Models of extended Pati-Salam gauge symmetry

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The possibility of constructing nonminimal models of the Pati-Salam type is investigated. The most interesting examples are found to have an $SU(6)\otimes SU(2)_L\otimes SU(2)_R$ gauge invariance. Two interesting symmetry-breaking patterns are analyzed: one leading to the theory of SU(5) color at an intermediate scale, the other to the quark-lepton symmetric model. In both cases there is ample room for new physics in the interesting energy range of 100 GeV to a few TeV. We also identify a new candidate for dark matter.

There have been a number of hypotheses concerning the unification of quarks and leptons. The earliest and one of the simplest of these is due to Pati and Salam [1]. They start with gauge group $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ and the following structure for one fermion generation:

$$F_L \sim (4,2,1), \quad F_R \sim (4,1,2) . \tag{1}$$

A discrete left-right Z_2 symmetry is also usually assumed. The Pati-Salam model therefore unifies quarks with leptons and left-handed fermions with right-handed fermions. One motivation for this type of model is that it has a simpler fermion spectrum than the standard model (SM). Another is that it provides a solution for the charge quantization problem of the SM, although the simplest solution of this problem is the introduction of neutrino masses [2].

The simplest model of Pati-Salam type has symmetry breaking as follows:

$$\begin{gathered} SU(4)\otimes SU(2)_L\otimes SU(2)_R \\ \downarrow M_1 \\ SU(3)_c\otimes SU(2)_L\otimes U(1)_Y \\ \downarrow M_W \\ SU(3)_c\otimes U(1)_Q . \end{gathered} \tag{2}$$

While it has some theoretical appeal, the model in its simplest form does not allow for much experimentally testable new physics. For example, if one calculates the Pati-Salam-symmetry-breaking scale M_1, using the experimental values for the three SM gauge-coupling constants at the W scale, then one finds a high value of about 10^{15} GeV. This value for M_1 is certainly consistent, since this scale implies very small values for the light-neutrino masses (via the seesaw mechanism), which is well within experimental bounds and also the more stringent limits from cosmology [3]. However, this scale also implies that $\Delta B=2$ processes such as $N-\bar{N}$ oscillations and other nonstandard-model phenomena are too small to be observable experimentally.

An interesting question, therefore, is whether the minimal Pati-Salam model can be extended in such a way as to incorporate new physics in the TeV region. There are two broad ways in which one might modify the Pati-Salam theory. One way is to change the symmetry-breaking sector [4]. Alternatively, one may investigate enlarging the gauge group. The latter possibility is entertained in this article.

In what way can we extend the gauge group of the Pati-Salam model? A priori, we can either extend the "weak" part [i.e., $SU(2)_L\otimes SU(2)_R$], or the "strong" part [i.e., SU(4)]. For example, we might consider a model with gauge group $SU(4)\otimes SU(3)_L\otimes SU(3)_R$. However, such a model will suffer from gauge anomalies [assuming the naive extension of the fermion content of Eq. (1) i.e., $F_L\sim(4,3,1)$, $F_R\sim(4,1,3)$] and therefore require extra unspecified fermions. This unfortunately makes the model less predictive and less interesting. A more interesting possibility is to extend the "strong" part. The simplest extension of this kind is a model with gauge group G_5, where

$$G_5 = SU(5)\otimes SU(2)_L\otimes SU(2)_R , \tag{3}$$

and the fermion spectrum

$$F_L\sim(5,2,1), \quad F_R\sim(5,1,2) . \tag{4}$$

This model is free of gauge anomalies [5]. However, if the model breaks down to the SM at some scale, then irrespective of the symmetry-breaking pattern, the fermions with exotic color will contain charge-$\frac{1}{2}$ states. To see this we note that the exotic colored fermions will transform under the SM gauge group G_{SM} [where $G_{SM}=SU(3)\otimes SU(2)_L\otimes U(1)_Y$] as $(1,2,y_1)$, $(1,1,y_2)$ $(1,1,y_3)$, where $y_{1,2,3}$ are general hypercharges. Since the usual fermions are anomaly-free, the exotic fermions must also be anomaly-free. This in turn implies that $y_1=0$ and $y_2=-y_3$. Since electric charge is given by $Q=I_3+Y/2$, we see that the $(1,2,y_1)$ multiplet (with $y_1=0$) contains $Q=\pm\frac{1}{2}$ fermions [6]. The prediction of charge-$\pm\frac{1}{2}$ fermions appears to be a problem experimentally since from electric charge conservation they should be absolutely stable, and no stable charge-$\pm\frac{1}{2}$ particles have been discovered. Even if they are very heavy, one might expect significant terrestrial abundances of these unusual fermions on cosmological grounds, since their annihilation cross section will decrease with increasing mass. This perhaps should not be regarded as a completely rigorous argument, since many aspects of the standard cosmological picture (such as the existence of dark matter) have not been tested and thus standard cosmology may still be wrong.

However, one version of such a theory can also be ruled out from collider experiments. The most phenomenologically interesting symmetry-breaking pattern of the $SU(5)\otimes SU(2)_L\otimes SU(2)_R$ model features an intermediate theory of SU(4) color [7]. However, for three-fermion generations this model predicts a light charge-$\pm\frac{1}{2}$ fermion (mass less than 21 GeV) [8], which is ruled out from collider experiments.

We now move on to the next simplest extension, which has gauge group G_6 where $G_6=SU(6)\otimes SU(2)_L\otimes SU(2)_R$. The fermions are

$$F_L\sim(6,2,1),\quad F_R\sim(6,1,2)\ . \tag{5}$$

The most phenomenologically interesting symmetry-breaking pattern down to $G_{SM}\otimes SU(2)'$ is

$$\begin{gathered} SU(6)\otimes SU(2)_L\otimes SU(2)_R \\ \downarrow M_1 \\ SU(5)\otimes SU(2)_L\otimes U(1)_{Y'} \\ \downarrow M_2 \\ SU(3)_c\otimes SU(2)'\otimes SU(2)_L\otimes U(1)_Y\ . \end{gathered} \tag{6}$$

The symmetry-breaking scale M_1 is associated with the Higgs fields $H_{L,R}$, which may be defined through the Yukawa Lagrangian

$$L_{\rm Yuk}=\lambda_1\bar{F}_L H_L (F_L)^c+\lambda_1\bar{F}_R H_R (F_R)^c+\text{H.c.}\ , \tag{7}$$

and transform as follows:

$$H_L\sim(21,3,1),\quad H_R\sim(21,1,3)\ . \tag{8}$$

H_L and H_R interchange under left-right symmetry, and so there is only one independent Yukawa coupling constant. These Higgs fields, which may be represented by symmetric matrices, are assumed to gain the vacuum expectation values (VEV's)

$$\langle H_L\rangle=0,\quad \langle H_R\rangle=v\begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&1\end{bmatrix}\otimes\begin{bmatrix}1&0\\0&0\end{bmatrix}\ . \tag{9}$$

This induces the spontaneous symmetry breaking at scale M_1 displayed above. Note that left-right symmetry is broken together with SU(6) at this scale. Thus the right-handed neutrinos gain Majorana masses of order M_1. The equality of the Pati-Salam and left-right-symmetry-breaking scales is inevitable when one employs the minimal Higgs sector. The U(1) symmetry generator Y' annihilates the H_R VEV and is given by

$$Y'=\tfrac{1}{5}T_{51}+2I_{3R}\ , \tag{10}$$

where $T_{51}=\text{diag}(1,1,1,1,1,-5)$ in SU(6) space and $I_{3R}=\text{diag}(\frac{1}{2},-\frac{1}{2})$ in $SU(2)_R$ space. The scale M_2 is generated by a VEV for the Higgs field

$$\chi\sim(15,1,1)\ , \tag{11}$$

which may be defined by the Yukawa Lagrangian term

$$L_{\rm Yuk}=\lambda_2\bar{F}_L\chi(F_L)^c+\lambda_2\bar{F}_R\chi(F_R)^c+\text{H.c.} \tag{12}$$

The Higgs field χ may be represented by an antisymmetric matrix and is assumed to gain the VEV

$$<\chi>=\begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&w&0\\0&0&0&-w&0&0\\0&0&0&0&0&0\end{bmatrix}\ . \tag{13}$$

The generator Y of standard hypercharge is given by

$$Y=Y'+\tfrac{1}{15}T_{32}\ , \tag{14}$$

where $T_{32}=$ diag $(2,2,2,-3,-3)$ in SU(5) space. As a consequence of the nonzero VEV of χ, the exotic fermionic degrees of freedom, which come from the fourth and fifth color (and which we call "quirks" [9]), gain masses from this Yukawa Lagrangian.

Finally, as in the minimal Pati-Salam model, electroweak breaking results from a biodoublet $\phi\sim$ (1,2,2). At this point we make some observations.

(1) The models we construct in this paper have the usual serious problems characteristic of grand unified theories (GUT's). These serious problems include both the gauge hierarchy problem and, when the minimal Higgs sector is used, phenomenologically unsuccessful fermion mass relations. We will not in this paper pursue solutions to these well-known problems of GUT's, except to point out that the correct fermion masses can be obtained at the expense of predictivity by enlarging the Higgs sector.

(2) Below the SU(6)-breaking scale M_1, the model reduces to the $SU(5)_c$ model [10]. As we will soon demonstrate, $SU(5)_c\otimes SU(2)_L\otimes U(1)_{Y'}$ may break down to the $G_{SM}\otimes SU(2)'$ symmetry at energies not far above the weak scale (i.e., the scale M_2). The existence of this intermediate scale makes possible an abundance of new physics occurring in the soon-to-be-probed energy region below a few TeV.

(3) The exotic fermionic degrees of freedom have the following $SU(2)'\otimes G_{SM}$ transformation properties:

$$Q_L\sim(2,1,2,0)\ ,\quad U_R\sim(2,1,1,1)\ ,\quad D_R\sim(2,1,1,-1)\ . \tag{15}$$

(We denote the corresponding standard quarks by lowercase letters.) These fermions have electric charges equal to $\pm\frac{1}{2}$, and are expected to be confined by the unbroken and asymptotically free SU(2)′ force. One can easily see that this means the model contains no absolutely stable bound states of the quirks [10].

(4) Unlike the case of the $SU(4)_c$ model at the intermediate scale, this model has no phenomenological problems associated with light-mass fermions (the difference is that the χ field is antisymmetric in color space) [8].

(5) The model contains leptoquark scalar bosons which can mediate rare processes such as $K_L \to \bar{\mu} e$. To get agreement with experiment, these bosons have to be heavy (greater than 30 TeV typically).

(6) As is well known, the origin of symmetry breaking in nature is untested. We use elementary Higgs bosons as *an example* of how nature might work.

Since the Lagrangian has $SU(6)\otimes SU(2)_L\otimes SU(2)_R$ gauge symmetry and there is a Z_2 discrete symmetry, there are only two fundamental gauge-coupling constants in the model. A consequence of this is that if we input the measured values of the SM gauge-coupling constants at the W-scale then we can get one relation for the symmetry-breaking scales.

Denote the two independent gauge-coupling constants of the model as g_3 and g_2. Between M_1 and M_2 there are three gauge-coupling constants g', g_2, and g_3. These coupling constants can be defined by the covariant derivative

$$D_\mu = \partial_\mu + i g_3 G^a_\mu \frac{\Lambda_a}{2} + i g_2 W^a_\mu \frac{\tau_a}{2} + i g' B'_\mu \frac{Y'}{2} , \quad (16)$$

where $\Lambda_a/2$ denote the generators of the SU(5) Lie algebra [normalized so that $\mathrm{Tr}\,(\Lambda_a\Lambda_b) = 2\delta_{ab}$], τ_a are the usual Pauli matrices and Y' is the $U(1)_{Y'}$ generator. Under the group $SU(5)_c\otimes SU(2)_L\otimes U(1)_{Y'}$, the fermions transform as

$$f_L \sim (1,2,-1) , \quad e_R \sim (1,1,-2) , \quad \nu_R \sim (1,1,0) ,$$
$$Q_L \sim (5,2,\tfrac{1}{5}) , \quad u_R \sim (5,1,\tfrac{6}{5}) , \quad d_R \sim (5,1,-\tfrac{4}{5}) . \quad (17)$$

By diagonalizing the gauge-boson mass matrix, one can obtain the relation

$$\frac{1}{\alpha'} = \frac{1}{\alpha_2} + \frac{3}{5\alpha_3} , \quad (18)$$

which holds at the scale M_1 [note that $\alpha' = (g')^2/4\pi$, etc.]. Between M_2 and M_W we have the standard fine-structure constants α_1, α_2, and α_3. One can show that α_1 is related to α' at M_2 by

$$\frac{1}{\alpha_1} = \frac{1}{\alpha'} + \frac{1}{15\alpha_3} . \quad (19)$$

Experimentally, we know the quantities α_1, α_2, and α_3 at M_W [11]:

$$\alpha_3(M_W) = 0.107^{+0.013}_{-0.008} ,$$
$$\alpha_2(M_W) = 0.0344 \pm 0.0007 , \quad (20)$$
$$\alpha_1(M_W) = 0.0101 \pm 0.0001 .$$

Using the renormalization-group evolution equations for the gauge-coupling constants together with Eqs. (18) and (19), we can derive a relation between the three scales M_1, M_2, and M_W. Evolution between M_W and M_2 is given to leading order in perturbation theory by the equations

$$\frac{1}{\alpha_{1,2,3}(M_2)} = \frac{1}{\alpha_{1,2,3}(M_W)} - \frac{b_{1,2,3}}{2\pi} \ln \frac{M_2}{M_W} . \quad (21)$$

The b functions $b_{1,2,3}$ are determined by the quantum numbers of contributing particles via the well-known formula

$$b = -\tfrac{11}{3} T(\text{gauge boson}) + \tfrac{2}{3} T(\text{Weyl fermion})$$
$$+ \tfrac{1}{3} T(\text{complex scalar}) , \quad (22)$$

where $T(R)$ for the representation R is given by $T(R)\delta_{ab} = \mathrm{Tr}(t_a t_b)$ (t_a being a representation matrix). Between M_2 and M_1 evolution is described by

$$\frac{1}{\alpha'(M_1)} = \frac{1}{\alpha'(M_2)} - \frac{b'_1}{2\pi} \ln \frac{M_1}{M_2} , \quad (23a)$$

$$\frac{1}{\alpha_{2,3}(M_1)} = \frac{1}{\alpha_{2,3}(M_2)} - \frac{b'_{2,3}}{2\pi} \ln \frac{M_1}{M_2} . \quad (23b)$$

Using the above, we obtain the relation

$$\zeta = \frac{10b_3 + 15b_2 - 15b_1}{2\pi} \ln \frac{M_2}{M_W} + $$
$$+ \frac{9b'_3 + 15b'_2 - 15b'_1}{2\pi} \ln \frac{M_1}{M_2} , \quad (24)$$

where

$$\zeta = -\frac{15}{\alpha_1(M_W)} + \frac{15}{\alpha_2(M_W)} + \frac{10}{\alpha_3(M_W)} . \quad (25)$$

If we neglect the (small) Higgs contribution to the b functions, then we have, for three generations,

$$b_1 = \tfrac{20}{3} , \quad b_2 = -\tfrac{22}{3} + 4 , \quad b_3 = -11 + 4 ,$$
$$b'_1 = \tfrac{42}{5} , \quad b'_2 = -\tfrac{22}{3} + 6 , \quad b'_3 = -\tfrac{55}{3} + 4 . \quad (26)$$

Note that we are assuming that the exotic fermions, the quirks, gain masses of about the symmetry-breaking scale M_2.

For the interesting case where $M_2 \sim M_W$, we find that

$$M_1 \approx 2\times 10^{11} \ \text{GeV}. \quad (27)$$

In fact, as M_2 varies from M_W to M_1, M_1 varies from 2×10^{11} GeV to 10^{15} GeV. Thus, like the minimal Pati-Salam model, the model is self-consistent, predicting very small $\Delta B = 2$ processes and tiny masses for the light Majorana neutrinos, both within experimental and cosmological bounds.

However, unlike the minimal Pati-Salam model, the $SU(6)\otimes SU(2)_L\otimes SU(2)_R$ model is compatible with the existence of new low-energy physics. This comes mainly in the form of exotic charge $\pm\frac{1}{2}$ fermions (quirks) with masses which may be of the order of the weak scale, which are confined by the unbroken-SU(2)′ gauge interactions. One can easily calculate that the confinement scale for these interactions is about equal to the QCD scale of 200 MeV. The quirks would therefore form heavy but nonrelativistic bound states. In addition, a second neutral gauge boson is predicted whose phenomenology within the SU(5)-color model has recently been studied [12].

We now turn to another interesting possibility arising from Pati-Salam $SU(6)\otimes SU(2)_L\otimes SU(2)_R$, namely, that it breaks first to the quark-lepton symmetric model [13] rather than to $SU(5)_c\otimes SU(2)_L\otimes U(1)_{Y'}$. This requires the adjoint Higgs field

$$\Phi\sim(35,1,1) \tag{28}$$

to be added to the model, together with $H_{L,R}$ and χ.

A nonzero VEV for Φ at scale M_1 performs the breaking

$$SU(6)\otimes SU(2)_L\otimes SU(2)_R \rightarrow SU(3)_l\otimes SU(3)_c\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_V\ , \tag{29}$$

under which the fermions transform as

$$F_L\sim(3,1,2,1)(-\tfrac{1}{3}),\quad F_R\sim(3,1,1,2)(-\tfrac{1}{3})\ ,$$
$$q_L\sim(1,3,2,1)(\tfrac{1}{3}),\quad q_R\sim(1,3,1,2)(\tfrac{1}{3})\ . \tag{30}$$

In addition to the left-right discrete symmetry, there is also a Z_2 discrete symmetry left over from SU(6) under which $F_{L,R}\leftrightarrow q_{L,R}$ and $C_\mu\leftrightarrow -C_\mu$, where C_μ is the $U(1)_V$ gauge field. This symmetry interchanges quarks with the fermions F. The multiplet F contains the standard leptons together with two exotic fermion partners (per Weyl lepton), which we call "liptons". The quark-lepton symmetric model thus defined was actually introduced as a novel way of unifying the properties of quarks with those of leptons. In this regard, it performs a similar task to orthodox Pati-Salam theory, but of course in a different manner. The embedding of discrete quark-lepton symmetry into Pati-Salam SU(6) thus represents the marriage of two alternative approaches to the unification of quarks with leptons. Pati-Salam SU(6) also has the theoretical significance of providing a connection between two hitherto disparate gauge models: color SU(5) and quark-lepton symmetric theories.

The surviving degrees of freedom from the Higgs fields [14] $H_{L,R}$ are $\Delta_{1L}\sim$ (6,1,3,1) plus its left-right and quark-lepton symmetric partners. The second stage of symmetry breaking proceeds via a nonzero VEV for Δ_{1L}. The residual gauge symmetry is now

$$SU(2)'\otimes SU(3)_c\otimes SU(2)_L\otimes U(1)_Y\otimes U(1)_X\ , \tag{31}$$

where Y, which is standard hypercharge, and X are given by

$$Y=2I_{3R}+V+\frac{T_{12}}{3} \tag{32a}$$

and

$$X=2I_{3R}+(3-\sqrt{3}\beta)V+\frac{\beta}{2\sqrt{3}}T_{12},\quad \beta\neq\frac{2}{\sqrt{3}}\ , \tag{32b}$$

where $T_{12}=\mathrm{diag}(1,1,-2)$. The charge X annihilates the VEV of Δ_{1L} for all values of β [15]. Note that left-right-symmetry breaking is decoupled from SU(6) breaking in this model. This is possible because we have the additional Higgs field Φ. Right-handed neutrinos pick up Majorana masses at this scale, but the liptons are still massless.

The third stage of symmetry breaking is induced by a nonzero VEV for the surviving field from the Higgs multiplet χ. This survivor is neutral under G_{SM}, but not under $U(1)_X$. The $U(1)_X$ is therefore broken and liptons acquire nonzero masses.

Finally, electroweak symmetry breaking is again performed by the remains of the $SU(2)_L\otimes SU(2)_R$ Higgs bidoublet. It is easy to see that the liptons are charge-$\pm\frac{1}{2}$ fermions confined by SU(2)′ gauge forces, as were the quirks.

To summarize, we have the symmetry-breaking pattern

$$\begin{gathered} SU(6)\otimes SU(2)_L\otimes SU(2)_R \\ \downarrow M_1 \\ SU(3)_l\otimes SU(3)_c\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_V \\ \downarrow M_2 \\ SU(2)'\otimes SU(3)_c\otimes SU(2)_L\otimes U(1)_Y\otimes U(1)_X \\ \downarrow M_3 \\ SU(2)'\otimes SU(3)_c\otimes SU(2)_L\otimes U(1)_Y \\ \downarrow M_W \\ SU(2)'\otimes SU(3)_c\otimes U(1)_Q\ . \end{gathered} \tag{33}$$

As for the $SU(5)_c$ scenario, a renormalization-group analysis can be performed to obtain one relationship between the SM coupling constants at M_W and the symmetry-breaking scales $M_{1,2,3}$. We denote the various b functions in this case as follows:

between M_W and M_3 : b_3,b_2, and b_Y for $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, respectively;

between M_3 and M_2 : b_3,b'_2, and b'_Y for $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$, respectively;

between M_2 and M_1 : b'_3 and b_V for $SU(3)_{l,c}$ and $U(1)_V$, respectively. (34)

Adopting the normalization that the coupling constant of $U(1)_V$ is $g_V/2$ [cf. normalization of g' in Eq. (16)], we obtain the relation

$$\xi=\frac{1}{6\pi}(3b_V-b'_3)\ln\frac{M_1}{M_W}+\frac{1}{6\pi}(b'_3+3b'_Y-2b_3-3b'_2-3b_V)\ln\frac{M_2}{M_W}+\frac{1}{2\pi}(b_Y-b'_Y+b'_2-b_2)\ln\frac{M_3}{M_W}\ , \tag{35}$$

where

$$\xi=\frac{1}{\alpha_1(M_W)}-\frac{1}{\alpha_2(M_W)}-\frac{2}{3\alpha_3(M_W)}\ . \tag{36}$$

By including the fermion contributions [17] to the b functions, one obtains

$$\xi=\frac{33}{18\pi}\ln\frac{M_1}{M_W}+\frac{99}{18\pi}\ln\frac{M_2}{M_W}\ . \tag{37}$$

Note that ξ does *not* depend on the scale M_3 (to this level of approximation). The quantum numbers of the liptons are such that the coefficient of $\ln(M_3/M_W)$ in Eq. (35) is zero.

This result immediately implies that the lipton masses, whose scale is set by M_3, can be light (in the 40 GeV–1 TeV range, say). It also means that the Z' boson associated with $U(1)_X$ can be relatively light, although it is expected to be heavier than about 700 GeV from neutral-current data [16]. This represents interesting nonstandard-model physics that may manifest itself at relatively low energies, just as in the previous model.

Equations (37) and (36) constrain *both* M_1 and M_2 to be large. Two representative cases are

$$\begin{aligned}&M_1=M_2\Longrightarrow M_1=10^{13}\ \text{GeV}\ ,\\&M_1=M_{\text{Planck}}=10^{19}\ \text{GeV}\Longrightarrow M_2=10^{12}\ \text{GeV}\ .\end{aligned} \tag{38}$$

This is once again self-consistent, because it implies tiny masses for the light neutrinos and unobservably small N-$\bar{N}$ oscillations [18]. In contrast with the SU(5)-color case, the gauge bosons in the coset space $SU(3)_l/SU(2)'$ will be heavy ($\sim M_2$).

An important difference between the present model and the $SU(5)_c$ case is that the unbroken asymptotically free gauge group SU(2)′ breaks off from its parent group [$SU(3)_l$ in this case] at the high scale of about 10^{12}–10^{13} GeV [from Eq. (38)]. [Remember that the SU(2)′ fine-structure constant α'_2 is equal to α_3 in both models at the breaking-off scale.] In the $SU(5)_c$ scenario, the SU(2)′ confinement radius (Λ') is about the same as the QCD scale, because of the numerical fact that the b functions for pure SU(2) gauge theory and for QCD with three quark-lepton generations are approximately equal. By contrast, in the present model the lipton fields remain massless between M_2 and M_3 and thus contribute to the b function for SU(2)′. They slow down the running of the α'_2 relative to α_3, thus causing SU(2)′ to confine at a much lower scale.

A straightforward calculation shows that

$$\Lambda'=M_W\exp\left[\frac{-3\pi}{11\alpha_3(M_W)}-\frac{1}{2}\ln\frac{M_2}{M_W}+\frac{6}{11}\ln\frac{M_3}{M_W}\right]\ . \tag{39}$$

For $M_2\approx10^{13}$ GeV, and for $M_3\approx M_W$, this yields

$$\Lambda'\approx\frac{1}{10}\ \text{keV}\ . \tag{40}$$

This is about 6 orders of magnitude lower than the QCD scale, and is of course much lower than the typical mass M_3 of the fermions which feel the SU(2)′ force, namely, the liptons. Lipton bound states are thus expected to be heavy ($\approx M_3$) and highly nonrelativistic. It also means that the SU(2)′ glueball will be the lightest state associated with this exotic color group.

Since the exotic glueballs are so light, one can easily show that they are almost stable. They will decay via loop diagrams to the ordinary fermions, with a lifetime which is longer than the age of the Universe. These glueballs may therefore be relevant to cosmology. In the standard cosmological picture (for reviews see [19]) it is easy to see that they will not affect the successful nucleosynthesis calculations. This is because the exotic gluons will decouple from the ordinary particles very early, i.e., when the temperature of the Universe cools to just below the lipton mass (see below). However, they may contribute to the missing mass of the Universe. Since the exotic gluons are relativistic when they decouple from the photons, it is straightforward to calculate their present abundance. To estimate the decoupling temperature of the exotic gluons, we note that the exotic gluons will be in equilibrium with the ordinary particles, provided that the number density of the liptons is large enough. The relevant reactions which maintain equilibrium are (i) the tree-level process between the liptons (L) and exotic gluons (g), $Lg\leftrightarrow Lg$, and (ii) $L\bar{L}\leftrightarrow e^+e^-$, $L\gamma\leftrightarrow L\gamma$, and so on. When the temperature drops just below the lipton mass, the liptons rapidly annihilate and consequently the exotic gluons drop out of equilibrium [20]. Since the decoupling temperature for the exotic gluons is high (greater than about 40 GeV), all three generations of quarks and leptons (except possibly the top quark) contribute to the heating of the photons relative to the exotic gluons. A simple calculation shows that the ratio of the photon temperature to the exotic gluon temperature just after the electrons and positrons annihilate is

$$\frac{T_\gamma}{T_g}=\left[\frac{4521}{172}\right]^{1/3}\ . \tag{41}$$

Calculating the present energy density of the glueballs, we find

$$\rho_g\approx3a\Lambda' t_g^3/\eta k=3a\Lambda'\left[\frac{172}{4521}\right]T_\gamma^3/\eta k\ , \tag{42}$$

where $a=8\pi^5k^4/(15h^3c^5)$ and $\eta=\pi^4/30\zeta(3)$. It should be noted that the calculations leading to Eqs. (41) and (42) have employed the usual simplified assumptions of cosmology. For example, we have neglected to consider phase transitions, two-body to three-body processes, and so on. Using the observation that the photon temperature (T_γ) is now about 2.7 K, we find that

$$\rho_g/\rho_c \approx 5.7\Lambda' \,(\mathrm{keV})/h_0^2 \,, \tag{43}$$

where $\rho_c = 3H_0^2/8\pi G$ is the critical density and h_0 is the measure of the uncertainty in the Hubble constant H_0 ($0.4 < h_0 < 1$). Thus we find that the glueballs may close the Universe [see Eq. (40)] and represent a good candidate for dark matter [21]. We find this to be an interesting result, because both the quark-lepton symmetric model and its SU(6) extension were motivated by other concerns. The dark-matter candidate emerges as a necessary by-product [22].

In conclusion, we have investigated the possibility that nature utilizes an extended gauge model of the Pati-Salam type. The model with gauge symmetry $SU(6)\otimes SU(2)_L\otimes SU(2)_R$ was shown to be the most interesting extended Pati-Salam model. We have shown that these models allow for observable new physics at currently available, or soon to be available, collider energies.

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[15] The most convenient choice for β is one which ensures a diagonal covariant derivative and diagonal kinetic-energy terms for the gauge fields of Y and X. For the purposes of this paper, we only need to know that there is a value for β for which this occurs, not its explicit expression. See Ref. [16] for a more detailed analysis.
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[17] Note that if surviving Higgs fields are included, then they do not change things much, because of various cancellations among them. If we include the Higgs contributions, then we would obtain Eq. (37) with $33 \to 29$ and $99 \to 103$.
[18] Note that there are leptoquark scalar bosons from the χ Higgs multiplet which mediate proton decay ($p \to \pi^0 e^+$). Thus these bosons have to be much heavier than the scale M_3. This is a similar problem to the minimal SU(5) GUT case, since the **5** of SU(5) contains a color-triplet leptoquark boson which mediates proton decay.
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[20] There are processes directly connecting exotic gluons with ordinary particles via loop Feynman diagrams involving virtual liptons. However, it is straightforward to show that the reaction rates for these processes are too slow to maintain equilibrium at temperatures below the lipton mass.
[21] The possibility that the missing mass of the Universe is composed of some type of exotic glueball has been previously considered in E. Carlson, S. L. Glashow, and U. Sarid (unpublished); S. Nussinov (unpublished); E. Carlson and L. Hall (unpublished), and possibly others.
[22] Note that one can turn this argument around, and use the cosmological requirement that there must not be too much mass in the Universe to place an upper bound of a few TeV on the lipton mass scale. Also note that the cosmological impact of the exotic glueballs in the $SU(5)_c$ model has been studied by E. Carlson *et al.*, Phys. Rev. D (to be published).