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Implications of precision electroweak experiments for m_t, ρ_0, $\sin^2\theta_W$, and grand unification

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The implications of precision Z-pole, W-mass, and weak-neutral-current data for $SU(2)\times U(1)$ models are described. Within the minimal model one finds $\sin^2\hat{\theta}_W(M_Z)=0.2334\pm0.0008$ in the modified minimal subtraction scheme or $\sin^2\theta_W\equiv 1-M_W^2/M_Z^2=0.2291\pm0.0034$ in the on-shell scheme, where the uncertainties include the m_t and M_H dependence. The top-quark mass is predicted to be 124^{+28+20}_{-34-15} GeV, where the second uncertainty is from M_H, with $m_t<174$ (182) GeV at 90 (95)% C.L. For the first time subleading effects and vertex corrections allow a significant separation of m_t and ρ_0 in models with a nonminimal Higgs structure. Allowing arbitrary m_t and Higgs representations one obtains $\sin^2\hat{\theta}_W(M_Z)=0.2333\pm0.0008$, $\rho_0=0.992\pm0.011$, and $m_t<294$ (310) GeV. The implications of these results for ordinary and supersymmetric grand unified theories are considered. Supersymmetric theories with a grand desert between the supersymmetry and unification scales are in striking agreement with data for $M_{\rm SUSY}$ in the M_Z–1 TeV range. Ordinary grand unified theories breaking to the standard model in more than one step are also discussed.

Weak-neutral-current data and W and Z properties have been a major quantitative test of the standard $SU(2)\times U(1)$ electroweak model [1–7]. In 1987 a systematic analysis of the implications of all existing data was carried out [2], which has been updated regularly [5,7]. There is now a considerable amount of high-precision data on the mass, total and partial widths, and asymmetries of the Z from CERN LEP [8–10], as well as important new results on the W mass [11,12], atomic parity violation in cesium [13,14], and $\overset{(-)}{\nu}_\mu e$ scattering [15]. There are also new direct 95%-C.L. lower limits $m_t>89$ GeV [16] and $M_H>48$ GeV [17] on the top-quark and Higgs boson masses, and new experimental constraints on the charm-quark threshold [18] relevant to the interpretation of deep-inelastic neutrino scattering.

It is therefore an appropriate time to reconsider the implications of all these results for testing the standard model, constraining m_t, and comparing the experimental value of $\sin^2\theta_W$ with ordinary and supersymmetric grand unified theories [19–23]. In this paper we will consider the standard $SU(2)\times U(1)$ model and extensions involving higher-dimensional representations of Higgs field [24]. Other extensions of the standard model will be considered elsewhere [25,26].

The data used in the analysis are summarized in Table I. The LEP results are averages of the four LEP experiments ALEPH, DELPHI, L3, and OPAL [10], which include all of the 1989–1990 data, with a proper treatment of common systematic errors. Γ_Z, $\Gamma_{l\bar{l}}$, $\Gamma_{\rm had}$, and $\Gamma_{\rm inv}$ refer, respectively, to the total, leptonic (average of e,μ,τ), hadronic, and invisible Z widths; $N_\nu\equiv\Gamma_{\rm inv}/\Gamma_{\nu\bar{\nu}}$ is the number of light neutrino flavors; $A_{\rm FB}(\mu)$ is the forward-backward asymmetry for muons; $\sigma_p^h=12\pi\Gamma_{e\bar{e}}\Gamma_{\rm had}/M_Z^2\Gamma_Z^2$ is the hadronic cross section on the pole; and $\bar{g}_A^2,\bar{g}_V^2$ are effective couplings related to $\Gamma_{l\bar{l}}$ and $A_{\rm FB}(\mu)$ by

$$\Gamma_{l\bar{l}}=\frac{G_F M_Z^3}{6\sqrt{2}\pi}(\bar{g}_A^2+\bar{g}_V^2)\ ,\qquad A_{\rm FB}(\mu)=\frac{3\bar{g}_V^2\bar{g}_A^2}{(\bar{g}_V^2+\bar{g}_A^2)^2}\ . \tag{1}$$

Only M_Z, Γ_Z, $\Gamma_{l\bar{l}}$, $R\equiv\Gamma_{\rm had}/\Gamma_{l\bar{l}}$, and $A_{\rm FB}(\mu)$ are used. The other LEP observables are not independent, but are displayed for completeness. Recent measurements of the W mass and weak-neutral-current data are also displayed in Table I. Older neutral-current results, included in the analysis, are described in [2,7]. The standard-model predictions for each quantity other than M_Z are also shown. These are computed using $M_Z=91.174\pm0.021$ GeV as input, using the range of m_t determined from the global fit, and 50 GeV $<M_H<$ 1 TeV.

In the standard model,

$$M_Z^2=\frac{A_0^2}{\hat{\rho}\hat{c}^2\hat{s}^2(1-\Delta\hat{r}_W)}=\frac{A_0^2}{c^2s^2(1-\Delta r)}\ ,\qquad M_W^2=\hat{\rho}\hat{c}^2M_Z^2=c^2M_Z^2\ , \tag{2}$$

where

$A_0^2=\pi\alpha/\sqrt{2}G_F=(37.2803\ \text{GeV})^2$, $\hat{s}^2\equiv\sin^2\hat{\theta}_W(M_Z)$

refers to the weak angle in the modified minimal-subtraction scheme ($\overline{\rm MS}$) scheme [27], $s^2\equiv\sin^2\theta_W=1-M_W^2/M_Z^2$ refers to the on-shell scheme [28], $\hat{c}^2\equiv1-\hat{s}^2$, and $c^2\equiv1-s^2$. The radiative correction parameters $\Delta\hat{r}_W$, $\hat{\rho}-1$, and Δr are taken from the calculation of Degrassi, Fanchiotti, and Sirlin [29]. As is well known, $\hat{\rho}\sim1+\Delta\rho_t$, where

$$\Delta\rho_t=\frac{3G_Fm_t^2}{8\sqrt{2}\pi^2}\simeq0.0031\left[\frac{m_t}{100\ \text{GeV}}\right]^2 \tag{3}$$

has a strong m_t dependence, while $\Delta r\simeq\Delta r_0$

TABLE I. Experimental values for LEP observables [10], M_W/M_Z [11], M_W [12], the weak charge in cesium Q_W [13,14], and the tree-level $\sin^2\theta^0$ from $\overset{(-)}{\nu}_\mu e \to \overset{(-)}{\nu}_\mu e$ [15], compared with the standard-model predictions for $M_Z = 91.174 \pm 0.021$ GeV, $m_t = 124^{+28}_{-34}$ GeV, and 50 GeV $< M_H <$ 1 TeV. Only the first five LEP observables are independent. The two errors for Q_W (Cs) are experimental and theoretical (in parentheses). The first uncertainty in the predictions is from the uncertainties in M_Z and Δr, the second is from m_t and M_H, and the third (in parentheses) is the theoretical QCD uncertainty. The older neutral-current quantities described in [2,7] are also used in the analysis.

Quantity	Value	Standard model
M_Z (GeV)	91.174 ± 0.021	
Γ_Z (GeV)	2.487 ± 0.009	$2.485 \pm 0.0021 \pm 0.008 (\pm 0.011)$
$\Gamma_{l\bar{l}}$ (MeV)	83.3 ± 0.4	$83.5 \pm 0.1 \pm 0.2$
$R = \Gamma_{\text{had}}/\Gamma_{l\bar{l}}$	20.94 ± 0.12	$20.78 \pm 0.003 \pm 0.016 (\pm 0.13)$
$A_{\text{FB}}(\mu)$	0.0154 ± 0.0048	$0.0142 \pm 0.0002 \pm 0.0015$
Γ_{had} (MeV)	1744 ± 10	$1735 \pm 1.6 \pm 5.6 (\pm 11)$
Γ_{inv} (MeV)	493 ± 10	$499.4 \pm 0.3 \pm 1.3$
N_ν	$2.97 \pm 0.04 \pm 0.04$	3
σ_p^b (nb)	41.44 ± 0.28	$41.44 \pm 0.02 \pm 0.02 (\pm 0.21)$
$\bar{g}_A^2$	0.250 ± 0.001	$0.251 \pm 0 \pm 0.001$
$\bar{g}_V^2$	0.0013 ± 0.0004	$0.0012 \pm 0 \pm 0.0001$
M_W (GeV)	79.91 ± 0.39	$80.05 \pm 0.03 \pm 0.19$
M_W/M_Z	0.8831 ± 0.0055	$0.8780 \pm 0.0001 \pm 0.0022$
Q_W (Cs)	$-71.04 \pm 1.58 (\pm 0.88)$	$-73.12 \pm 0.08 \pm 0.05$
$\sin^2\theta^0$	$0.240 \pm 0.009 \pm 0.008$	$0.232 \pm 0.0003 \pm 0.001$

$-\Delta\rho_t/\tan^2\theta_W$ is even more sensitive. $\Delta\hat{r}_W \sim \Delta r_0 \sim 0.07$ has no quadratic m_t dependence. There is additional logarithmic dependence on m_t and M_H in $\hat{\rho}$, $\Delta\hat{r}_W$, and Δr, as well as $O(\alpha)$ effects associated with low-energy physics. These effects are important and are fully incorporated in the analysis, but will not be displayed here.

For the Z widths, we use the results of Hollik [30], which include the m_t dependence of the $Z \to b\bar{b}$ vertex corrections [31]. For the hadronic widths we include QCD corrections to $O(\alpha_s^2)$ using an effective $\alpha_s(M_Z) = 0.12 \pm 0.02$. The uncertainty is chosen larger than the average of the LEP determination of α_s (e.g., $\alpha_s = 0.123 \pm 0.007$ [32]) in order to roughly incorporate other QCD uncertainties associated with higher-order effects. For $A_{\text{FB}}(\mu)$ we use the calculation of Degrassi and Sirlin [33], which agrees with Hollik [30].

The most precise measurement of atomic parity violation is in the cesium atom [14]. Cesium is a hydrogenlike atom, allowing a clean theoretical interpretation of the results. Recent calculations have reduced the theoretical uncertainty to the 1% level [13]. The current value of Q_W differs from the standard-model prediction by slightly over one standard deviation.

Deep-inelastic neutrino reactions are still an important constraint. The ratio of neutral-to-charged-current cross sections has been measured at the 1% level [2–4]. The most important theoretical uncertainty is the charm-quark threshold in the charged-current denominator. In [2] the threshold was parametrized [34] in terms of an effective charm-quark mass $m_c^{\text{eff}} = 1.5 \pm 0.3$ GeV determined from the experimental cross section for neutrino-induced dimuon production [35]. The m_c^{eff} range is important because a much lower value (e.g., 0.8 GeV [36]) would considerably weaken the upper limit on m_t. Recently, the Chicago-Columbia-Fermilab-Rochester (CCFR) group has measured the threshold precisely, obtaining [18] $m_c^{\text{eff}} = 1.34 \pm 0.21(\text{stat})^{+0.25}_{-0.04}(\text{syst})$ GeV, in agreement with the old result based on data from the CERN-Dortmund-Heidelberg-Saclay (CDHS) Collaboration [37]. The lower central value is offset by the smaller lower error bar in obtaining upper limits on m_t.

The fits of various data sets to the standard model are displayed in Fig. 1 and Table II. Figure 1 shows the $\pm 1\sigma$ limits on $\sin^2\hat{\theta}_W$ for fixed m_t as a function of m_t for various combinations of data. It is apparent that the combination of M_Z with M_W, Γ, $A_{\text{FB}}(\mu)$, and deep-inelastic

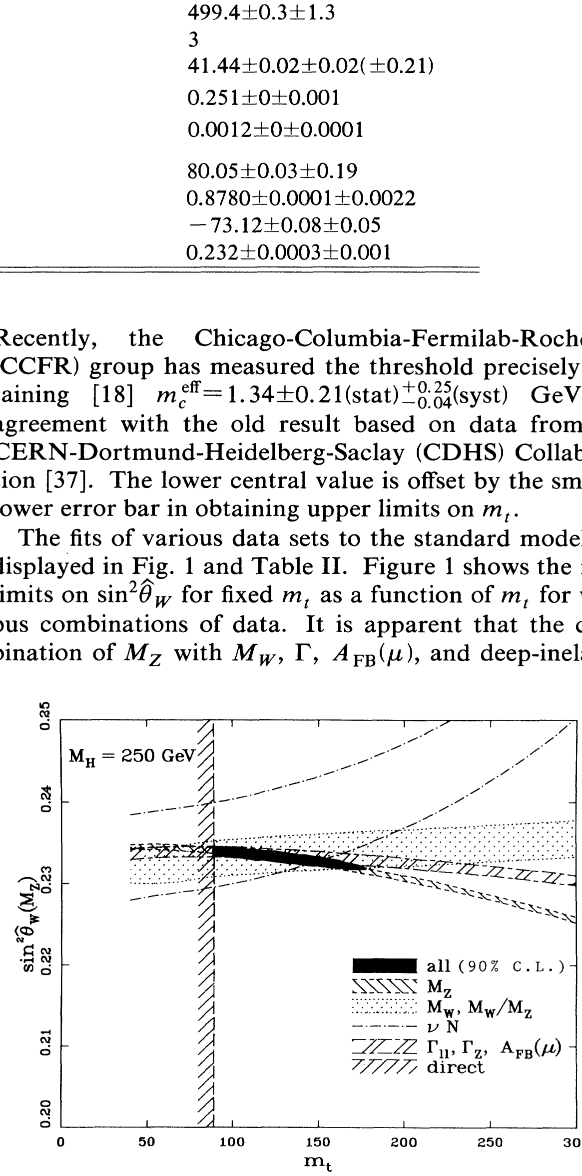

FIG. 1. Values of $\sin^2\hat{\theta}_W(M_Z)$ with $\pm 1\sigma$ errors for fixed m_t as a function of m_t for $M_H = 250$ GeV. Also shown is the 90%-C.L. region allowed in a fit to all data. The direct lower limit [16] $m_t > 89$ GeV is also shown.

TABLE II. Values of $\sin^2\hat{\theta}_W(M_Z)$ and m_t obtained for various data sets. The $\sin^2\hat{\theta}_W(M_Z)$ error includes m_t and M_H. The first error for m_t includes experimental and theoretical uncertainties for $M_H=250$ GeV. The second error is the variation for $M_H \to 50$ GeV ($-$) and $M_H \to 1000$ GeV ($+$). The last column lists the upper limits on m_t at 90% (95%) C.L. for $M_H=1000$ GeV, which gives the weakest upper limit. The direct CDF constraint $m_t > 89$ GeV is included.

Data	$\sin^2\hat{\theta}_W(M_Z)$	m_t (GeV)	$m_t^{\max}$ (GeV)
M_Z, M_W	0.2329 ± 0.0014	$139 \pm 47 \pm 16$	213 (225)
M_Z, Γ	$0.2340^{+0.0003}_{-0.0015}$	91^{+55+34}_{-2-1}	176 (188)
$M_Z, A_{\text{FB}}(\mu)$	0.2323 ± 0.0023	$158 \pm 68 \pm 23$	250 (266)
$M_Z, \Gamma, A_{\text{FB}}(\mu)$	$0.2337^{+0.0006}_{-0.0012}$	$110^{+43}_{-21} \pm 22$	179 (190)
$M_Z, M_W, \Gamma, A_{\text{FB}}(\mu)$	0.2334 ± 0.0009	$124 \pm 33 \pm 17$	179 (188)
$M_Z, \nu N$	$0.2339^{+0.0005}_{-0.0016}$	99^{+58+23}_{-10-9}	184 (197)
All	0.2334 ± 0.0008	124^{+28+20}_{-34-15}	174 (182)

νN scattering places upper limits of ~ 200 GeV on m_t. This is quantified in Table II, where the values of $\sin^2\hat{\theta}_W$ and m_t are shown for simultaneous fits to various combinations of data. The $\sin^2\hat{\theta}_W$ errors are almost entirely due to the m_t uncertainty [for fixed m_t, the Z mass would yield $\sin^2\hat{\theta}_W(M_Z)$ to ± 0.0001 (± 0.0003), where the first error is from ΔM_Z and the second is theoretical, from Δr]. The central values are for a Higgs-boson mass $M_H=250$ GeV. The uncertainties induced by allowing 50 GeV $< M_H <$ 1 TeV are folded into the $\sin^2\hat{\theta}_W$ errors and are shown explicitly for m_t. Also listed are the 90%- and 95%-C.L. upper limits on m_t from each data set. The latter are for $M_H=1$ TeV, which gives the weakest upper limit. It is seen that the LEP results alone, as well as LEP combined with M_W, yield 90%-C.L. upper limits ~ 180 GeV on m_t, confirming previous limits which were dominated by M_Z and νN scattering. The νN scattering, atomic parity violation (which does not contribute significantly to the m_t limit), and other neutral-current observables are still very important, however, because of their role in constraining new physics beyond the standard model [25].

The fit to all data yields

$$\sin^2\hat{\theta}_W(M_Z)=0.2334\pm 0.0008\ ,$$
$$\sin^2\theta_W=0.2291\pm 0.0034\ , \qquad (4)$$
$$m_t=124^{+28+20}_{-34-15}\ \text{GeV}\ ,$$

where the $\sin^2\theta_W$ errors include m_t and M_H (the on-shell $\sin^2\theta_W$ is about 4 times as sensitive to m_t), and the second error in m_t is from M_H. One can also regard $\Delta\hat{r}_W$ (or Δr) as free parameters. From a fit to all data in which $\sin^2\theta_W$, m_t, and Δr are free, one obtains $\Delta r=0.056^{+0.006}_{-0.010}$, $\Delta\hat{r}_W=0.061^{+0.010}_{-0.005}$, in good agreement with standard-model expectations [29].

The upper limit on m_t from all data is (174,182,197) GeV at (90,95,99)% C.L., respectively, for $M_H=1$ TeV. For $M_H=250$ GeV, the corresponding limits are (157,165,180) GeV, while $m_t<(144,154,170)$ GeV for $M_H=50$ GeV. The χ^2 distributions as functions of m_t are shown in Fig. 2(a) for the three values of M_H. It is apparent from Table II and Fig. 2 that the central value and upper limit on m_t are rather sensitive to M_H. However, until m_t is known independently, no significant constraint can be obtained on M_H from these types of experiments provided one assumes $M_H \le 1$ TeV. (It is not clear that it makes any sense to discuss the minimal standard model perturbatively for larger $M_H > 1$ TeV.) Even if m_t were known, present data would only constrain M_H weakly: For example, for $m_t=100$ GeV, present data would favor $M_H \sim 48$ GeV, but would still allow $M_H \sim 810$ (1150) GeV at 90% (95%) C.L.

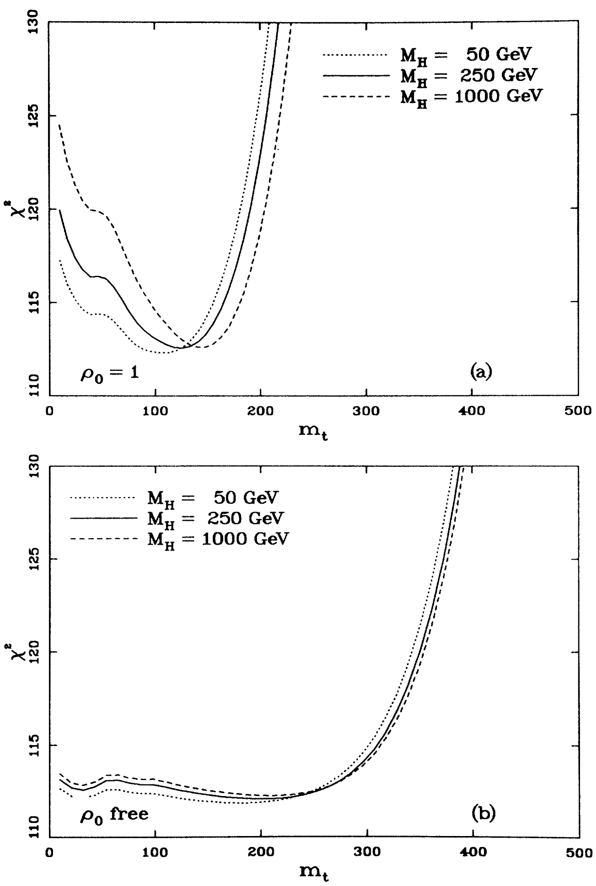

FIG. 2. (a) χ^2 distribution as a function of m_t for 134 df in a fit to all data for $M_H=50$, 250, and 1000 GeV. The χ^2 is minimized with respect to $\sin^2\hat{\theta}_W(M_Z)$ for each m_t. (b) Same as (a), except ρ_0 is also a free parameter (133 df).

Now that $\sin^2\hat{\theta}_W$ is well determined, it is useful to compare with the predictions of grand unification [19–22]. Using as inputs $\alpha^{-1}(M_Z)=127.9\pm0.2$ [29], $\alpha_s(M_Z)=0.12\pm0.012$ [8], and $\sin^2\hat{\theta}_W(M_Z)=0.2334\pm0.0008$, one obtains

$$\begin{aligned} \alpha_1^{-1} &\equiv \frac{3}{5}\alpha^{-1}\cos^2\hat{\theta}_W(M_Z)=58.83\pm0.11 , \\ \alpha_2^{-1} &\equiv \alpha^{-1}\sin^2\hat{\theta}_W(M_Z)=29.85\pm0.11 , \\ \alpha_3^{-1} &\equiv \alpha_s^{-1}=8.33\pm0.83 , \end{aligned} \tag{5}$$

at M_Z. These can be propagated to high renormalization scales using the standard two-loop renormalization-group equations [20]. Of course, in simple grand unified theories breaking in one step to the standard model (SM) or the minimal supersymmetric standard model (MSSM), one expects the three couplings to meet at the unification scale M_X. (One actually expects a calculable discontinuity in the $\overline{\text{MS}}$ scheme [20]. This is included in the analysis but is too small to see in the figures.)

The running couplings in the standard model are shown in Fig. 3(a). It is clearly seen that they do not meet at a point, thus ruling out simple grand unified theories such as SU(5), SO(10), or E_6 which break in a single step to the standard model [19]. Of course, such models are also excluded by the nonobservation of proton decay, but this independent evidence is welcome.

On the other hand, in the minimal supersymmetric extension of the standard model, the couplings do meet within the experimental uncertainties. This is illustrated in Fig. 3(b) for the case in which all of the new particles have a common mass $M_{\text{SUSY}}=M_Z$. Almost identical curves are obtained for larger M_{SUSY}, such as 1 TeV. The unification scale M_X is sufficiently large ($>10^{16}$ GeV) that proton decay by dimension-6 operators is adequately suppressed, although there may still be a problem with dimension-5 operators [23]. This success is encouraging for supersymmetric grand unified theories such as SUSY-SU(5) or SUSY-SO(10). However, some caution

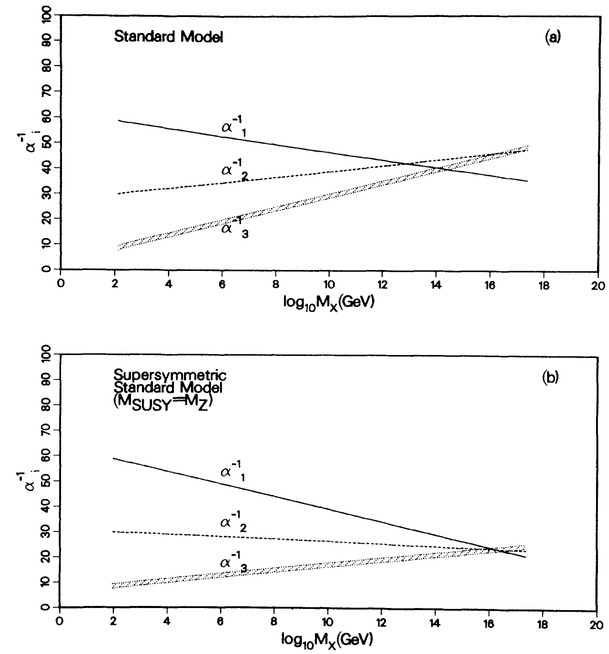

FIG. 3. (a) Running couplings in the standard model. (b) Running couplings in the MSSM with two Higgs doublets for $M_{\text{SUSY}}=M_Z$. The corresponding figure for $M_{\text{SUSY}}=1$ TeV is almost identical.

is in order, because such models have many uncertainties, such as those involving possible splitting of the superheavy-Higgs-boson masses from the unification scale [19,22].

One can also use $\alpha(M_Z)$ and $\alpha_s(M_Z)$ to predict $\sin^2\hat{\theta}_W(M_Z)$. The resulting prediction, the scale (M_X) at which $(5\alpha_1^{-1}+3\alpha_2^{-1})/8$ and α_3^{-1} unify, and the value of $\alpha_3^{-1}(M_X)$ are listed in Table III for a number of models. It is seen in Table III and Fig. 4 that ordinary grand unified theories such as SU(5) with one or two Higgs doublets predict $\sin^2\hat{\theta}_W$ considerably below the experimental

TABLE III. Predictions of $\sin^2\hat{\theta}_W$ and the unification scale in the non-SUSY and SUSY SU(5) at two-loop order, and for non-SUSY SO(10) grand unification at one loop. (The same predictions hold for many larger groups.) For SO(10) we assume $SO(10)\to SU(3)\times SU(2)_L\times SU(2)_R\times U_{B-L}$ at the unification scale M_X, while $SU(3)\times SU(2)_L\times SU(2)_R\times U_{B-L}\to SU(3)\times SU(2)_L\times U_Y$ at M_R. The inputs are $\alpha^{-1}=127.9\pm0.2$, $\alpha_S=0.12\pm0.012$, and $\sin^2\hat{\theta}_W(M_Z)=0.2334\pm0.0008$. N_H is the number of Higgs doublets. δ and Δ refer to two classes of $SU(2)_L\times SU(2)_R\times U_{B-L}$ models.

Model	SUSY scale	$\sin^2\hat{\theta}_W$	M_X	$\alpha_3^{-1}(M_X)$
SU(5)($N_H=1$)		$0.2102^{-0.0031}_{+0.0037}$	$4.5^{+3.4}_{-2.2}\times10^{14}$	$41.2^{-0.1}_{+0.2}$
SU(5)($N_H=2$)		$0.2142^{-0.0030}_{+0.0036}$	$2.9^{+2.0}_{-1.4}\times10^{14}$	$40.7^{-0.2}_{+0.2}$
SUSY SU(5)($N_H=2$)	M_Z	$0.2334^{-0.0027}_{+0.0031}$	$1.9^{+1.7}_{-1.0}\times10^{16}$	$25.2^{-0.4}_{+0.5}$
SUSY SU(5)($N_H=4$)	M_Z	$0.2561^{-0.0022}_{+0.0027}$	$9.2^{+7.0}_{-0.46}\times10^{14}$	$23.9^{-0.4}_{+0.5}$
SUSY SU(5)($N_H=2$)	1 TeV	$0.2315^{-0.0026}_{+0.0032}$	$1.4^{+1.2}_{-0.7}\times10^{16}$	$26.5^{-0.4}_{+0.5}$
SUSY SU(5)($N_H=4$)	1 TeV	$0.2525^{-0.0022}_{+0.0027}$	$8.3^{+6.3}_{-4.2}\times10^{14}$	$25.2^{-0.4}_{+0.5}$
	M_R	$\sin^2\hat{\theta}_W$	M_X	$\alpha_3^{-1}(M_X)$
SO(10)($\delta_{L,R}$)	1 TeV	$0.2811^{-0.0016}_{+0.0020}$	$2.0^{+2.1}_{-1.2}\times10^{18}$	$50.25^{-0.07}_{+0.04}$
SO(10)($\Delta_{L,R}$)	1 TeV	$0.2765^{-0.0017}_{+0.0022}$	$5.6^{+6.6}_{-3.5}\times10^{19}$	$53.97^{-0.18}_{+0.11}$
SO(10)($\delta_{L,R}$)	$4.5^{-2.0}_{+5.0}\times10^{10}$	0.2334 (input)	$1.2^{+1.8}_{-0.8}\times10^{16}$	$44.57^{-0.27}_{+0.31}$
SO(10)($\Delta_{L,R}$)	$2.2^{-1.1}_{+3.2}\times10^{10}$	0.2334 (input)	$5.3^{+10.0}_{-3.9}\times10^{16}$	$46.22^{-0.44}_{+0.56}$

value. [These predictions actually hold for many grand unification groups larger than SU(5).] On the other hand, the predictions of the supersymmetric extension of SU(5) and similar groups predicts $\sin^2\hat{\theta}_W$ in agreement with observations for two Higgs doublets and $M_{\rm SUSY}$ in the range $M_Z - 1$ TeV. The predicted value is too high for four Higgs doublets.

We have seen that the observed low-energy couplings do not unify at a single scale if the running is due to the standard-model particles only, i.e., if there is a desert between M_Z and M_X. Adding supersymmetry in the desert is one way to achieve unification. Another possibility is to allow a group larger than SU(5) to break to the standard model in two or more stages. For example, an ordinary SO(10) model can break [38] first to left-right symmetric [39] $SU(3)\times SU(2)_L \times SU(2)_R \times U_{B-L}$ at a scale M_X and then to the standard model at M_R.

The predictions of such models for a fixed M_R of 1 TeV are shown in Table III for two popular versions of the model: one in which $SU(2)_R$ breaking is accomplished by introducing $SU(2)_L$ and $SU(2)_R$ doublets δ_L and δ_R [39], and one in which Higgs triplets [40] $\Delta_{L,R}$ are introduced. The latter version can generate large Majorana masses for right-handed neutrinos, but are hard to incorporate into a superstring framework [24]. As is well known [41,19], the models generate much too high a $\sin^2\hat{\theta}_W$ for $M_R \simeq 1$ TeV. However, one can obtain a viable model if M_R is left as a free parameter. In this case, $\sin^2\hat{\theta}_W$ is an input rather than a prediction. One obtains $M_R \sim 10^{10}-10^{11}$ GeV for the two versions, with a high enough M_X to avoid problems with proton decay. This scale of M_R is of relevance to seesaw models of neutrino mass that are suggested by the solar-neutrino problem [42].

A simple extension of the standard model is to allow higher-dimensional Higgs representations. It is convenient to define $\rho_0 \equiv M_W^2/(\hat{\rho}\hat{c}^{\,2} M_Z^2)$, which is unity in the standard model or in extensions involving Higgs doublets or singlets only. At the tree level,

$$\rho_0 = \frac{\sum_i (t_i^2 - t_{3i}^2 + t_i)|\langle \varphi_i \rangle|^2}{\sum_i 2t_{3i}^2 |\langle \varphi_i \rangle|^2} , \tag{6}$$

where t_i and t_{3i} are the total and third component of the weak isospin of φ_i. One expects $\rho_0 = 1$ in most superstring models [24], while ρ_0 may differ from unity in models involving compositeness [43]. ρ_0 modifies the SM expressions for observables by $M_Z \rightarrow M_Z^{\rm SM}/\sqrt{\rho_0}$, $\Gamma_Z \rightarrow \rho_0 \Gamma_Z^{\rm SM}$, and $L_{\rm NC} \rightarrow \rho_0 L_{\rm NC}^{\rm SM}$, where $L_{\rm NC}$ is an effective four-Fermi neutral-current operator. If m_t were known independently, ρ_0 would be determined very precisely. For example, current data yield $\rho_0 = 1.001 \pm 0.002$ (0.992 ± 0.002) for $m_t = 100$ (200) GeV. However, ρ_0 affects the observables in the same way as the quadratic terms in m_t (except for the $Z \rightarrow b\bar{b}$ vertex [31]), and so it is hard to separate the two. To first approximation one determines the combination

$$\rho_{\rm eff} = \rho_0 (1 + \Delta\rho_t) , \tag{7}$$

which takes the value 1.005 ± 0.0024. However, ρ_0 and m_t can be separated [44] by the $\ln m_t$ terms in the radiative corrections to M_W and M_Z and by the $Z \rightarrow b\bar{b}$ vertex (which most sensitively affects the total Z width at present).

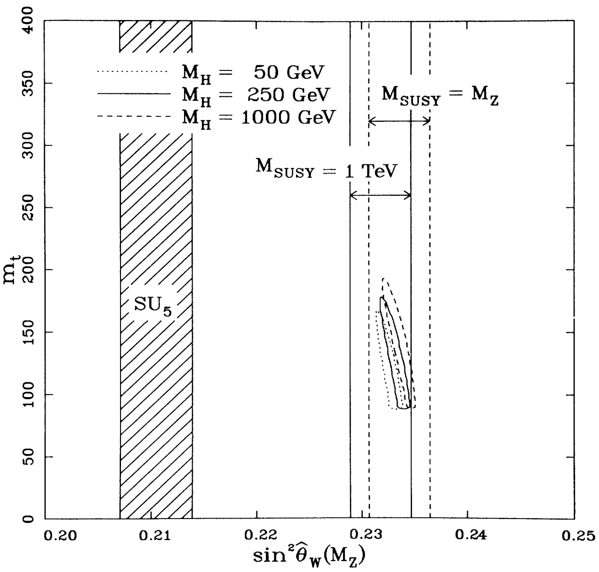

FIG. 4. Predictions for $\sin^2\hat{\theta}_W(M_Z)$ vs m_t in ordinary and supersymmetric grand unified theories, compared with the regions allowed by the data at 90% C.L. for $M_H = 50$, 250, and 1000 GeV.

For the first time the data are sufficiently accurate to allow a reasonably precise separation. A fit to all data yields

$$\begin{aligned} &\sin^2\hat{\theta}_W(M_Z) = 0.2333 \pm 0.0008 , \\ &\rho_0 = 0.992 \pm 0.011 , \\ &m_t = 203^{+73}_{-113} \text{ GeV} , \end{aligned} \tag{8}$$

where the errors in $\hat{s}^{\,2}$ and ρ_0 include m_t and M_H, and those in m_t include the M_H effect. That is, $\sin^2\hat{\theta}_W$ is determined just as well with ρ_0 free as in the standard model, while ρ_0 is determined to $\simeq 1\%$ even with m_t free. One also obtains the upper limit $m_t < 294$ (310) GeV at 90% (95%) C.L. with ρ_0 free. The χ^2 distribution as a function of m_t is shown in Figs. 2(b). From (8) one obtains $\rho_0 < 1.004$ and $\rho_0 > 0.979$, both at 90% C.L. Using the tree-level expression (6), this implies $|\langle \varphi^0 \rangle| < 25$ GeV for the triplet $(\varphi^{++}\,\varphi^{+}\,\varphi^0)$, and $|\langle \chi^0 \rangle| < 7.9$ GeV for $(\chi^+\,\chi^0\,\chi^-)$, to be compared with the doublet expectation value $\langle \varphi_{1/2} \rangle \sim 246$ GeV. As long as $\rho_0 \simeq 1$, it is safe to neglect the effects of nondoublet vacuum expectations values on radiative corrections, which are of order $\alpha(\rho_0 - 1)$. However, new scalar or fermion SU(2) multiplets with large mass splittings can lead to loop effects that are identical in form to ρ_0 [43]. In the presence of such multiplets, one can interpret the results on ρ_0 as applying to $\rho_0/(1-\alpha T)$, where T is the isospin-breaking parameter defined in [26].

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