PHYSICAL REVIEW D VOLUME 44, NUMBER 3 1 AUGUST 1991

Left-right-symmetric electroweak models with triplet Higgs field

N.G. Deshpande
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403

J.F. Gunion
Department of Physics, University of California–Davis, Davis, California 95616

B. Kayser
National Science Foundation, Division of Physics, Washington, D.C. 20550

Fredrick Olness*
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403
(Received 02 April 1990; revised manuscript received 09 April 1991)

We examine the predictions of the conventional $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ left-right-symmetric model in the case where the minimal Higgs sector (containing one bidoublet, one L-triplet, and one R-triplet Higgs field) and the standard lepton representations (incorporating right-handed partners for the observed neutrinos) are adopted. We show that a complete analysis of spontaneous symmetry breaking for the Higgs sector leads to a highly restrictive range of possibilities for global minima that are simultaneously consistent with all experimental observations (such as lepton masses, K_L-K_S mixing, etc.). As a result, the possible phenomenologies for the gauge and Higgs bosons of the model are very limited. For instance, we demonstrate that in the absence of explicit CP violation in the Higgs potential, spontaneous CP violation does not arise and the fermion couplings exhibit "manifest" left-right symmetry. Further, we find no entirely natural solutions other than ones in which all of the extra (non-standard-model) gauge and Higgs bosons associated with the left-right-symmetric extension are extremely heavy (typically, more massive than 10^7 GeV). Only by "fine-tuning" certain parameters of the Higgs potential is it possible to bring these extra particles down to an observable mass scale. Alternatively, symmetries can be introduced to eliminate the terms in the Higgs potential associated with these parameters, but only at the sacrifice of introducing undesirable consequences for fermion masses. Many of the pitfalls and problems are illustrated using a simplified model. Overall, we emphasize the necessity of performing a complete minimization of the Higgs sector *before* extracting phenomenology.

I. OVERVIEW

Left-right symmetry for physics at very high energy is an extremely attractive possibility. In the conventional $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ left-right- (LR-)symmetric model [1], implementation of LR symmetry requires the introduction of right-handed partners for the observed gauge bosons and neutrinos and a Higgs sector containing at least one bidoublet, one right-handed triplet and one left-handed triplet Higgs field. Triplet representations for the latter Higgs fields are chosen so that they can couple to lepton-lepton channels, thereby allowing for the generation of neutrino masses via the "seesaw" mechanism. This minimal left-right-symmetric model has been analyzed extensively [2–5], and many constraints have been derived which restrict the character of the model (cf. Ref. [6–11]). In these papers, increasing attention has been focused on the Higgs sector and the role that spontaneous symmetry breaking plays in determining the possible phenomenological features of the model, in particular the observability of the extra gauge and Higgs bosons. Our paper is devoted to systematizing and extending these considerations.

We show that the simultaneous requirements of (1) a completely self-consistent global minimum for the Higgs sector and (2) consistency with current experimental observations for Z decays, K_L-K_S mixing, lepton masses, etc., are even more restrictive than previously realized. Let us state clearly at the outset that many of the individual constraints we impose have been previously investigated in the literature; the primary contribution of this work is to synthesize these separate pieces into a coherent framework to determine the manifestations of a realistic LR model. Once all such constraints are imposed, the phenomenological structure that emerges for the minimal LR model is remarkably inflexible.

The chain of choices and phenomenological branches for the minimal LR model are summarized succinctly in Fig. 1; our notation will become apparent as we proceed. The reader should find it helpful to refer to this figure as we develop the more detailed discussions.

Let us now outline in more detail the content of the model and the organization of the paper. We review the minimal left-right-symmetric model, including Higgs content, quark-Higgs-boson and lepton-Higgs-boson couplings, and generation of quark and lepton masses. We write the Higgs-boson couplings in a manner such that the flavor-diagonal and flavor-changing couplings are explicitly displayed. The most general LR-symmetric Higgs potential is also presented, while the minimization is carried out in the Appendix. We analyze the phase degrees of freedom, and confirm that, for a Higgs potential with-

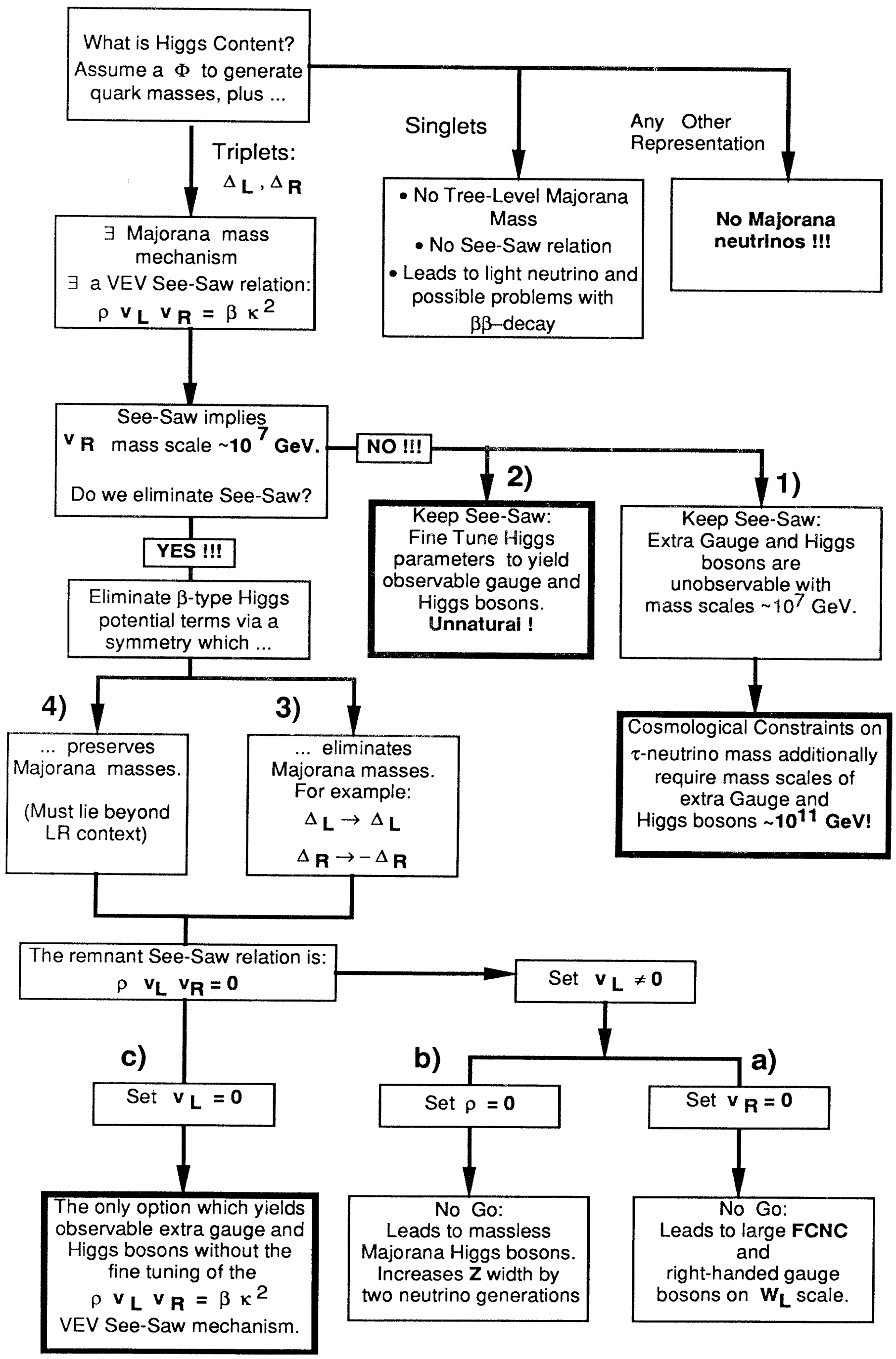

FIG. 1. Flow chart for the minimal LR model involving triplet Higgs fields. This diagram summarizes the chain of choices possible, and the phenomenological constraints relevant for each choice. Note that the *only* phenomenologically viable manifestations of this class of models are represented by the three cells with the heavy borders, all but one of which requires either fine-tuning or the introduction of additional symmetries. The classifications (1)–(4) and (a)–(c) correspond to those of Sec. IV A.

out explicit CP violation, spontaneous CP violation does not occur — that is, all the vacuum expectation values (VEV's) of the Higgs fields can be chosen to be real. As a consequence, in the absence of explicit CP violation, the gauge and Higgs-boson sectors of the full model will be CP conserving. In such a case, one can demonstrate that the LR model must be manifestly or quasimanifestly (as defined later) left-right symmetric, which in turn has important consequences for the constraints on the model.

We then consider the VEV seesaw relationship which takes the generic form $\rho v_L v_R = \beta\kappa_+^2$, where $\kappa_+ = \sqrt{|\kappa_1|^2 + |\kappa_2|^2}$, v_L, v_R, κ_1, and κ_2 are VEV's, and ρ and β are combinations of Higgs potential parameters whose ratio should be of order 1 in the absence of fine-tuning or additional symmetries. For the model to be consistent with the observed phenomena, the symmetry-breaking pattern that should arise is $v_R \gg \kappa_+ \gg v_L$. Clearly, such a solution is consistent with the VEV seesaw. (Without placing the LR-symmetric model in the context of a grand-unification scheme, we cannot, however, explain why $v_R \gg v_L$.) We also derive the severe restrictions obtained as a result of the VEV seesaw when the lepton masses are required to agree with experiment. The necessity of fine-tuning and/or extra symmetries in order to satisfy such restrictions is reviewed.

The restrictions related to lepton and neutrino masses are severe. We will see that, given the observed magnitude of the charged-lepton masses, small neutrino masses are very difficult to obtain unless (a) the mass scale of new physics is very large, (b) certain parameters of the Higgs potential are fine-tuned, or (c) the terms in the Higgs potential associated with these parameters are eliminated by an additional symmetry or via embedding the LR theory in a grand-unification scheme. Indeed, barring possibilities (b) and (c), if we demand that the electron-neutrino satisfy the current experimental bound ($m_{\nu_e} \lesssim 10$ eV), we are forced to require $v_R \gtrsim 10^7$ GeV, thus raising the scale of the W_R and Z' to $10^3 - 10^4$ TeV. The only observable consequences of a LR-symmetric model with such a large value of v_R would be the existence of a Majorana neutrino mass.

A far more interesting mass range for the extra W_R and Z' bosons is 1 – 10 TeV; this region is accessible to exploration by future colliders [3, 5]. (Note that this region is also above the lower bounds arising from flavor-changing neutral-current (FCNC) considerations in LR models with manifest left-right symmetry [9].) In light of the VEV seesaw relation, we find that it is necessary to "fine-tune" the $\gamma = \beta/\rho$ ratio of Higgs potential parameter combinations to high precision ($\gamma \lesssim 10^{-7}$) in order to obtain boson masses which are observable at a Superconducting Super Collider (SSC) type of facility. We examine the question of whether imposing a discrete symmetry on the Lagrangian can force $\gamma = 0$ in a natural manner. If we require realistic quark and lepton masses, *and* Majorana masses for the neutrinos, then the symmetries we explore fail to achieve this goal. But, if we give up having Majorana masses and the neutrino-mass seesaw mechanism, a symmetry that requires $\gamma = 0$ can be found.

To have complete freedom in adjusting quark masses, we shall see (in Sec. II) that the Higgs bidoublet (Φ and $\tilde{\Phi}$) must couple to fermions with unconstrained couplings. If only one of these fields couples to fermions, then at the tree level the Cabibbo-Kobayashi-Maskawa (CKM) matrices are diagonal and the up-type-quark masses are proportional to the down-type-quark masses. However, we note that avoiding a zero quark coupling for the Φ or $\tilde{\Phi}$, while desirable, may not be absolutely essential so long as radiative corrections generate off-diagonal elements in the CKM matrix and the predicted m_t/m_b ratio is within the range obtained for top-quark masses between the current experimental lower limit of about 80 GeV and the upper limit (based on keeping deviations in $\rho_{\rm EW}$ small) of about 200 GeV.

Section IV is devoted to presenting the two types of models in which the β-type Higgs potential terms are absent. These models are obtained either by constructing a simple symmetry which requires their absence (so long as we are willing to give up the Majorana lepton couplings) or by assuming that they are required to be zero within the context of embedding the LR theory in a grand-unified-theory (GUT) scheme (in which Majorana couplings could in general still be allowed to be present). The latter case will be given the most emphasis, since it is the *only* version of the minimal left-right-symmetric model which will yield observable extra bosons, acceptable quark masses, *and* Majorana neutrino masses without resorting to fine-tuning. Removing the β-type terms yields a model with two interesting possibilities: (a) $v_L = 0$, and (b) $v_L \neq 0$. In the latter case, there are (tree-level) massless bosons in the theory which affect neutrino counting [12].

To illustrate the very limited freedom remaining after the phenomenological constraints have been imposed, we consider a "toy" model. This model is closely related to the general $v_L = 0$ case above (in a sense, it is a subset of the general case), but it has the important virtue that it is analytically solvable. One quickly discovers that, even though we have (by hand) evaded the problems associated with the VEV seesaw and lepton masses, there are still more experimental constraints that must be satisfied. Unlike the standard model, the Higgs fields can mediate flavor-changing reactions in the LR theories, and we must require that these interactions are sufficiently suppressed. At the same time, the Higgs boson that plays the role of the standard-model Higgs boson must be light enough to satisfy unitarity constraints. These constraints, as reviewed in Sec. II, are applied to the case of this toy model. We find that to suppress the FCNC we must require that either κ_1 or κ_2 must be very small or equal to zero. (This requirement is actually more general than the specific toy model discussed.) While this could result from standard evolution in a GUT scheme, in the minimal LR context it must be input by hand. The consequences of having one of the κ's small or equal to zero include the prediction of small or zero W_L-W_R mixing. We also find that, in this model, the quark, lepton and neutrino masses are acceptable. Further detailed consequences are explored.

Finally, we turn our discussion toward general properties of the Higgs sector which are common to all of these different branches independent of any additional symmetries or special choices of the parameters; this analysis applies to the most general case. (Additionally, much of this discussion is qualitatively applicable to many of the extensions of the LR models, of which this minimal LR model is a subset.)

Certainly there are many exciting features and potential signatures for LR models with triplet Higgs fields. We hope that our presentation of the various scenarios will prove sufficiently transparent to allow the consumer of extended electroweak models to judge for himself the degree of skepticism that is appropriate when considering the phenomenology of these theories with extended and very complicated Higgs sectors.

II. REVIEW OF LR-SYMMETRIC MODEL

As a preface to our analysis, we begin by reviewing the salient features and constraints of the minimal left-right model. Many of these points have been addressed previously in the literature in an isolated context. The purpose of this work is to synthesize these individual pieces into a coherent and interdependent set of constraints, and then impose this set of constraints upon the minimal left-right model to determine the most realistic phenomenology we can expect which is consistent with current experimental observations.

A. The Higgs content of the model

We consider the general class of left-right models which are invariant under the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry, with the following Higgs content [1, 2, 5, 11, 13]:

$$\phi(1/2, 1/2^*, 0), \quad \Delta_L(1,0,2), \quad \Delta_R(0,1,2), \tag{2.1}$$

where the $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ quantum numbers are indicated in parentheses [1, 3]. A convenient representation of the fields is given by the 2×2 matrices:

$$\phi \equiv \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \tag{2.2}$$

$$\Delta_L \equiv \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix}, \tag{2.3}$$

$$\Delta_R \equiv \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix}. \tag{2.4}$$

In our convention, a neutral field ϕ^0 is written in terms of correctly normalized real and imaginary components as $\phi^0 = (1/\sqrt{2})(\phi^{0\,r} + i\phi^{0\,i})$. These fields transform according to the relation

$$\begin{aligned} &\phi \to U_L \phi U_R^\dagger\ , \quad \tilde{\phi} \to U_L \tilde{\phi} U_R^\dagger\ , \\ &\Delta_L \to U_L \Delta_L U_L^\dagger\ , \quad \Delta_L^\dagger \to U_L \Delta_L^\dagger U_L^\dagger\ , \\ &\Delta_R \to U_R \Delta_R U_R^\dagger\ , \quad \Delta_R^\dagger \to U_R \Delta_R^\dagger U_R^\dagger\ , \end{aligned} \tag{2.5}$$

where $U_{L,R}$ are the general $SU(2)_L$ and $SU(2)_R$ unitarity transformations, and $\tilde{\phi} \equiv \tau_2 \phi^* \tau_2$.

The VEV v_R breaks the $SU(2)_R$ symmetry and sets the mass scale for the extra gauge bosons (W_R and Z') and for the right-handed neutrino field (ν_R). The VEV's κ_1 and κ_2 serve the twin purpose of breaking the remaining $SU(2)_L \otimes U(1)_{B-L}$ symmetry down to the usual $U(1)_{EM}$, thereby setting the mass scale for the observed W_L and Z bosons, and of providing quark and lepton Dirac masses. Clearly, v_R must be significantly larger than the larger of κ_1 and κ_2 in order than the W_R and Z' be significantly heavier than the W_L and Z. Δ_L is the LR-symmetric counterpart of the Δ_R; v_L plays an important role in the VEV seesaw relation which is characteristic of LR models. The triplet VEV v_L must be substantially smaller than the larger of κ_1 and κ_2 in order that $\rho_{EW} = m_{W_L}^2/(m_Z^2 \cos^2\theta_W)$ be within 1% of unity, as observed experimentally.

B. The Higgs potential

We now consider the most general Higgs potential [2, 11, 3, 5]. For our theory to be left-right symmetric, it is necessary that the Lagrangian be invariant under the (discrete) left-right symmetry defined by

$$\Psi_L \leftrightarrow \Psi_R, \quad \Delta_R \leftrightarrow \Delta_L, \quad \phi \leftrightarrow \phi^\dagger, \tag{2.6}$$

where $\Psi_{L,R}$ are column vectors containing the left-handed and right-handed fermion fields of the theory. The global phases of the field matrices or vectors appearing in Eq. (2.6) can be chosen in such a way that phases do not appear in Eq. (2.6) (cf. the Appendix). In this way, the most general form of the Higgs potential V obeying Eq. (2.6) contains mostly real parameters, and this potential is displayed in the Appendix.

The discrete LR symmetry of Eq. (2.6) ensures that all the Higgs-boson couplings are real, except for α_2. If we assume that the potential is CP conserving, we would then require α_2 to be real. *A priori*, the real potential coefficients can be either positive or negative; stability conditions at the minimum will require certain combinations of them to be positive. Note that the potential is not invariant under $\phi_1^0 \leftrightarrow \phi_2^0$ (Ref. [14]). (One can restore the $\phi_1^0 \leftrightarrow \phi_2^0$ symmetry by setting $\beta_2 = \beta_3$ and $\alpha_2 = 0$.)

The neutral Higgs fields δ_R^0, δ_L^0, ϕ_1^0, and ϕ_2^0 can potentially acquire VEV's, v_R, v_L, κ_1, and κ_2, respectively. Explicitly, we have

$$\langle\phi\rangle \equiv \begin{pmatrix} \frac{\kappa_1}{\sqrt{2}} & 0 \\ 0 & \frac{\kappa_2}{\sqrt{2}} \end{pmatrix}, \quad \langle\Delta_{L,R}\rangle \equiv \begin{pmatrix} 0 & 0 \\ \frac{v_{L,R}}{\sqrt{2}} & 0 \end{pmatrix}. \tag{2.7}$$

In general, the VEV's can be complex. Global phase rotations have already been employed to make the discrete LR symmetry of Eq. (2.6) phase free. However, we do still have the freedom to use an $SU(2)_L$ and an $SU(2)_R$ rotation to remove two phases. A symmetric choice is to set v_L and v_R real. Or, if $v_L = 0$, then we can choose v_R and κ_1 to be real. [Note that, in general, the relative phase of κ_1 and κ_2 is physical and cannot be eliminated. However, $m_{W_L}^2 = \frac{1}{4}g^2(|\kappa_1|^2 + |\kappa_2|^2)$ does not depend on this relative phase.] Since we shall consider cases in which v_L

and κ_2 are zero, we shall use these phase transformations to set $v_R, \kappa_1 \in \mathbb{R}^+$. A further $\mathrm{U}(1)_{B-L}$ transformation cannot be used to remove the phase of v_L without introducing a phase for v_R [which transforms in the same way as v_L under $\mathrm{U}(1)_{B-L}$].

For the potential to be at a minimum when all the neutral fields are evaluated at their respective VEV's, we must require that

$$\frac{\partial V}{\partial v_R} = \frac{\partial V}{\partial \kappa_1} = \frac{\partial V}{\partial\, \mathrm{Re}\, \kappa_2} = \frac{\partial V}{\partial\, \mathrm{Re}\, v_L} = \frac{\partial V}{\partial\, \mathrm{Im}\, \kappa_2} = \frac{\partial V}{\partial\, \mathrm{Im}\, v_L} = 0 \ . \tag{2.8}$$

Typically, the first three conditions above can be used to eliminate the μ_3^2, μ_1^2, and μ_2^2 parameters of the potential, respectively. As one adjusts the other parameters of the potential, and the VEV's of the Higgs fields, these μ_i^2's must be altered accordingly. A basic presumption of all attacks on this (and, indeed, any) Higgs sector is that evolution from some very-high-energy scale boundary conditions of the larger (GUT) theory, from which the LR-symmetric model emerges, actually determines these parameters to have definite values such that the resulting phenomenology of the model is acceptable. However, delicate adjustments of the remaining parameters of the potential are regarded as generally unlikely to be the result of a GUT scenario, and fall into the category of fine-tuning that one hopes to avoid. In addition, at a true local minimum all the physical Higgs bosons must have positive squared masses for a solution of Eq. (2.8). This implies that various combinations of the potential parameters must be positive. (Detailed expressions are given in the Appendix.) Of the twenty real degrees of freedom contained in this Higgs sector, six are absorbed in giving mass to the left- and right-handed gauge bosons, $W_L^\pm$, $W_R^\pm$, Z, and Z'.

The minimization conditions of Eq. (2.8), coupled with the reality of all the coefficients except α_2 in V and our ability to choose κ_1 and v_R to be real, imply additional phase information for κ_2 and v_L. In particular, if we remove explicit CP violation from the potential by taking α_2 to be real, then, barring certain types of extreme fine-tuning, κ_2 and v_L must be real (but can be either positive or negative, so long as stability conditions for the potential minimum are satisfied). If the β parameters of the Higgs potential are zero (a case we shall consider) then κ_2 must again be real and, in the absence of another fine-tuning, v_L must be zero so that its phase is irrelevant. The proof of these statements appears in the Appendix. Some of these same results were suggested by Masiero, Mohapatra, and Pecci [7], and obtained by Basecq *et al.* [8]. Of course, if explicit CP invariance is demanded of the entire Lagrangian, including the Yukawa couplings, then this absence of spontaneous CP violation will imply that the theory will be CP conserving in its entirety. Further, we will demonstrate that one implication of having both κ_1 and κ_2 real is that the model exhibits manifest or, at least, quasimanifest left-right symmetry (MLRS or QMLRS). That the absence of spontaneous CP violation only implies QMLRS, but need not imply MLRS, seems to have been overlooked previously, and will be demonstrated shortly. If we now allow for a small amount of CP violation through a small phase for α_2, the minimization conditions allow small phases for κ_2 and v_L, and there will be a small violation of QMLRS and small phases in the CKM matrix emerging from the Yukawa couplings. However, for much of our discussion we will take the VEV's to be real. Corrections due to small CP violation phases should be relatively minor.

C. The quark-Higgs-boson couplings

The most general Yukawa interaction invariant separately under $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ transformations (under which $\Psi_L \to U_L\Psi_L$ and $\Psi_R \to U_R\Psi_R$) is [11, 4, 5]

$$\mathcal{L}_Y = \overline{\Psi}_L^i(\mathcal{F}_{ij}\phi + \mathcal{G}_{ij}\tilde{\phi})\Psi_R^j + \mathrm{H.c.} \quad , \tag{2.9}$$

where

$$\Psi^i = \begin{pmatrix} \widehat{\mathbf{U}}^i \\ \widehat{\mathbf{D}}^i \end{pmatrix} \quad , \tag{2.10}$$

and the caret over the quark fields indicates that these are the gauge eigenstates. $\mathcal{F}$ and $\mathcal{G}$ are the Yukawa coupling matrices, and the i, j indices are family indices summed over the quark flavors. Imposing the discrete left-right-symmetry requirement of Eq. (2.6) on $\mathcal{L}_Y$, we find: $\mathcal{F} = \mathcal{F}^\dagger$, and $\mathcal{G} = \mathcal{G}^\dagger$. We can rotate the gauge eigenstates into the mass eigenstates with unitary matrices $\mathbf{V}$:

$$\begin{matrix}\text{gauge}\\ \text{eigenstates}\end{matrix} \left\{ \begin{matrix} \widehat{\mathbf{U}}_{L,R} = \mathbf{V}^{\mathbf{U}}_{L,R}\ \mathbf{U}_{L,R} \\ \widehat{\mathbf{D}}_{L,R} = \mathbf{V}^{\mathbf{D}}_{L,R}\ \mathbf{D}_{L,R} \end{matrix} \right\} \begin{matrix}\text{mass}\\ \text{eigenstates}\end{matrix} \ , \tag{2.11}$$

where $\widehat{\mathbf{U}}$ and $\widehat{\mathbf{D}}$ are vectors representing the up- and down-type quarks. In terms of these matrices, the usual Cabibbo-Kobayashi-Maskawa (CKM) matrix in the left sector, and the corresponding matrix in the right sector, are given by

$$\mathbf{V}^{\mathrm{CKM}}_{L,R} = \mathbf{V}^{\mathbf{U}\,\dagger}_{L,R}\ \mathbf{V}^{\mathbf{D}}_{L,R} \ , \tag{2.12}$$

Note that, *a priori*, there is no reason for $\mathbf{V}_L^{\mathrm{CKM}}$ to equal $\mathbf{V}_R^{\mathrm{CKM}}$.

For the up-type quarks, we have (henceforth, we will assume the Hermitian conjugate terms)

$$\overline{\widehat{\mathbf{U}}}_L(\mathcal{F}\phi_1^0 + \mathcal{G}\phi_2^{0\,*})\widehat{\mathbf{U}}_R = \overline{\mathbf{U}}_L\mathbf{V}_L^{\mathbf{U}\,\dagger}(\mathcal{F}\phi_1^0 + \mathcal{G}\phi_2^{0\,*})\mathbf{V}_R^{\mathbf{U}}\mathbf{U}_R \ . \tag{2.13}$$

Taking the VEV of the ϕ fields, we can determine the up-type-quark mass matrix:

$$\frac{1}{\sqrt{2}}\,\overline{\mathbf{U}}_L\mathbf{V}_L^{\mathbf{U}\,\dagger}(\mathcal{F}\kappa_1 + \mathcal{G}\kappa_2^*)\mathbf{V}_R^{\mathbf{U}}\mathbf{U}_R \equiv \overline{\mathbf{U}}_L\mathbf{M}^{\mathbf{U}}\mathbf{U}_R \ , \tag{2.14}$$

where $\mathbf{M}^{\mathbf{U}}$ represents the diagonal matrix of physical quark masses. For the down-type quarks, we have, similarly,

$$\overline{\widehat{\mathbf{D}}}_L(\mathcal{F}\phi_2^0+\mathcal{G}\phi_1^{0\,*})\widehat{\mathbf{D}}_R=\overline{\mathbf{D}}_L\mathbf{V}_L^{\mathbf{D}\,\dagger}(\mathcal{F}\phi_2^0+\mathcal{G}\phi_1^{0\,*})\mathbf{V}_R^{\mathbf{D}}\mathbf{D}_R\ . \tag{2.15}$$

Again, taking the VEV of the ϕ fields, we find

$$\frac{1}{\sqrt{2}}\,\overline{\mathbf{D}}_L\mathbf{V}_L^{\mathbf{D}\,\dagger}(\mathcal{F}\kappa_2+\mathcal{G}\kappa_1^*)\mathbf{V}_R^{\mathbf{D}}\mathbf{D}_R\equiv\overline{\mathbf{D}}_L\mathbf{M}^{\mathbf{D}}\mathbf{D}_R\ , \tag{2.16}$$

where $\mathbf{M}^{\mathbf{D}}$ represents the diagonal down-quark mass matrix. We note that in our convention, $\mathbf{M}^{\mathbf{U}}$ and $\mathbf{M}^{\mathbf{D}}$ have positive-definite entries.

1. Manifest and quasimanifest left-right symmetry

Note that, in general, $\mathbf{V}_L^{\mathbf{U}}\neq\mathbf{V}_R^{\mathbf{U}}$ since the matrix $(\mathcal{F}\kappa_1+\mathcal{G}\kappa_2^*)$ need not be Hermitian, and similarly, $\mathbf{V}_L^{\mathbf{D}}\neq\mathbf{V}_R^{\mathbf{D}}$. However, we have argued that κ_1 can always be defined to be real, while if there is no explicit CP violation in the Higgs potential (i.e., α_2 real) then the minimization conditions imply that κ_2 is also real. In this case, or should one of the κ's vanish, it is immediately clear that $\mathcal{F}\kappa_1+\mathcal{G}\kappa_2^*$ and $\mathcal{F}\kappa_2+\mathcal{G}\kappa_1^*$, appearing in Eqs. (2.14) and (2.16), are Hermitian (given the hermiticity of $\mathcal{F}$ and $\mathcal{G}$) and can each be diagonalized by a unitary transformation. If after such diagonalization in the up *and* down sectors, all the entries on the diagonals are positive, then we may take $\mathbf{V}_L^{\mathbf{U}}=\mathbf{V}_R^{\mathbf{U}}$ *and* $\mathbf{V}_L^{\mathbf{D}}=\mathbf{V}_R^{\mathbf{D}}$. In this case, one has $\mathbf{V}_R^{\mathrm{CKM}}=\mathbf{V}_L^{\mathrm{CKM}}$, which has been referred to as manifest left-right symmetry [1]. More generally, however, some entries on the diagonals in each sector will be negative. In this case, we must write $\mathbf{V}_R^{\mathbf{U}}=\mathbf{V}_L^{\mathbf{U}}\mathbf{W}^{\mathbf{U}}$ and $\mathbf{V}_R^{\mathbf{D}}=\mathbf{V}_L^{\mathbf{D}}\mathbf{W}^{\mathbf{D}}$, where $\mathbf{W}^{\mathbf{U}}$ and $\mathbf{W}^{\mathbf{D}}$ are diagonal matrices with entries of $+1$ or -1 on the diagonal as required to yield all entries in $\mathbf{M}^{\mathbf{U}}$ and $\mathbf{M}^{\mathbf{D}}$ positive. In this case, we have $\mathbf{V}_R^{\mathrm{CKM}}=\mathbf{W}^{\mathbf{U}}\mathbf{V}_L^{\mathrm{CKM}}\mathbf{W}^{\mathbf{D}}$. In other words, $\mathbf{V}_{Rij}^{\mathrm{CKM}}=\pm\mathbf{V}_{Lij}^{\mathrm{CKM}}$. To our knowledge, the fact that corresponding elements of $\mathbf{V}_R^{\mathrm{CKM}}$ and $\mathbf{V}_L^{\mathrm{CKM}}$ can differ in sign even when the up- and down-quark mass matrices are Hermitian has not been previously noticed; we shall refer to the relation $\mathbf{V}_{Rij}^{\mathrm{CKM}}=\pm\mathbf{V}_{Lij}^{\mathrm{CKM}}$ as quasimanifest left-right symmetry. Note that relative phases between the $\mathbf{V}_L^{\mathrm{CKM}}$ and $\mathbf{V}_R^{\mathrm{CKM}}$ matrix entries are potentially observable. For instance, suppose unitary diagonalization leads to all positive entries except for the top quark. We must then introduce $\mathbf{W}^{\mathbf{U}}=\mathrm{diag}\{+1,+1,-1\}$. In the standard model this has no observable consequence; but in the LRS theory we have $\mathbf{V}_{Rti}^{\mathrm{CKM}}=-\mathbf{V}_{Lti}^{\mathrm{CKM}}$, so that, for instance, W_R exchange interferes destructively with W_L exchange in $t\to be^+\nu_e$ decay. However, for our purposes, QMLRS is just as good as MLRS; in either case, the bound on the W_R mass of $m_{W_R}\gtrsim 1.7$ TeV coming from K-$\overline{K}$ mixing will hold so long as the magnitude of the CKM entries are the same in the R sector as in the L sector *and* the main contribution from the mixed W_L-W_R box diagrams comes from just one choice of internal (up-) quark lines. In fact, one finds that the box diagram with two internal charm-quark lines always dominates [15]. To summarize, we find that a consistent treatment of phases and minimization conditions in a model with no explicit CP violation implies that there will be no spontaneous CP violation, and that the minimal LR model is either MLRS or QMLRS, thereby ensuring the applicability of the strong bound, $m_{W_R}\gtrsim 1.7$ TeV.

2. An important special case: $\mathcal{F}=0$ or $\mathcal{G}=0$

A special case which will turn out to be of particular importance in our considerations is that where either $\mathcal{F}$ or $\mathcal{G}$ is zero (for example, due to some type of symmetry). Then the transformations required to diagonalize the down-quark mass matrix can be taken to be the same (up to a possible overall phase depending upon the phase of κ_2 relative to κ_1) as those required to diagonalize the up-quark mass matrix: i.e., $\mathbf{V}_L^{\mathbf{D}}=\mathbf{V}_L^{\mathbf{U}}$ and $\mathbf{V}_R^{\mathbf{D}}=\mathbf{V}_R^{\mathbf{U}}$. In this case, $\mathbf{V}_L^{\mathrm{CKM}}$ and $\mathbf{V}_R^{\mathrm{CKM}}$ are unit matrices, again aside from a possible overall phase. In the case where no explicit CP violation is introduced, since we have defined our phases so that κ_1 is real and positive, κ_2 must also be real. If $\kappa_2>0$ then the CKM matrices are unit matrices. If $\kappa_2<0$ then assuming that $\mathbf{V}_R^{\mathbf{U}}$ and $\mathbf{V}_L^{\mathbf{U}}$ have been defined to produce positive up-quark masses, positive down-quark matrices can be obtained, for example, by writing $\mathbf{V}_L^{\mathbf{D}}=\mathbf{V}_L^{\mathbf{U}}$ and $\mathbf{V}_R^{\mathbf{D}}=-\mathbf{V}_R^{\mathbf{U}}$, in which case $\mathbf{V}_L^{\mathrm{CKM}}=I$ and $\mathbf{V}_R^{\mathrm{CKM}}=-I$.

In addition to trivial CKM matrices, setting $\mathcal{G}=0$ forces the up- and down-type-quark masses to be proportional: the ratio is fixed by $\kappa_1/|\kappa_2|$. This result arises because both the up- and down-type-quark masses now are derived from the same Yukawa coupling matrix $\mathcal{F}$. Furthermore, it is interesting to note that this relation among the quark masses will not be altered by soft-breaking terms of dimension 2. Such a relation among up- and down-type-quark masses, while not in agreement with experiment in the case of the u and d quarks, could be regarded as acceptable if there are small corrections to quark masses from additional new physics and/or radiative effects [16]. However, the ratio of the two bidoublet VEV's is no longer an adjustable parameter; it is fixed by the third family: $\kappa_1/|\kappa_2|=m_t/m_b$. Radiative correction effects would also have to be responsible for the off-diagonal elements of the CKM matrix observed in the L sector. Overall, having $\mathcal{G}=0$ (or $\mathcal{F}=0$) leads to very awkward phenomenology.

3. Flavor-changing couplings

For $|\kappa_1|^2\neq|\kappa_2|^2$, we can invert Eqs. (2.14) and (2.16) to solve for $\mathcal{F}$ and $\mathcal{G}$ in terms of the physical masses of the up and down quarks:

$$\mathcal{F}=\frac{\sqrt{2}}{\kappa_-^2}(\kappa_1^*\mathbf{V}_L^{\mathbf{U}}\mathbf{M}^{\mathbf{U}}\mathbf{V}_R^{\mathbf{U}\,\dagger}-\kappa_2^*\mathbf{V}_L^{\mathbf{D}}\mathbf{M}^{\mathbf{D}}\mathbf{V}_R^{\mathbf{D}\,\dagger})\ ,$$
$$\mathcal{G}=\frac{\sqrt{2}}{\kappa_-^2}(-\kappa_2\mathbf{V}_L^{\mathbf{U}}\mathbf{M}^{\mathbf{U}}\mathbf{V}_R^{\mathbf{U}\,\dagger}+\kappa_1\mathbf{V}_L^{\mathbf{D}}\mathbf{M}^{\mathbf{D}}\mathbf{V}_R^{\mathbf{D}\,\dagger})\ , \tag{2.17}$$

where we define

$$\kappa_\pm^2=|\kappa_1|^2\pm|\kappa_2|^2\quad . \tag{2.18}$$

We can now write the general interaction term for the quark mass eigenstate with the neutral ϕ-type Higgs fields:

$$\frac{\sqrt{2}}{\kappa_-^2}\overline{\mathbf{U}}_L[\,\mathbf{M^U}(\kappa_1^*\phi_1^0-\kappa_2\phi_2^{0\,*}) +\mathbf{V}_L^{\rm CKM}\mathbf{M^D}\mathbf{V}_R^{\rm CKM\,\dagger}(-\kappa_2^*\phi_1^0+\kappa_1\phi_2^{0\,*})]\mathbf{U}_R, \tag{2.19}$$

$$\frac{\sqrt{2}}{\kappa_-^2}\overline{\mathbf{D}}_L[\,\mathbf{M^D}(\kappa_1\phi_1^{0\,*}-\kappa_2^*\phi_2^0) +\mathbf{V}_L^{\rm CKM\,\dagger}\mathbf{M^U}\mathbf{V}_R^{\rm CKM}(-\kappa_2\phi_1^{0\,*}+\kappa_1^*\phi_2^0)]\mathbf{D}_R\,.$$

To identify the flavor-changing and flavor-conserving combinations, we define the orthogonal neutral fields:

$$\phi_+^0=\frac{1}{\kappa_+}(-\kappa_2^*\phi_1^0+\kappa_1\phi_2^{0\,*})\,,\qquad \phi_-^0=\frac{1}{\kappa_+}(\kappa_1^*\phi_1^0+\kappa_2\phi_2^{0\,*})\,. \tag{2.20}$$

The inverse transformation is

$$\phi_1^0=\frac{1}{\kappa_+}(-\kappa_2\phi_+^0+\kappa_1\phi_-^0)\,,\qquad \phi_2^0=\frac{1}{\kappa_+}(\kappa_1\phi_+^{0*}+\kappa_2\phi_-^{0*})\,. \tag{2.21}$$

In terms of $\phi_\pm^0$, the couplings to the quarks are

$$\frac{\sqrt{2}}{\kappa_-^2}\overline{\mathbf{U}}^L\left[\phi_-^0\frac{\kappa_-^2}{\kappa_+}\mathbf{M^U}+\phi_+^0\left(\frac{-2\kappa_1^*\kappa_2}{\kappa_+}\mathbf{M^U}+\kappa_+\mathbf{V}_L^{\rm CKM}\mathbf{M^D}\mathbf{V}_R^{\rm CKM\,\dagger}\right)\right]\mathbf{U}^R\,, \tag{2.22}$$

$$\frac{\sqrt{2}}{\kappa_-^2}\overline{\mathbf{D}}^L\left[\phi_-^{0*}\frac{\kappa_-^2}{\kappa_+}\mathbf{M^D}+\phi_+^{0*}\left(\frac{-2\kappa_1\kappa_2^*}{\kappa_+}\mathbf{M^D}+\kappa_+\mathbf{V}_L^{\rm CKM\,\dagger}\mathbf{M^U}\mathbf{V}_R^{\rm CKM}\right)\right]\mathbf{D}^R\,. \tag{2.23}$$

Note that these couplings are not diagonal since the CKM matrices are not diagonal. This non-diagonality always yields powerful constraints, especially in this minimal model for which we have MLRS or QMLRS in the limit where explicit CP violation is absent. Indeed, it is obvious from Eq. (2.23) that only the two components, i.e., the real and imaginary parts, of the complex ϕ_-^0 field can have flavor-diagonal couplings. Thus, the real component of the ϕ_-^0 must correspond to the "standard model" Higgs boson, and the imaginary component must be the massless Goldstone field absorbed by the light Z. In order that the flavor-changing couplings of the ϕ_+^0 in Eqs. (2.22) and (2.23) not conflict with experiment, the mass eigenstates containing significant mixtures of ϕ_+^0 must have a large mass. The exact requirement will be presented shortly.

D. The lepton-Higgs-boson couplings

We now turn to the lepton-Higgs-boson couplings. These are more complicated than the quark-Higgs-boson couplings because the leptons have both Dirac- and Majorana-type Yukawa couplings. The most general Lagrangian invariant under $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$ transformations, and the discrete LR-symmetry operation of Eq. (2.6) is [17]

$$\mathcal{L}_{\rm mass}=f_{ij}\,\overline{\Psi}_L^i\,\phi\,\Psi_R^j+g_{ij}\,\overline{\Psi}_L^i\,\tilde{\phi}\,\Psi_R^j+\text{H.c.} +i\,(\,h_M\,)_{ij}(\Psi_L^{iT}C\tau_2\,\Delta_L\,\Psi_L^j+\Psi_R^{jT}C\tau_2\,\Delta_R\,\Psi_R^j) +\text{H.c.}\,, \tag{2.24}$$

where, as in the quark Yukawa case, f and g must be Hermitian matrices, and in component form

$$\Psi_{L,R}=\begin{pmatrix}\nu_{L,R}\\ e_{L,R}^-\end{pmatrix}\,. \tag{2.25}$$

We shall focus on a single generation, and we shall ignore generation mixing; therefore we shall drop the $\{i,j\}$ indices on f, g and h_M. In this one-generation approximation, LR symmetry [Eq. (2.6)] requires that f and g be real. h_M, the Majorana coupling, can be taken to be real and positive as a result of our ability to rotate Ψ_L and Ψ_R by a common phase without changing the phase of the f_{ij}'s and g_{ij}'s.

To identify the mass contributions, we insert the VEV's for the Higgs fields. (As discussed, v_R and κ_1 can be taken to be real and positive, while κ_2 will have only a small phase if only a small explicit CP violation is introduced into the Higgs potential. Thus we work in the approximation where all VEV's are real.) The charged-lepton mass comes only from the f and g terms:

$$m(\ell^+)=\frac{1}{\sqrt{2}}\,(f\kappa_2+g\kappa_1)\,. \tag{2.26}$$

Neutrino mass derives both from the f and g terms, which lead to Dirac mass, and from the h_M term, which leads to Majorana mass. Defining, as usual, $\psi^c\equiv C(\overline{\psi})^T$, it is convenient to employ the self-conjugate spinors

$$\nu\equiv\frac{\nu_L+\nu_L{}^c}{\sqrt{2}},\quad N\equiv\frac{\nu_R+\nu_R{}^c}{\sqrt{2}}\,. \tag{2.27}$$

We also define

$$h_D=\frac{1}{\sqrt{2}}\frac{f\kappa_1+g\kappa_2}{\kappa_+}. \tag{2.28}$$

This quantity governs the size of the Dirac-type neutrino mass term. Note its close relation to the combination appearing in the charged-lepton mass of Eq. (2.26). In terms of h_D and h_M, the neutrino mass matrix can be written in the form

$$\begin{pmatrix} \bar{\nu} \\ \bar{N} \end{pmatrix}^T \begin{pmatrix} \sqrt{2}\, h_M\, v_L & h_D \kappa_+ \\ h_D \kappa_+ & \sqrt{2}\, h_M\, v_R \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix} . \quad (2.29)$$

Normally, we expect h_M and h_D to be similar in size, $h_D \sim h_M \sim m_\ell/\kappa_+$. In such a case, since we require $v_R \gg \kappa_+, v_L$ (for $m_{W_R} \gg m_{W_L}$), ν and N will be approximate mass eigenstates with masses:

$$m_N \simeq \sqrt{2}\, h_M\, v_R ,$$
$$m_\nu \simeq \sqrt{2} \left(h_M\, v_L - \frac{h_D^2 \kappa_+^2}{2\, h_M\, v_R} \right) . \quad (2.30)$$

One can ask if these systematics must always apply. One possibility might be to have $h_M = 0$ and h_D nonzero. However, then if f and g are comparable, we would have a Dirac neutrino with mass of order m_ℓ, a clearly unacceptable scenario. The very small ratio

$$m_{\nu_e}/m_e = \frac{(f\kappa_1 + g\kappa_2)}{(g\kappa_1 + f\kappa_2)} \lesssim \frac{(10 \text{ eV})}{(0.5 \text{ MeV})} \sim 10^{-5} , \quad (2.31)$$

requires, for example, $f \ll g$ and $\kappa_2 \ll \kappa_1$ (assuming the absence of a finely tuned cancellation between $f\kappa_1$ and $g\kappa_2$). In the other extreme, h_D could be zero. This is certainly acceptable so long as m_N is sufficiently large. From Eq. (2.30) we see that in this case $m_N = m_\nu v_R/v_L$. For the electron case, where $m_{\nu_e} \lesssim 10$ eV this clearly requires a very large value for v_R/v_L in order that the N_e have a large mass. Constraints on the N_e mass arise from neutrinoless double-β decay. The bound obtained in Ref. [5] takes the form

$$m_{N_e} > 63 \text{ GeV} \left(\frac{1.6 \text{ TeV}}{m_{W_R}} \right)^4 . \quad (2.32)$$

This yields an important constraint. For instance, suppose that $m_{W_R} \sim 1.4$ TeV, so that Eq. (2.32) requires $m_{N_e} > 100$ GeV. Then $v_R/v_L = m_{N_e}/m_{\nu_e}$ must exceed 10^{10} for $m_{\nu_e} \lesssim 10$ eV. For $m_{W_R} \sim 1.4$ TeV, $v_R \sim 3$ TeV ($m_{W_R} = g v_R/\sqrt{2}$), and we see that $v_L \lesssim 1$ keV would be required. This is an even stronger constraint than that which emerges from ρ_{EW}. In particular, for $\rho_{\text{EW}} = (\kappa_+^2 + 2v_L^2)/(\kappa_+^2 + 4v_L^2)$ to be within 1% of unity only requires $v_L \lesssim 0.07\kappa_+ \sim 15$ GeV. Further, it is obvious that if the h_D term (rather than the h_M term) dominates m_ν, then $m_N < m_\nu v_R/v_L$ and the constraint becomes even stronger. This type of strong constraint on v_L can only be avoided if there is a very finely tuned cancellation between the h_M and h_D terms contributing to m_ν in Eq. (2.30).

E. Review of FCNC bounds

Here, we briefly review the FCNC constraints relevant for our analysis. From Eqs. (2.22) and (2.23) we see that it is the ϕ_+^0 Higgs boson which has the flavor-violating couplings. Actually, there are two real components, ϕ_+^{0r} and ϕ_+^{0i}, which will give rise to FCNC transitions; we denote these generically as ϕ_{FCNC}^0. These have the potential to generate large $\overline{K^0}$-K^0 transitions in contradiction to experiment. The relation between the $\overline{K^0}$-K^0 transition amplitude and the experimentally measured K mass splitting is determined by

$$\Delta M_K = 2\, \text{Re} \left(\langle \overline{K^0} | \mathcal{H}_{\text{eff}} | K^0 \rangle \langle K^0 | \mathcal{H}_{\text{eff}} | \overline{K^0} \rangle \right)^{1/2} . \quad (2.33)$$

The problem is that the standard-model box diagram already yields the bulk of the transition amplitude necessary to generate the proper mass splitting; thus, the contribution from the FCNC Higgs boson must be limited. A reasonable condition is [6, 10, 9]

$$(\Delta M_K)_{\text{expt}} \geq 2\, \text{Re} \left(\langle \overline{K^0} | \mathcal{H}_{\text{eff}} | K^0 \rangle \langle K^0 | \mathcal{H}_{\text{eff}} | \overline{K^0} \rangle \right)^{1/2} . \quad (2.34)$$

From Eq. (2.23), we find the effective Hamiltonian for the Higgs-boson-exchange diagram to be

$$\mathcal{H}_{\text{eff}}(\phi_{\text{FCNC}}^0) = \left(\frac{\sqrt{2}\kappa_+}{\kappa_-^2} \right)^2 \frac{1}{M(\phi_{\text{FCNC}}^0)^2} \times (\mathbf{V}_{is}^{L*}\, \mathbf{M}_{ii}^{\text{U}}\, \mathbf{V}_{id}^{R})\, (\mathbf{V}_{js}^{R*}\, \mathbf{M}_{jj}^{\text{U}}\, \mathbf{V}_{jd}^{L}) \times (\overline{\Psi}_{\bar{s}} \mathbf{R} \Psi_d) \otimes (\overline{\Psi}_{\bar{s}} \mathbf{L} \Psi_d) . \quad (2.35)$$

[$\mathbf{V}_L^{\text{CKM}}$ has been abbreviated as $\mathbf{V}^L$ and $\mathbf{V}_R^{\text{CKM}}$ as $\mathbf{V}^R$; $\mathbf{R} = (1+\gamma_5)/2$ and $\mathbf{L} = (1-\gamma_5)/2$.] From the above Hamiltonian, a bound on the ϕ_{FCNC}^0 mass can be obtained [10, 6, 18]. This is found to be

$$M(\phi_{\text{FCNC}}^0) \gtrsim 10 \text{ TeV} . \quad (2.36)$$

Escape from this bound is completely impossible in our minimal model with MLRS or QMLRS, whether approximate or exact.

F. Review of unitarity bounds

We now examine the constraints coming from the condition that the W-W scattering amplitude satisfy unitarity. Recall the bound derived by Lee, Quigg, and Thacker [19], for the standard-model Higgs boson (ϕ_{SM}) is

$$M(\phi_{\text{SM}}) \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} \simeq 1 \text{ TeV} . \quad (2.37)$$

What is the corresponding limit in the LR-symmetric model?

The term in the Lagrangian which contributes to the W_L-W_L scattering is

$$\mathcal{L} = \frac{g^2}{2}\, W_L^+ W_L^- \, (\phi_1^0 \phi_1^{0\,*} + \phi_2^0 \phi_2^{0\,*}) . \quad (2.38)$$

Transforming to the (ϕ_+^0, ϕ_-^0) basis and shifting the fields by their VEV's, yields a trilinear coupling of the form

$$\mathcal{L} = \frac{g^2}{2} W_L^+ W_L^- (\kappa_+ \phi_-^{0r}) , \tag{2.39}$$

where ϕ_-^{0r} is defined by $\phi_-^0 = (\phi_-^{0r} + i\phi_-^{0i})/\sqrt{2}$. Recall that ϕ_-^0 has flavor-diagonal couplings, and note that the FCNC Higgs boson ϕ_+^0 does not couple to the $W_L^+ W_L^-$ vertex. Since $m_{W_L} = g\kappa_+/2$, we see that the $W_L^+ W_L^- \phi_-^{0r}$ coupling is igm_{W_L}, just as in the standard model. It is now apparent that W_L-W_L unitarity requires that

$$M(\phi_-^{0r}) \leq\sim 1 \text{ TeV} \quad . \tag{2.40}$$

[Note, some analyses argue that the value is lower, i.e., $\sim$ 630 GeV (Ref. [20]).] Most importantly, the Higgs bosons with flavor-changing couplings, i.e., the real and imaginary components of the ϕ_+^0, are not constrained by unitarity in W-W scattering. Thus, they are *a priori* allowed to be as heavy as required for phenomenological consistency with the FCNC bounds discussed earlier.

III. THE SPONTANEOUS BREAKING OF THE HIGGS SECTOR

A. The vacuum expectation value seesaw mechanism

The requirement that the potential have a minimum implies, in part, that

$$\frac{\partial V}{\partial v_L} = \frac{\partial V}{\partial v_R} = 0 \ . \tag{3.1}$$

We can solve the above two equations, eliminate μ_3^2, and thus find

$$\beta_2\kappa_1^2 + \beta_1\kappa_1\kappa_2 + \beta_3\kappa_2^2 = (2\rho_1 - \rho_3)v_L v_R \quad . \tag{3.2}$$

This is the infamous VEV seesaw constraint which implies a relation among the widely varying VEV scales. (As noted in the Appendix, this is simply the zero-phase limit of the real part of a more general VEV seesaw relation. This relation, in combination with other minimization conditions, implies that the allowed phases of the κ_2 and v_L VEV's must be zero in the absence of explicit CP violation in the Higgs potential introduced via an imaginary part to α_2.) Let us examine this relation to see if it will generate the proper mass scale for the physical particles.

We take the Higgs potential parameters β_i and ρ_i to be of order unity. If these parameters were too large, they would violate unitarity [19, 20, 11] and lead to a nonperturbative theory. These parameters can be very small, but that would require fine-tuning. We must also keep in mind that β_i and ρ_i, as well as κ_2 and v_L, can have any sign, so long as conditions of vacuum stability are satisfied (i.e., all Higgs bosons have positive mass squared); however, we can define v_R and κ_1 to be positive definite. We would like v_R to be $\sim$ 3 TeV so that the W_R mass is in a range observable at the SSC ($W_R \lesssim 10$ TeV). The κ's are of order $\sim$ 250 GeV, as determined from the W_L mass. Therefore, we expect that the natural scale for v_L arising from the seesaw mechanism is $\sim$ 10 GeV. Values of this size or lower are acceptable from the point of view of keeping corrections to $\rho_{\rm EW}$ below the 1% level. However, it turns out that the more critical question is whether this scenario yields sufficiently small values for the neutrino masses.

By introducing γ, where

$$\gamma = \frac{\beta_2\kappa_1^2 + \beta_1\kappa_1\kappa_2 + \beta_3\kappa_2^2}{(2\rho_1 - \rho_3)\ \kappa_+^2} \quad , \tag{3.3}$$

we abbreviate the seesaw relation

$$v_L = \gamma \frac{\kappa_+^2}{v_R} \quad , \tag{3.4}$$

and we can express the neutrino mass from Eq. (2.30) as

$$m_\nu \simeq \sqrt{2} \left(h_M \gamma - \frac{1}{2} \frac{h_D^2}{h_M} \right) \frac{\kappa_+^2}{v_R} \quad , \tag{3.5}$$

where h_M are Majorana-type Yukawa couplings, and h_D, defined in Eq. (2.28), relates to Dirac-type Yukawa couplings. From its definition, we expect γ to be a parameter of order ~ 1; however, a priori, it can be either positive or negative. [For instance, the $2\rho_1 - \rho_3$ combination in the denominator of Eq. (3.3) must generally be negative for vacuum stability, while β_2 can easily be positive and κ_1 can be significantly larger than κ_2.] We will now examine whether the above relations can yield consistent mass values for both the charged lepton and its neutrino. For simplicity, our discussion will neglect mixing between lepton families, and γ will be assumed to be positive.

Let us begin by noting that it is by no means trivial to obtain acceptable values for both m_N and m_ν. In particular, one must be careful that m_N does not become too small [21]. From Eq. (3.5) we find

$$m_N = \frac{m_\nu}{\gamma - \dfrac{h_D^2}{2\, h_M{}^2}} \frac{v_R^2}{\kappa_+^2} \ . \tag{3.6}$$

From Eq. (2.32) we learn that $v_R \sim 3$ TeV ($m_{W_R} \sim$ 1.4 TeV) requires $m_{N_e} \gtrsim 100$ GeV. For $m_{\nu_e} \sim 10$ eV, Eq. (3.6) then implies that $\gamma - h_D^2/(2\, h_M{}^2) \lesssim 10^{-8}$. Barring a highly tuned cancellation, this requires both $\gamma \lesssim 10^{-8}$ and $h_D^2/(2h_M^2) \lesssim 10^{-8}$ (equivalent to $h_M \gtrsim 10^{-2}$ using $h_D \sim m_e/\kappa_+$). In short, whether or not the γ term dominates m_{ν_e}, the only way of avoiding too small a value for m_{N_e} is to have $\gamma \lesssim 10^{-8}$. This same type of restriction on γ can be obtained by examining just m_ν itself, without reference to m_N. We describe the procedure in the following.

We begin by noting that a comparison of Eqs. (2.28) and (2.26) makes it apparent that unless f and g are very different in size, h_D^e will be of order $m_e/\kappa_+ \simeq 2 \times 10^{-6}$. (While such a small value for the Yukawa couplings associated with the lepton sector has no fundamental explanation, either in the LR model or in the standard model, this small scale could result from some higher theory. In contrast, fine-tuning of the Higgs potential parameters is generally regarded as a much more serious problem.) Let us now suppose that γ is order ~ 1. Neglecting (for the moment) the possibility of cancellations between the two terms in Eq. (3.5), we have the smallest m_{ν_e} values when $h_M^e \simeq h_D^e$. If m_{ν_e} is required to be less than $\sim$ 10 eV, we find that v_R must be $\gtrsim 10^7$ GeV. Consequently m_{W_R} and $m_{Z'}$ also will be in the range $10^6 \sim 10^7$ GeV. In this limit, the model reduces to the standard model with no observable consequences other than the Majorana neutrinos. We shall now examine the constraints placed on γ if the scale of v_R is to be much lower.

First, it can be easily checked that for γ near 1, extreme fine-tuning is required in order to obtain sufficient cancellation between the two terms of Eq. (3.5) so that v_R values in the 3 to 10 TeV range would be obtained. We exclude this possibility as being highly unnatural. Having eliminated such cancellation, we now ask what γ values could give us $v_R \sim 3$ TeV (for example). In Eq. (3.5) we have two choices; either the first term ($h_M \gamma$) or the second term ($h_D^2/2\,h_M$) can dominate. We examine these possibilities in turn. In the first case, we assume $\gamma\, h_M > h_D^2/2\, h_M$ and $v_R \sim 3$ TeV, which implies (in the case of the electron generation)

$$\gamma h_M^e = \frac{m_{\nu_e} v_R}{\sqrt{2}\kappa_+^2} \leq \frac{\sim 10 \text{ eV} \times 3 \text{ TeV}}{\sqrt{2}(250 \text{ GeV})^2} \sim 10^{-9} . \quad (3.7)$$

But, from our assumption, we have

$$h_M^{e\,2}\gamma > \frac{1}{2} h_D^{e\,2} \simeq \frac{1}{2}\left(\frac{m_e}{\kappa_+}\right)^2 \simeq 10^{-12} \quad , \quad (3.8)$$

where we have used the assumption that $m_e \simeq h_D^e \kappa_+$. Together, these require

$$h_M^e \gtrsim 10^{-3}, \quad \gamma \lesssim 10^{-6} ; \quad (3.9)$$

that is, we must require the fine-tuning of γ to 6 or 7 orders of magnitude.

For the second case, we assume $\gamma < h_D^2/2\, h_M{}^2$, which implies (note that a negative fermion mass can always be rotated to positive mass through redefinition of the fermion field)

$$m_{\nu_\ell} = \frac{h_D^{\ell\,2} \kappa_+^2}{\sqrt{2}\, h_M^\ell v_R} \quad . \quad (3.10)$$

Again, we use $m_\ell \simeq h_D \kappa_+$ to find (in the electron generation)

$$h_M^e = \frac{m_e^2}{\sqrt{2} m_{\nu_e} v_R} \gtrsim \frac{(0.5 \text{ MeV})^2}{\sqrt{2}(10 \text{ eV})(\sim 3 \text{ TeV})} \simeq 5 \times 10^{-3} . \quad (3.11)$$

This relation, together with our initial assumption for γ implies

$$\gamma < \frac{h_D^{\ell\,2}}{2 h_M^{\ell\,2}} = \frac{m_e^2}{2\kappa_+^2 h_M^{e\,2}} = \frac{(0.5 \text{ MeV})^2}{2(250 \text{ GeV})^2(5 \times 10^{-3})^2} \sim 10^{-7} . \quad (3.12)$$

We conclude that if we require the m_{W_R} to be in the few TeV range, it is necessary to fine-tune γ to 6 or 7 orders of magnitude to obtain a small enough m_{ν_e} value.

If we carry this analysis one step further and consider the limits arising from cosmological constraints, we can demand [22, 23]:

$$\sum_\ell m_{\nu_\ell} \lesssim 100 \text{ eV} . \quad (3.13)$$

If we accept this constraint (which can be evaded should the ν_τ have a very short lifetime), we can then repeat the above analysis for the τ neutrino. The restriction that $m_{\nu_\tau} \lesssim 100$ eV allows us to enhance the limits by a large factor. Briefly, when γh_M^τ dominates m_{ν_τ} we find $h_M^\tau \gtrsim 10^4$, $\gamma \lesssim 10^{-12}$, while if the h_D^τ term is dominant we find $h_M^\tau \gtrsim 5 \times 10^3$, $\gamma \lesssim 10^{-12}$. This certainly represents *extreme* fine-tuning for γ; in addition, the values required for h_M^τ are much too large for perturbation theory to be valid for the lepton Majorana couplings.

One situation that we have not explored in the above analysis is that which arises as a result of the fact that κ_1 and κ_2 enter differently into m_ℓ, Eq. (2.26), and h_D, Eq. (2.28). Were there some reason to have $\kappa_2 \ll \kappa_1$ and $f \ll g$, then h_D^ℓ would be much smaller than m_ℓ/κ_+. Writing $h_D^e = \epsilon\,(m_e/\kappa_+)$, we find that the fine-tuning requirements on γ obtained above are reduced by a factor of $1/\epsilon^2$. Thus, if $\epsilon \sim 10^{-3}$ the fine-tuning requirement for γ would be largely eliminated (unless we employ the cosmologically based fine-tuning requirement). However, such a small value for ϵ requires fine-tuning for the f, g, and κ_2 values. In later sections, we shall see that simple symmetries can be invoked to set $f = 0$, but that a small value for κ_2 in this situation is likely to lead to too large a m_t/m_b ratio. Further, we would still not have escaped the problem of predicting too small a value for the N mass unless we fine-tune γ.

This dilemma allows for two definite branches: (1) we can fine-tune so as to obtain Higgs potential parameters β_1, β_2, and β_3 that are 6 or 7 orders of magnitude smaller than is natural; or (2) we can look for new symmetries that dispose of the problem altogether by eliminating the terms in the Higgs potential associated with the β's, thereby eliminating the VEV seesaw relation.

B. Eliminating the VEV seesaw via additional symmetries

We can avoid the severe restrictions of the seesaw relation of Eq. (3.2) if we can find a symmetry which requires the β Higgs potential parameters involved to vanish.

1. Symmetries which retain the Majorana coupling.

We look for a symmetry which eliminates the β_i-type terms in the Higgs potential while leaving invariant the Majorana Yukawa couplings of the leptons to the Higgs triplet. We begin by considering a general set of transformations of the Higgs fields:

$$\Delta_L \to e^{2i\varepsilon_L}\Delta_L \ , \quad \Delta_R \to e^{2i\varepsilon_R}\Delta_R, \quad \phi \to e^{i\varepsilon_\phi}\phi \ , \tag{3.14}$$

and investigate whether these will achieve our purpose. The invariance of the lepton Majorana mass term of the Lagrangian

$$\mathcal{L}_M = i(h_M)_{ij}(\Psi_{iL}^T \mathcal{C}\tau_2\Delta_L\Psi_{jL} + \Psi_{iR}^T\mathcal{C}\tau_2\Delta_R\Psi_{jR}) + \text{H.c.} \tag{3.15}$$

requires that the lepton fields transform as

$$\Psi_L \to e^{-i\varepsilon_L}\Psi_L \ , \quad \Psi_R \to e^{-i\varepsilon_R}\Psi_R \ . \tag{3.16}$$

However, the invariance of both the ϕ and $\tilde{\phi}$ parts of the Dirac lepton mass term,

$$\mathcal{L}_D = f\overline{\Psi}_L\phi\Psi_R + g\overline{\Psi}_L\tilde{\phi}\Psi_R + \text{H.c.} \ , \tag{3.17}$$

requires ϕ to transform as $\phi \to \pm\phi$. (Note, $\phi \to -\phi$ would yield a discrete symmetry, not a continuous one.) If we suppose for the moment that ϕ transforms in this fashion, we find

$$\mathcal{L}_D \sim \overline{\Psi}_L\phi\Psi_R \to (\pm 1)e^{i\varepsilon_L}e^{-i\varepsilon_R}\overline{\Psi}_L\phi\Psi_R \ , \tag{3.18}$$

which implies

$$e^{2i\varepsilon_R} = e^{2i\varepsilon_L} \quad . \tag{3.19}$$

Thus, there is no symmetry which will allow us to eliminate the β-type terms in the Higgs potential while retaining the Majorana *and* Dirac Yukawa couplings.

Thus, we temporarily abandon the idea of keeping both the ϕ and $\tilde{\phi}$ terms in $\mathcal{L}_D$, and we allow ϕ to transform nontrivally such that $f = 0$, and $g \neq 0$. [This choice is such that, even if $\kappa_2 = 0$, $m(\ell)$ will still be nonzero; cf. Eq. (2.26).] We now see that

$$\mathcal{L}_D \sim \overline{\Psi}_L\tilde{\phi}\Psi_R \to e^{i\varepsilon_L}e^{-i\varepsilon_\phi}e^{-i\varepsilon_R}\overline{\Psi}_L\tilde{\phi}\Psi_R \ , \tag{3.20}$$

which implies

$$e^{i\varepsilon_L}e^{-i\varepsilon_\phi}e^{-i\varepsilon_R} = 1 \ . \tag{3.21}$$

If we now examine the transformation properties of the β_i terms of the Higgs potential,

$$\begin{aligned} &\beta_1[\text{Tr}(\phi\Delta_R\phi^\dagger\Delta_L^\dagger) + \text{Tr}(\phi^\dagger\Delta_L\phi\Delta_R^\dagger)] \\ &+\beta_2[\text{Tr}(\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger) + \text{Tr}(\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger)] \\ &+\beta_3[\text{Tr}(\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger) + \text{Tr}(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger)] \end{aligned} \tag{3.22}$$

we find that β_3 will be left invariant by the symmetry

$$\begin{aligned} &\beta_3[\text{Tr}(\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger) + \text{Tr}(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger)] \\ &\longrightarrow \beta_3[\ e^{-2i\varepsilon_L}e^{2i\varepsilon_R}e^{2i\varepsilon_\phi}\text{Tr}(\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger) \\ &\qquad +e^{2i\varepsilon_L}e^{-2i\varepsilon_R}e^{-2i\varepsilon_\phi}\text{Tr}(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger)] \\ &\quad = \beta_3[\text{Tr}(\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger) + \text{Tr}(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger)] \ , \end{aligned} \tag{3.23}$$

where we have used the identity of Eq. (3.21). Note that if we had chosen to retain the f term of the lepton Dirac Yukawa Lagrangian, we would have required [instead of Eq. (3.21)] $e^{i\varepsilon_L}e^{i\varepsilon_\phi}e^{-i\varepsilon_R} = 1$, and it would have been the β_2 term that was left invariant. Thus, any symmetry of this simple U(1) class will fail to eliminate all of the β terms.

Nonetheless, in the case, for example, where the g term is retained and the β_3 term survives the symmetry, we could still eliminate the role of β_3 in the VEV seesaw relation by taking κ_2 to be very small or zero. This presents no problems for the lepton sector; however, we must still examine the quark Yukawa couplings. Since the latter involve no Majorana-type term of the form Eq. (3.15), it is clear that the quark Ψ's can transform in any manner that we wish. Thus, we can choose to retain either the $\mathcal{F}$ or $\mathcal{G}$ term of Eq. (2.9), but not both. ($\mathcal{F} \neq 0$ is preferable for a small κ_2 limit. This would require the quark Ψ's to transform oppositely to the lepton Ψ's.) This leads to two awkward phenomenological predictions: (a) we predict proportionality between up- and down-type-quark masses, with the ratio being determined by $\kappa_1/|\kappa_2|$; and (b) the CKM matrix is predicted to be a unit matrix. Both predictions could be altered by one-loop corrections. While proportionality between up- and down-quark masses is not in perfect agreement with observation, the mismatch with experiment could easily be overcome by small corrections from additional new physics or radiative effects. The appropriate value for $\kappa_1/|\kappa_2|$ is of order 30 for a top-quark mass of 145 GeV. For such a ratio, the contribution of the β_3 term in the VEV seesaw would be suppressed by a factor of 10^{-3} compared to naive expectations, and β_3 would only have to be fine-tuned at the level of 10^{-2} to 10^{-3}. Though this is a considerable improvement, such a level of fine-tuning would still be unnatural. Of course, if the cosmological constraint of Eq. (3.13) is accepted, then the fine-tuning constraint becomes a factor of 10^{-5} worse, and the very small size required for β_3 could not possibly be explained in any natural way. Thus, this possibility seems quite contrived and we will not examine its phenomenology further.

Thus, we have failed to find any additional symmetry that will allow us to eliminate *all* the β-type Higgs potential terms, thereby completely escaping the restrictions of the VEV seesaw relation, while maintaining the phenomenologically attractive lepton Majorana coupling. Nonetheless, one can certainly imagine embedding the LR model in a larger GUT scheme. Then, it can easily be imagined that there is a hidden group under which the various fields transform in such a way that the β terms are eliminated, while leaving invariant all the Yukawa couplings and all the other Higgs potential terms. Certainly, these β-type terms in the potential are distinct from all the others in that the Δ_R and Δ_L fields each enter linearly (cf. the Appendix). We shall pursue the phenomenology for this type of model shortly.

2. Symmetries that zero the Majorana couplings

It is straightforward to construct a symmetry operation that forces all the β terms to be zero, provided we allow the Majorana coupling to be noninvariant under the symmetry (and thereby zeroed when the symmetry is imposed). However, it is essentially mandatory that any symmetry which zeroes the Majorana coupling must allow for simultaneous invariance of *both* the ϕ and $\tilde{\phi}$ Dirac couplings, thereby allowing retention of both the $\mathcal{F}$ and $\mathcal{G}$ couplings in the quark sector. (The f and g lepton sector couplings are also both allowed in such a case.)

The reason for needing to retain both the $\mathcal{F}$ and $\mathcal{G}$ Dirac quark couplings is as follows. In the lepton sector, Eq. (2.31) must be satisfied in the absence of a Majorana coupling. Satisfying this equation requires a very small ratio for, say, f/g, and even then a sufficiently small ratio for m_{ν_e}/m_e is achieved only for $|\kappa_2|/\kappa_1 \lesssim 10^{-5}$. However, this yields completely unacceptable results for the quark sector unless both $\mathcal{F}$ and $\mathcal{G}$ are nonzero. For instance, if $\mathcal{G}$ is required to be zero in the quark sector, then the ratio of up-type to down-type quark masses would be $\kappa_1/|\kappa_2| \gtrsim 10^5$, which is much too large. (The CKM matrix would also be trivial, but higher order corrections in some extended model might fix this up.) This problem does not arise if $\mathcal{F}$ and $\mathcal{G}$ can both be nonzero. Even though obtaining the correct m_{ν_e}/m_e ratio will continue to require that f/g and $|\kappa_2|/\kappa_1$ be very small, so long as $\mathcal{G}$ and $\mathcal{F}$ are not too different in size, acceptable quark masses can be obtained. Indeed, values for these quark Yukawas that yield the experimentally determined quark masses are easily found, and require no more fine-tuning than is typical of the Yukawa couplings to the Higgs boson in the standard model.

The conditions for retaining all the Dirac Yukawa couplings have already been given in the previous subsection: in the simplest case one need only define the symmetry so that the fermion Ψ's (both lepton and quark) and the bidoublet field ϕ remain invariant. It is then easy to specify transformation properties for the Δ_R and Δ_L that force the β's to be zero, while retaining all other Higgs potential terms. For example, we can define the symmetry operation as

$$\Delta_L \to \Delta_L \ , \quad \Delta_R \to -\Delta_R \ . \tag{3.24}$$

Since the β terms have one Δ_L and one Δ_R field, they will not be invariant under this symmetry, whereas all the other Higgs potential terms have an even number of Δ_L fields and an even number of Δ_R fields. As expected, this symmetry does, however, imply that the Majorana Yukawa coupling of Eq. (3.15) must be absent.

IV. LEFT-RIGHT PHENOMENOLOGY

A. A detailed look at "realistic" models

In light of the discussion of the previous section, it is apparent that the VEV seesaw relation forces us into a rather narrow set of possibilities. These may be summarized as follows: (1) the β_i terms are of order 1, and the extra gauge and Higgs bosons have mass scales on the order of $\sim 10^7$ GeV; (2) the β_i terms are "fine-tuned" by hand (order by order) to satisfy the see-saw relation with $\gamma \sim 10^{-7}$ [or $\gamma \sim 10^{-12}$ assuming Eq. (3.13)]; (3) the β_i terms are forced to vanish by the symmetry of Eq. (3.24), leading to a Dirac neutrino, and thereby to the requirement of a very small value for κ_2/κ_1; or (4) the β_i terms are constrained to vanish in a context beyond our model [25]; Regarding the second possibility, we acknowledge that if one is allowed to fine-tune the set of β parameters, the seesaw constraint can be obviated and light-boson masses can arise. This is reminiscent of the Yukawa couplings in the standard model which are adjusted to 1 part in $\sim 10^5$. Although this fine-tuning can be done to the set of β parameters in the LR model, we note that the tuning necessary to effectively remove the seesaw constraint is 1 part in $\sim 10^7$ or $\sim 10^{12}$ depending on the cosmological limits incorporated. For these reasons, we set aside the "fine-tuned" model; once one resorts to fine-tuning the parameters, there is little predictive power left in the model. Although the first case is the most natural, it contains no new particles of observable mass scale. We shall return to this case in the next section. Here, we briefly investigate the characteristics of the third and fourth cases.

Assuming that the β_1, β_2, and β_3 terms can be eliminated, the remnant of the seesaw relation of Eq. (3.2) which arises from the first derivative conditions is

$$0 = (2\rho_1 - \rho_3) v_L v_R \quad . \tag{4.1}$$

We have three obvious choices to satisfy Eq. (4.1): (a) set $v_R = 0$, (b) set $(2\rho_1 - \rho_3) = 0$, or (c) set $v_L = 0$. We immediately dismiss choice (a), as this would yield a mass for W_R in the range of m_{W_L}, in contradiction to observation. In the case where both v_L and v_R are nonzero, and $2\rho_1 - \rho_3 = 0$, diagonalization of the mass matrices (see the Appendix) reveals the fact that the neutral bosons of the left-handed triplet, $\delta_L^{0\,r}$ and $\delta_L^{0\,i}$, are massless at the tree level. If they are truly massless, then they will be invisible simply because they will not decay. However, it is quite likely that small masses for the $\delta_L^{0\,r}$ and $\delta_L^{0\,i}$ are generated at one loop, in particular through the Yukawa Majorana term of the Lagrangian. Nonetheless, even if they develop a small mass from such one-loop corrections, their only couplings and decays are to $\nu\nu$-type channels which are also invisible. Since the known Z would decay into a pair of δ_L^0 particles with a width equivalent to that of two neutrino generations [22, 5], this possibility appears to be eliminated by the recent neutrino counting limits reported by the Mark II group at the SLAC Linear Collider (SLC) and by the various CERN LEP experiments [12]. Thus, choice (c) ($v_L = 0$) is the only phenomenologically viable scenario among those that arise when all the β parameters of the Higgs potential are zero. Hence, we will examine the two cases, (3) and (4) defined above, for choice (c).

For case (4c), where Majorana neutrino mass terms are allowed, $v_L = 0$ leads to an attractive scenario for lepton and neutrino masses. In particular, we retain a mechanism for generating neutrino masses — a primary motivation for selecting the triplet Higgs representation — without predicting too small a N_e mass given the small ν_e mass. This can be seen from Eq. (2.30) by setting $v_L = 0$ and noting that h_D^e and h_M^e are nonzero (and relatively unconstrained) in this case. Indeed, we had already seen in the arguments following Eq. (2.30) that consistency of N_e and ν_e masses with experiment tended to require very small values of v_L. Case (3c) is possibly less attractive because of the small value required for $|\kappa_2|/\kappa_1$, and the absence of a seesaw mechanism in neutrino mass generation. However, there is no fundamental difficulty for such a model so long as both of the Dirac couplings $\mathcal{F}$ and $\mathcal{G}$ are nonzero in the quark sector. Thus, we have arrived at two models which potentially yield reasonable phenomenology, but *only* for a highly constrained set of Higgs-boson couplings and VEV's. We find this result to be particularly interesting, given the apparently large number of free parameters in this model.

In fact, it is worth reemphasizing what has occurred in our analysis up to this point. We have required that a phenomenologically acceptable minimum of the Higgs potential (1) allow for generation of proper fermion masses, (2) allow for generation of proper boson masses, and (3) respect charge conservation and vacuum stability. The task of finding such a minimum is greatly complicated by relations such as the VEV seesaw which arises from the first-derivative conditions. These relations must be satisfied in order to generate a minimum of the Higgs potential, but they have the (unnatural) property of relating parameters across widely differing scales, and can lead to (intuitively) unexpected results, as will again become apparent in the following section. We have seen that it is far from trivial to satisfy these relations without encountering severe phenomenological difficulties. Indeed, even in some subclasses of the (4c) and (3c) models that have survived to this point, additional phenomenological constraints will arise from flavor-changing neutral-current limitations. Satisfying these constraints will require additional restrictions upon these two models. We study an example of such a subclass in the following section.

Certainly, to demonstrate that our two class (c) LR-symmetric models, with v_L and all β's equal to zero, are not necessarily free of phenomenological disaster requires further analysis. The phenomenology of this class of models has been examined [5]; however, a complete analytical analysis of these models is far too complex to reasonably consider. We shall illustrate the precarious position of this class of models by examining the following "toy" model which is very similar to the general "realistic" case.

B. An illustrative look at a "toy" model of the $\beta_i = 0$, $v_L = 0$ class

In this section, we briefly look at the class (4c) and (3c) models of the above section ($\beta_i = 0, v_L = 0$) with one slight variation: we eliminate the μ_2^2, λ_4, and α_2 terms from the Higgs potential. We make this selection because (1) this choice allows us to solve this model analytically, (2) it is similar to the "realistic" case discussed above (only differing by three terms in the Higgs potential) and, therefore, we might expect some features of our "toy" model to teach us about the more general class of LR-symmetric models, and (3) this "toy" model has some interesting properties which merit investigation. Specifically, we shall show that this model has significant difficulties in matching experimental phenomenology, requiring further restrictions on the model parameters. Such difficulties may be indicative of problems that one must be careful to avoid in the more general type (4c) and (3c) "realistic" LR-symmetric models.

The above-mentioned parameters can be eliminated in the Higgs potential by applying the symmetry $\phi \to i\phi$. However, this symmetry cannot be extended consistently to the entire Lagrangian without eliminating one of the two Dirac Yukawa coupling terms, yielding a trivial tree-level CKM matrix. More importantly, the predicted quark masses would be totally unsatisfactory. Indeed, since we shall also discover that the resulting model requires κ_2 to be zero, it is clear from previous discussions that we would obtain vanishing down-type-quark masses. In particular, for $\mathcal{G} = 0$ Eqs. (2.14) and (2.16) predict that $m_b = m_t|\kappa_2|/\kappa_1 = 0$ for $\kappa_2 = 0$; radiative corrections to this result are unlikely to be sufficiently large that a reasonable b-quark mass could be obtained. Thus, this ϕ symmetry has to be restricted to the Higgs potential if we are to have reasonable phenomenology. Consequently, we label this special parameter case a "toy" model.

We shall focus our discussion on the imaginary Higgs-boson mass matrix to illustrate one potential phenomenological difficulty of this restricted subclass of the (4c) and (3c) type models. This difficulty arises in the general case where *both* $\kappa_1, \kappa_2 \neq 0$. In this case, we can simplify the mass matrix using the full set of first derivative conditions; the matrix in the $\{\phi_-^{0i}, \phi_+^{0i}, \delta_R^{0i}, \delta_L^{0i}\}$ basis is (for the case where the β_i are zero, we use the first-derivative conditions to eliminate the μ_i^2 parameters and α_3 in the mass matrix elements given in Appendix section 3)

$$\mathcal{M}_{0i}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -8\kappa_+^2\lambda_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_R^2(\rho_3 - 2\rho_1)/2 \end{pmatrix} . \tag{4.2}$$

We focus on the first two entries on the diagonal. We see that the imaginary component of ϕ_-^0 (ϕ_-^{0i}) is the massless Goldstone mode, while the flavor-changing ϕ_+^{0i} Higgs boson mass squared is given by $M^2(\phi_+^0) = 8(\kappa_1^2 + \kappa_2^2)(-\lambda_2) =\sim 32 m_{W_L}^2 |\lambda_2|/g^2$. We immediately see that the ϕ_+^{0i} Higgs boson will violate the FCNC bound of Eq. (2.36) unless λ_2 is very large (~ 200); but, a large λ_2 will lead to a violation of unitarity, and a breakdown of perturbation theory [11].

What this model illustrates is that, despite the large number of (apparently) free parameters in the Higgs potential, one is in great danger of losing the freedom to decouple the mass scales of the FCNC Higgs bosons (which must be $\gtrsim 10$ TeV), from the mass scale of the standard-model Higgs boson (which must be $\lesssim 1$ TeV). A detailed analysis of the "realistic" model of the previous section shows that it is possible to decouple these mass scales, without further restrictions on the model, but only by careful choices of the Higgs potential parameters.

In the case of our "toy model," there is also a way to avoid the FCNC problem, but it requires a special scenario for the VEV's. (See Fig. 2.) This scenario leads to restrictive but, not necessarily unacceptable, phenomenology. In particular, we take (for example) $\kappa_2 = 0$. When μ_2^2, α_2, λ_4, and the β_i terms are set to zero, the first-derivative conditions become homogeneous; i.e.,

$$\frac{\partial V}{\partial \kappa_i} = \kappa_i \; f_{\kappa_i}(\ldots) \; , \tag{4.3}$$

where $f_{\kappa_i}(\ldots)$ is a general quadratic function of the VEV's, and κ_i represents any of the VEV's. Therefore, we can satisfy the first-derivative conditions by setting *either* $f_{\kappa_i}(\ldots) = 0$ or $\kappa_i = 0$. We can escape from the FCNC difficulties by taking $\kappa_2 = 0$, so that the associated constraint $f_{\kappa_2}(\ldots) = 0$ is removed. This, it turns out, restores our freedom to decouple the mass scale of the standard-model Higgs boson from the mass scale of the FCNC Higgs boson. (This can be checked using the equations for the imaginary components of the neutral mass matrix of the Appendix obtained prior to first-derivative substitutions.)

What are the phenomenological implications of this? First, we remind the reader that even if there is some (small) amount of explicit CP violation in the Higgs potential, thereby in general allowing for a small amount of violation of manifest or quasimanifest left-right symmetry in the model, the restriction of $\kappa_2 = 0$ will restore MLRS or QMLRS. See the discussion following

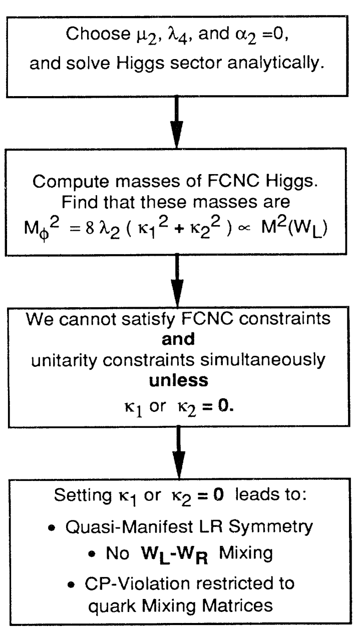

FIG. 2. Flow chart for the "toy" model of Sec. IV. This diagram summarizes the phenomenological constraints relevant to this specific class of models.

Eq. (2.16). If MLRS or QMLRS holds, we have already noted that the lower bound [9] on m_{W_R} of 1.6 TeV is iron clad. [Without the manifest left-right-symmetry constraint, the lower bound on m_{W_R} is reduced [10], depending (in the minimal LR models we are exploring) upon the amount of explicit CP violation allowed.] Additionally, since the W_L-W_R mixing angle (ξ) is approximately given by [26, 27]

$$\tan(2\xi) \simeq \frac{2\kappa_1\kappa_2}{v_R^2} \equiv \frac{4m_{W_L}^2}{m_{W_R}^2} \left(\frac{\kappa_1\kappa_2}{\kappa_+^2}\right) \; , \tag{4.4}$$

setting κ_2 (or κ_1) equal to zero implies that there is no W_L-W_R mixing. This implies that there can be no phases in W_L-W_R mixing, thus eliminating this source of CP violation. Consequently, even if CP violation is introduced into the Higgs potential explicitly, if $\kappa_2 = 0$ the only sources of CP violation in the interactions of the W_L will be those of the standard model. In particular, the W_L contributions to the electric dipole moment of the neutron and the parameter ε' arise from the complex phase in the CKM matrix, and are identical to the standard-model results [27, 26]. Of course, Higgs-boson- and W_R-, etc., exchange diagrams could give additional contributions.

So far, all that we have said is the same whether we consider the "toy" model as falling into class (4c) or class (3c). Since $\kappa_2 = 0$ is required to remove FCNC Higgs-boson problems in any case, the fact that the class (3c) model (with no Majorana Yukawa coupling) requires small or zero κ_2 in order to achieve an acceptably small value for m_{ν_e}/m_e presents no additional restriction. In fact, the main distinction between the two models is the fact that in class (3c) the N and ν combine to form a (light) Dirac neutrino, while in class (4c) we have (by fiat) retained the Majorana couplings and the neutrino-mass seesaw mechanism.

Although these two "toy" models that we examined formed a very small subset of the (4c) and (3c) general LR-symmetric models, they clearly emphasize the point that the parameters of the general model are severely constrained by experimental phenomenology in a complex way that can only be accurately described by analytically solving the fully general case — a formidable task that we do not attempt here. The analytic solution of this "toy" model highlights potential phenomenological difficulties that may arise for the general case when multiple constraints are applied simultaneously. However, we *have* examined the general case to ensure that it has sufficient freedom to decouple the masses of the "standard-model" and the FCNC Higgs bosons for some choices of the parameters. But, it is not feasible to completely map out the region of parameter space [within the general class (4c) and (3c) models] in which the FCNC and other potential phenomenological problems do not arise.

V. THE GENERAL HIGGS SECTOR

Having examined the possible branches of the minimal LR model outlined in Fig. 1, we now turn our discussion toward general properties of the Higgs sector which are common to all of these different branches. It is important to note that the material discussed in this section does not depend upon any additional symmetries or special choices of the parameters; this analysis applies to the most general case. In particular, it applies even to the case where all the parameters (in particular the β's) of the most general Higgs potential are nonzero, and have not been fine-tuned to be small. Recall that in this latter case, v_R must be very large, and we shall verify that all extra Higgs and gauge bosons (beyond the SM) will be very heavy. Additionally, much of this discussion is qualitatively applicable to many of the extensions of the LR models.

We have performed the complete minimization of the general Higgs potential, and outlined the results in the Appendix. Using these results, it is possible to characterize the probable ranges of masses and mixings. In this analysis, we will assume that none of the VEV's are exactly zero. However, we will assume that explicit CP violation is small, implying that the complex phases of the VEV's are likely to be small, i.e., of order v_L/v_R. Thus, we have performed the analysis in the approximation where we take the VEV's to be real. (This simplification is assumed so that the results are comprehensible. The previously mentioned analysis of the FCNC constraints was performed in the most general context with complex α_2 and complex VEV's.) This has the effect of decoupling the real and imaginary components of the Higgs fields, and thus the general 8×8 neutral mass matrix reduces to two separate 4×4 pieces.

When estimating the scale of the boson masses, it is very important to first impose the minimization conditions because these conditions give rise to subtle relations between the extreme mass scales involved. For example, one cannot ignore all terms of order v_L relative to v_R; in particular, the VEV seesaw relation $\beta_2\kappa_1^2+\beta_1\kappa_1\kappa_2+\beta_3\kappa_2^2=(2\rho_1-\rho_3)v_Lv_R$ connects these scales. Therefore, we must substitute all conditions arising from the minimization *before* we analyze the physical mass scales of the Higgs bosons and their mixings.

In the Appendix, we first present the general form of the mass matrices, and then substitute the minimization conditions. In addition to substituting the relations arising from the first-derivative conditions, one must also ensure that the second derivatives are positive so that the physical mass values squared are positive; this step involves a large set of complicated inequalities. Once again, we have not attempted to map out the entire region of parameter space such that these inequalities are satisfied. We have, however, verified that choices for the parameters do exist such that there are no obvious conflicts with phenomenology. For instance, it is clear that taking v_R to be very large avoids many potential difficulties.

Having utilized the minimization conditions, we can safely neglect terms of order (v_L/v_R). For the neutral sector, it is useful to perform a rotation of the fields from the gauge-eigenstate basis to what we shall call the flavor-eigenstate basis. This rotation is the same as the rotation discussed in Sec. II, and it identifies the flavor-conserving and flavor-changing components of the ϕ fields. A schematic form of the mass matrix for the real components of the neutral field in the $\{\phi_-^{0r},\phi_+^{0r},\delta_R^{0r},\delta_L^{0r}\}$ basis is

$$\begin{pmatrix} \lambda\kappa^2 & \lambda\kappa^2 & \alpha v_R\kappa & 0 \\ \lambda\kappa^2 & \alpha v_R^2 & \alpha v_R\kappa & \beta v_R\kappa \\ \alpha v_R\kappa & \alpha v_R\kappa & 2\rho_1 v_R^2 & 0 \\ 0 & \beta v_R\kappa & 0 & \frac{1}{2}(\rho_3-2\rho_1)v_R^2 \end{pmatrix}. \tag{5.1}$$

Here, we have introduced a shorthand notation where the parameters $\{\alpha,\beta,\lambda,\rho,\kappa\}$ without subscripts stand for generic parameters of their class, and have indicated for each entry only the largest contributing type of term. The exact entries are presented in the Appendix. (This same generic notation will be used for the other mass matrices that follow.) Note that after we have dropped the (v_L/v_R) terms, the δ_R^{0r} and δ_L^{0r} fields do not mix directly. Also, it is interesting to note that although the ϕ_-^{0r} and ϕ_+^{0r} fields are not mass eigenstates, their mixing is *doubly* suppressed; i.e., the mixing angle is $\sim(\kappa/v_R)^4$, not $\sim(\kappa/v_R)^2$ as naively expected. We see that in the limit that the β couplings (and v_L) vanish, the δ_L^{0r} field decouples from the other Higgs bosons. Furthermore, the mass squared of this boson is proportional to $(\rho_3-2\rho_1)$, a combination we have seen and shall continue to see frequently. Additionally, the mass squared of the δ_R^{0r} field is proportional to (ρ_1). Therefore, we find that $\rho_1\geq 0$ while $(\rho_3-2\rho_1)\geq 0$. This implies that significant cancellation in $\rho_3-2\rho_1$ is not altogether improbable, which in turn implies that the masses of the $\{\delta_L\}$ bosons could easily be lighter than the naive estimate of $v_R\sim m_{W_R}$.

For the neutral imaginary mass matrix in the $\{\phi_-^{0i},\phi_+^{0i},\delta_R^{0i},\delta_L^{0i}\}$ basis, we find

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha v_R^2 & 0 & \beta v_R\kappa \\ 0 & 0 & 0 & 0 \\ 0 & \beta v_R\kappa & 0 & \frac{1}{2}(\rho_3-2\rho_1)v_R^2 \end{pmatrix}. \tag{5.2}$$

Note that in this form the Goldstone bosons $\{\phi_-^{0i},\delta_R^{0i}\}$ are readily apparent. As before, we see that the δ_L^{0i} field decouples as the β couplings (and v_L) vanish. The mass scales of the various Higgs bosons are as expected. The neutral real mass matrix is such that the ϕ_-^{0r} can be light, while the other Higgs bosons are likely to have masses set by the scale v_R. The neutral imaginary mass matrix has the two required zero eigenvalues, but the other two masses will again be set by v_R. Thus, if v_R is very large, as required if the β's are not fine-tuned or required to be zero, all the non-standard-model Higgs bosons in the neutral sector will be very heavy.

For the singly charged Higgs sector, we will exhibit the results in the $\{(\kappa_1\phi_1^+ + \kappa_2\phi_2^+)/\kappa_+,(\kappa_1\phi_2^+ - \kappa_2\phi_1^+)/\kappa_+,\delta_R^+,\delta_L^+\}$ basis. We find

$$\begin{pmatrix} \alpha v_R^2 & 0 & \alpha v_R\kappa & \beta v_R\kappa \\ 0 & 0 & 0 & 0 \\ \alpha v_R\kappa & 0 & \alpha\kappa^2 & \beta\kappa^2 \\ \beta v_R\kappa & 0 & \beta\kappa^2 & \frac{1}{2}(\rho_3-2\rho_1)v_R^2 \end{pmatrix}. \tag{5.3}$$

In this form, the Goldstone corresponding to W_L is obvious. The Goldstone corresponding to W_R is a complicated mix of the remaining fields; it is predominantly δ_R^+ with mixings of order (κ/v_R). As was the case for the neutral δ_L fields, we see that the δ_L^+ field has a mass-squared matrix entry of order v_R^2 (that could be suppressed for small $\rho_3 - 2\rho_1$) and that it decouples as the β couplings (and v_L) vanish.

For the doubly charged sector, in the $\{\delta_R^{++}, \delta_L^{++}\}$ basis, we find a mass-squared matrix of the form

$$\begin{pmatrix} 2\rho_2 v_R^2 & \beta\kappa^2 \\ \beta\kappa^2 & \frac{1}{2}(\rho_3 - 2\rho_1)v_R^2 \end{pmatrix} . \quad (5.4)$$

The eigenstates will generally have mass of order v_R, and, in the pattern of previous δ_L fields, we see that the δ_L^{++} field decouples as the β couplings (and v_L) vanish.

In conclusion, we see that, as $v_R \to \infty$, the masses of all the extra gauge bosons and the Higgs bosons will also approach infinity *except* for the (LR analogue of the) standard-model Higgs boson whose mass scale is of order $\sim \kappa \sim m_{W_L}$, independent of the magnitude of v_R. As such, it could well happen that the only signature of an underlying LR-symmetric theory that will be accessible at present and foreseeable machines, will be a Majorana-type neutrino in addition to the neutral Higgs boson that plays the role of the SM Higgs boson in the LR model. Conversely, if the extra gauge bosons and Higgs bosons are within reach of the future colliders, this LR model will exhibit some very interesting phenomenology, as has been discussed, for example, in Refs. [1, 3, 5, 23, 28, 24]. In particular, Ref. [5] discusses signatures and production mechanisms for the various Higgs bosons at e^+e^- and hadron colliders, focusing especially on the experimentally accessible signals for the left-handed triplet Δ_L members as a function of the parameter combination $\rho_3 - 2\rho_1$ which we have seen controls the magnitude of the v_R^2 entry in the mass squared of the δ_L's.

VI. CONCLUSIONS

In this paper, we have presented a detailed analysis of the spontaneous symmetry breaking and the Higgs sector of the conventional minimal $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ left-right-symmetric theory. Specifically, we performed a critical assessment of the phenomenological viability of such models, and indicated the degree of "fine-tuning" necessary to satisfy experimental observations.

We have demonstrated that it is nontrivial to obtain a minimum of the Higgs potential which yields phenomenologically acceptable boson and fermion mass values; this task is further complicated by relations such as the VEV seesaw conditions which have the (unnatural) property of relating parameters across widely differing scales. There are many attractive aspects to a left-right-symmetric gauge theory, including (i) a mechanism for neutrino mass generation, (ii) a VEV seesaw relation which naturally requires $v_R \gg \kappa$ if $v_L \ll \kappa$, and (iii) the identification of the U(1) quantum number with $(B - L)$; and (iv) a collection of (potentially) observable Higgs and gauge bosons including doubly charged Higgs boson as well as many Majorana-type Higgs boson with only leptonic couplings and thus interesting purely leptonic decay signatures. However, substantial "fine-tuning" or extra physics is necessary to allow v_R and m_{W_R} to be small enough that the phenomenology of the extra Higgs and gauge bosons can be in an experimentally accessible energy range. Whether this makes the LR-symmetric models unattractive is a judgment that we leave to the individual reader.

In the absence of such fine-tuning we have seen that v_R must be very large ($\gtrsim 10^7$ GeV). In this case, all of the new particles associated with the underlying left-right-symmetric theory will have masses set by the scale v_R, and thus be experimentally inaccessible. In light of our illustration with this minimal LR model, it is essential that the consumer of extended electroweak models should retain a degree of skepticism when considering the phenomenology of theories with extended and very complicated Higgs sectors that have not been analyzed using a complete and internally consistent Higgs potential minimization.

ACKNOWLEDGMENTS

We would like to thank V. Barger, M. E. Ebel, A. J. Grifols, P. Langacker, A. Mendez, R. N. Mohapatra, S. Pakvasa, P. B. Pal, and P. Zoglin for valuable discussions. N.G.D, B.K, and F.O. would like to acknowledge the gracious hospitality of the University of Washington Summer Institute in Theoretical Physics where part of this work was performed. This work was supported in part by the Department of Energy under Grants Nos. DE-FG-06-40224, DE-AS03-76ER70191, and DE-AC03076SF00098.

APPENDIX

1. The most general Higgs potential and vacuum expectation value choices

For our theory to be left-right symmetric, it is necessary that our Lagrangian be invariant under the (discrete) left-right symmetry defined by

$$\Psi_L \leftrightarrow \Psi_R, \quad \Delta_R \leftrightarrow \Delta_L, \quad \phi \leftrightarrow \phi^\dagger . \quad (A1)$$

A priori, it is possible that one could allow for the possibility of phases in the above left-right transformations e.g., $\phi \leftrightarrow e^{i\theta_\phi}\phi^\dagger$ or $\Delta_R \leftrightarrow e^{i\theta_\Delta}\Delta_L$, etc. However, one may always absorb such phases by appropriate global phase rotations of the fields. We shall assume that this has been done. As a result, we are not free to use global phase rotations to remove other phases that may appear in the theory. However, the phase-free form of left-right symmetry as stated in Eq. (A1) does imply that many parameters in the Higgs potential that might otherwise be complex will be required to be real. Alternatively, one could imagine allowing phases in Eq. (A1) in which case many of the Higgs potential parameters could be complex, but then global phase rotations of the fields could be employed to make them real. Following the strict form of Eq. (A1), the most general form of the Higgs potential is

$$
\begin{aligned}
V = & -\mu_1^2[\mathrm{Tr}(\phi^\dagger\phi)] - \mu_2^2[\mathrm{Tr}(\tilde\phi\phi^\dagger) + \mathrm{Tr}(\tilde\phi^\dagger\phi)] - \mu_3^2[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)] + \lambda_1\{[\mathrm{Tr}(\phi\phi^\dagger)]^2\} \\
& +\lambda_2\{[\mathrm{Tr}(\tilde\phi\phi^\dagger)]^2 + [\mathrm{Tr}(\tilde\phi^\dagger\phi)]^2\} + \lambda_3[\mathrm{Tr}(\tilde\phi\phi^\dagger)\mathrm{Tr}(\tilde\phi^\dagger\phi)] \\
& +\lambda_4\{\mathrm{Tr}(\phi\phi^\dagger)[\mathrm{Tr}(\tilde\phi\phi^\dagger) + \mathrm{Tr}(\tilde\phi^\dagger\phi)]\} + \rho_1\{[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)]^2 + [\mathrm{Tr}(\Delta_R\Delta_R^\dagger)]^2\} \\
& +\rho_2[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger)] + \rho_3[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger)] \\
& +\rho_4[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger) + \mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R)] + \alpha_1\{\mathrm{Tr}(\phi\phi^\dagger)[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)]\} \\
& +\alpha_2[\mathrm{Tr}(\phi\tilde\phi^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\phi^\dagger\tilde\phi)\mathrm{Tr}(\Delta_L\Delta_L^\dagger)] + \alpha_2^*[\mathrm{Tr}(\phi^\dagger\tilde\phi)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\tilde\phi^\dagger\phi)\mathrm{Tr}(\Delta_L\Delta_L^\dagger)] \\
& +\alpha_3[\mathrm{Tr}(\phi\phi^\dagger\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\phi^\dagger\phi\Delta_R\Delta_R^\dagger)] + \beta_1[\mathrm{Tr}(\phi\Delta_R\phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\phi^\dagger\Delta_L\phi\Delta_R^\dagger)] \\
& +\beta_2[\mathrm{Tr}(\tilde\phi\Delta_R\phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\tilde\phi^\dagger\Delta_L\phi\Delta_R^\dagger)] + \beta_3[\mathrm{Tr}(\phi\Delta_R\tilde\phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\phi^\dagger\Delta_L\tilde\phi\Delta_R^\dagger)] \ . \qquad \text{(A2)}
\end{aligned}
$$

We have explicitly written out each term completely to display the full parity symmetry of Eq. (A1). Note that because we eliminated, as discussed above, any phases in Eq. (A1), all terms in the potential are self-conjugate *except* for the α_2 term; as such, α_2 is the only parameter which may be complex. At first glance, it would appear that there are many types of terms missing from this most general potential such as $\mathrm{Tr}(\Delta_L\Delta_L^\dagger\Delta_L\Delta_L^\dagger)$. A straightforward (and lengthy) calculation has shown that the set we use in our Higgs potential is the minimal independent set from which all other terms can be constructed. (The results contained in this appendix were generated using the symbolic manipulation programs REDUCE and MATHEMATICA, and the typescript was generated directly from this output so as to minimize the possibility of introducing errors.)

Let us now discuss the phases of the VEV's that may be assumed by the neutral components of Δ_R, Δ_L, and ϕ. Since we have employed our global phase degrees of freedom in eliminating phases from the left-right transformation symmetry of Eq. (A1), our only remaining freedom in choosing VEV's is that allowed by the underlying U_L and U_R transformations. Of these, only the T_L^3 and T_R^3 components are useful for the VEV's of the neutral Higgs fields. Using

$$U_L = \begin{pmatrix} e^{i\theta_L} & 0 \\ 0 & e^{-i\theta_L} \end{pmatrix} , \qquad \text{(A3)}$$

and the corresponding form for U_R, one finds

$$
\begin{aligned}
&\kappa_1 \to \kappa_1 e^{i(\theta_L-\theta_R)}, \quad \kappa_2 \to \kappa_2 e^{-i(\theta_L-\theta_R)} \\
&v_L \to v_L e^{-2i\theta_L}, \quad v_R \to v_R e^{-2i\theta_R}.
\end{aligned} \qquad \text{(A4)}
$$

Clearly, we have two degrees of freedom. We use these to choose $v_R, \kappa_1 \in \mathbb{R}^+$. It is these two VEV's that are always nonzero in the various different scenarios that we examine in this paper.

We are now in a position to consider the minimization of the potential. As outlined in the text, there are six minimization conditions when two of the VEV's (κ_2 and v_L) are, a priori, complex. Writing $\kappa_2 = |\kappa_2|e^{i\theta_2}$ and $v_L = |v_L|e^{i\theta_L}$, we may think of these six minimization conditions as resulting from the first derivatives with respect to v_R, κ_1, $|\kappa_2|$, $|v_L|$, θ_2, and θ_L. As given in more detail later (for the case of real VEV's), the first three first-derivative equations can be used to determine μ_3^2, μ_1^2, and μ_2^2, respectively. The required values for these parameters will be assumed to be an output of some GUT scenario. The remaining three first-derivative equations impose strong constraints on quadratic couplings appearing in the Higgs potential, and on the relative phases of the VEV's. We shall analyze the case in which the Higgs potential does not have any *explicit CP* violation; i.e., the single possibly complex coupling of the potential α_2 will be taken to be real. The results for the second trio of first derivatives can then be given in compact form. (A substitution for μ_3^2 from the v_R derivative will be made in the $|v_L|$ derivative equation, and the θ_2 derivative equation will be simplified using results from the θ_L derivative equation.) We find:

$$
\begin{aligned}
(2\rho_1-\rho_3)v_R v_L = {} & \beta_1\kappa_1\kappa_2\cos(\theta_L-\theta_2) + \beta_2\kappa_1^2\cos\theta_L \\
& +\beta_3\kappa_2^2\cos(\theta_L-2\theta_2) \ ,
\end{aligned} \qquad \text{(A5)}
$$

$$
\begin{aligned}
0 = {} & \beta_1\kappa_1\kappa_2\sin(\theta_L-\theta_2) + \beta_2\kappa_1^2\sin\theta_L \\
& +\beta_3\kappa_2^2\sin(\theta_L-2\theta_2) \ ,
\end{aligned} \qquad \text{(A6)}
$$

$$
\begin{aligned}
0 = {} & v_R v_L\{2\kappa_1\kappa_2\sin(\theta_L-\theta_2)(\beta_2+\beta_3) \\
& + [\kappa_1^2\sin\theta_L + \kappa_2^2\sin(\theta_L-2\theta_2)]\beta_1\} \\
& + \sin\theta_2\kappa_1\kappa_2\left[\alpha_3(v_R^2+v_L^2) + (4\lambda_3-8\lambda_2)(\kappa_1^2-\kappa_2^2)\right] \ ,
\end{aligned} \qquad \text{(A7)}
$$

where these equations come from the v_L, θ_L, and θ_2 derivatives, respectively. In these equations and the ensuing discussion, v_L refers to the magnitude $|v_L|$ and similarly for κ_2. Clearly, Eqs. (A5) and (A6) are simply the real and imaginary components of a single complex equation. A solution of this complex equation requires that the three complex plane vectors obtained from the β terms add together to give a real number. This observation is crucial to the phase arguments to appear below. The relation contained in Eq. (A5) is that which we refer to as the VEV seesaw relation in the limit where the angles are zero. The above equations are similar to those obtained in Ref. [8] in a different notation, but their angle factors appear to enter differently (and in such a way that the v_L and θ_L derivative equations are not obviously the real and imaginary components of a single complex equation); further, we have included an important substitution in order to simplify the θ_2 derivative equation.

Let us now consider the solution of the above equations. The key observation follows from Eq. (A7). Recall first that $v_R \gg \kappa_1, \kappa_2 \gg v_L$ is required for correct phenomenology. Suppose all the couplings are comparable; in particular let us first assume that the β's are not anomalously small (contrary to our eventual conclusion that they must be fine-tuned to be small in order that the neutrino mass be small) and that neither of the κ's is very small or zero. Then, assuming that the Higgs potential parameters appearing in the coefficient of $\sin\theta_2$ in Eq. (A7) are not fine-tuned so that this coefficient is tiny, Eq. (A7) implies that $\sin\theta_2$ is at most of order v_L/v_R, a very small number. Thus, θ_2 must be very near zero or π. Substituting this result into the complex vector equation, whose real and imaginary components are Eqs. (A5) and (A6), implies that the complex plane β vectors can only add to give a real number if $\sin\theta_L$ is also of order v_L/v_R. (We assume the absence of fine-tuning such that the β's have closely correlated magnitudes and opposite signs.) But, when substituted into Eq. (A7), this in turn implies that $\sin\theta_2 \sim v_L^2/v_R^2$, and so forth. Thus, it is clear that the only solutions to these two equations are those where $\theta_L = 0$ or π and $\theta_2 = 0$ or π. Of course, if $v_L = 0$ identically this result follows trivially from these equations. Next, suppose that $\kappa_2 = 0$. Then Eq. (A6) implies immediately that $\sin\theta_L = 0$; the angle θ_2 is irrelevant in this case. Next, let us imagine that the β's are all simply very small (as required for good lepton masses, see text). Then Eq. (A7) gives an even smaller result for $\sin\theta_2$, and Eqs. (A5) and (A6) continue to require that $\theta_L, \theta_2 = 0, \pi$. Finally, suppose that the β's are exactly zero as the result of some higher symmetry. Clearly, Eq. (A7) implies that $\sin\theta_2 = 0$ (unless $\kappa_2 = 0$, in which case the phase of κ_2 is irrelevant). The angle θ_L cannot be determined, but from Eq. (A5) we see that either $2\rho_1 - \rho_3 = 0$ or $v_L = 0$ is required. The former possibility is discarded on phenomenological grounds, as discussed in the text, and would, in any case, be regarded as extreme fine-tuning in the absence of a symmetry leading to the relation. Hence, for this case, we must have $v_L = 0$ and θ_L becomes irrelevant.

Even an extremely specialized fine-tuning of the parameters of the Higgs potential, where there are very strong cancellations in the coefficients of the angle factors appearing in Eqs. (A5)–(A7), fails to yield a phenomenologically viable option. For example, taking $\alpha_3 = (8\lambda_2 - 4\lambda_3)\kappa_-^2/(v_R^2 + v_L^2)$, it will be possible for phases to develop for the VEV's v_L and κ_2; however, since α_3 determines the mass scale of the FCNC Higgs boson, we lose the ability to decouple the FCNC Higgs-boson-mass scale from the analogue of the standard model Higgs-boson-mass scale. Therefore, such a fine-tuning constraint will lead to large FCNC's in contradiction with experimental observation (cf. Sec. V and Appendix sections 6 and 7). This example demonstrates the subtle interplay of the various constraints on the LR model, and these are precisely the type of constraints that we deal with in a general framework in Sec. IV. To summarize, we find that in *all* cases spontaneous CP violation does not arise. This conclusion is the same as that reached in Refs. [7] and [8] on the basis of similar arguments.

With this background, we are now in a position to compute the components of the various mass matrices. We shall continue to take α_2 to be real, so as to avoid explicit CP violation; all the other couplings are real, as already discussed in the text. Further, as a result of the phase arguments above, when α_2 is real all the VEV's will be real as well. We will present the results both before and after the first-derivative constraints have been substituted; both expressions will be useful when examining the different manifestations of the Higgs sector displayed in Fig. 1.

2. The real components of the neutral mass matrices

We first compute the components of the real mass matrix in the $\{\phi_1^{0r}, \phi_2^{0r}, \delta_R^{0r}, \delta_L^{0r}\}$ basis. The mass matrices are symmetric matrices which we require to have positive eigenvalues. We recall the shorthand $\kappa_\pm^2 = (\kappa_1^2 \pm \kappa_2^2)$:

$$M_{11}^{\rm Re} = \lambda_1(3\kappa_1^2 + \kappa_2^2) + 2\kappa_2^2(2\lambda_2 + \lambda_3) + 6\kappa_1\kappa_2\lambda_4 - \mu_1^2 + \alpha_1(v_L^2 + v_R^2)/2 + \beta_2 v_L v_R \,, \quad \text{(A8)}$$

$$M_{12}^{\rm Re} = M_{21}^{\rm Re} = 2\kappa_1\kappa_2(\lambda_1 + 4\lambda_2 + 2\lambda_3) + 3\lambda_4\kappa_+^2 - 2\mu_2^2 + \alpha_2(v_L^2 + v_R^2) + (\beta_1 v_L v_R)/2 \,, \quad \text{(A9)}$$

$$M_{13}^{\rm Re} = M_{31}^{\rm Re} = v_L(2\beta_2\kappa_1 + \beta_1\kappa_2)/2 + v_R(\alpha_1\kappa_1 + 2\alpha_2\kappa_2) \,, \quad \text{(A10)}$$

$$M_{14}^{\rm Re} = M_{41}^{\rm Re} = v_L(\alpha_1\kappa_1 + 2\alpha_2\kappa_2) + v_R(2\beta_2\kappa_1 + \beta_1\kappa_2)/2 \,, \quad \text{(A11)}$$

$$M_{22}^{\rm Re} = \lambda_1(\kappa_1^2 + 3\kappa_2^2) + 2\kappa_1^2(2\lambda_2 + \lambda_3) + 6\kappa_1\kappa_2\lambda_4 - \mu_1^2 + (\alpha_1 + \alpha_3)(v_L^2 + v_R^2)/2 + \beta_3 v_L v_R \,, \quad \text{(A12)}$$

$$M_{23}^{\rm Re} = M_{32}^{\rm Re} = v_L(\beta_1\kappa_1 + 2\beta_3\kappa_2)/2 + v_R[2\alpha_2\kappa_1 + \kappa_2(\alpha_1 + \alpha_3)] \,, \quad \text{(A13)}$$

$$M_{24}^{\rm Re} = M_{42}^{\rm Re} = v_L[2\alpha_2\kappa_1 + \kappa_2(\alpha_1 + \alpha_3)] + v_R(\beta_1\kappa_1 + 2\beta_3\kappa_2)/2 \,, \quad \text{(A14)}$$

$$M_{33}^{\rm Re} = (\alpha_1\kappa_+^2 + 4\alpha_2\kappa_1\kappa_2 + \alpha_3\kappa_2^2)/2 - \mu_3^2 + \rho_3 v_L^2/2 + 3\rho_1 v_R^2 \,, \quad \text{(A15)}$$

$$M_{34}^{\rm Re} = M_{43}^{\rm Re} = (\beta_2\kappa_1^2 + \beta_1\kappa_1\kappa_2 + \beta_3\kappa_2^2)/2 + \rho_3 v_L v_R \,, \quad \text{(A16)}$$

$$M_{44}^{\rm Re} = (\alpha_1\kappa_+^2 + 4\alpha_2\kappa_1\kappa_2 + \alpha_3\kappa_2^2)/2 - \mu_3^2 + 3\rho_1 v_L^2 + \rho_3 v_R^2/2 \,. \quad \text{(A17)}$$

3. The imaginary components of the neutral mass matrices

In a manner similar to the previous section, we compute the components of the imaginary mass matrix in the

$\{\phi_1^{0i}, \phi_2^{0i}, \delta_R^{0i}, \delta_L^{0i}\}$ basis:

$$M_{11}^{\rm Im} = \lambda_1\kappa_+^2 - 2\kappa_2^2(2\lambda_2 - \lambda_3) + 2\kappa_1\kappa_2\lambda_4 - \mu_1^2 + \alpha_1(v_L^2 + v_R^2)/2 - \beta_2 v_L v_R \ , \tag{A18}$$

$$M_{12}^{\rm Im} = M_{21}^{\rm Im} = -8\kappa_1\kappa_2\lambda_2 - \lambda_4\kappa_+^2 + 2\mu_2^2 - \alpha_2(v_L^2 + v_R^2) + \beta_1 v_L v_R/2 \ , \tag{A19}$$

$$M_{13}^{\rm Im} = M_{31}^{\rm Im} = v_L(2\beta_2\kappa_1 + \beta_1\kappa_2)/2 \ , \tag{A20}$$

$$M_{14}^{\rm Im} = M_{41}^{\rm Im} = -v_R(2\beta_2\kappa_1 + \beta_1\kappa_2)/2 \ , \tag{A21}$$

$$M_{22}^{\rm Im} = \lambda_1\kappa_+^2 - 2\kappa_1^2(2\lambda_2 - \lambda_3) + 2\kappa_1\kappa_2\lambda_4 - \mu_1^2 + (\alpha_1 + \alpha_3)(v_L^2 + v_R^2)/2 - \beta_3 v_L v_R \ , \tag{A22}$$

$$M_{23}^{\rm Im} = M_{32}^{\rm Im} = -v_L(\beta_1\kappa_1 + 2\beta_3\kappa_2)/2 \ , \tag{A23}$$

$$M_{24}^{\rm Im} = M_{42}^{\rm Im} = v_R(\beta_1\kappa_1 + 2\beta_3\kappa_2)/2 \ , \tag{A24}$$

$$M_{33}^{\rm Im} = (\alpha_1\kappa_+^2 + 4\alpha_2\kappa_1\kappa_2 + \alpha_3\kappa_2^2)/2 - \mu_3^2 + \rho_3 v_L^2/2 + \rho_1 v_R^2 \ , \tag{A25}$$

$$M_{34}^{\rm Im} = M_{43}^{\rm Im} = (\beta_2\kappa_1^2 + \beta_1\kappa_1\kappa_2 + \beta_3\kappa_2^2)/2 \ , \tag{A26}$$

$$M_{44}^{\rm Im} = (\alpha_1\kappa_+^2 + 4\alpha_2\kappa_1\kappa_2 + \alpha_3\kappa_2^2)/2 - \mu_3^2 + \rho_1 v_L^2 + \rho_3 v_R^2/2 \ . \tag{A27}$$

4. First-derivative conditions

We now compute first derivative constraints for the ϕ Higgs fields:

$$\frac{\partial V}{\partial\phi_1^{0r}} = \kappa_1^3\lambda_1 + 3\kappa_1^2\kappa_2\lambda_4 + \kappa_2^3\lambda_4 + \kappa_1\kappa_2^2(\lambda_1 + 4\lambda_2 + 2\lambda_3) + \kappa_1[-\mu_1^2 + (\alpha_1 v_L^2)/2 + \beta_2 v_L v_R + (\alpha_1 v_R^2)/2] + \kappa_2[-2\mu_2^2 + \alpha_2 v_L^2 + (\beta_1 v_L v_R)/2 + \alpha_2 v_R^2] \ , \tag{A28}$$

$$\frac{\partial V}{\partial\phi_2^{0r}} = \kappa_2^3\lambda_1 + \kappa_1^3\lambda_4 + 3\kappa_1\kappa_2^2\lambda_4 + \kappa_1^2\kappa_2(\lambda_1 + 4\lambda_2 + 2\lambda_3) + \kappa_1[-2\mu_2^2 + \alpha_2 v_L^2 + (\beta_1 v_L v_R)/2 + \alpha_2 v_R^2] + \kappa_2[-\mu_1^2 + (\alpha_1 v_L^2)/2 + (\alpha_3 v_L^2)/2 + \beta_3 v_L v_R + (\alpha_1 v_R^2)/2 + (\alpha_3 v_R^2)/2] \ . \tag{A29}$$

Using the above relations, we can solve for μ_1^2 and μ_2^2:

$$\mu_1^2 = [2v_L v_R(\beta_2\kappa_1^2 - \beta_3\kappa_2^2) + (v_L^2 + v_R^2)(\alpha_1\kappa_-^2 - \alpha_3\kappa_2^2)]/(2\kappa_-^2) + (\kappa_+^2\lambda_1 + 2\kappa_1\kappa_2\lambda_4) \ , \tag{A30}$$

$$\mu_2^2 = \{v_L v_R[\beta_1\kappa_-^2 - 2\kappa_1\kappa_2(\beta_2 - \beta_3)] + (v_L^2 + v_R^2)(2\alpha_2\kappa_-^2 + \alpha_3\kappa_1\kappa_2)\}/(4\kappa_-^2) + \kappa_1\kappa_2(2\lambda_2 + \lambda_3) + (\lambda_4\kappa_+^2)/2 \ . \tag{A31}$$

Likewise, for the Δ Higgs fields, we have

$$\frac{\partial V}{\partial\delta_R^{0r}} = (\rho_3 v_L^2 v_R)/2 + \rho_1 v_R^3 + v_L[(\beta_2\kappa_1^2)/2 + (\beta_1\kappa_1\kappa_2)/2 + (\beta_3\kappa_2^2)/2] + v_R[(\alpha_1\kappa_1^2)/2 + 2\alpha_2\kappa_1\kappa_2 + (\alpha_1\kappa_2^2)/2 + (\alpha_3\kappa_2^2)/2 - \mu_3^2] \ , \tag{A32}$$

$$\frac{\partial V}{\partial\delta_L^{0r}} = \rho_1 v_L^3 + (\rho_3 v_L v_R^2)/2 + v_R[(\beta_2\kappa_1^2)/2 + (\beta_1\kappa_1\kappa_2)/2 + (\beta_3\kappa_2^2)/2] + v_L[(\alpha_1\kappa_1^2)/2 + 2\alpha_2\kappa_1\kappa_2 + (\alpha_1\kappa_2^2)/2 + (\alpha_3\kappa_2^2)/2 - \mu_3^2] \ . \tag{A33}$$

With the above two equations, we can solve for μ_3^2, and a parameter of our choice, which we pick to be β_2:

$$\mu_3^2 = [\alpha_1\kappa_+^2 + 4\alpha_2\kappa_1\kappa_2 + \alpha_3\kappa_2^2 + 2\rho_1(v_L^2 + v_R^2)]/2 \ , \tag{A34}$$

$$\beta_2 = [-\beta_1\kappa_1\kappa_2 - \beta_3\kappa_2^2 + (2\rho_1 - \rho_3)v_L v_R]/\kappa_1^2 \ . \tag{A35}$$

5. Rotation to the flavor-diagonal basis

Referring back to Sec. II, we shall find it useful to rotate the neutral fields into what we shall call the flavor diagonal basis. That is, we go from the $\{\phi_1, \phi_2, \delta_R, \delta_L\}$ basis to the $\{\phi_-, \phi_+, \delta_R, \delta_L\}$ basis. Recall that it is the ϕ_- which is flavor-diagonal, and therefore must dominantly comprise the LR analogue of the light "standard-model" Higgs boson.

To this end, we define rotation matrix

$$R = \begin{pmatrix} \kappa_1/\kappa_+ & \kappa_2/\kappa_+ & 0 & 0 \\ -\kappa_2/\kappa_+ & \kappa_1/\kappa_+ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \tag{A36}$$

which will accomplish our change of basis.

6. The rotated real matrix

We now examine the real components of the mass matrix in the flavor-diagonal basis, $\{\phi_-^{0r}, \phi_+^{0r}, \delta_R^{0r}, \delta_L^{0r}\}$, with the first-derivative conditions substituted and the simplifying condition: $v_L = 0$. Note, the rotation necessary to change the basis is: $\widetilde{\mathcal{M}}^{\rm Re} = RM^{\rm Re}R^T$, where $M^{\rm Re}$ is in the mass matrix in the $\{\phi_1^{0r}, \phi_2^{0r}, \delta_R^{0r}, \delta_L^{0r}\}$ basis, and $\widetilde{\mathcal{M}}^{\rm Re}$ is in the flavor-diagonal basis:

$$\widetilde{M}_{11}^{\rm Re} = 2\lambda_1\kappa_+^2 + 8\kappa_1^2\kappa_2^2(2\lambda_2+\lambda_3)/\kappa_+^2 + 8\kappa_1\kappa_2\lambda_4 \ , \tag{A37}$$

$$\widetilde{M}_{12}^{\rm Re} = \widetilde{M}_{21}^{\rm Re} = 4\kappa_1\kappa_2\kappa_-^2(2\lambda_2+\lambda_3)/\kappa_+^2 + 2\lambda_4\kappa_-^2 \ , \tag{A38}$$

$$\widetilde{M}_{13}^{\rm Re} = \widetilde{M}_{31}^{\rm Re} = \alpha_1 v_R\kappa_+ + \kappa_2 v_R(4\alpha_2\kappa_1 + \alpha_3\kappa_2)/\kappa_+ \ , \tag{A39}$$

$$\widetilde{M}_{14}^{\rm Re} = \widetilde{M}_{41}^{\rm Re} = 0 \ , \tag{A40}$$

$$\widetilde{M}_{22}^{\rm Re} = (4\lambda_2 + 2\lambda_3)\kappa_-^4/\kappa_+^2 + \alpha_3 v_R^2\kappa_+^2/(2\kappa_-^2) \ , \tag{A41}$$

$$\widetilde{M}_{23}^{\rm Re} = \widetilde{M}_{32}^{\rm Re} = v_R(2\alpha_2\kappa_-^2 + \alpha_3\kappa_1\kappa_2)/\kappa_+ \ , \tag{A42}$$

$$\widetilde{M}_{24}^{\rm Re} = \widetilde{M}_{42}^{\rm Re} = v_R(\beta_1\kappa_1 + 2\beta_3\kappa_2)\kappa_+/(2\kappa_1) \ , \tag{A43}$$

$$\widetilde{M}_{33}^{\rm Re} = 2\rho_1 v_R^2 \ , \tag{A44}$$

$$\widetilde{M}_{34}^{\rm Re} = \widetilde{M}_{43}^{\rm Re} = 0 \ , \tag{A45}$$

$$\widetilde{M}_{44}^{\rm Re} = -v_R^2(2\rho_1 - \rho_3)/2 \ . \tag{A46}$$

7. The rotated imaginary matrix

We now examine the imaginary components of the mass matrix in the flavor-diagonal basis, $\{\phi_-^{0i}, \phi_+^{0i}, \delta_R^{0i}, \delta_L^{0i}\}$, with the first-derivative conditions substituted and the simplifying condition: $v_L = 0$. Note, the rotation necessary to change the basis is $\widetilde{\mathcal{M}}^{\rm Im} = R^T M^{\rm Im} R$, where $M^{\rm Im}$ is in the mass matrix in the $\{\phi_1^{0i}, \phi_2^{0i}, \delta_R^{0i}, \delta_L^{0i}\}$ basis, and $\widetilde{\mathcal{M}}^{\rm Im}$ is in the flavor-diagonal basis. Also note that the rotation for the imaginary matrix differs from that for the real case:

$$\widetilde{M}_{11}^{\rm Im} = \widetilde{M}_{12}^{\rm Im} = \widetilde{M}_{13}^{\rm Im} = \widetilde{M}_{14}^{\rm Im} = 0 \ , \tag{A47}$$

$$\widetilde{M}_{22}^{\rm Im} = -2\kappa_+^2(2\lambda_2 - \lambda_3) + \alpha_3 v_R^2\kappa_+^2/(2\kappa_-^2) \ , \tag{A48}$$

$$\widetilde{M}_{23}^{\rm Im} = 0 \ , \tag{A49}$$

$$\widetilde{M}_{24}^{\rm Im} = \widetilde{M}_{42}^{\rm Im} = v_R(\beta_1\kappa_1 + 2\beta_3\kappa_2)\kappa_+/(2\kappa_1) \ , \tag{A50}$$

$$\widetilde{M}_{33}^{\rm Im} = \widetilde{M}_{34}^{\rm Im} = 0 \ , \tag{A51}$$

$$\widetilde{M}_{44}^{\rm Im} = -v_R^2(2\rho_1 - \rho_3)/2 \ , \tag{A52}$$

8. The singly charged Higgs sector

We now present the singly charged mass matrix with no minimization conditions imposed in the $\{\phi_1^+, \phi_2^+, \delta_R^+, \delta_L^+\}$ basis:

$$M_{11}^+ = [\,2\lambda_1\kappa_+^2 + 4\kappa_1\kappa_2\lambda_4 - 2\mu_1^2 + \alpha_1(v_L^2 + v_R^2) + \alpha_3 v_R^2]/2 \ , \tag{A53}$$

$$M_{12}^+ = M_{21}^+ = -2\kappa_1\kappa_2(2\lambda_2+\lambda_3) - \lambda_4\kappa_+^2 + 2\mu_2^2 - \alpha_2(v_L^2 + v_R^2) \ , \tag{A54}$$

$$M_{13}^+ = (-2\beta_2\kappa_1 v_L - \beta_1\kappa_2 v_L + \alpha_3\kappa_1 v_R)/(2\sqrt{2}) \ , \tag{A55}$$

$$M_{14}^+ = M_{41}^+ = (\alpha_3\kappa_2 v_L + \beta_1\kappa_1 v_R + 2\beta_3\kappa_2 v_R)/(2\sqrt{2}) \ , \tag{A56}$$

$$M_{22}^+ = \lambda_1\kappa_+^2 + 2\kappa_1\kappa_2\lambda_4 - \mu_1^2 + [\alpha_1(v_L^2 + v_R^2)]/2 + (\alpha_3 v_L^2)/2 \ , \tag{A57}$$

$$M_{23}^+ = M_{32}^+ = (\beta_1\kappa_1 v_L + 2\beta_3\kappa_2 v_L + \alpha_3\kappa_2 v_R)/(2\sqrt{2}) \ , \tag{A58}$$

$$M_{24}^+ = M_{42}^+ = (\alpha_3\kappa_1 v_L - 2\beta_2\kappa_1 v_R - \beta_1\kappa_2 v_R)/(2\sqrt{2}) \ , \tag{A59}$$

$$M_{33}^+ = (\alpha_1\kappa_+^2)/2 + (\alpha_3\kappa_+^2)/4 + 2\alpha_2\kappa_1\kappa_2 - \mu_3^2 + (\rho_3 v_L^2 + 2\rho_1 v_R^2)/2 \ , \tag{A60}$$

$$M_{34}^+ = M_{43}^+ = (\beta_1\kappa_+^2 + 2\beta_2\kappa_1\kappa_2 + 2\beta_3\kappa_1\kappa_2)/4 \ , \tag{A61}$$

$$M_{44}^+ = (\alpha_1\kappa_+^2)/2 + (\alpha_3\kappa_+^2)/4 + 2\alpha_2\kappa_1\kappa_2 - \mu_3^2 + (2\rho_1 v_L^2 + \rho_3 v_R^2)/2 \ . \tag{A62}$$

9. The rotated singly charged Higgs sector

We now examine the imaginary components of the mass matrix in the rotated basis, with the first-derivative conditions substituted and the simplifying condition: $v_L = 0$. Note, the rotation necessary to change the basis is $\widetilde{\mathcal{M}}^+ = RM^+R^T$, where M^+ is in the mass matrix in the $\{\phi_1^+, \phi_2^+, \delta_R^+, \delta_L^+\}$ basis, and $\widetilde{\mathcal{M}}^+$ is in the rotated basis, $\{(\kappa_1\phi_1^+ + \kappa_2\phi_2^+)/\kappa_+, (\kappa_1\phi_2^+ - \kappa_2\phi_1^+)/\kappa_+, \delta_R^+, \delta_L^+\}$:

$$\widetilde{M}_{11}^+ = (\alpha_3\kappa_+^2 v_R^2)/(2\kappa_-^2) \ , \tag{A63}$$

$$\widetilde{M}_{12}^+ = \widetilde{M}_{21}^+ = 0 \ , \tag{A64}$$

$$\widetilde{M}_{13}^+ = \widetilde{M}_{31}^+ = (\alpha_3\kappa_+ v_R)/(2\sqrt{2}) \ , \tag{A65}$$

$$\widetilde{M}_{14}^+ = \widetilde{M}_{41}^+ = [v_R\kappa_+(\beta_1\kappa_1 + 2\beta_3\kappa_2)]/(2\sqrt{2}\kappa_1) \ , \tag{A66}$$

$$\widetilde{M}^{+}_{22} = \widetilde{M}^{+}_{23} = \widetilde{M}^{+}_{32} = \widetilde{M}^{+}_{24} = \widetilde{M}^{+}_{42} = 0 \ , \quad \text{(A67)}$$

$$\widetilde{M}^{+}_{33} = (\alpha_3 \kappa_-^2)/4 \ , \quad \text{(A68)}$$

$$\widetilde{M}^{+}_{34} = \widetilde{M}^{+}_{43} = [\kappa_-^2(\beta_1\kappa_1 + 2\beta_3\kappa_2)]/(4\kappa_1) \ , \quad \text{(A69)}$$

$$\widetilde{M}^{+}_{44} = [\alpha_3\kappa_-^2 - 2v_R^2(2\rho_1 - \rho_3)]/4 \ . \quad \text{(A70)}$$

10. The doubly charged Higgs sector

We now present the doubly charged mass matrix with no minimization conditions imposed in the $\{\delta_R^{++}, \delta_L^{++}\}$ basis:

$$M^{++}_{11} = [\ \alpha_1\kappa_+^2 + \alpha_3\kappa_1^2 + 4\alpha_2\kappa_1\kappa_2 - 2\mu_3^2 + \rho_3 v_L^2 + 2v_R^2(\rho_1 + 2\rho_2)]/2 \ , \quad \text{(A71)}$$

$$M^{++}_{12} = M^{++}_{21} = (\beta_3\kappa_1^2 + \beta_1\kappa_1\kappa_2 + \beta_2\kappa_2^2 + 4\rho_4 v_L v_R)/2 \ , \quad \text{(A72)}$$

$$M^{++}_{22} = [\ \alpha_1\kappa_+^2 + \alpha_3\kappa_1^2 + 4\alpha_2\kappa_1\kappa_2 - 2\mu_3^2 + 2v_L^2(\rho_1 + 2\rho_2) + \rho_3 v_R^2]/2 \ . \quad \text{(A73)}$$

Again, we substitute first-derivative conditions and set $v_L = 0$:

$$\widetilde{M}^{++}_{11} = (\alpha_3\kappa_-^2 + 4\rho_2 v_R^2)/2 \ , \quad \text{(A74)}$$

$$\widetilde{M}^{++}_{12} = \widetilde{M}^{++}_{21} = [\kappa_-^2(\beta_3\kappa_1^2 + \beta_1\kappa_1\kappa_2 + \beta_3\kappa_2^2)]/(2\kappa_1^2) \ , \quad \text{(A75)}$$

$$\widetilde{M}^{++}_{22} = \{\alpha_3\kappa_-^2 - [v_R^2(2\rho_1 - \rho_3)]\}/2 \ . \quad \text{(A76)}$$

---

* Address after August 1, 1991: Department of Physics, Southern Methodist University, Dallas, Texas 75275.

[1] G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975); R. N. Mohapatra, F. E. Paige, and D. P. Sidhu, *ibid.* **17**, 2462 (1978); R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981).

[2] C. S. Lim and T. Inami, Prog. Theor. Phys. **67**, 1569 (1982).

[3] J. F. Gunion, B. Kayser, R. N. Mohapatra, N. G. Deshpande, J. Grifols, A. Mendez, P. B. Pal, and F. Olness, *Physics of the Superconducting Super Collider, Snowmass, 1986*, Proceedings of the Summer Study, Snowmass, Colorado, 1986, edited by R. Donaldson and J. Marx (Division of Particles and Fields of the APS, New York, 1987), p. 197. See also references contained therein.

[4] J. F. Gunion, A. Mendez, and F. Olness, Int. J. Mod. Phys. A **2**, 195 (1987).

[5] J. F. Gunion, J. Grifols, A. Mendez, B. Kayser, and F. Olness, Phys. Rev. D **40**, 1546 (1989).

[6] G. Ecker, W. Grimus, and H. Newfeld, Phys. Lett. **127B**, 365 (1983); R. N. Mohapatra, G. Senjanović, and M. D. Tran, Phys. Rev. D **28**, 546 (1983); F. J. Gilman and M. H. Reno, *ibid.* **29**, 937 (1984); Phys. Lett. **127B**, 426 (1983).

[7] A. Masiero, R. N. Mohapatra, and R. D. Peccei, Nucl. Phys. **B192**, 66 (1981).

[8] J. Basecq, J. Liu, J. Milutinovic, and L. Wolfenstein, Nucl. Phys. **B272**, 145 (1986); J. Basecq and D. Wyler, Phys. Rev. D **39**, 870 (1989).

[9] G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **48**, 848 (1982).

[10] F. I. Olness and M. E. Ebel, Phys. Rev. D **30**, 1034 (1984).

[11] H. A. Weldon, Phys. Rev. D **30**, 1547 (1984); Phys. Lett. **146B**, 59 (1984); F. I. Olness and M. E. Ebel, Phys. Rev. D **32**, 1769 (1985).

[12] See, for example, the talks by B. Richter (SLC) and J. Thresher (LEP) presented at the Annual Meeting of the Division of Particles and Fields of the APS, Houston, Texas, 1990.

[13] For an interesting *LR* model which yields pseudo-Dirac neutrinos without a Higgs triplet, see P. B. Pal, University of Oregon Institute of Theoretical Science Report No. OITS-434, 1990 (unpublished).

[14] G. Senjanović, Nucl. Phys. **B153**, 334 (1979).

[15] We thank P. Langacker for a useful conversation on these points.

[16] E. Ma, Phys. Rev. Lett. **59**, 607 (1987); **62**, 1228 (1989); **63**, 1042 (1989); Phys. Rev. D **39**, 1922 (1989); K.S. Babu and Ernest Ma, Phys. Lett. B **228**, 508 (1989); Phys. Rev. Lett. **61**, 674 (1988); Mod. Phys. Lett. A **4**, 1975 (1989); D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58**, 1600 (1987); R. N. Mohapatra, Phys. Lett. B **198**, 69 (1987); **201**, 517 (1988); K.S. Babu, Ernest Ma, and J. Pantaleone, *ibid.* **218**, 233 (1989); K.S. Babu and X.-G. He, *ibid.* **219**, 342 (1989); Mod. Phys. Lett. A **4**, 61 (1989); K.S. Babu and V. S. Mathur, Phys. Rev. D **38**, 3550 (1988); K.S. Babu, Phys. Lett. B **203**, 132 (1988); B. S. Balakrishna, Phys. Rev. Lett. **60**, 1602 (1988); B. S. Balakrishna, A. L. Kagan, and R. N. Mohapatra, Phys. Lett. B **205**, 345 (1988); T. Banks, Nucl. Phys. **B303**, 172 (1988); G. C. Branco, W. Grimus, and L. Lavoura, *ibid.* **B312**, 492 (1989); G. C. Branco and C. Q. Geng, Phys. Rev. Lett. **58**, 969 (1987); S. T. Petcov and S. T. Toshev, Phys. Lett. **143B**, 175 (1984).

[17] B. Kayser, F. Gibrat-Debu, and F. Perrier, *The Physics of Massive Neutrinos*, Lecture Notes in Physics Vol. 25 (World Scientific, Singapore, 1989).

[18] Since the right CKM matrix is unknown in principle, this is nontrivial; however, limits can still be set. See Ref. [10].

[19] D. A. Dicus and V. S. Mathur, Phys. Rev. D **7**, 3111 (1973); M. Veltman, Acta Phys. Pol. **B8**, 475 (1977); B. W. Lee, C. Quigg, and H. B. Thacker, Phys. Rev. Lett. **38**, 883 (1977); Phys. Rev. D **16**, 1519 (1977).

[20] W. Marciano, G. Valencia, and S. Willenbrock, Phys. Rev. D **40**, 1725 (1989).

[21] M. Gronau, C. Leung, and J. Rosner, Phys. Rev. D **29**, 2539 (1984); J. Gunion and B. Kayser, in *Design and Utilization of the Superconducting Super Collider*, Proceed-

ings of the Summer Study, Snowmass, Colorado, 1984, edited by R. Donaldson and J. Morfin (Division of Particles and Fields of the APS, New York, 1985), p. 153.

[22] P. B. Pal, Nucl. Phys. **B227**, 237 (1983); R. N. Mohapatra and P. B. Pal, Phys. Lett. B **179**, 105 (1987); Phys. Rev. D **38**, 2226 (1988).

[23] H. Harari and Y. Nir, Nucl. Phys. **B292**, 251 (1987); Phys. Lett. B **188**, 163 (1987); M. Roncadelli and G. Senjanović, Phys. Lett. **107B**, 59 (1981).

[24] Chang and Mohapatra have examined an extension of this model that includes an additional *parity-odd singlet* field η. In the context of this extended theory, it is possible for the parity-breaking and the $SU(2)_R$-breaking scales to be decoupled such that we can achieve the goal of light neutrino masses *and* observable (mass scale $\sim$TeV) extra gauge and Higgs bosons.

[25] P. Cea and G. L. Fogli, Phys. Lett. **102B**, 22 (1981); D. Cocolicchio and G. L. Fogli, Phys. Rev. D **30**, 2391 (1984); **32**, 3020 (1985).

[26] S. M. Barr and W. J. Marciano, in *CP Violation*, edited by C. Jarlskog, Advanced Series on Directions in High Energy Physics, Vol. 3 (World Scientific, Singapore, 1989), p. 455.

[27] X.-G. He, B. McKellar, and S. Pakvasa, Phys. Rev. Lett. **61**, 1267 (1988); Int. J. Mod. Phys. A **4**, 5011 (1989).

[28] P. Langacker and S. U. Sankar, Phys. Rev. D **40**, 1596 (1989); P. Langacker, University of Pennsylvania Report No. UPR-0401T, 1989 (unpublished); P. Langacker, S. U. Sankar, and K. Schilcher, Phys. Rev. D **38**, 2841 (1988).