Consistent analysis of the $\Delta I = \frac{1}{2}$ rule in strange particle decays

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A comprehensive summary of recent progress in the understanding of nonleptonic weak decays of strange particles is presented. The dynamics of these processes is strongly influenced by nonperturbative correlations between two quarks in scalar color-antitriplet states. A coherent description of the $\Delta I = \frac{1}{2}$ rule in kaon and hyperon decays as well as the K_L-K_S mass difference emerges, resolving so far mysterious puzzles. In all cases, the calculations are free of undetermined parameters. Extensive use is made of chiral symmetry and chiral perturbation theory. Relations of our ideas to other approaches, namely, the $1/N_c$ expansion, dual QCD sum rules, and lattice gauge theory, are pointed out.

I. INTRODUCTION

Soon after the discovery of strange particles it was observed that their weak decays exhibit an unexpected pattern. Nonleptonic hyperon as well as kaon decay processes associated with isospin change $\Delta I = \frac{1}{2}$ strikingly dominate other transitions, indicating a complex interplay of strong and weak interactions. A prominent example of this so-called $\Delta I = \frac{1}{2}$ rule [1] is manifest in the experimental ratio [2]

$$\frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K^+ \to \pi^+\pi^0)} \simeq 450 \ , \tag{1.1}$$

which should be close to one without the influence of QCD. Effects of similar strength also occur in hyperon decays. Theoretical investigations of nonleptonic weak transitions have a long history. With the advance of SU(3) and current algebra, several relations between decay amplitudes have been obtained. Development of the standard model and QCD brought further important progress. The renormalization-group summation of hard-gluon corrections to the weak vertex leads to an enhancement of $\Delta I = \frac{1}{2}$ amplitudes and a small suppression of $\Delta I = \frac{3}{2}$ ones. But although this effect shows the correct tendency, sizable factors are still missing. More than 30 years after its discovery, the explanation of the $\Delta I = \frac{1}{2}$ rule in the framework of the standard model is therefore still a challenge. In particular, the magnitude of hyperon and kaon decay amplitudes remained unexplained [3, 4].

On the other hand, the dominant mechanism responsible for nonleptonic two-body decays of D and B mesons could be established soon after the first experimental data were available [5, 6]. These decays predominantly proceed via the direct production of one of the final mesons by a color singlet quark current present in the effective weak Hamiltonian. In this process, nonperturbative QCD plays an essential role. It is taken care of by the use of meson decay constants and form factors for single current matrix elements. That this so-called "new factorization" in energetic transitions is a consequence of QCD is to some extent supported by the $1/N_c$ expansion [7, 8] and QCD sum rules [9], but a deeper understanding is necessary and a challenge for further investigations. Since confinement forces are important even in energetic heavy-quark decay processes, they are expected to be dominant in transitions of strange particles. The $\Delta I = \frac{1}{2}$ puzzle has, therefore, to be resolved in dealing with nonperturbative QCD in the low-energy domain.

In a series of recent papers [10–12] we have shown that the concept of diquarks as effective degrees of freedom is the important key for the understanding of nonleptonic weak decays at low energies. The strong nonperturbative forces between two quarks in a color antitriplet state lead to quark-quark correlations comparable in strength to the quark-antiquark attraction inside mesons. The main part of the effective weak Hamiltonian does nothing but to transform a quasibound spin-zero (su) diquark into a (ud) diquark. This picture can be directly applied to nonleptonic hyperon decays and yields an excellent description of all S- and P-wave amplitudes. In the case of kaon decays, the chiral dynamics of pseudoscalar mesons in interaction with diquarks determines the structure and strength of $\Delta I = \frac{1}{2}$ transitions. The calculation involves the same combination of coupling constants as in hyperon decays. We, therefore, obtain a coherent understanding of nonleptonic weak decays of strange particles in terms of a single dynamical effect.

This paper is organized as follows. In Sec. II we shortly review the concept of an effective weak Hamiltonian for nonleptonic decays. It incorporates perturbative QCD corrections to the weak vertex. The pattern of these corrections already gives important hints about the underlying physics. It leads us to the notion of diquarks as effective degrees of freedom of QCD in the low-energy regime. The basic properties of these quasiparticles, namely their masses and couplings to local currents, are investigated with QCD sum rules. Section III reviews the treatment of nonleptonic hyperon decays using these ideas. This application is most natural since diquarks are part of baryon wave functions. In the main part of this paper we focus on the more elaborate case of kaon decays. In Sec. IV, the phenomenology of $K \to \pi\pi$ amplitudes is summarized. We compute the factorization contribution to these decays and emphasize the strong restrictions on weak amplitudes imposed by chiral symmetry. The technique necessary to incorporate effects of long-distance quark-quark correlations and their contribution to kaon decay amplitudes is developed in Sec. V. The calculation of the $K \to \pi\pi$ amplitude is discussed in detail. Using functional integration techniques our model is cast into a form suitable for the consistent application of chiral perturbation theory. This approach is particularly useful in higher order calculations since it allows the determination of all the coupling constants of the effective chiral Lagrangian. It is subject of Sec. VI. Relations to other approaches, namely to the $1/N_c$ expansion, dual QCD sum rules, and lattice gauge theory, are pointed out in this context. The consistent treatment of higher-order chiral corrections to the $\Delta I = \frac{1}{2}$ amplitude is presented in Sec. VII. It includes meson loops and contributions from higher dimensional chiral operators. An alternative treatment of the final-state interactions using dispersion relations is also discussed. Including these corrections we obtain an excellent theoretical description of both isospin amplitudes relevant in $K \to \pi\pi$ decays. Finally, in Sec. VIII recent progress in the understanding of the long-distance part of the K_L-K_S mass difference is reviewed. This issue is intimately related to the $\Delta I = \frac{1}{2}$ enhancement in nonleptonic kaon decays. Section IX contains a summary and our conclusions.

II. THE EFFECTIVE WEAK HAMILTONIAN

To lowest order in the standard model, nonleptonic weak decays are governed by a single W-exchange diagram. Strong interactions affect this simple picture in a twofold way. Hard-gluon corrections can be accounted for by perturbative methods and give rise to new effective weak vertices. Long-distance confinement forces are responsible for the binding of quarks inside the asymptotic hadronic states. The theoretical tools to separate these two regimes are provided by the operator-product expansion [13] and renormalization-group techniques [14]. The result is the well-known effective Hamiltonian which, in the case of strange-particle decays, reads [15]

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=\pm,3,5,6} c_i(\mu)\, Q_i + \text{H.c.} \tag{2.1}$$

It consists of a product of local four-quark operators Q_i and scale dependent Wilson coefficients $c_i(\mu)$. V_{ud} and V_{us} are elements of the quark mixing matrix [16]. They are real in the standard parametrization [2]. G_F is Fermi's constant. In the basis of Gilman and Wise, the operators are

$$\begin{aligned} Q_\pm &= \tfrac{1}{2}[(\bar s\, u)_{V-A}\, (\bar u\, d)_{V-A} \pm (\bar s\, d)_{V-A}\, (\bar u\, u)_{V-A}] , \\ Q_{3,5} &= (\bar s\, d)_{V-A} \sum_{q=u,d,s} (\bar q\, q)_{V\mp A} , \\ Q_6 &= -2 \sum_{q=u,d,s} (\bar s\, q)_{S+P}\, (\bar q\, d)_{S-P} , \end{aligned} \tag{2.2}$$

where $(\bar s\, u)_{V-A} = \bar s\, \gamma_\mu (1-\gamma_5)\, u$, etc. Long-distance physics becomes apparent in hadronic matrix elements of these operators. On the other hand, the Wilson coefficients take into account perturbative corrections associated with the evolution from the scale of the W boson to a hadronic scale μ. For our numerical estimates we choose the scale such that $\alpha_s(\mu) = 0.5$. The corresponding values are [17]

$$\begin{aligned} &c_+ \simeq 0.65 , \quad c_- \simeq 2.42 , \quad c_3 \simeq -0.01 , \\ &c_5 \simeq 0.006 , \quad c_6 \simeq -0.02 \end{aligned} \tag{2.3}$$

as compared to $c_+ = c_- = 1$, $c_3 = c_5 = c_6 = 0$ without QCD corrections. Obviously, c_3 and the coefficients of the penguin operators $Q_{5,6}$ are very small even at low scales.

In order to understand the reason for the strong perturbative correction of $c_\pm$, it is instructive to consider the effective Hamiltonian as describing a scattering process $s + u \to u + d$. According to the decomposition $\mathbf{3} \otimes \mathbf{3} = \mathbf{3}^* \oplus \mathbf{6}$, the initial and final states can be in an antitriplet or sextet with respect to color. These two possibilities correspond to Q_- and Q_+, respectively. The fact that the perturbative gluon exchange is attractive in the $\mathbf{3}^*$ and repulsive in the $\mathbf{6}$ channel explains the enhancement of c_- and the suppression of c_+, respectively. By means of a Fierz transformation this can be made more explicit [18]:

$$\begin{aligned} \mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} [\, & c_-(\mu)\, (u\, d)^\dagger_{3^*}\, (s\, u)_{3^*} \\ & + c_+(\mu)\, (u\, d)^\dagger_6\, (s\, u)_6 + \cdots + \text{H.c.}] , \end{aligned} \tag{2.4}$$

where $(s\, u)_{3^*} = \epsilon_{kij}\, s_i^T C\, (1-\gamma_5)\, u_j$ is a scalar and pseudoscalar color-antitriplet diquark current, and the other currents are given by similar expressions. C is the charge-conjugation matrix. The ellipses stand for contributions of Q_3 and penguin operators.

From the very existence of baryons, it follows that also the long-distance color force between two quarks in an antitriplet state is attractive. Consequently, one expects a further enhancement of matrix elements of Q_-. If the attractive potential is strong enough to build quasibound two-quark states, the currents in (2.4) can be interpolated by local diquark fields. This is likely to be the case for the $\mathbf{3}^*$ channel only, where both the short- and long-distance forces are attractive. After all, diquarks do exist inside baryons. (The product of sextet currents behaves differently. Since in this case the short-distance force is repulsive, bound color-sextet diquarks seem very unlikely. To handle this term, it should therefore be rewritten in its usual form.) Clearly, because of their open color, diquarks cannot be asymptotic states of QCD. They may, however, act as quasiparticles at some intermediate scales. Their status is like that of constituent quarks which dissolve into smaller partons at high q^2.

In hadron physics and spectroscopy, the binding of quarks to diquarks has important implications and has, therefore, often been discussed [19]. Because of the particular form (2.4) of the effective Hamiltonian, weak interactions are even more suited to see and study the corresponding effects. The operator Q_- can act in a very special and intuitive way. It annihilates a color-antitriplet (su) diquark and creates a (ud) state (or vice versa). The crucial question is, however, whether or not this $\Delta I = \frac{1}{2}$ process is really the dominant one. This question can be answered by considering the coupling strength of a diquark state to the associated local current [20], e.g., for a scalar (ud) diquark with color index l:

$$\epsilon_{kij} \langle \Omega \,|\, u_i^T C \gamma_5 \, d_j \,|\, (ud)_l \rangle = \left(\frac{2}{3}\right)^{1/2} \delta_{kl} \; g_{(ud)}(\mu) \; , \tag{2.5}$$

where $\sqrt{2/3}$ is a color factor. $g_{(ud)}$ has been defined in analogy to g_π, the coupling of a pion to the pseudoscalar current, and is a scale-dependent quantity. "Diquark decay constants" such as this have been investigated by various methods. The approach from QCD sum rules starts from the time-ordered product of two diquark currents:

$$G(q) = i \int \mathrm{d}^4x \, e^{iqx} \, \langle \Omega \,|\, \mathcal{T} \{ : j_i(x) \, S_{ik}[x,0] \; j_k^\dagger(0) : \} \,|\, \Omega \rangle \; , \tag{2.6}$$

where the path-ordered Schwinger string

$$S_{ik}[x,0] = \left[\mathcal{P} \exp \left(i g_s \int_0^x A_\mu \, \mathrm{d}x^\mu \right) \right]_{ik} \tag{2.7}$$

ensures gauge covariance. A careful analysis of correlators such as this gives large coupling constants [20, 21] (as large as g_π) indicating strong binding inside the two-quark system. Stability of the sum rules requires that pseudoscalar ($J^P = 0^-$) diquarks are considerably heavier than their scalar ($J^P = 0^+$) counterparts, in agreement with quark model expectations. An important result of this analysis is that the anomalous dimension of the diquark decay constants is minus one-half that of the Wilson coefficient c_-. The renormalization-group-improved perturbative scaling is [21]

$$g_{\mathrm{di}} \sim [\alpha_s(\mu)]^{-2/9} \tag{2.8}$$

and is not significantly modified by nonperturbative contributions. Consequently, $c_-(\mu)\, g_{(ud)}(\mu)\, g_{(su)}(\mu)$ becomes scale independent. It is precisely this product which enters all our computations. In Table I we quote numerical results for different combinations of flavor and parity. The equal parity product is relevant for hyperon decays, while the mixed parity combinations govern the $K \to \pi\pi$ amplitudes. These numbers refer to a mass of 500 MeV for the scalar (ud) diquark. They increase by 25% (decrease by 20%) if this mass is 600 (400) MeV.

Alternatively, one may extract $g_{(ud)}$ and $g_{(su)}$ from the analysis of hyperon decays [12], or study the coupling of a proton, described as a $u\,(ud)$ bound state, to a baryonic current [20]. The results of these different methods nicely agree and thereby reduce the uncertainty inherent in sum rule estimates. In the following computations we use

$$\begin{aligned} c_- \, g^+_{(ud)} \, g^+_{(su)} &= (0.075 \pm 0.015)\ \mathrm{GeV}^4 \; , \\ c_- \, g^\pm_{(ud)} \, g^\mp_{(su)} &= (0.090 \pm 0.015)\ \mathrm{GeV}^4 \; , \end{aligned} \tag{2.9}$$

consistent with little explicit chiral-symmetry breaking.

The fact that the diquark couplings are large gives us confidence that the picture of the effective Hamiltonian acting on diquark states is an appropriate one. It annihilates and creates these states in a very effective way, and the combinations (2.9) incorporate, as we shall see, the major nonperturbative effects active in nonleptonic transitions at low energies. More precisely, this is true as long as $g_{\mathrm{di}} \gg f_\pi \, m_q$, with m_q being the mass of the decaying quark. Matrix elements of scalar diquark currents are then much larger than those of vector currents which arise in factorization of the weak Hamiltonian in its usual form (2.1). This relation is well satisfied in strange-particle decays. Since Q_- transforms like an $\Delta I = \frac{1}{2}$ operator, this will lead us to a natural understanding of the $\Delta I = \frac{1}{2}$ enhancement. In heavy quark decays, on the other hand, matrix elements of vector and axial-vector currents are clearly dominant. This is the reason why (new) factorization works quite well in energetic D and, even better, in B decays [5, 22].

III. NONLEPTONIC HYPERON DECAYS

Substituting the color-antitriplet currents in the first part of (2.4) by normalized, scalar and pseudoscalar diquark fields

$$\phi_L = \frac{1}{\sqrt{2}} \{(ds), (su), (ud)\}_{S-P} \tag{3.1}$$

TABLE I. Combinations of diquark coupling constants with the Wilson coefficient c_- as obtained from QCD sum rules [21].

μ^2 (GeV^2)	$\alpha_s(\mu)$	$c_- \, g^+_{(ud)} \, g^+_{(su)}$ (GeV^4)	$c_- \, g^+_{(ud)} \, g^-_{(su)}$ (GeV^4)	$c_- \, g^-_{(ud)} \, g^+_{(su)}$ (GeV^4)
1.00	0.37	0.070	0.088	0.090
0.50	0.45	0.072	0.090	0.090
0.25	0.58	0.075	0.091	0.091
0.09	1.00	0.082	0.093	0.097
	error	0.014	0.025	0.024

one obtains the effective Hamiltonian of the diquark model

$$\mathcal{H}_{\rm eff}^{\rm di} = \frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}\, \frac{4}{3}\, c_-\, g_{(ud)}\, g_{(su)}\, \phi_L^\dagger\, \lambda_6\, \phi_L\ . \qquad (3.2)$$

It simulates the strong attractive quark-quark correlations active in nonleptonic weak transitions. This form of the Hamiltonian can be directly applied to hyperon decays, since diquarks are part of the asymptotic baryon wave functions.

The theoretical description of hyperon decays starts from baryon-pole formulas and current algebra relations [4]. One defines dimensionless amplitudes A and B by

$$\langle\, B_j\, \pi\, |\, \mathcal{H}_{\rm eff}(0)\, |\, B_i\, \rangle = i\, \bar{u}_j\, (A - \gamma_5\, B)\, u_i\ . \qquad (3.3)$$

They describe parity violating S-wave and parity conserving P-wave transitions between $J^P = \frac{1}{2}^+$ baryon states B_i and B_j. (For the case of Ω^- decays, see Ref. [12].) As an example, Fig. 1(a) shows the contributions to the P-wave amplitude for the decay $\Lambda \to p\,\pi^-$ in the pole model. Weak interaction can either change the Λ into a virtual neutron which then emits a pion, or the pion may be emitted first leading to an intermediate Σ^+ state. The P-wave amplitudes are therefore determined by the baryon matrix elements of the effective weak Hamiltonian. Since in a constituent quark model for baryons there cannot be color-sextet diquarks, Eq. (3.2) directly applies and simply replaces an $(su)_{0^+}$ state in the initial baryon by a $(ud)_{0^+}$ diquark leaving the remaining quark unaffected [23]. The calculation depends very little on details of the baryon wave functions [12]. The first combination of coupling constants quoted in (2.9) fixes the magnitude of all baryon matrix elements of the effective Hamiltonian. For octet baryons the d/f ratio turns out to be precisely (-1), with $d = -f \simeq 0.37 \times 10^{-7}$ GeV.

The same matrix elements also determine the soft-meson limit of the S-wave amplitudes. To obtain the S-wave amplitudes for on-shell pions, however, a trick is required. One writes the amplitudes as a sum of pole terms using the fact that the relevant $\frac{1}{2}^-$ baryon intermediate states, which contain 0^- diquarks, are heavier by about 700 MeV than $\frac{1}{2}^+$ baryons. Finally, consistency with the soft-pion limit is enforced to fix the coupling constants.

With this input, the diquark contributions to both S- and P-wave amplitudes are determined in magnitude and sign. They have to be supplemented by the amplitudes obtained from factorization of the full effective Hamiltonian (2.1). The latter account for the effect of the strong QCD attraction between quark and antiquark instead of two quarks and, in particular, take care of the $\Delta I = \frac{3}{2}$ parts of the decay amplitudes and of the contributions from penguin operators. They are generally not large since matrix elements of vector currents are suppressed for small particle momenta. Nevertheless, the addition of the factorization amplitudes is necessary, in particular, in those decays where the diquark contribution is small. Because of the completely different decay mechanism there is very little danger of double counting. (See also the discussion in Sec. VI about the different behavior of factorization and diquark contributions with respect to the large-N_c expansion.) The relative sign of both amplitudes, which is determined from theory, turns out to be crucial then. In Tables II and III, we compare the theoretical and experimental results. The S-wave amplitudes for decays to $\pi\,N$ states include a correction for final-state interactions suggested by the measured phase shifts [12]. It is quite remarkable that in spite of the approximations used the theoretical amplitudes calculated in this way are in very satisfactory agreement with experiment. No undetermined parameters have to be introduced. The old puzzle of the P waves and their connection to the S waves, which was a headache for many

TABLE II. S-wave hyperon decay amplitudes A in units 10^{-7}, taken from Ref. [12].

Decay	Theory	Experiment
$\Lambda \to n\,\pi^0$	-2.11	-2.36 ± 0.03
$\Lambda \to p\,\pi^-$	2.74	3.25 ± 0.02
$\Xi^0 \to \Lambda\,\pi^0$	3.46	3.43 ± 0.06
$\Xi^- \to \Sigma^0\,\pi^-$	-4.58	-4.49 ± 0.02
$\Sigma^+ \to n\,\pi^+$	0.03	0.14 ± 0.03
$\Sigma^+ \to p\,\pi^0$	-3.30	-3.26 ± 0.11
$\Sigma^- \to n\,\pi^-$	4.29	4.27 ± 0.01

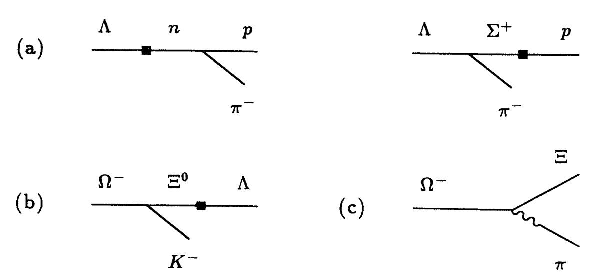

FIG. 1. Pole contributions to the parity-conserving P-wave amplitudes for the decays $\Lambda \to p\,\pi^-$ (a) and $\Omega^- \to \Lambda\,K^-$ (b) in the diquark model. The black square indicates the weak vertex from (3.2). The transition $\Omega^- \to \Xi\,\pi$ in (c) can proceed via factorization only.

TABLE III. P-wave hyperon decay amplitudes B in units 10^{-7}, taken from Ref. [12].

Decay	Theory	Experiment
$\Lambda \to n\,\pi^0$	-15.59	-15.61 ± 1.40
$\Lambda \to p\,\pi^-$	23.20	22.40 ± 0.54
$\Xi^0 \to \Lambda\,\pi^0$	-14.43	-12.13 ± 0.71
$\Xi^- \to \Sigma^0\,\pi^-$	19.78	17.45 ± 0.58
$\Sigma^+ \to n\,\pi^+$	44.22	41.83 ± 0.17
$\Sigma^+ \to p\,\pi^0$	30.74	26.74 ± 1.32
$\Sigma^- \to n\,\pi^-$	-1.18	-1.44 ± 0.17
$\Omega^- \to \Lambda\,K^-$	7.70	5.37 ± 0.13
$\Omega^- \to \Xi^0\,\pi^-$	1.46	1.80 ± 0.08
$\Omega^- \to \Xi^-\,\pi^0$	-0.75	-1.10 ± 0.07

years, is now resolved.

In Ω^- decays, a clear separation of diquark and factorization amplitudes occurs. The transition $\Omega^- \to K^- \Lambda$, shown in Fig. 1(b), proceeds via the strong process $\Omega^- \to K^- \Xi^0$ followed by the weak transition $\Xi^0 \to \Lambda$ whose strength is determined by the large diquark decay constants in (2.9). Because of the flavor quantum numbers there is no factorization contribution in this case. The decays $\Omega^- \to \Xi \pi$, on the other hand, cannot proceed via a diquark transition but do allow for factorization, see Fig. 1(c). To our satisfaction, these amplitudes are strongly suppressed although they can proceed via $\Delta I = \frac{1}{2}$ transitions. The pattern emerging in the low-energy regime is that weak decays are always large if they can proceed via a diquark transition, and small otherwise. We know of no exception of this general rule. It will prove to be particularly valid in the nonleptonic decays of kaons, to which we turn now.

IV. THE $\Delta I = \frac{1}{2}$ RULE IN KAON DECAYS

The decays $K \to \pi\pi$ are described by three independent amplitudes which, in the presence of final-state interactions, have the isospin decomposition [3]

$$\begin{aligned} i\,\mathcal{A}(K_S \to \pi^+\pi^-) &= \left(\frac{2}{3}\right)^{1/2} A_0\, e^{i\delta_0} + \left(\frac{1}{3}\right)^{1/2} A_2\, e^{i\delta_2} \equiv A_{+-}\ , \\ \frac{i}{\sqrt{2}}\mathcal{A}(K_S \to \pi^0\pi^0) &= \left(\frac{1}{3}\right)^{1/2} A_0\, e^{i\delta_0} - \left(\frac{2}{3}\right)^{1/2} A_2\, e^{i\delta_2} \equiv \frac{1}{\sqrt{2}} A_{00}\ , \\ i\,\mathcal{A}(K^+ \to \pi^+\pi^0) &= \left(\frac{3}{4}\right)^{1/2} A_2\, e^{i\delta_2} \equiv A_{+0}\ . \end{aligned} \tag{4.1}$$

Electromagnetically induced $\Delta I = \frac{5}{2}$ weak transitions are negligible. A_0 and A_2 are real amplitudes corresponding to isospin $I = 0$ and $I = 2$ in the final state, respectively. δ_0 and δ_2 denote the S-wave $\pi\pi$ scattering phases. Experimentally, one has [2]

$$\begin{aligned} |A_{+-}| &= (3.911 \pm 0.007) \times 10^{-7}\ \text{GeV}\ , \\ |A_{00}| &= (3.714 \pm 0.015) \times 10^{-7}\ \text{GeV}\ , \\ |A_{+0}| &= (1.831 \pm 0.006) \times 10^{-8}\ \text{GeV}\ . \end{aligned} \tag{4.2}$$

Accordingly,

$$\begin{aligned} A_0 &= \frac{1}{\sqrt{6}} |2A_{+-} + A_{00}| = (4.71 \pm 0.01) \times 10^{-7}\ \text{GeV}\ , \\ A_2 &= \frac{2}{\sqrt{3}} |A_{+0}| = (2.11 \pm 0.01) \times 10^{-8}\ \text{GeV}\ , \\ |\delta_0 - \delta_2| &= (56.5 \pm 3.0)^\circ\ . \end{aligned} \tag{4.3}$$

These numbers should be compared to the amplitudes obtained from factorization of the effective weak Hamiltonian (2.1). They are

$$\begin{aligned} i\,\mathcal{A}_f(K_S \to \pi^+\pi^-) &= \frac{\kappa}{\sqrt{2}}[\ (1-\xi)\,c_- + (1+\xi)\,c_+ \\ &\qquad +2\xi\,c_3 - 2\chi\,(c_6 + \xi\,c_5)], \\ i\,\mathcal{A}_f(K_S \to \pi^0\pi^0) &= \frac{\kappa}{\sqrt{2}}[\ (1-\xi)\,c_- - (1+\xi)\,c_+ \\ &\qquad +2\xi\,c_3 - 2\chi\,(c_6 + \xi\,c_5)], \\ i\,\mathcal{A}_f(K^+ \to \pi^+\pi^0) &= \frac{\kappa}{\sqrt{2}}(1+\xi)\,c_+\ . \end{aligned} \tag{4.4}$$

The parameter ξ has been introduced in Refs. [5]. The abbreviations are

$$\begin{aligned} \kappa &= \frac{G_F}{\sqrt{2}} V_{ud} V_{us} f_\pi (m_K^2 - m_\pi^2) \simeq 0.53 \times 10^{-7}\ \text{GeV}\ , \\ \chi &= \frac{2v^2}{m_K^2 - m_\pi^2} \left(\frac{m_{f_0}^2}{m_{f_0}^2 - m_K^2} \frac{f_K}{f_\pi} - 1 \right) \simeq 6.6 \pm 1.2\ . \end{aligned} \tag{4.5}$$

In κ we have neglected the $K\pi$ form factor which experimentally is very close to one. For the estimate of χ, pole dominance for the $\pi\pi$ form factor of the $\bar{d}d$ current has been assumed, with $m_{f_0} \simeq 976$ MeV being the mass of the nearest isospin-zero scalar meson. The constant

$$v = \frac{m_\pi^2}{m_u + m_d} = \frac{m_K^2}{m_s + m_u} \tag{4.6}$$

is related to the quark condensate. In estimating its value, the running quark masses have to be evaluated at the same scale as the Wilson coefficients. In fact, the product $\chi\, c_6$ is almost scale independent. For $\alpha_s(\mu) = 0.5$ one has (Ref. [26]) $(m_s + m_d) \simeq 230 \pm 20$ MeV, and thus $v \simeq 1.07 \pm 0.10$ GeV.

Using the values of the Wilson coefficients as given in (2.3), we find

$$\begin{aligned} |\mathcal{A}_f(K_S \to \pi^+\pi^-)| &\simeq (1.25 - 0.70\,\xi) \times 10^{-7}\ \text{GeV}\ , \\ |\mathcal{A}_f(K_S \to \pi^0\pi^0)| &\simeq (0.76 - 1.18\,\xi) \times 10^{-7}\ \text{GeV}\ , \\ |\mathcal{A}_f(K^+ \to \pi^+\pi^0)| &\simeq 2.44\,(1+\xi) \times 10^{-8}\ \text{GeV}\ . \end{aligned} \tag{4.7}$$

Note that, in factorization, the contribution of penguin operators is very small and does not exceed 3% of the experimental amplitudes. The parameter ξ takes into account our insufficient knowledge how to handle the so-called Fierz terms in factorization. In the conventional prescription, $\xi = 1/N_c$, but this neglects matrix elements of color-octet currents and is therefore not justified. "New factorization" requires $\xi \simeq 0$ and is phenomenologically favored in exclusive decays of D and B mesons [5, 22], indicating that quark and antiquark from a color-singlet current like to stay together to form an asymptotic hadron state. It is important to note that new factorization gives the exact result for the decay amplitudes in the $N_c \to \infty$ limit. From (4.7) it is apparent that also in K decays $\xi = 0$ is favored compared to $\xi = \frac{1}{3}$. In particular, this brings the $\Delta I = \frac{3}{2}$ factorization amplitude already in rough agreement with experiment. Also the $\Delta I = \frac{1}{2}$ amplitude becomes larger [8], but the main part is still missing.

The application of the ideas presented in Sec. II to kaon decays is less straightforward than in the case of hyperons, since diquarks are not part of meson wave functions in an apparent way. One may, however, make use of the fact that the pseudoscalar mesons are Goldstone-bosons associated with the spontaneously broken chiral symmetry of the QCD Lagrangian. Chiral symmetry completely determines the structure and, to a large extent, also the strength of the Goldstone boson interactions among themselves and with external fields. Chiral perturbation theory is the systematic expansion of these interactions in powers of the meson momenta and masses [27–29] (in the remainder commonly denoted by p). It is the most powerful nonperturbative (in the sense of perturbation theory in α_s) approach to QCD at low energies and provides a convenient framework for the investigation of the long-distance contributions to nonleptonic weak decays of kaons.

As usual we introduce a nonlinear representation in flavor space

$$\Sigma(x) = \exp\left(\frac{i\sqrt{2}}{f}\,\lambda_a\,P_a(x)\right) , \tag{4.8}$$

where λ_a are the Gell-Mann matrices, and $P_a(x)$ denote the pseudoscalar fields. In lowest order of the chiral expansion, f coincides with the pion decay constant $f_\pi \simeq 132$ MeV. Under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, the fields transform according to $\Sigma \to U_L \Sigma U_R^\dagger$. To order p^2, the most general strong-interaction Lagrangian is uniquely determined and reads ($\langle \ldots \rangle$ denotes a trace in flavor space)

$$\mathcal{L}_{\rm strong} = \frac{f^2}{8}\,\langle\, \partial_\mu \Sigma\, \partial^\mu \Sigma^\dagger \,\rangle + v\,\frac{f^2}{4}\,\langle\, m_q\,\Sigma^\dagger + \Sigma\, m_q^\dagger \,\rangle \, . \tag{4.9}$$

It is "invariant" under the chiral group if the fictitious transformation $m_q \to U_L\, m_q\, U_R^\dagger$ of the quark mass matrix is implemented. The constant v, defined in (4.6), gives masses to the Goldstone bosons.

In order to obtain the structure chiral symmetry imposes on weak decays, one needs to find, to a given order of the chiral expansion, the set of all meson operators with the appropriate transformation properties under SU(3) and isospin. It is well known [30] that to lowest order there are only two operators transforming like an SU(3) octet. This part of the chiral Lagrangian is conveniently written as

$$\mathcal{L}_8 = -\frac{G_F}{\sqrt{2}}\,V_{ud}\,V_{us}\left(\, 4\,g_8\,\langle\, \lambda_6\, L_\mu\, L^\mu \,\rangle + h_8\, v\,\frac{f^4}{2}\langle\, \lambda_6\,(m_q\,\Sigma^\dagger + \Sigma\, m_q^\dagger)\,\rangle \right) . \tag{4.10}$$

The components of

$$L_\mu = -i\,\frac{f^2}{4}\,\Sigma\,\partial_\mu \Sigma^\dagger \;, \quad \langle\, L_\mu \,\rangle = 0 \tag{4.11}$$

are the lowest-order chiral realizations of the $V-A$ color-singlet quark currents appearing in the effective Hamiltonian (2.1). This is readily derived applying the Noether procedure to (4.9). The dimensionless parameters g_8 and h_8 are not restricted by chiral symmetry. They have to be determined from experiment or within the context of a given model. The second operator in (4.10) is, by the equations of motion, a total derivative. It does not contribute to physical (on-shell) decay amplitudes [29–31].

The single operator transforming like **27** is again bilinear in L_μ and has components

$$\begin{aligned}\mathcal{L}_{27} = -\frac{G_F}{\sqrt{2}}\,V_{ud}\,V_{us}\,\big[\, & 4\,g_{27}^{1/2}\,(L_{23}L_{11} + L_{13}L_{21} + 2\,L_{23}L_{22} - 3\,L_{23}L_{33} + \text{ H.c.}) \\ & + 4\,g_{27}^{3/2}\,(L_{23}L_{11} + L_{13}L_{21} - L_{23}L_{22} + \text{ H.c.})\big] \, ,\end{aligned} \tag{4.12}$$

where contraction over μ is implicitly assumed. The two parts mediate transitions with $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$, respectively.

The on-shell $K \to \pi\,\pi$ amplitudes computed from these Lagrangians read

$$\begin{aligned} i\,\mathcal{A}(K_S \to \pi^+\pi^-) &= \sqrt{2}\,\kappa\,(g_8 + g_{27}^{1/2} + g_{27}^{3/2}) \, , \\ i\,\mathcal{A}(K_S \to \pi^0\,\pi^0) &= \sqrt{2}\,\kappa\,(g_8 + g_{27}^{1/2} - 2\,g_{27}^{3/2}) \, , \\ i\,\mathcal{A}(K^+ \to \pi^+\pi^0) &= \frac{3}{\sqrt{2}}\,\kappa\, g_{27}^{3/2} \end{aligned} \tag{4.13}$$

with κ given in (4.5). We find it convenient to define the phenomenological parameters in terms of the on-shell decay amplitudes to all orders of the chiral expansion, i.e., to absorb all higher order corrections into a redefinition of these parameters. In this way, they become directly related to measurable quantities. From the experimental amplitudes (4.2) one deduces the values

$$\begin{aligned} |\, g_8 + g_{27}^{1/2} \,| &= 5.13 \pm 0.07 \, , \\ |\, g_{27}^{3/2} \,| &= 0.163 \pm 0.002 \, . \end{aligned} \tag{4.14}$$

Because of the SU(3) relation $g_{27}^{1/2} = \frac{1}{5}\, g_{27}^{3/2}$, the **27**-contribution to $\Delta I = \frac{1}{2}$ amplitudes is tiny, i.e., $|\, g_{27}^{1/2}\,| \ll |\, g_8\,|$. The $\Delta I = \frac{1}{2}$ rule therefore reflects octet enhancement in nonleptonic weak transitions.

By naively replacing the color-singlet quark currents in the effective weak Hamiltonian (2.1) by the hadronic currents (4.11), on the other hand, one obtains [3]

$$\begin{aligned} g_8^f &= \frac{c_-}{2} + \frac{c_+}{10} - \chi\, c_6 \simeq 1.41 \ , \\ h_8^f &= \frac{c_6\, v}{m_s - m_d} \left(1 - \frac{\langle \bar{s}\, s \rangle}{\langle \bar{d}\, d \rangle}\right) \simeq -0.05 \ , \\ g_{27}^{1/2,f} &= \frac{c_+}{15} \simeq 0.04 \ , \\ g_{27}^{3/2,f} &= \frac{c_+}{3} \simeq 0.22 \ . \end{aligned} \tag{4.15}$$

This procedure precisely corresponds to new factorization, as can be inferred from comparison of (4.13) and (4.4). The reason is that the replacement

$$\bar{q}_i\, \gamma_\mu\, \frac{1 - \gamma_5}{2}\, q_j \longrightarrow \left(L_\mu\right)_{ij} \tag{4.16}$$

is strictly valid in the $N_c \longrightarrow \infty$ limit where the color-singlet quark currents hadronize independently. Corrections to this limit are formally suppressed in the $1/N_c$ expansion, although obviously quite large phenomenologically. It seems likely, therefore, that the large-N_c expansion breaks down in the $\Delta I = \frac{1}{2}$ channel. There are, in fact, strong indications for this also from other investigations [32, 33].

It will turn out that, due to dominant long-distance effects, the pattern of the $1/N_c$ corrections is completely different than that arising from conventional factorization of the weak Hamiltonian. As in the case of hyperon decays, our strategy will be to supplement the new factorization contribution (4.15) by the long-distance contribution from quark-quark correlations. The observation that the coefficient $g_{27}^{3/2}$, which remains unaffected from diquark effects, is only slightly overestimated in new factorization supports this treatment.

V. THE CHIRAL DIQUARK MODEL

The strong attractive quark-quark interaction in color-antitriplet states certainly influences nonleptonic kaon decays. In order to calculate this contribution one needs to know how pseudoscalar mesons couple to diquark states. The basic idea here is to generalize the standard approach from chiral perturbation theory [10]. In addition to the octet of pseudoscalar mesons, we consider the normalized local diquark fields ϕ_L and ϕ_R in (3.1) as effective degrees of freedom of QCD at low-energy scales. They transform as antitriplets with respect to both color and flavor SU(3). The diquark part of the weak interaction Lagrangian is simply given by (3.2). It has the form of an off-diagonal mass term and could in principle be diagonalized by a suitable field redefinition. This greatly simplifies the calculation of Feynman diagrams, but otherwise does not affect the decay amplitudes [35]. For reasons of transparency we stick here to the unrotated fields.

The strong-interaction Lagrangian (4.9) is supplemented by kinetic and mass terms for the diquark fields [34]

$$\begin{aligned} \mathcal{L}_\phi = &\, D_\mu \phi_L^\dagger\, D^\mu \phi_L + D_\mu \phi_R^\dagger\, D^\mu \phi_R \\ & - m^2\, (\phi_L^\dagger\, \phi_L + \phi_R^\dagger\, \phi_R) \\ & + \frac{\tilde{v}}{2}\, (\phi_L^\dagger\, m_q^*\, \phi_R + \phi_R^\dagger\, m_q^T\, \phi_L) \end{aligned} \tag{5.1}$$

as well as interaction terms between mesons and diquarks. We shall restrict ourselves to nonderivative couplings here. This guarantees the best possible ultraviolet behavior. If not removable by field redefinitions, derivative interactions would imply Noether currents not separable into meson and diquark parts and would signify an unwanted internal structure of these particles. The most general interaction Lagrangian bilinear in ϕ, and of up to second order in the chiral expansion, is given by

$$\begin{aligned} \mathcal{L}_{\rm int} = &\, \frac{\Delta m^2}{2}\, (\phi_L^\dagger\, \Sigma^*\, \phi_R + \phi_R^\dagger\, \Sigma^T\, \phi_L) - \frac{v''}{2}\, \langle\, m_q\, \Sigma^\dagger + \Sigma\, m_q^\dagger\, \rangle\, (\phi_L^\dagger\, \phi_L + \phi_R^\dagger\, \phi_R) \\ & + \frac{v'}{2}\, [\phi_L^\dagger\, (m_q^*\, \Sigma^T + \Sigma^*\, m_q^T)\, \phi_L + \phi_R^\dagger\, (m_q^T\, \Sigma^* + \Sigma^T\, m_q^*)\, \phi_R] \ . \end{aligned} \tag{5.2}$$

We note that the first term is the only operator of zeroth order (p^0) in the chiral expansion.

The coupling parameters in (5.1) and (5.2) are related to the diquark mass spectrum. For reasons of simplicity we shall assume $\tilde{v} = 0$, i.e., the same SU(3) mass splitting for $J^P = 0^+$ and 0^- diquarks. The corresponding term in (5.1) does not anyway give rise to interactions with mesons. Expanding Σ one obtains the mass matrices

$$\begin{aligned} \mathcal{M}_\pm^2 = &\left(m^2 \mp \frac{\Delta m^2}{2} + (v'' - v')\, \langle\, m_q\, \rangle\right) \mathbb{1} \\ & + v' \begin{pmatrix} m_d + m_s & 0 & 0 \\ 0 & m_s + m_u & 0 \\ 0 & 0 & m_u + m_d \end{pmatrix} \end{aligned} \tag{5.3}$$

for diquarks of positive and negative intrinsic parity ($J^P = 0^\pm$), respectively, where $\mathbb{1}$ is the unit matrix in flavor space. Flavor eigenstates of different parity differ in their mass squares by a constant amount Δm^2, while the SU(3) mass splitting is proportional to the current quark masses:

$$\delta m^2 \equiv m_{(su)}^2 - m_{(ud)}^2 = v'\, (m_s - m_d) = \frac{v'}{v}\, (m_K^2 - m_\pi^2) \tag{5.4}$$

as it is the case for mesons. In the spirit of the quark model one expects the ratio v'/v to be close to one. From (5.3) the choice $v'' = v'$ appears natural. Then only current masses of quarks being constituents of a diquark contribute to the diquark mass. It turns out that the precise value of v'' is almost irrelevant for the applications. Its contribution to the $K \to 2\pi$ amplitude is of order p^4 and furthermore suppressed by m_π^2. Already at this stage we note that in lowest order of the chiral expansion only the interaction term proportional to Δm^2 contributes to weak decay amplitudes [35]. The terms proportional to v' and v'' give rise to total divergences of the type of the second operator in (4.10). Although necessary for a realistic diquark mass spectrum, they are of minor importance for the numerical analysis. In the following we set $v'' = v' = v$ whenever numbers are quoted.

In the applications it will be useful to expand the decay amplitudes in terms of the SU(3) mass splitting δm^2. We therefore define averaged masses $m_\pm$ and their ratio by

$$m_\pm^2 \equiv \frac{1}{2}\,(m_{(ud)\pm}^2 + m_{(su)\pm}^2)\ , \quad z \equiv \frac{m_-^2}{m_+^2} \tag{5.5}$$

such that

$$m_{(ud)\pm}^2 = m_\pm^2 - \frac{\delta m^2}{2}\ , \quad m_{(su)\pm}^2 = m_\pm^2 + \frac{\delta m^2}{2}\ . \tag{5.6}$$

On phenomenological grounds, one expects a rather large mass splitting between different parity diquark states, i.e.,

$$m_- - m_+ \simeq m_{a_0} - m_\pi \simeq m_{K^*(1430)} - m_K \simeq 900 \text{ MeV}. \tag{5.7}$$

This pattern is nicely confirmed by the QCD sum-rule analysis [21]. Since diquarks only appear as internal lines in Feynman diagrams, it is tempting to consider the limit $m_- \to \infty$, thereby removing the heavy fields from the model. This limit can readily be performed after the nonlinear field redefinition

$$\begin{aligned} \psi_L &= \zeta^\dagger\,\phi_L = \frac{1}{\sqrt{2}}\,(\psi_+ - \psi_-)\ , \\ \psi_R &= \zeta\,\phi_R = \frac{1}{\sqrt{2}}\,(\psi_+ + \psi_-) \end{aligned} \tag{5.8}$$

with $\zeta = (\Sigma^*)^{1/2}$. This simplified version of the diquark model has been discussed in detail in our earlier papers [10]. We shall point out the implications of the $m_- \to \infty$ limit below.

The chiral diquark model presented here has very nice features. First of all, one can prove that all weak decay amplitudes are ultraviolet convergent [35] (as long as no meson loops are involved). This is not trivial, since already the lowest-order diagrams involve diquark loops some of which diverge individually. The sum of all graphs is always finite, however. As in conventional chiral perturbation theory, current algebra relations and the Gell-Mann–Cabibbo theorem are satisfied by construction. Finally, to lowest order in the chiral expansion weak decay amplitudes only depend on the single quantity

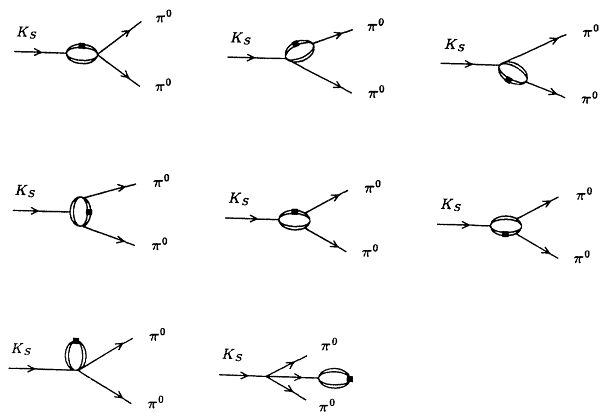

FIG. 2. Feynman diagrams for the decay $K_S \to \pi^0\pi^0$ in the diquark model.

$$z_0 \equiv z(m_q = 0) = \frac{m^2 + \dfrac{\Delta m^2}{2}}{m^2 - \dfrac{\Delta m^2}{2}} \tag{5.9}$$

which corresponds to the chiral limit $m_q = 0$ of the mass ratio z introduced in (5.5).

We are now in a position to compute the nonperturbative contribution to weak decay amplitudes of kaons arising from long-distance quark-quark correlations in color-antitriplet states. The set of loop diagrams contributing to the decay $K_S \to 2\pi^0$ in lowest order of the chiral diquark model is shown in Fig. 2. The double lines are diquark propagators, and a black square represents the weak vertex from (3.2). The evaluation of these graphs is straightforward, although rather tedious. It involves the scalar two- and three-point functions Q_{ij} and C_{ijk} defined in the Appendix. Single diagrams diverge logarithmically, but the sum of all contributions is finite. The resulting on-shell decay amplitude reads

$$\begin{aligned} i\,\mathcal{A}_{\rm di}(K_S \to 2\pi^0) = \frac{G_F}{16\pi^2}\,V_{ud}\,V_{us} \\ \times \frac{2\,c_-\,g_{(ud)}\,g_{(su)}}{f_\pi^3}\,(A + B + C) \end{aligned} \tag{5.10}$$

with

$$\begin{aligned} A &= (\Delta m^2)^2\,(C_{222}^{-++} + C_{222}^{+--} - C_{223}^{+-+} - C_{223}^{-+-})\ , \\ B &= \Delta m^2\,[Q_{22}^{++}(m_K^2) - Q_{22}^{--}(m_K^2) - Q_{23}^{++}(m_\pi^2) + Q_{23}^{--}(m_\pi^2) - Q_{23}^{+-}(m_K^2) + Q_{23}^{-+}(m_K^2)]\ , \\ C &= \frac{v'}{v}\,m_\pi^2 \left(Q_{22}^{++}(m_K^2) + Q_{22}^{--}(m_K^2) - Q_{23}^{+-}(m_K^2) - Q_{23}^{-+}(m_K^2) + \frac{2}{1-\rho^2}\,Q_{22}^{+-}(m_\pi^2) - \frac{1}{1+\rho}\,Q_{23}^{++}(m_\pi^2) \right. \\ &\quad \left. - \frac{1}{1-\rho}\,Q_{23}^{--}(m_\pi^2) \right) + \frac{2\,v''}{v}\,m_\pi^2\,[Q_{33}^{++}(m_K^2) + Q_{33}^{--}(m_K^2) - Q_{22}^{++}(m_K^2) - Q_{22}^{--}(m_K^2)]\ . \end{aligned} \tag{5.11}$$

Here, $\rho = \delta m^2/\Delta m^2$ is the ratio of the SU(3) and parity mass splitting. The lower index on the two- and three-point functions denotes the diquark flavor $(2 = (su), 3 = (ud))$ while the upper signs refer to parity. Note that, as promised, only differences of the logarithmic divergent two-point integrals Q_{ij} enter in (5.11). Therefore, the divergent parts cancel.

The reader may convince her- or himself that the total amplitude vanishes in the SU(3) symmetry limit [where the masses of (ud) and (su) diquarks become degenerate], in accordance with the Gell-Mann–Cabibbo theorem. In order to bring the complicated expression (5.11) in a more transparent form, we expand the amplitude in powers of meson masses. Using (5.4), a cumbersome calculation gives a simple result valid to fourth order [35]:

$$i\,\mathcal{A}_{\rm di}(K_S \to 2\,\pi^0) = G_F\,V_{ud}\,V_{us}\,f_\pi\,(m_K^2 - m_\pi^2)\frac{c_-\,g_{(ud)}\,g_{(su)}}{8\pi^2 f_\pi^4}\,\Phi(z)\,(1+\Delta_4) + O(p^6)\ , \tag{5.12}$$

where

$$\Phi(z) = \frac{z+1}{z-1}\ln z - 2 \tag{5.13}$$

is a function of the ratio of averaged diquark masses introduced in (5.5). Note that to lowest order (p^2) the amplitude only depends on the single parameter $z_0 = z(m_q = 0)$, and is in accordance with the general structure (4.13) required by chiral symmetry. The parameters v' and v'' of the diquark Lagrangian enter in the fourth-order (p^4) term Δ_4. It reads

$$\Phi(z)\,\Delta_4 = \frac{m_\pi^2}{2\,m_+^2}\left[\frac{(4v''-v')\,v'}{v^2}\left(1+\frac{1}{z}\right) + \frac{v'}{v}\,F(z) + 2\,G(z)\right] - \frac{m_K^2}{12\,m_+^2}\left(\frac{2v'}{v} - 1\right)[F(z) + 2\,G(z)] \tag{5.14}$$

with functions

$$F(z) = 1 + \frac{1}{z} - \frac{2}{z-1}\ln z\ ,$$
$$G(z) = \frac{z^2+4z+1}{(z-1)^3}\ln z - 3\,\frac{z+1}{(z-1)^2}\ . \tag{5.15}$$

This fourth-order term enhances the amplitude by $(16 \pm 4)\%$, depending on the values of the mass ratio z and m_+. Contributions of even higher orders in (5.12) are small.

Table IV contains numerical values for the diquark-model amplitude computed from (5.10) for different mass parameters. For comparison, the results of the approximation (5.12) are shown in parentheses. It is an accurate approximation over the full range of parameters. In this table, the mass dependence of the diquark coupling constants as obtained from QCD sum rules has been taken into account. This stabilizes our results, i.e., decreases the dependence on the mass parameters. For the remainder of this paper, the parameter values of Table IV are used as a representative range. We average the amplitude over this range and take the variation as an estimate of the intrinsic error of our model. This gives

$$i\,\mathcal{A}_{\rm di}(K_S \to 2\,\pi^0) = (1.73 \pm 0.24 \pm 0.29) \times 10^{-7}\ \text{GeV}\ , \tag{5.16}$$

where the first error takes into account the variation with these parameters, while the second one reflects the uncertainty in the diquark coupling constants (2.9).

In the limit $z \to \infty$ corresponding to infinitely heavy pseudoscalar diquarks, the function $\Phi(z)$ diverges logarithmically, while $\Delta_4 \to 0$. This shows that the heavy

TABLE IV. $i\,\mathcal{A}_{\rm di}(K_S \to 2\,\pi^0)$ in units of 10^{-7} GeV for different diquark masses (given in GeV), as computed from (5.10). The SU(3) averaged mass m_+ has been defined in (5.5), and $\Delta m = m_- - m_+$. Numbers in parentheses refer to the approximation (5.12).

$m^+_{(ud)}$	$m^+_{(su)}$	m_+	$\Delta m = 0.7$	$\Delta m = 0.8$	$\Delta m = 0.9$	$\Delta m = 1.0$
0.40	0.62	0.52	1.438	1.626	1.814	1.999
			(1.308)	(1.493)	(1.677)	(1.860)
0.50	0.69	0.60	1.398	1.599	1.802	2.004
			(1.326)	(1.525)	(1.725)	(1.925)
0.60	0.77	0.69	1.423	1.643	1.867	2.091
			(1.380)	(1.598)	(1.820)	(2.042)

mass m_- of the pseudoscalar diquarks acts as an ultraviolet momentum cutoff Λ in the loop integrals. Thereby, we recover the result of the simplified diquark model [10]

$$i\,\mathcal{A}_{\rm di}(K_S\to 2\,\pi^0)\simeq G_F\,V_{ud}\,V_{us}\,f_\pi\,(m_K^2-m_\pi^2)\times\frac{c_-\,g_{(ud)}\,g_{(su)}}{8\pi^2 f_\pi^4}\,\ln\left(1+\frac{\Lambda^2}{m_+^2}\right)\,. \tag{5.17}$$

As in the case of hyperon decays, the diquark amplitude has to be supplemented by (new) factorization. Both amplitudes add with the same sign. In Sec. VI we shall discuss in detail that (5.10) is formally suppressed with respect to factorization in the $1/N_c$ expansion. Therefore there is no danger of double counting. Since $\mathcal{H}_{\rm eff}^{\rm di}$ in (3.2) is a $\Delta I=\frac{1}{2}$ operator, the diquark mechanism contributes to the isospin amplitude A_0 only. Hence, we find the theoretical results [see (4.3) and (4.13)]

$$\begin{aligned}A_0^{\rm th}&=\left(\frac{3}{2}\right)^{1/2}[i\,\mathcal{A}_{\rm di}(K_S\to 2\,\pi^0)+\sqrt{2}\,\kappa\,(g_8^f+g_{27}^{1/2,f})]\\&=[(2.12\pm0.46)+(1.33\pm0.27)]\times10^{-7}\ \text{GeV}\\&=(3.45\pm0.53)\times10^{-7}\ \text{GeV}\,,\end{aligned} \tag{5.18}$$

$$A_2^{\rm th}=\sqrt{6}\,\kappa\,g_{27}^{3/2,f}=(2.81\pm0.56)\times10^{-8}\ \text{GeV}\,,$$

where a 20% uncertainty has been associated with the factorization amplitudes to account for the uncertainty in the factorization scale. These numbers have to be compared with the experimental values quoted in (4.3). The pattern already looks quite promising, although some corrections are still needed. They are mainly due to meson loop effects (final-state interactions among them), which have not yet been taken into account. This is the subject of Sec. VII.

VI. CONNECTION TO CHIRAL PERTURBATION THEORY

The previous section dealt with an explicit calculation of a decay amplitude in the diquark model. Already the lowest-order (one diquark loop) result turned out to have quite a complex structure. Although the requirements of chiral symmetry are obeyed, it is hard to see how they come about, e.g., how the common factor $(m_K^2-m_\pi^2)$ in (5.12) arises. It would be instructive to see the connection with the conventional formulation of chiral perturbation theory, where this structure is a direct consequence of the derivative interactions in (4.10) and (4.12). This can be achieved in an elegant path-integral formulation of the diquark model [35]. We evaluate the generating functional, formally written as

$$Z[j_a]\sim\int\mathcal{D}P_a\,\mathcal{D}\phi_L\,\mathcal{D}\phi_R\exp\left(i\int{\rm d}x\,[\mathcal{L}(x)+j_a\,P_a(x)]\right), \tag{6.1}$$

where j_a are external sources for the pseudoscalar meson fields. The total Lagrangian $\mathcal{L}(x)$ consists of (4.9), (5.1), (5.2), and the weak interaction (3.2). The diquark degrees of freedom can be integrated out explicitly in (6.1). This gives rise to an effective, nonlocal Euclidean action involving the meson fields only:

$$\begin{aligned}\Gamma_{\rm di}^E[\Sigma]=&-\text{tr}\,\ln(1-G\,M_L)-\text{tr}\,\ln(1-G\,M_R)\\&-\text{tr}\,\ln\Bigg(1+\frac{G_F}{\sqrt{2}}\,V_{ud}\,V_{us}\,\frac{4}{3}\,c_-\,g_{(ud)}\,g_{(su)}\,\lambda_6\,G\,(1-M_L\,G)^{-1}\\&\qquad-\frac{(\Delta m^2)^2}{4}\,\Sigma^*\,G\,(1-M_R\,G)^{-1}\,\Sigma^T\,G\,(1-M_L\,G)^{-1}\Bigg)\,.\end{aligned} \tag{6.2}$$

The symbol tr means trace in flavor, color, and coordinate space. The matrices M_L and M_R are given by

$$\begin{aligned}M_L&=\frac{v'}{2}\,(m_q^*\,\Sigma^T+\Sigma^*\,m_q^T)-\frac{v''}{2}\,\langle\,m_q\,\Sigma^\dagger+\Sigma\,m_q^\dagger\,\rangle\,\mathbb{1}\,,\\M_R&=\Sigma^T\,M_L\,\Sigma^*\,.\end{aligned} \tag{6.3}$$

Furthermore,

$$G(x-y)=\int\frac{{\rm d}^4p}{(2\pi)^4}\,\frac{e^{ip\cdot(x-y)}}{p^2+m^2} \tag{6.4}$$

is the free Euclidean diquark propagator with m being the common diquark mass introduced in (5.1).

The effective action (6.2) can be expanded in terms of local meson operators with an increasing number of derivatives on the Σ field or powers of M_L and M_R. Up to order p^4 this program is carried out in Refs. [35]. It yields the standard form of the effective chiral Lagrangian, but the otherwise undetermined coupling constants are fixed in terms of the parameters of the diquark model. Here we just quote the resulting weak interaction Lagrangian to lowest order (p^2). It is of the general form (4.10) with coefficients

$$\begin{aligned}g_8^{\rm di}&=\frac{c_-\,g_{(ud)}\,g_{(su)}}{8\pi^2 f_\pi^4}\,\Phi(z_0)\,,\\h_8^{\rm di}&=\frac{c_-\,g_{(ud)}\,g_{(su)}}{8\pi^2 f_\pi^4}\,\frac{v'}{v}\,[Q_{m_+m_+}(0)+Q_{m_-m_-}(0)]\,.\end{aligned} \tag{6.5}$$

The diquark contribution to g_8 could have also been read off from the decay amplitude (5.12) when compared to (4.13). Note, however, that the ratio z_0 of diquark masses in the chiral limit, as defined in (5.9), appears as argument in the function $\Phi(z_0)$ while the ratio z of SU(3) averaged masses is to be used in (5.12). Both differ, of course, in higher orders of the chiral expansion. The effective-action approach immediately shows that, to lowest order, all weak amplitudes in the diquark model are determined by the same coefficient $g_8^{\rm di}$, as it is required by chiral symmetry.

As a side remark, we note that our approach shows that, unlike commonly believed, the coefficient h_8 of the weak mass operator in (4.10) gets contributions from nonpenguin operators. In our case $h_8^{\rm di}$ involves the logarithmically divergent two-point functions Q_{ij} (Ref. [36]) evaluated at zero external momentum and with $0^{\pm}$ diquark masses in the limit $m_q = 0$. They can be regularized with a cutoff in the loop momentum. For reasonable values of this cutoff, $h_8^{\rm di}$ is larger than the penguin contribution h_8^f in (4.15) by more than an order of magnitude and has a different sign. Although this coefficient is irrelevant for the decay amplitudes considered here, it would be important, for instance, in decays of pseudoscalar mesons into a light Higgs particle [37].

Equation (6.5) shows that nonperturbative quark-quark correlations in color-antitriplet states give a sizable long-distance contribution to the phenomenological parameter g_8 of the lowest-order effective chiral Lagrangian for nonleptonic weak decays. Adding to this the factorization result from (4.15), we obtain the total coefficient [38]

$$\begin{aligned} g_8 &= \frac{c_-}{2}\left(1+\frac{g_{(ud)}\,g_{(su)}}{4\pi^2 f_\pi^4}\,\Phi(z_0)\right)+\frac{c_+}{10}-\chi\, c_6 \\ &= (2.85\pm0.71)+(1.41\pm0.28)=4.26\pm0.76\ . \end{aligned} \qquad (6.6)$$

The first number refers to the diquark contribution, the second one to factorization. The errors have been determined as in (5.18).

Alternatively, one may define an effective, scale-independent coefficient $c_-^{\rm eff}$ by

$$c_-^{\rm eff} = c_-(\mu_f) + (5.7\pm1.0)\left(\frac{c_-\, g_{(ud)}\, g_{(su)}}{0.09\ {\rm GeV}^4}\right)\ , \qquad (6.7)$$

where the factorization contribution has to be understood at the factorization point μ_f, the diquark contribution is scale independent. The last two equations are central results of our analysis. They clearly show how matrix elements of the octet operator Q_- between pseudoscalar mesons are, in addition to the perturbative enhancement of the coefficient c_-, further amplified by long-distance phenomena. The strength of this effect is measured by the nonperturbative diquark coupling constants. Note that this very welcome enhancement differs drastically from the conventional factorization prescription giving $c_-^{\rm eff} = c_-\,(1-1/N_c)$.

At this point it is worthwhile to discuss the N_c dependence of g_8 in (6.6). The pion decay constant f_π is of order $\sqrt{N_c}$, whereas the diquark coupling constants $g_{\rm di}$ [more precisely, the matrix element (2.5)] do not depend on the number of colors. Each diquark loop in the diagrams of Fig. 2 is associated with a color multiplicity factor $\frac{1}{2}\,N_c\,(N_c-1)$ according to the dimension of the irreducible antisymmetric representation contained in $N_c \otimes N_c$. In addition, at least one gluon exchange is required because of the color mismatch between mesons and diquarks. We illustrate this in Fig. 3 for one of the diagrams contributing to $\bar{K}^0 \to \pi^0$ transitions. Quite generally, diquark loops are therefore suppressed by one power of $\alpha_s \sim 1/N_c$ and count like $O(N_c)$ in the large-N_c expansion. Consequently, the diquark contribution to g_8 is formally suppressed by $1/N_c$ compared to factorization

$$g_8^{\rm di} = O\!\left(\frac{1}{N_c}\right)\ , \quad g_8^f = O(1)\ . \qquad (6.8)$$

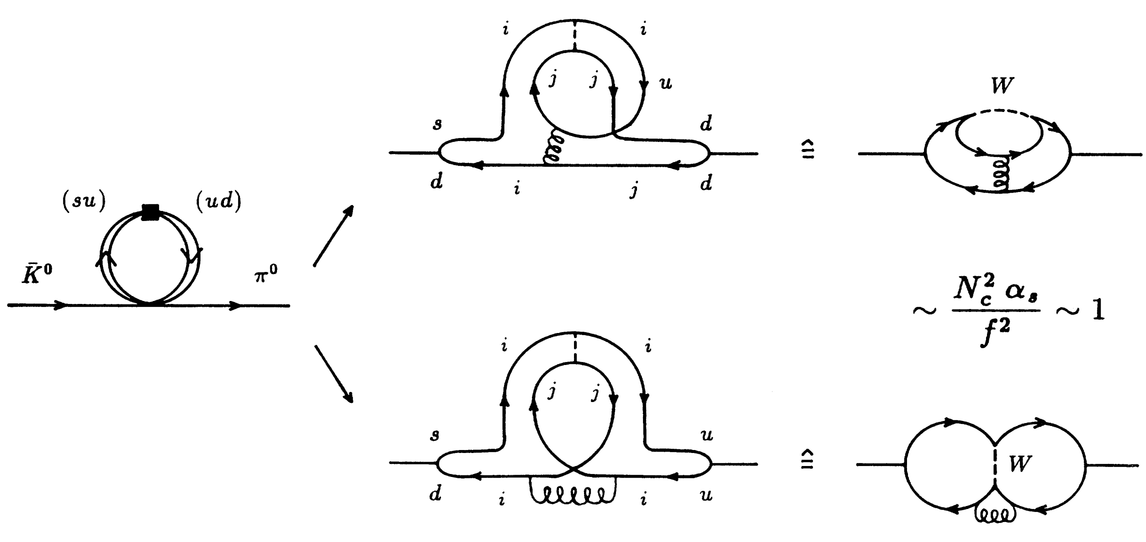

FIG. 3. Typical quark-line diagrams arising in the diquark model. $\bar{K}^0 \to \pi^0$ transitions represented by this Feynman graph can proceed in two ways corresponding to "eye" and "figure eight" topologies. The formation of an intermediate diquark pair in color-antitriplet states requires the exchange of at least one gluon. Generally, in weak interactions, diquark amplitudes are suppressed by $1/N_c$ compared to factorization. For $K \to \pi$ transitions the factorized amplitude is proportional to $f_\pi\, f_K \sim N_c$.

This is in accordance with the statement that factorization becomes exact in the $N_c \to \infty$ limit [7], but apparently g_8^{di} is by far the dominant contribution at the physical value $N_c = 3$. One is, nevertheless, forced to add factorization since it gives the only term which survives in the formal limit $N_c \to \infty$. With the help of the N_c power counting one can unambiguously distinguish between both contributions. There is no danger of double counting.

The above results are particularly suited for a comparison to other approaches. We want to stress that (6.6) describes a dynamical enhancement of $\Delta I = \frac{1}{2}$ amplitudes already in lowest order of chiral perturbation theory. Such an effect is not observed in the detailed analysis of the $\Delta I = \frac{1}{2}$ rule by Bardeen, Buras, and Gérard [8] who start from the factorization ansatz $\mathcal{H}_{\text{eff}} \sim L_\mu L^\mu$, where L_μ is the hadronic current operator introduced in (4.11), and compute nonfactorizable $1/N_c$ corrections arising from meson loops. All of these corrections are of next-to-leading order (p^4) in the chiral expansion. In our opinion, this ansatz does not fully take into account the possibility of quark reordering before hadronization. Contributions of this type are strongly enhanced by the diquark mechanism. They are caused by the complex structure of the composite four-quark operator $\mathcal{H}_{\text{eff}}$.

In this context, it is interesting that the behavior indicated by (6.6) is also found in dual QCD sum rules [32]. In this approach one finds an enormously large gluonic radiative correction to the coefficient g_8 provided and only if $1/N_c$ corrections are taken into account. This effect indicates a break down of both perturbative QCD and the $1/N_c$ expansion. The diquark model allows for the first time an estimate of the size of this nonperturbative effect.

Finally, we mention that recently $K \to \pi$ and $K \to \pi\,\pi$ matrix elements have also been investigated in numerical lattice calculations [39]. The quark flow diagrams arising in our model are very similar to those considered there, the so-called "eye" and "figure eight" topologies. For $K \to \pi$ transitions, examples of both cases are shown in Fig. 3. From (6.6) it follows that in our model matrix elements of the operator Q_- give the dominant contribution to $\Delta I = \frac{1}{2}$ amplitudes. If lattice results become more conclusive in the future, it should be possible to test the ideas presented here in numerical calculations.

VII. HIGHER-ORDER CHIRAL CORRECTIONS

Up to now, the theoretical isospin amplitudes (5.18) are still in slight disagreement with the experimental ones given in (4.3). In addition, they have no relative phase, while the experimental phase difference $|\delta_0 - \delta_2| \simeq 56°$ is quite substantial. The fact that the $K \to \pi\,\pi$ amplitudes must have absorptive parts is a consequence of unitarity. It is due to the possibility of $\pi\,\pi$ rescattering in the final state. In order to account for this one has to consider higher-order corrections in the chiral expansion. They arise from meson loops as well as from higher-dimensional operators in the effective chiral Lagrangian.

The general framework for higher-order calculations in chiral perturbation theory has been worked out by Gasser and Leutwyler [28], and has later been applied to nonleptonic weak interaction [29]. It is convenient to use the method of dimensional regularization for the ultraviolet divergences arising from meson loop diagrams. The poles in $(D-4)$ (D denotes the space-time dimension) can be absorbed into a renormalization of the coupling constants associated with operators of order p^4 in the effective chiral Lagrangian. These operators have been classified for the strong [28], and recently also for weak interactions [29]. Unfortunately, their number is quite large, and consequently many *a priori* unknown phenomenological parameters, the renormalized coupling constants, enter the theory. Whereas it is possible to determine the coefficients of the strong-interaction Lagrangian experimentally, this seems not feasible for weak interactions since there are as many as 38 independent octet and 28 independent **27**-plet operators in next-to-leading order.

Therefore, one has to rely on phenomenological models to provide estimates for these coupling constants. $1/N_c$ inspired arguments can give a rough pattern, but they are not very predictive in this case since most of the relevant weak operators involve the same power of N_c. Furthermore, we have just seen that there are good arguments that the $1/N_c$ expansion is not useful in the particular case of $\Delta I = \frac{1}{2}$ nonleptonic weak decays of kaons. Recently, some geometrical methods relating the coefficients of the strong and weak Lagrangians have been suggested [40], but the success of such ideas still has to be demonstrated. We shall use a different approach here. With the method developed in Sec. VI it is possible to determine all the coupling constants of both the strong and the octet part of the weak-interaction Lagrangian in the framework of the diquark model [35]. Weak couplings get an additional contribution from factorization. It is obtained by replacing the color singlet quark currents in the effective Hamiltonian (2.1) by hadronic currents L_μ derived from the next-to-leading-order strong-interaction Lagrangian.

Let us illustrate the renormalization procedure with an example from strong interactions. Next-to-leading-order corrections to the matrix elements of the axial-vector current between a meson and the vacuum modify the lowest-order relation $f_\pi = f_K = f$. In dimensional regularization with $D = 4 + 2\epsilon$ one finds, for the contribution of meson loops ($\hat{\epsilon}^{-1} = \epsilon^{-1} + \gamma_E - \ln 4\pi - 1$, where $\gamma_E = 0.5772\ldots$ is Euler's constant),

$$\frac{f_K}{f_\pi} = 1 - \frac{3}{2}\,\frac{m_K^2 - m_\pi^2}{(4\pi f)^2}\,\frac{1}{\hat{\epsilon}} + \frac{5}{4}\,\mu_\pi - \frac{1}{2}\,\mu_K - \frac{3}{4}\,\mu_\eta\ . \quad (7.1)$$

The chiral logarithms μ_i are defined as

$$\mu_i = \frac{m_i^2}{(4\pi f)^2}\,\ln\frac{m_i^2}{\mu^2}\ , \quad i = \pi, K, \eta\ , \quad (7.2)$$

where μ is an arbitrary subtraction point. The contribution of $O(p^4)$ operators to (7.1) reads [28]

$$\frac{8}{f^2}\,(m_K^2-m_\pi^2)\,L_5 \equiv \frac{8}{f^2}\,(m_K^2-m_\pi^2) \times\left(\frac{3}{16\,(4\pi)^2}\,\frac{1}{\hat{\epsilon}}+L_5^r(\mu)\right)\,, \tag{7.3}$$

and the renormalized ratio of decay constants becomes

$$\frac{f_K}{f_\pi}=1+\frac{5}{4}\,\mu_\pi-\frac{1}{2}\,\mu_K-\frac{3}{4}\,\mu_\eta+\frac{8}{f^2}\,(m_K^2-m_\pi^2)\,L_5^r(\mu)\,. \tag{7.4}$$

The scale dependence of the renormalized coefficient $L_5^r(\mu)$ cancels the scale dependence of the logarithms.

The diquark-model prediction for this coefficient reads [35]

$$L_5^{\rm di}=\frac{3}{16}\,\frac{v'}{v}\,\frac{\Phi(z_0)}{(4\pi)^2} \tag{7.5}$$

involving the same function as appearing in the decay amplitude (5.12). Assuming that the diquark model gives the dominant contribution to L_5 at a scale of 1 GeV, i.e., $L_5^r(1\,\text{GeV})\simeq L_5^{\rm di}=(0.75\pm0.14)\times10^{-3}$, one finds $f_K/f_\pi=1.215\pm0.015$ in excellent agreement with the experimental ratio (Ref. [2]) $f_K/f_\pi=1.22\pm0.01$. Note that this scale is very reasonable and roughly coincides with both the factorization point and the masses of pseudoscalar diquarks which effectively cutoff the diquark loop integrals.

Similarly, other phenomenological parameters L_i of the strong interaction Lagrangian [28] can be understood in terms of quark-quark correlations. It turns out that, with exception of L_1, L_2, L_3 which are known to be saturated by the ρ resonance [41], the diquark model reproduces the experimental values within few percent at the scale $\mu\simeq1$ GeV. This can be seen as a dual approach to Ref. [42] where the couplings L_i are saturated with meson resonances. But in addition, the diquark model gives predictions also for the octet part of the weak interaction chiral Lagrangian, and for long-distance contributions to the effective $\Delta S=2$ interaction (see Sec. VIII and Ref. [11]). The corresponding coupling constants are called (Ref. [29]) E_i and D_i, respectively. Knowledge of E_i at the scale $\mu\simeq1$ GeV builds the basis for a consistent treatment of next-to-leading-order corrections to the amplitude A_0, to which we turn now.

The meson loop diagrams contributing to the process $K_S\to2\,\pi^0$ are shown in Fig. 4. The black squares now denote weak vertices from the effective Lagrangian (4.10) with $g_8=g_8^f+g_8^{\rm di}$ as given in (6.6). For the amplitude A_0 one can safely neglect the contribution proportional to $g_{27}^{1/2}$ and absorb the next-to-leading order chiral correction into a redefinition of the coupling parameter g_8. To this end, we define the renormalized coupling g_8^r by means of the on-shell decay amplitudes (4.13), i.e.,

$$A_0\equiv\left(\frac{3}{2}\right)^{1/2}G_F\,V_{ud}\,V_{us}\,(m_K^2-m_\pi^2)\,f_\pi\,g_8^r\,e^{i\delta_0} \tag{7.6}$$

to all orders in chiral perturbation theory. Here, f_π is the renormalized pion decay constant, and δ_0 the strong-interaction phase of the amplitude. Note that g_8^r is no longer associated with a single operator but gets contributions from all orders in the chiral expansion. It is convenient to separate the contributions from factorization and diquarks to g_8^r,

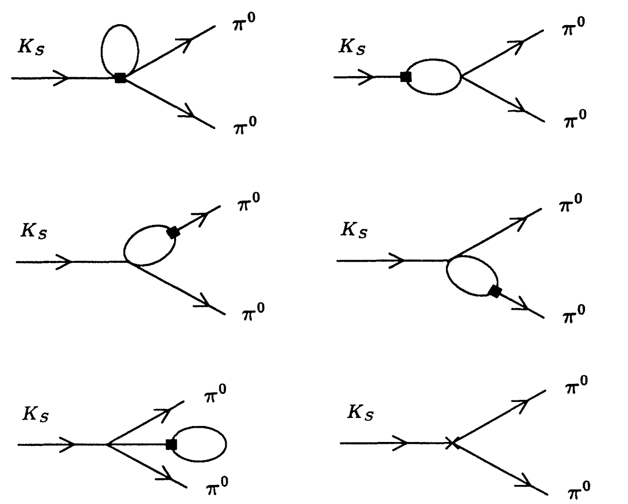

FIG. 4. Meson loop corrections to the $K_S\to\pi^0\pi^0$ amplitude. The last diagram represents the renormalization of meson fields and decay constants as well as contributions from the next-to-leading-order weak chiral Lagrangian.

$$g_8^r=(1+\Delta_f)\,g_8^f+(1+\Delta_{\rm di})\,g_8^{\rm di}\,, \tag{7.7}$$

where the "bare couplings" g_8^f and $g_8^{\rm di}$ have been given in (4.15) and (6.5), respectively. Both differ in the power of the (unrenormalized) decay constant f and are therefore renormalized differently. The complete $O(p^4)$ calculation of the chiral corrections Δ_i is the subject of Refs. [35]. We just quote the result here. One finds

$$1+\Delta_f=\frac{f_\pi}{f_K}\left[1+\frac{157}{18}\,\mu_\pi-\frac{15}{2}\,\mu_K+\frac{9}{2}\,\mu_\eta +\frac{m_K^2}{(m_K^2-m_\pi^2)}\left(\frac{f_K}{f_\pi}-1\right)+C_1\right],$$

$$1+\Delta_{\rm di}=\frac{\Phi(z)}{\Phi(z_0)}\,\frac{f_\pi}{f_K}\left(1+\frac{13}{18}\,\mu_\pi-13\,\mu_K+\frac{9}{2}\,\mu_\eta +\Delta_4+C_2\right), \tag{7.8}$$

$$\delta_0=2\pi\,\frac{m_K^2}{(4\pi\,f_\pi)^2}\left(1-\frac{m_\pi^2}{2\,m_K^2}\right)\left(1-\frac{4\,m_\pi^2}{m_K^2}\right)^{1/2}\simeq26^\circ\,.$$

The contribution of chiral logarithms in Δ_f agrees with the result given in Ref. [8]. Here, however, we present for the first time expressions for the full correction. Apart from the chiral logarithms these contain terms arising from tree diagrams of the next-to-leading-order (p^4) weak chiral Lagrangian. In the case of Δ_f they are proportional to L_5 and have been expressed in terms of $(f_K/f_\pi-1)$ with the help of (7.4). We see that part of the correction $\Delta_{\rm di}$ results in the replacement of the "bare" ratio z_0 appearing in (6.5) by the physical ratio z of averaged diquark masses [42]. The remaining fourth-order correction from p^4 operators is contained in the quantity Δ_4 such that, apart from meson loops, we recover (5.12). Finally, the constants C_1 and C_2 are rather complicated functions of the meson masses [35]. They have values $C_1 \simeq 0.16$ and $C_2 \simeq 0.11$.

At the matching scale $\mu = 1$ GeV, the next-to-leading-order chiral corrections amount to an enhancement of g_8 by about 25%:

$$\Delta_f \simeq 0.23 \; , \quad \Delta_{\rm di} \simeq 0.26 \pm 0.14 \; . \tag{7.9}$$

The error in $\Delta_{\rm di}$ reflects the variation with the diquark mass parameters. Below we will show that an increase of the amplitude A_0 of this size is already expected from $\pi\,\pi$ final-state interaction alone. However, the total chiral correction includes more, namely, vertex corrections and the renormalization of the meson decay constants. From (6.6) one finds the renormalized coefficient

$$g_8^r = (5.25 \pm 0.93) + (2.15 \pm 0.48) \ln \frac{\mu}{\rm GeV} \; , \tag{7.10}$$

where the dependence on the scale μ, where the matching of meson loops and p^4 operators is done, has been shown explicitly. Two thirds of this value are accounted for by the diquark contribution. Adding the small contribution of $g_{27}^{1/2}$ from (4.15), and taking into account a 30% uncertainty in the scale μ, we finally obtain

$$g_8 + g_{27}^{1/2} = 5.3 \pm 1.1 \; . \tag{7.11}$$

The accuracy of the next-to-leading-order calculation can be estimated from the result for the scattering phases, which have to reproduce the large value of the phase difference given in (4.3). To order p^4 one finds $\delta_2 \simeq -\delta_0/2 \simeq -13^\circ$, and hence $\delta_0 - \delta_2 \simeq 40^\circ$. The value of δ_2 is indeed consistent with the experimental value for the $I = 2$, S-wave $\pi\,\pi$ scattering phase (Ref. [43]) $\delta_2^{\rm expt} \simeq -(11 \pm 4)^\circ$. In the case of δ_0 one finds agreement with (7.8) after subtraction of the phase shift associated with the strongly inelastic $f_0(975)$ resonance [44]. It is therefore sufficient to stick to the one-loop approximation.

The full next-to-leading-order calculation of the $\Delta I = \frac{3}{2}$ amplitude A_2 in chiral perturbation theory has not yet been done. It would require a model treatment of the order-p^4 chiral Lagrangian for $\Delta I = \frac{3}{2}$ transitions. From the fact that the $\pi\,\pi$ scattering phase is small and negative in this case, one may expect a moderate suppression of A_2. For an estimate of this effect we follow the approach of Ref. [45] where, starting from a Muskhelishvilli-Omnès-type dispersion relation, the effect of final-state interactions to the $K \to 2\,\pi$ amplitudes can be determined from the measured S-wave $\pi\,\pi$ scattering phases $\delta_0(s)$ and $\delta_2(s)$. To this end one considers the isospin amplitudes as analytic functions in $s = p_K^2$, satisfying $A(s = m_\pi^2) = 0$ as required by the Gell-Mann–Cabibbo theorem. Assuming that the derivative of $A(s)$ with respect to s is known at $s = m_\pi^2$, a once-subtracted dispersion relation can be solved. It gives a multiplicative final-state correction factor which, in the case of A_2, reads [45]

$$G_2(s) = \exp\left(i\,\delta_2(s) + \frac{s - m_\pi^2}{\pi} \times \mathcal{P} \int_{4m_\pi^2}^{\infty} {\rm d}s' \, \frac{\delta_2(s')}{(s' - m_\pi^2)\,(s' - s)} \right) . \tag{7.12}$$

$\mathcal{P}$ denotes the principle value of the integral. We have evaluated this function using the experimental data for the scattering phase (Ref. [43]) $\delta_2(s)$. The result is

$$G_2(m_K^2) \simeq (0.89 \pm 0.03)\, e^{-i\,(11 \pm 4)^\circ} \; . \tag{7.13}$$

As expected, the $\Delta I = \frac{3}{2}$ amplitude is moderately suppressed by final-state interactions. We shall take this number as an estimate of the full next-to-leading-order correction of A_2. [In the case of A_0, the estimated correction is (Ref. [45]) $|\, G_0(m_K^2) \,| \simeq 1.40$. It is of the same order of magnitude as the total chiral correction given in (7.9).]

We are now in a position to present our final results for both isospin amplitudes. The $\Delta I = \frac{1}{2}$ coefficient (7.11) governs the amplitude A_0, while the factorization result for A_2 has to be corrected for final-state interactions by means of (7.13). This yields

$$A_0 = (4.86 \pm 1.01) \times 10^{-7} \; {\rm GeV} = (1.03 \pm 0.21)\, A_0^{\rm expt} \; , \tag{7.14}$$
$$A_2 = (2.53 \pm 0.51) \times 10^{-8} \; {\rm GeV} = (1.20 \pm 0.24)\, A_2^{\rm expt}$$

in excellent agreement with the experimental values given in (4.3). The $\Delta I = \frac{1}{2}$ rule is no longer mysterious.

VIII. THE K_L-K_S MASS DIFFERENCE

As a further example, we focus on the effects of long-range quark-quark correlations on the neutral kaon mass matrix. The observation of K^0-$\bar{K}^0$ mixing was the first example of particle-antiparticle oscillations and had important implications for the development of the standard model. It still gives the tightest constraint on flavor-changing neutral currents, is responsible for the mass difference Δm_K between the weak eigenstates K_L and K_S, and determines the CP impurity parameter ϵ. The measurement of a nonzero ϵ by Christenson *et al.* provided the first and up to now only evidence for CP violation [46]. Today it is still not possible to firmly constrain parameters of the standard model from K^0-$\bar{K}^0$ mixing, since the theoretical description involves hadronic matrix elements, causing sizable uncertainties. In particular, Δm_K is very sensitive to long-distance physics.

In the standard model, the short-distance K^0-$\bar{K}^0$ transitions are described by box diagrams with double W-boson exchange, as shown in Fig. 5(a). From the associated amplitude with external quarks, an effective $\Delta S = 2$ Hamiltonian is derived by subsequently integrating out the W bosons, top and charm quarks [47]. Even for a heavy top quark, the real part of the mixing amplitude is almost insensitive to the third family. The dominant short-distance contribution to the K_L-K_S mass difference

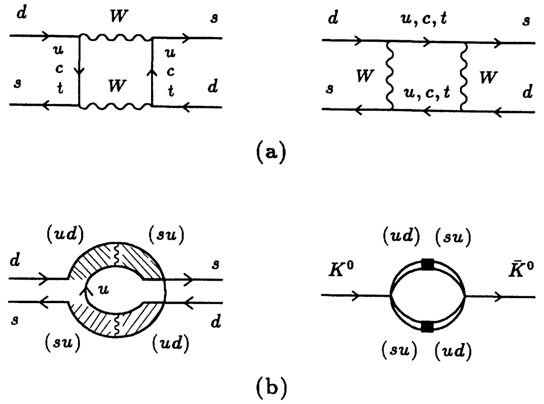

FIG. 5. Different contributions to K^0-$\bar{K}^0$ mixing: (a) short-distance box diagram transitions in the standard model; (b) long-distance quark-quark correlations simulated by intermediate diquark states. The Feynman graph of the diquark model is also shown.

comes from the charm quark

$$m_K \, \Delta m_K^{\rm box} \simeq \frac{G_F^2}{16\pi^2} \, \eta_c(\mu) \, V_{ud}^2 \, V_{us}^2 \, m_c^2 \times \langle \bar{K}^0 | \, (\bar{s}\, d)_{V-A} (\bar{s}\, d)_{V-A} \, | K^0 \rangle \; . \qquad (8.1)$$

The scale-dependent coefficient η_c takes into account short-distance QCD corrections to the box diagrams. It is roughly independent of the top-quark mass and has the value (Ref. [47]) $\eta_c \simeq 0.7$ for $\alpha_s(\mu) \simeq 1$. The observation that Δm_K is proportional to m_c^2 led to an estimate of the charm quark mass by Galliard and Lee [48] already in 1974.

The hadronic matrix element in (8.1) is conveniently written in terms of the so-called B_K parameter,

$$\langle \bar{K}^0 | \, (\bar{s}\, d)_{V-A} (\bar{s}\, d)_{V-A} \, | K^0 \rangle = \tfrac{8}{3} \, B_K(\mu) \, f_K^2 \, m_K^2 \; , \qquad (8.2)$$

such that the combination $\eta_c(\mu) \, B_K(\mu)$ is scale independent. There are several methods to calculate this parameter, most of which give values significantly smaller than the naive vacuum-insertion result $B_K = 1$. While QCD sum rules [32, 49] and chiral perturbation theory [50] yield rather small values $B_K \sim 0.3 - 0.6$, numerical lattice computations [39] predict $B_K \sim 0.8 - 1.0$. In the large-N_c limit, $B_K = \frac{3}{4}$. We shall use $B_K = 0.6 \pm 0.3$ [at the scale where $\alpha_s(\mu) = 1$] for an estimate, and obtain, with $m_c \simeq 1.35$ GeV,

$$\Delta m_K^{\rm box} \simeq (1.0 \pm 0.5) \times 10^{-15} \text{ GeV} \simeq (0.14 \pm 0.07) \, \Gamma_S \qquad (8.3)$$

while experimentally [2]

$$\Delta m_K = (3.522 \pm 0.016) \times 10^{-15} \text{ GeV} \; , \qquad \frac{\Delta m_K}{\Gamma_S} = 0.477 \pm 0.002 \; . \qquad (8.4)$$

$\Gamma_S \simeq 7.377 \times 10^{-15}$ GeV is the experimental decay width of K_S. Obviously, the short-distance analysis can only account for a small fraction of the mass difference.

The situation becomes even worse if mixing via single-particle intermediate states $K^0 \leftrightarrow \pi^0, \eta, \eta' \leftrightarrow \bar{K}^0$ is taken into account. It is well known that the sum of the contributions from π^0 and the octet state η_8 vanishes in the SU(3)-symmetry limit [51]. Symmetry-breaking corrections are quite large in this case, however, and the result is particularly sensitive to the $\eta\,\eta'$ mixing angle and to the coupling of the singlet η_1 to K_L. A recent detailed analysis can be found in Ref. [11]. Assuming a nonet symmetry for matrix elements of the effective weak Lagrangian between pseudoscalar mesons, one obtains (for $\vartheta_{\eta\eta'} = -20^\circ \pm 4^\circ$)

$$\frac{\Delta m_K^{\pi^0,\eta,\eta'}}{\Gamma_S} \simeq -0.15 \pm 0.15 \qquad (8.5)$$

as a rough estimate. It shows the tendency to cancel the result (8.3) of the short-distance analysis. Consequently, the main part of the mass difference must come from further long-distance contributions.

In the diquark model, due to flavor conservation in strong interactions only the single diagram shown in Fig. 5(b) induces K^0-$\bar{K}^0$ mixing. It accounts for the attractive long-range forces between light quarks in the box diagrams. The corresponding contribution to the mass difference is of the form [10]

$$m_K \, \Delta m_K^{\rm di} = \left(\frac{G_F}{\sqrt{2}} \, V_{ud} \, V_{us} \right)^2 \, (c_- \, g_{(ud)} \, g_{(su)})^2 \, \frac{4}{3} \, \frac{I(m_{\rm di})}{(4\pi \, f_\pi)^2} \qquad (8.6)$$

with $I(m_{\rm di})$ being a rather complicated, finite function of the diquark masses. In terms of the scalar two-point integral defined in the Appendix it reads [11]

$$I(m_{\rm di}) = \left\{ Q_{33}^{++} - 2\, Q_{33}^{+-} + Q_{33}^{--} + Q_{22}^{++} - 2\, Q_{22}^{+-} + Q_{22}^{--} - 2 \left[\left(\frac{\Delta m^2}{\delta m^2} \right)^2 - 2 \right] \left(Q_{22}^{+-} + Q_{33}^{+-} - Q_{23}^{+-} - Q_{32}^{+-} \right) \right\}_{p^2 = m_K^2} , \qquad (8.7)$$

where δm^2 has been defined in (5.4). The total expression for $I(m_{\rm di})$ stays finite in the SU(3)-symmetry limit $\delta m^2 = 0$.

In Table V we show numerical results for the diquark contribution to Δm_K for different mass parameters. As previously, we have assumed the same SU(3) mass splitting for mesons and diquarks. If the scalar (ud) diquark is not too light, the dependence on the parameters is rather moderate. For sensible values $m_{(ud)}^{+} \simeq 0.5 - 0.6$ GeV we obtain

$$\Delta m_K^{\rm di} \simeq (3.5 \pm 1.2) \times 10^{-15} \text{ GeV} = (0.47 \pm 0.16) \, \Gamma_S \; , \qquad (8.8)$$

TABLE V. $\Delta m_K^{\rm di}$ in units of 10^{-15} GeV. The mass parameters (given in GeV) are the same as in Table IV.

Δm	$m^+_{(ud)} = 0.4$	$m^+_{(ud)} = 0.5$	$m^+_{(ud)} = 0.6$
0.6	4.803	3.679	3.274
0.7	4.777	3.637	3.218
0.8	4.811	3.653	3.220
0.9	4.883	3.709	3.264
1.0	4.979	3.790	3.337

where the error takes into account the uncertainty in the diquark coupling constants. It is exciting that the single diagram of the diquark model can perfectly account for the experimental value of the mass difference given in (8.4). The conclusion is that Δm_K is completely dominated by long-distance physics and is caused by the dynamical mechanism also responsible for the $\Delta I = \frac{1}{2}$ enhancement in kaon and hyperon decays.

It is possible to analyse K^0-$\bar{K}^0$ mixing not in terms of intermediate diquark states, but in the framework of an effective meson theory characterized by a weak chiral Lagrangian of the form

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\Delta S=1} + \mathcal{L}_{\Delta S=2} \; . \qquad (8.9)$$

Contributions to the mass difference then arise from diagrams involving either two separate $\Delta S = 1$ vertices or a contact $\Delta S = 2$ vertex induced by long-distance effects. For details of the rather lengthy next-to-leading-order calculation we refer the interested reader to Ref. [11]. The result agrees with the prediction of the diquark model. We thus conclude that the single graph shown in Fig. 5(b) combines the effects of a huge set of meson loop diagrams.

IX. CONCLUSIONS

Although the general pattern of weak decays is well described by the standard model of strong and electroweak interactions, a coherent understanding of nonleptonic transitions and, in particular, of the $\Delta I = \frac{1}{2}$ enhancement observed in strange-particle decays, was always missing. The intricate interplay between strong and weak interactions in these processes has remained a mystery for decades. During the last years, important new information came from detailed experimental and theoretical studies of decays of D and B mesons. Soon it was recognized that nonperturbative aspects of QCD are important even in these energetic transitions. In two-body decays the dominant effects arise from the confining color forces between quarks and antiquarks during the hadronization. They can be taken care of by an appropriate factorization prescription, by hadronic form factors and meson decay constants. In this way a satisfactory semiquantitative description of energetic transitions of heavy mesons could be established.

In the case of strange-particle decays, on the other hand, it has been known for a long time that factorization cannot account for the large $\Delta I = \frac{1}{2}$ amplitudes. This has often been considered as an argument against this approximation, but it is actually not. The point is simply that factorized amplitudes involve matrix elements of vector and axial-vector currents, which become small for kinematical reasons at low momentum transfers. In addition to the attractive interaction between quarks and antiquarks, the strong-binding forces between two quarks in color-antitriplet states must also be considered then. They lead to momentum independent matrix elements of scalar currents and, consequently, become important at low energies. We have shown, in fact, that strongly correlated quark pairs play the key role in the understanding of nonleptonic weak decays at low energies.

A measure of the strength of the attraction between two quarks are the so-called diquark decay constants $g_{(\rm di)}$. Their calculation from QCD sum rules gives large values, as large as the coupling of the pion to a pseudoscalar current. It is, therefore, justified to simulate the strongly correlated quark pairs by local diquark fields. The fundamental nonperturbative quantity which enters the effective weak Hamiltonian is the product

$$c_- \, g_{(ud)} \, g_{(su)} \simeq 0.09 \text{ GeV}^4 \; . \qquad (9.1)$$

This combination is scale independent. It reflects a dynamical enhancement of matrix elements of the $\Delta I = \frac{1}{2}$ operator Q_-.

The calculation of hyperon decay amplitudes is the most natural application of this concept, since quark pairs in color-antitriplet states are part of baryon wave functions. The weak Hamiltonian simply replaces an (su) diquark in the initial (or intermediate) baryon by a (ud) diquark, with an amplitude determined in magnitude and sign by the above product of coupling constants. If the resulting amplitudes are supplemented by those obtained from factorization of the weak Hamiltonian, a very satisfactory quantitative description of all S- and P-wave amplitudes is obtained. A simple pattern emerges. In strange-particle decays, amplitudes are always large if they can proceed via a diquark transition, and small if not. An interesting confirmation of this rule occurs in Ω^- decays. The process $\Omega^- \to \Xi\,\pi$ is strongly suppressed compared to $\Omega^- \to \Lambda\, K^-$ although it involves a $\Delta I = \frac{1}{2}$ transition. The flavor quantum numbers simply do not allow for a diquark transition in this case.

The main part of this paper focuses on nonleptonic decays of kaons. We have shown that a quantitative treatment of these processes becomes possible by a generalization of conventional chiral perturbation theory. Chiral symmetry fixes to a large extent the structure and strength of the interaction between scalar diquarks and pseudoscalar mesons. The coupling constants of the effective Lagrangian are determined in terms of mass parameters for which little freedom exists. The diquark fields are then integrated out in the generating functional. The chiral expansion of the effective action yields the standard form of the chiral Lagrangians for strong and weak interactions of pseudoscalar mesons. All couplings are calculable in terms of diquark mass parameters. In particular, we have shown that the octet part of the effective weak interaction is strongly enhanced by nonperturbative effects already in lowest order of the chiral expansion. Matrix elements of the operator Q_- get, in addition to the factorization amplitude, a long-distance contribution from quark-quark correlations which is nonleading in the $1/N_c$ expansion but nevertheless dominant for the physical value $N_c = 3$. For large N_c, factorization becomes exact, but it fails completely to describe the $\Delta I = \frac{1}{2}$ amplitude for $K \longrightarrow \pi\pi$. A breakdown of the $1/N_c$ expansion in the $\Delta I = \frac{1}{2}$ channel accompanied by a failure of perturbative QCD has also been found in dual QCD sum-rule studies. The diquark model is the first nonperturbative approach which predicts the size of this effect.

Including next-to-leading-order chiral corrections, our theoretical results for the two isospin amplitudes describing $K \longrightarrow \pi\pi$ decays are in excellent agreement with experiment

$$\frac{A_0^{\rm th}}{A_0^{\rm expt}} = 1.0 \pm 0.2 \; , \quad \frac{A_2^{\rm th}}{A_2^{\rm expt}} = 1.2 \pm 0.2 \; . \qquad (9.2)$$

The resolution of the $\Delta I = \frac{1}{2}$ puzzle happens in various steps. Perturbative QCD corrections increase the amplitude A_0 by roughly a factor of 2, while A_2 is moderately suppressed. Evaluated in factorization it is only slightly larger than the experimental value. For the $\Delta I = \frac{1}{2}$ amplitude A_0 there is still a factor of 3 missing, however. It is this amplitude which is further amplified by the attractive long-distance forces between quark pairs in color-antitriplet states. Calculated in the diquark model, this contribution is larger than factorization by a factor of 2. Adding both is justified since they have a different physical origin and are of different order with respect to the $1/N_c$ expansion. Finally, meson loop effects and contributions from higher-dimensional operators lead to an additional increase of the $\Delta I = \frac{1}{2}$ amplitude by 25%, and account for the final-state interaction phases. They also reduce A_2 by 10% bringing it into agreement with the experimental value.

The analysis of the K_L-K_S mass difference provides a further test of the proposed diquark mechanism. Many processes influence the K^0-$\bar{K}^0$ mixing amplitude making the problem rather complex. The simplest contributions, however, namely those from short-distance box-diagram transitions and single-meson pole graphs, balance each other. Therefore, the main part of the mass difference has to be explained in terms of long-distance phenomena. We have shown that the single diagram of the chiral diquark model provides a successful description. It represents long-range color interactions between light quarks in the box diagrams, and gives

$$\Delta m_K^{\rm di} \simeq (3.5 \pm 1.2) \times 10^{-15} \text{ GeV} \qquad (9.3)$$

in agreement with the experimental value. This result is also in accordance with the complete next-to-leading-order analysis of the K_L-K_S mass difference in chiral perturbation theory. It shows that the origin of the mass difference is closely related to the dynamical mechanism responsible for the $\Delta I = \frac{1}{2}$ enhancement observed in strange-particle decays.

A coherent, self-consistent and intuitive physical picture of the major long-distance effects in nonleptonic weak transitions has emerged. Hyperon as well as kaon decay amplitudes and the long-range part of the K_L-K_S mass difference are intimately related and find their explanation in a single dynamical effect. The mystery of nonleptonic weak decays of strange particles is resolved.

From the knowledge of the physical origin of the $\Delta I = \frac{1}{2}$ rule, further conclusions can be drawn. The CP-violating quantity ϵ' is not strongly enhanced even though the relevant penguin operator Q_6 is a $\Delta I = \frac{1}{2}$ operator. The reason is that Q_6 cannot be written in terms of scalar diquark currents. Thus, ϵ' may be estimated using factorization [18, 52]. Our explanation of the $\Delta I = \frac{1}{2}$ enhancement is not in conflict with the small value of ϵ' found experimentally. In D decays, $f_\pi \, m_c$ is of about equal magnitude as $g_{\rm di}$, indicating that diquark effects can still be important, but are no longer dominant. In energetic two-body decays the diquark loop contribution cannot compete with a more direct meson generation. Factorization with asymptotic particle states gives the dominant contribution to the decay amplitudes and provides for a semiquantitative understanding of many energetic two-body D decays [5]. In specific channels, diquark effects may still be sizable, however. They certainly have an influence on inclusive charm decay rates [6]. This effect has to be taken into account even for a qualitative understanding of the lifetimes of D^+ and D^0. Because of the strong increase of factorization amplitudes with the available energy, quark-quark correlations are of almost no significance in B decays. An important exception are decays into baryons. The formation of the final baryon-antibaryon pair is greatly facilitated if diquarks are generated in an intermediate step. This mechanism implies numerous selection rules and the possibility to explore experimentally the way the additional quark-antiquark pair is created [6, 10, 53].

Note added in proof. The value quoted in (4.3) for the phase difference $\delta_0 - \delta_2$ obtained from $K \longrightarrow \pi\pi$ decays is rather uncertain because of isospin breaking effects, which have not been taken into account. In a detailed analysis of $\pi\pi$ scattering data, Devlin and Dickey [Rev. Mod. Phys. **51**, 237 (1979)] obtain $\delta_0 - \delta_2 = (41.4 \pm 8.1)^\circ$, in agreement with the result found in chiral perturbation theory. We are grateful to J. Gasser for pointing this out to us.

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APPENDIX

Here we collect the definitions and some useful formulas for the scalar two- and three-point integrals appearing in decay amplitudes of the diquark model. In D space-time dimensions, the two-point function $Q_{ij}(p^2)$ at external momentum p is defined as

$$\frac{Q_{ij}(p^2)}{(4\pi)^2} = -i\,\mu^{(4-D)}\int\frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{[(k-p)^2 - m_i^2 + i\eta]\,(k^2 - m_j^2 + i\eta)}.$$

It is logarithmically divergent for $D = 4$. An arbitrary mass scale μ has been introduced in order to make Q_{ij} dimensionless. For $D \to 4$ one finds

$$Q_{ij}(p^2) = -\left(\frac{2}{D-4} + \gamma_E - \ln 4\pi\right) - \int_0^1 \mathrm{d}x\,\ln\frac{M_{ij}^2(x,p^2)}{\mu^2} + O(D-4)\ ,$$

where

$$M_{ij}^2(x,p^2) = x\,m_i^2 + (1-x)\,m_j^2 - x\,(1-x)\,p^2 - i\eta\ .$$

The integration over the Feynman parameter x can be done analytically.

The scalar three-point integral

$$\frac{C_{ijk}(p_1,p_2)}{(4\pi)^2} = -i\int\frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{1}{(k^2 - m_i^2 + i\eta)\,[(k-p_1)^2 - m_j^2 + i\eta]\,[(k+p_2)^2 - m_k^2 + i\eta]}$$

is finite and can be evaluated in $D = 4$ dimensions. For simplicity, the dependence on the pion momenta p_1 and p_2 has not been explicitly shown in (5.11). Introducing Feynman parameters and setting $q^2 = (p_1 + p_2)^2$ one obtains

$$C_{ijk}(p_1,p_2) = -\int_0^1 \mathrm{d}y \int_0^y \mathrm{d}x\,(a\,y^2 + b\,x^2 + c\,x\,y + d\,y + e\,x + f)^{-1}$$

with coefficients

$$\begin{aligned} a &= p_2^2\ ,\\ b &= q^2\ ,\\ c &= p_1^2 - p_2^2 - q^2\ ,\\ d &= m_k^2 - m_i^2 - p_2^2\ ,\\ e &= m_j^2 - m_k^2 + p_2^2 - p_1^2\ ,\\ f &= m_i^2 - i\eta\ . \end{aligned}$$

The parameter integrals may be evaluated using the technique of 't Hooft and Veltman [54].

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