) , \tag{19a}$$

where $j=u,c,t,t'$ and

$$P'^2=\text{diag}[1,\ \exp[-2i(\beta_{us}-\beta_{ud})],\ \exp[-2i(\beta_{ub}-\beta_{ud})],\ \exp[-2i(\beta_{ub'}-\beta_{ud})]] , \tag{19b}$$

where β_{ij}'s are defined in Eq. (17). We are now in a position to determine the only remaining parameter $\lambda=r\sin\alpha$ by fitting the parameter for the neutral kaon system.

III. RESULTS AND DISCUSSIONS

We begin with the scenario with a heavy right-handed mass scale. The low-energy phenomenology in this scenario is essentially governed by left-handed currents. For the K_L-K_S mass difference (Δm_K) and the CP-violation parameter (ϵ) we use the standard box-diagram formulas available in the literature. Since QCD corrections to the box-diagram formulas are not known for quark masses larger than m_W, we do not include them in our analysis. However, since these corrections are usually of order 1, we hope this will not affect our conclusions qualitatively.

We represent our results for several choices of the parameters in Table II. It is clear from the table that the coupling between the third and the fourth generation is quite large in this model.

For a relatively low right-handed mass scale the right-handed KM matrix (K_R) plays an important role. In this case, the CP-violating phases in K_L turn out to be too small. In Table III we have presented K_R for $M_R=2$ TeV (Ref. 13) and $M_H=3$ TeV (Ref. 13) where M_R and M_H are the masses of the right-handed gauge boson and the additional nonstandard Higgs boson.

Since there is only one CP violating phase in this model which can be uniquely fixed from the ϵ parameter as shown above, one can predict, in principle, other CP violating quantities such as ϵ'/ϵ and the electric dipole moment of the neutron. It should be emphasized that the four-generation standard model with three arbitrary CP-violating phases does not have such predictive power. In practice, however, such predictions cannot be immediately tested since the present experimental results for these quantities are not sufficiently accurate. Moreover, there are large theoretical uncertainties in these predictions involving strong-interaction effects. Nevertheless for the purpose of illustration we present some crude estimates.

The preliminary result from the Fermilab E731 experiment[14] gives $\epsilon'/\epsilon=(-0.5\pm1.5)\times10^{-3}$, which is significantly different from the CERN NA31 result:[15] $\epsilon'/\epsilon=(3.3\pm1.1)\times10^{-3}$. Using the relevant formulas from Ref. 16, the KM matrices in Table II and assuming that contributions from W_R-induced diagrams are suppressed, we have estimated $\epsilon'/\epsilon=-0.76\times10^{-3}$, -0.91×10^{-3}, -0.12×10^{-2} from Tables II(a), II(b), and II(c), respectively. For a relatively low right-handed mass scale the dominant contributions to ϵ'/ϵ through W_L-W_R mixing can be estimated by using the formulas from Ref. 5 and Table III. We obtain $\epsilon'/\epsilon=0.39\times10^{-3}$, 0.41×10^{-3}, 0.42×10^{-3} from Tables III(a), III(b), III(c), respectively.

The published upper limit on the electric dipole moment of the neutron (d_n) is $\sim10^{-25}$ e cm (Ref. 17). The contribution of the standard model sector to d_n is known to be negligible compared to the above limit. Using the formulas from Ref. 18, we obtain the following estimates (in e cm) from table III: $d_n=0.15\times10^{-28}$ [III(a)], 0.16×10^{-28} [III(b)], and 0.19×10^{-28} [III(c)].

The above results are not in conflict with the data provided proper allowances are given for theoretical and experimental uncertainties.

Note added. After this work was completed the first round of data on the Z width from CERN LEP (unpublished) was available, which disfavors the existence of a fourth-generation model with a light neutrino. These results are now published and well known.[19] All four-generation models with the somewhat unnatural choice of heavy neutrinos ($m_{\nu4}\geq M_{Z/2}$) appear to be the only viable ones. It should, however, be emphasized that the mixing of the Z boson with an additional neutral gauge boson and/or the mixing between ordinary and exotic fermions may reduce the standard-model prediction for the Z width significantly, creating thereby room for additional light neutrinos.[20] In particular, as shown by Bhattacharyya *et al.*[20] the mixing between ordinary neutrinos and exotic neutral leptons reduces the Z width sufficiently to accommodate one more light-neutrino species. It is seen from the above discussions that modifications mainly restricted to the leptonic sector can accommodate four light neutrinos. How this affects the phenomenology of leptonic and semileptonic processes in a Z_4 model is a subject in its own right and we do not attempt to present it here. Some authors have also suggested models with naturally heavy neutrinos.[21] In such models due to the seesaw mechanism and appropriate choices of the Dirac and Majorana masses of the neutrinos, each neutrino mass turns out to be $O(m^2_{\text{lepton}}/M)$, where m_{lepton} is the mass of the charged-lepton partner and M is the weak scale. Obviously for a heavy fourth-generation charged-lepton mass $\approx M$, $m_{\nu4}\approx M$ and ν_4 does not contribute to the Z width.

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