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Exclusive decays of heavy and light mesons

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In the exclusive decay of the B meson into a K^* and a photon, the large recoil momentum creates a difficulty in making a connection between the quark level calculations and the meson states. In the quark model, depending on how the recoil momentum is handled, this can lead to differences of a factor of about 5 in the branching ratio. In order to explore this problem, we look at some other meson processes where there exist some experimental data. For the π form factor we show that, by imposing current conservation together with relativistic meson and spinor normalizations within the quark model, it is possible to remove a parameter κ which had previously been introduced to obtain agreement with the form-factor data. The data on the decay $\omega \to \pi^0\mu^+\mu^-$ and $\omega \to \pi^0\gamma$ also show the importance of retaining relativistic meson and spinor normalizations; however, we do not resolve the recoil problem. For the exclusive decays $B \to K^*\gamma$ and $B \to K^*e^+e^-$ we show the extent to which the heavy-quark symmetries determine these decays, even in regions of large recoil.

I. INTRODUCTION

The decays $b \to s + X$ form an important class of rare decays in which the standard model can be further tested. These involve the sequence of flavor transitions, $b \to u, c, t \to s$ in which two W vertices occur. The coupling to the massive virtual internal particles is the source of the interest in such decays. In the original attempt to exploit this [1], building on earlier work [2] in the decay $s \to d\gamma$ it was hoped that the strong dependence on the mass of the top would provide a means of detecting the top. In 1987 it was realized [3, 4] that the short-distance calculations [5] of the processes $s \to d\gamma$ could be taken over to the $b \to s\gamma$ and the branching ratios could be enhanced. Also the dependence on the top quark would be softened considerably. These papers did not include a full two-loop QCD-corrected calculation of the rare processes which have now been carried out by the Caltech group [6, 7] and by the Toronto group [8–11]. Although there was a difference between the latter two groups, arising from a technical point in the handling of divergences, this has now been resolved and apart from some minor details they are in agreement.

The next task is to find limits on the exclusive decays. It does not appear easy to go from the inclusive charmless nonleptonic decay $b \to s$+gluon to the exclusive nonleptonic charmless decays. From the inclusive decay $b \to s\gamma$ the exclusive rare decay that might be most easily calculated is $B \to K^*\gamma$ and there have been a number of attempts to calculate this branching ratio. However, the problem remains: in the published literature there seems to be an inherent difficulty between the application of the quark model and the sum-rule approach [12–15].

Even within the quark model there are contradictory results. Here the problem can be summarized as follows. The amplitude for the decay, $\langle K^*|\bar{s}i\sigma_{\mu\nu}q^\nu b_R|B\rangle$, may be written in terms of three Lorentz-covariant form factors: $f_1(q^2)$, $f_2(q^2)$, and $f_3(q^2)$. Then $\Gamma(B \to K^*\gamma)/\Gamma(b \to s\gamma) \approx |f_1(0)|^2$ gives the exclusive to inclusive branching ratio. The amplitude is calculated in the quark model using Gaussian momentum wave functions of B and K^*. The form factor $f_1(0)$ can be easily extracted in terms of the overlap integral between the B and K^* momentum wave functions. The overlap integral has the form $\exp[-O(1)(1 - m_{K^*}/m_B)^2]$ and is accompanied by a phase-space factor [16] $\sqrt{(E_{K*}/m_B)} \approx 1/\sqrt{2}$, coming from the noncovariant normalization of the meson wave functions, by comparison with the situation in atomic physics [17]. With relativistic normalization for the quark spinors, there are other factors which occur; these contribute another factor of $\sqrt{2}$. (This can be seen readily in the paper of Altomari [12].) The square of $f_1(0)$ then gives a branching ratio of approximately 25%, close to a value obtained in a calculation [14] using QCD sum rules and symmetry properties, $28 \pm 11\%$ (although a later paper [15] now appears to favor the higher value of about 40%; no estimate of the likely error is given, however).

However, there are other relativistic effects to be considered; these modify the factor of $O(1)$ in the exponent. For the two-body decay, the kinematics are such that the process take place far from threshold. In Altomari's paper, the kinematics are chosen so that the exponent is a function of the three-momentum of the K^* rather than of the factor $(1 - m_{K^*}/m_B)^2$. This leads to the introduction of a factor $[1 + (m_B - m_{K^*})^2/(4m_Bm_{K^*})]$ in the exponent which brings the estimated branching ratio down to the much smaller value of 4.5%. This difference between the two forms of exponent corresponds to keeping a nonrelativistic (25%) or relativistic (4.5%) kinematic structure.

At present, there is no consensus on the resolution of this problem. Our purpose here is to explore these problems by considering other exclusive processes for which data exist. These are the pion form factor and the process $\omega \to \pi^0\mu^+\mu^-$. Although these are very different in mass from the heavy B system there are similarities in a number of parts of the calculations to those of the heavy mesons. We also show how the heavy-quark symmetry limit enters into the decay of $b \to s$ processes. We use the quark model to show the extent to which the heavy quark symmetries hold even though the s quark may not be sufficiently heavy. The symmetry relations seem to hold very well over the full kinematic ranges of $b \to s$ processes. An explicit calculation also shows that care must be taken to distinguish between the kinematics and the symmetry relations.

II. THE B DECAY AND THE QUARK MODEL

A. Meson transitions in the quark model

For the meson transition $X(q_1\bar{q}_2) \to Y(q_3\bar{q}_2)$ shown in Fig. 1, the nonrelativistic quark model is used to calculate the hadronic matrix element $\langle Y(\mathbf{k})|J_\mu|X(\mathbf{0})\rangle$ in the rest frame of X. It begins with the construction of weakly bound quark-model states of X and Y as

$$\begin{aligned} |X(\mathbf{0})\rangle &= \sqrt{2M_X}\int d\mathbf{p}\sum \phi_X(\mathbf{p})\chi^{Jm_J}_{Lm_L\,Sm_S}\chi^{Sm_S}_{\frac{1}{2}\sigma\,\frac{1}{2}\bar\sigma}|q_1(\mathbf{p},\sigma)\bar{q}_2(-\mathbf{p},\bar\sigma)\rangle, \\ |Y(\mathbf{k})\rangle &= \sqrt{2E_Y}\int d\mathbf{p}\sum \phi_Y\left(\mathbf{p}+\frac{m_2}{m_2+m_3}\mathbf{k}\right)\chi^{Jm_J}_{Lm_L\,Sm_S}\chi^{Sm_S}_{\frac{1}{2}\sigma\,\frac{1}{2}\bar\sigma}|q_3(\mathbf{k}+\mathbf{p},\sigma)\bar{q}_2(-\mathbf{p},\bar\sigma)\rangle, \end{aligned} \tag{1}$$

where the χ functions are Clebsch-Gordan coefficients that couple the quark spins σ and $\bar\sigma$ to the meson spin S, and the meson spin S and orbital momentum L to its total angular momentum J. The functions ϕ_X and ϕ_Y are the $q\bar{q}$ relative momentum wave functions of the mesons X and Y. In the rest frame of X and with recoil momentum $\mathbf{k}$, the corresponding quark momenta of X and Y are $q_1(\mathbf{p})\bar{q}_2(-\mathbf{p})$ and $q_3(\mathbf{k}+\mathbf{p})\bar{q}_2(-\mathbf{p})$, respectively; $q_1\bar{q}_2$ has relative momentum $\mathbf{p}$ while $q_3\bar{q}_2$ has relative momentum $\mathbf{p}+m_2/(m_2+m_3)\mathbf{k}$. To calculate $\langle Y(\mathbf{k})|J_\mu|X(\mathbf{0})\rangle$ involves knowledge of the quark matrix element $\langle q_3\bar{q}_2|J_\mu|q_1\bar{q}_2\rangle$; this matrix element can be obtained exactly using full Dirac spinors and an overlap integral between the X and Y momentum wave functions.

There are, however, two problems to face in this use of the quark model. The quark-model states of X and Y in Eq. (1) are only valid when X and Y are weakly bound. This is, however, the case when $m_1+m_2 \approx M_X$ and $m_2+m_3 \approx M_Y$. To see this, let us define the binding energies of X and Y in the transition of $X(q_1\bar{q}_2) \to Y(q_3\bar{q}_2)$ as

$$\begin{aligned} \Delta_X(\mathbf{p}) &\equiv M_X - \sqrt{\mathbf{p}^2+m_1^2} - \sqrt{\mathbf{p}^2+m_2^2}, \\ \Delta_Y(\mathbf{k},\mathbf{p}) &\equiv \sqrt{\mathbf{k}^2+M_Y^2} - \sqrt{(\mathbf{k}+\mathbf{p})^2+m_3^2} \\ &\quad -\sqrt{\mathbf{p}^2+m_2^2}. \end{aligned} \tag{2}$$

In the nonrelativistic quark model, the Fermi motion of quarks inside a meson are assumed to be nonrelativistic. It is then obvious from Eq. (2) that $\Delta_X(\mathbf{p})$ is small and $|\Delta_Y(\mathbf{k},\mathbf{p})| < m_2$. Since in most cases m_2 is less than 50% of M_Y, both X and Y are then weakly bound and the use of the quark-model states of X and Y in Eq. (1) is justified in this case.

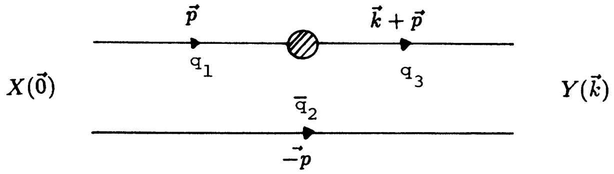

FIG. 1. The quark level diagram of the meson transition $X(q_1\bar{q}_2) \to Y(q_3\bar{q}_2)$.

The other problem comes from the relative momentum wave functions in Eq. (1); they are determined by solving the Schrödinger equation of the corresponding $q\bar{q}$ system with a Coulomb plus linear potential [18, 19] between the quarks. For L=0 meson states that we will consider here, they are chosen to be Gaussian wave functions of the form

$$\phi(\mathbf{p}) = (\pi\beta^2)^{-3/4}\,e^{-\mathbf{p}^2/2\beta^2}\ , \tag{3}$$

with a variational parameter β. The formulation of the relative momentum wave function is then obviously nonrelativistic. The $q_3(\mathbf{k}+\mathbf{p})\bar{q}_2(-\mathbf{p})$ system of meson Y becomes highly relativistic in the region of large recoil and the use of the above nonrelativistic momentum wave function for ϕ_Y is then questionable. In Ref. [18], this kind of problem was treated by fixing the meson and quark spinor normalizations at the zero-recoil point and ignoring all of the recoil dependence in the matrix element except for the momentum wave-function part. The recoil momentum can be written as

$$|\mathbf{k}| = \sqrt{E_Y^2 - M_Y^2} = \sqrt{1+\frac{t_m-q^2}{4M_XM_Y}}\sqrt{\frac{M_Y}{M_X}}\sqrt{t_m-q^2}, \tag{4}$$

where $q^2=(P_X-P_Y)^2$ and $t_m \equiv (M_X-M_Y)^2$. Since $q^2=t_m$ corresponds to the point of zero recoil, near this point $|\mathbf{k}|$ can take the nonrelativistic form: $|\mathbf{k}| = \sqrt{M_Y/M_X}\sqrt{t_m-q^2}$. In Ref. [18], the nonrelativistic form of the recoil momentum in the momentum wave function was adopted. The recoil dependence of the matrix element at large recoil was prescribed by multiplying $|\mathbf{k}|$ by a universal relativistic correction factor $1/\kappa$ (where

κ=0.7 was determined by fitting the quark-model result of the charge pion form factor to experiment [21]).

In Sec. III, we examine the pion form factor in the quark model and find instead that so long as the relativistic normalizations of meson and quark spinors are kept, the correction factor $1/\kappa$ seems to be unnecessary and the procedure for fixing various normalization factors at zero recoil is then inappropriate. We shall maintain fully relativistic normalizations of meson and quark spinors throughout our calculations; the recoil dependence of the matrix element will then come from both the momentum wave functions and normalization factors.

B. The decay $B \to K^*\gamma$

The exclusive decays $B \to X_s\gamma$ are assumed to be well modeled by the quark-level decay $b \to s\gamma$; i.e., the exclusive decay $B \to K^*\gamma$ is expected to proceed through the quark level decay [20]. The branching ratio is suppressed however by the overlap between the B and the K^* momentum wave functions. For this decay, we need to calculate the hadronic matrix element [12,13] $\langle K^*(\mathbf{k})|\bar{s}i\sigma_{\mu\nu}q^\nu b_R|B(0)\rangle$ at $q^2=0$. The matrix element has the covariant expansion:

$$\langle K^*(k)|\bar{s}i\sigma_{\mu\nu}q^\nu b_R|B(k')\rangle = f_1(q^2)i\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}k'^\lambda k^\sigma + \left[(m_B^2-m_{K^*}^2)\epsilon_\mu^* - \epsilon^*\cdot q(k'+k)_\mu\right] f_2(q^2) + f_3(q^2)\epsilon^*\cdot q(k'-k)_\mu \tag{5}$$

with three form factors $f_1(q^2)$, $f_2(q^2)$, and $f_3(q^2)$. The branching ratio of the exclusive to inclusive process, $R=\Gamma(B\to K^*\gamma)/\Gamma(b\to s\gamma)$, is written in terms of f_1 and f_2 as [12, 13]

$$R = \frac{\Gamma(B\to K^*\gamma)}{\Gamma(b\to s\gamma)} \cong \frac{m_b^3(m_B^2-m_{K^*}^2)^3}{m_B^3(m_b^2-m_s^2)^3}\frac{1}{2}\left[|f_1(0)|^2+4|f_2(0)|^2\right]. \tag{6}$$

The B and K^* mesons are weakly bound since $m_b+m_d \approx m_B$ and $m_s+m_d\approx m_{K^*}$. The hadronic matrix element is calculated in the quark model by constructing weakly bound quark-model states of B and K^* as in Eq. (1). The form factors $f_1(q^2)$ and $f_2(q^2)$ can be extracted easily from the matrix element using Eq. (5). They are given by (see the Appendix for details)

$$f_1(0) = -\sqrt{\frac{E_{K^*}}{m_B}}\int d\mathbf{p}\,\phi_B(\mathbf{p})\phi_{K^*}\left(\mathbf{p}+\frac{m_d}{m_s+m_d}\mathbf{k}\right)\sqrt{\frac{E_s+m_s}{2E_s}}\sqrt{\frac{E_b+m_b}{2E_b}}\left(1+\frac{E_\gamma+p_z}{E_s+m_s}\right)\left(1+\frac{p_z}{E_b+m_b}\right), \tag{7}$$

$$f_2(0) = -\frac{1}{2}f_1(0),$$

where the energy and momentum terms are given as follows:

$$E_{K^*} = \frac{m_B^2+m_{K^*}^2}{2m_B},\quad E_\gamma = |\mathbf{k}| = \frac{m_B^2-m_{K^*}^2}{2m_B},$$
$$E_b = \sqrt{\mathbf{p}^2+m_b^2},\quad E_s=\sqrt{(\mathbf{k}+\mathbf{p})^2+m_s^2}.$$

It is clear from Eq. (7) that we have kept the relativistic normalizations of meson and quark spinor; the recoil dependence of f_1 and f_2 come from the momentum wave functions as well as various normalization factors. In the numerical evaluation of the integral, we choose $m_d=0.33$ GeV, $m_s=0.55$ GeV, and $m_b=5.0$ GeV; the momentum wave functions ϕ_{K^*} and ϕ_B are taken to be Gaussian wave functions [Eq. (3)] with the variational parameters [18] $\beta_{K^*}=0.34$ GeV and $\beta_B=0.41$ GeV.

The recoil momentum $|\mathbf{k}|=(m_B^2-m_{K^*}^2)/2m_B$ is highly relativistic in this case, bringing into question the use of the nonrelativistic function ϕ_{K^*}. If we keep the quantity $|\mathbf{k}|$ explicitly and use the relativistic value [12] $|\mathbf{k}|=(m_B^2-m_{K^*}^2)/2m_B$ in ϕ_{K^*}, we obtain numerically $f_1(0)=-0.19$ and $f_2(0)=0.095$; this gives a branching ratio $\Gamma(B\to K^*\gamma)/\Gamma(b\to s\gamma)$ of 4.0%. Another possibility [18] is to use the nonrelativistic $|\mathbf{k}|=\sqrt{m_{K^*}/m_B}(m_B-m_{K^*})$ in ϕ_{K^*}. Since we keep fully relativistic meson and quark spinor normalizations, we drop the correction factor $1/\kappa$. This however produces a very different result with $f_1(0)=-0.43$ and $f_2(0)=0.21$ which gives a branching ratio [13] of 21% close to the result of the QCD sum-rule calculation.

Thus if the nonrelativistic form of the recoil momentum $|\mathbf{k}|$ is used in ϕ_{K^*} the branching ratio is enhanced by a factor of 5. An immediate question then is whether we should trust the relativistic 4% or the nonrelativistic 21% branching ratios as they both involve a continuation from a small recoil region where the calculations are presumably reliable to a region of large recoil. To help answer this question we look for guidance from data. The pion form factor and the decay $\omega\to\pi^0\mu^+\mu^-$ are exclusive processes for which there are data and which have been partly investigated in the quark model.

III. EXCLUSIVE LIGHT-MESON PROCESSES

In this section, we present our calculations of the pion form factor and the $\omega\pi^0$ transition form factor. First we must resolve the difficulty of dealing with light mesons. The physical masses of π and ω are both very much different from their total constituent masses; the weak binding requirements of constructing the quark model states of π and ω cannot be satisfied. We present here a prescription that we believe can correct this problem. We begin with the observation that in the transition $X(q_1\bar{q}_2) \to Y(q_3\bar{q}_2)$, shown in Fig. 1, the following identity relates quark energies and momenta in the rest frame of X:

$$\frac{\mathbf{k}\cdot(\mathbf{k}+\mathbf{p})}{E_{k+p}+m_3} + \frac{\mathbf{k}\cdot\mathbf{p}}{E_p+m_1} = (E_{k+p} - E_p + m_3 - m_1)\left(1 + \frac{(\mathbf{k}+\mathbf{p})\cdot\mathbf{p}}{(E_p+m_1)(E_{k+p}+m_3)}\right) + 2(m_1 - m_3). \tag{8}$$

The terms E_p and E_{k+p} are the energies of the decay and recoil quarks respectively. Recall that if X and Y are both weakly bound mesons, we have $m_1 + m_2 \approx M_X$, $m_2 + m_3 \approx M_Y$, and $E_Y - M_X \approx E_{k+p} - E_p$. Equation (8) then reads

$$\frac{\mathbf{k}\cdot(\mathbf{k}+\mathbf{p})}{E_{k+p}+m_3} + \frac{\mathbf{k}\cdot\mathbf{p}}{E_p+m_1} \approx (E_Y - M_X + \sqrt{t_m})\left(1 + \frac{(\mathbf{k}+\mathbf{p})\cdot\mathbf{p}}{(E_p+m_1)(E_{k+p}+m_3)}\right). \tag{9}$$

If X and Y are not weakly bound mesons, Eq. (9) no longer follows simply from (8). Nevertheless, as we shall show, it can be taken as a prescription for dealing with the binding problem of the quark model in light meson transitions.

The condition of current conservation in the $\pi^+ \to \pi^+$ transition matrix element is

$$\langle \pi^+(k)|q^\mu J_\mu^{\text{em}}|\pi^+(k')\rangle = 0, \quad q = k - k' . \tag{10}$$

which, in the $\pi^+(k')$ rest frame, becomes

$$\int d\mathbf{p}\,\phi_\pi(\mathbf{p})\phi_\pi(\mathbf{p}+\tfrac{1}{2}\mathbf{k})\sqrt{\frac{E_{k+p}+m}{2E_{k+p}}}\sqrt{\frac{E_p+m}{2E_p}}\left[\left(\frac{\mathbf{k}\cdot(\mathbf{k}+\mathbf{p})}{E_{k+p}+m} + \frac{\mathbf{k}\cdot\mathbf{p}}{E_p+m}\right) - (E_\pi - m_\pi)\left(1 + \frac{(\mathbf{k}+\mathbf{p})\cdot\mathbf{p}}{(E_p+m)(E_{k+p}+m)}\right)\right] = 0, \tag{11}$$

where ϕ_π is the relative momentum wave function of the π.

Numerically the left-hand side of Eq. (11) does not quite vanish (it peaks at about -0.2 GeV near zero recoil and dies off at large recoil); we will use it to specify the current conservation in Eq. (10). If we calculate the pion form factor without this condition, we find that it is an increasing function of $-q^2$ at $-q^2 = 0$ which means it will exceed one for a small range of $-q^2$. With the current-conservation condition, however, this is corrected. Note that Eq. (9) reproduces (11) for $\pi^+ \to \pi^+$; it is then reasonable to treat (9) as a prescription which serves the same role as (11) in other light-meson transition processes. Our procedure is then to correct for the binding problem by replacing the left-hand side of (9) by its right-hand side in any light-meson form factor calculated in the rest frame of the initial meson.

A. The pion form factor

The hadronic matrix element $\langle \pi^+(k)|J_\mu^{em}|\pi^+(k')\rangle$ has the general covariant expansion

$$\langle \pi^+(k)|J_\mu^{em}|\pi^+(k')\rangle = (k'+k)_\mu f_\pi(q^2) + (k'-k)_\mu g_\pi(q^2) \tag{12}$$

with two form factors $f_\pi(q^2)$ and $g_\pi(q^2)$ to be evaluated for $q^2 = (k-k')^2 < 0$. Current conservation of (10) requires $f_\pi(q^2) = \langle \pi^+(k)|J_0^{em}|\pi^+(k')\rangle/(E_\pi + m_\pi)$ and $g_\pi(q^2) = 0$. The nonvanishing form factor $f_\pi(q^2)$ is then given by

$$f_\pi(q^2) = e\frac{2\sqrt{m_\pi E_\pi}}{E_\pi + m_\pi}\int d\mathbf{p}\,\phi_\pi(\mathbf{p})\phi_\pi(\mathbf{p}+\tfrac{1}{2}\mathbf{k})\sqrt{\frac{E_{k+p}+m}{2E_{k+p}}}\sqrt{\frac{E_p+m}{2E_p}}\left(1 + \frac{(\mathbf{k}+\mathbf{p})\cdot\mathbf{p}}{(E_{k+p}+m)(E_p+m)}\right), \tag{13}$$

where $\mathbf{k}$ is the recoil momentum in the initial $\pi^+(k')$ rest frame and is chosen to be along the z direction. The quark masses m_u and m_d are given by $m_u = m_d = m = 0.33$ GeV, and the energy and momentum terms in (13) are given by

$$E_\pi = m_\pi + \frac{-q^2}{2m_\pi}, \quad |\mathbf{k}| = \sqrt{E_\pi^2 - m_\pi^2},$$

$$E_p = \sqrt{\mathbf{p}^2 + m^2}, \quad E_{k+p} = \sqrt{(\mathbf{k}+\mathbf{p})^2 + m^2}.$$

B. The $\omega\pi^0$ transition form factor

For the $\omega\pi^0$ transition, we need $\langle\pi^0(k)|J_\mu^{em}|\omega(k')\rangle$, the dipole transition matrix element for $q^2 = (k'-k)^2 = 0$ to $t_m = (m_\omega - m_\pi)^2$. It has the covariant expansion:

$$\langle\pi^0(k)|J_\mu^{em}|\omega(k')\rangle = f_{\omega\pi}(q^2) i\varepsilon_{\mu\nu\lambda\sigma}\epsilon^\nu(k'+k)^\lambda(k'-k)^\sigma \tag{14}$$

with one form factor $f_{\omega\pi}(q^2)$. Using the prescription given by (9), we obtain

$$f_{\omega\pi}(q^2) = \mu\sqrt{\frac{E_\pi}{m_\omega}}\int d\mathbf{p}\,\phi_\omega(\mathbf{p})\phi_\pi(\mathbf{p}+\tfrac{1}{2}\mathbf{k})\sqrt{\frac{E_{k+p}+m}{2E_{k+p}}}\sqrt{\frac{E_p+m}{2E_p}} \times\left[\frac{m_\omega+m_\pi}{E_\pi+m_\pi}\left(1+\frac{(\mathbf{k}+\mathbf{p})\cdot\mathbf{p}}{(E_{k+p}+m)(E_p+m)}\right)-\frac{p_x^2+p_y^2}{(E_{k+p}+m)(E_p+m)}\right], \tag{15}$$

where $\mu = 2.79e/(2M_{\text{proton}})$, $\mathbf{k}$ is the recoil momentum of π^0 in the ω rest frame and is again chosen to be along the z direction. The energy and momentum terms are given by

$$E_\pi = \frac{m_\omega^2 + m_\pi^2 - q^2}{2m_\omega}, \quad |\mathbf{k}| = \sqrt{E_\pi^2 - m_\pi^2},$$
$$E_p = \sqrt{\mathbf{p}^2 + m^2}, \quad E_{k+p} = \sqrt{(\mathbf{k}+\mathbf{p})^2 + m^2}.$$

C. Discussion

In Eqs. (13) and (15), ϕ_π and ϕ_ω are the relative momentum wave functions of the π and the ω; they are taken to be Gaussian wave functions with variational parameters [18] $\beta_\pi = \beta_\omega = 0.31$ GeV. Recall that the derivation of the relative momentum wave function of the meson is in the nonrelativistic and weak binding limits. We should therefore adopt, in the momentum wave function, an expression for the recoil momentum $|\mathbf{k}|$ with the weak-binding assumption explicitly put in it. We note that if ω and π are both weakly bound mesons, they will have masses equal to the total constituent masses $m + m = 0.66$ GeV. We shall use then, but only in the momentum wave function,

$$|\mathbf{k}| = \sqrt{1 + \frac{t_m - q^2}{16m^2}}\sqrt{t_m - q^2} \tag{16}$$

as the relativistic form of $|\mathbf{k}|$, and

$$|\mathbf{k}| = \sqrt{t_m - q^2} \tag{17}$$

as the nonrelativistic form of $|\mathbf{k}|$. The t_m term in (16) and (17) is defined with physical meson masses; this will maintain the full kinematic region of q^2 when it is timelike. In the case of the pion form factor $t_m = 0$ while for the $\omega\pi^0$ transition form factor $t_m = (m_\omega - m_\pi)^2$.

Figure 2(a) shows the quark-model calculations of $f_\pi(q^2)$ in (13) using the relativistic and the nonrelativistic forms of $|\mathbf{k}|$ [Eqs. (16) and (17)] in the momentum wave function. Also shown in Fig. 2(a) are the data [21] for $f_\pi(q^2)$. Note that this calculation of the pion form factor gives a result which is too small at large re-

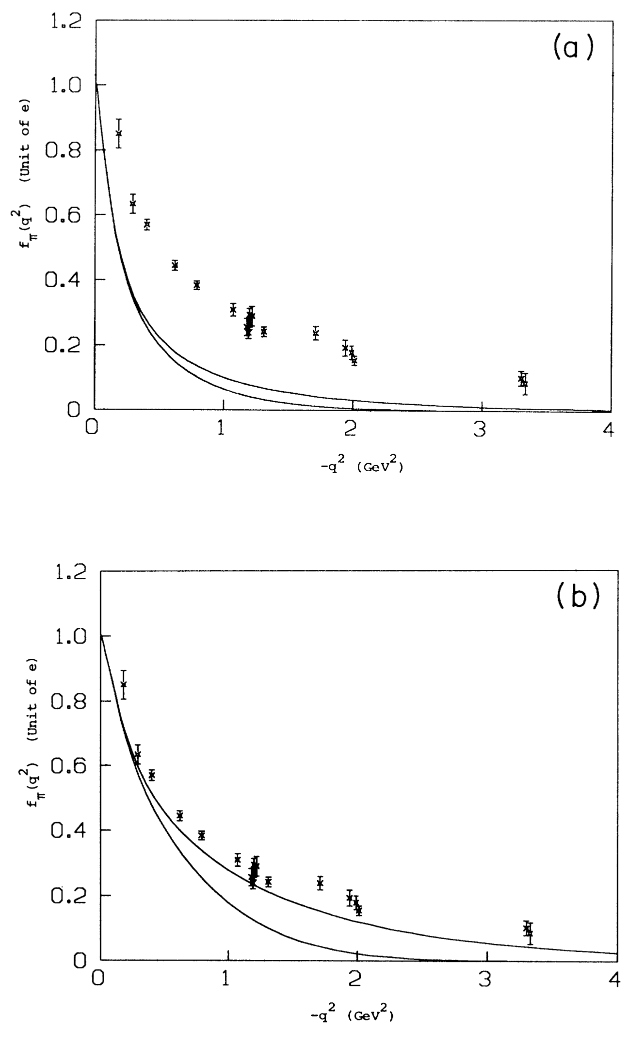

FIG. 2. The quark-model calculation of the pion form factor $f_\pi(q^2)$ compared to experiment [21]. The top curve employs the nonrelativistic form of the recoil momentum in the momentum wave function; the bottom curve employs the relativistic one. (a) shows $f_\pi(q^2)$ from Eq. (13). (b) shows $f_\pi(q^2)$ from Eq. (13) but with the factor $2\sqrt{m_\pi E_\pi}/(E_\pi + m_\pi) = 1$.

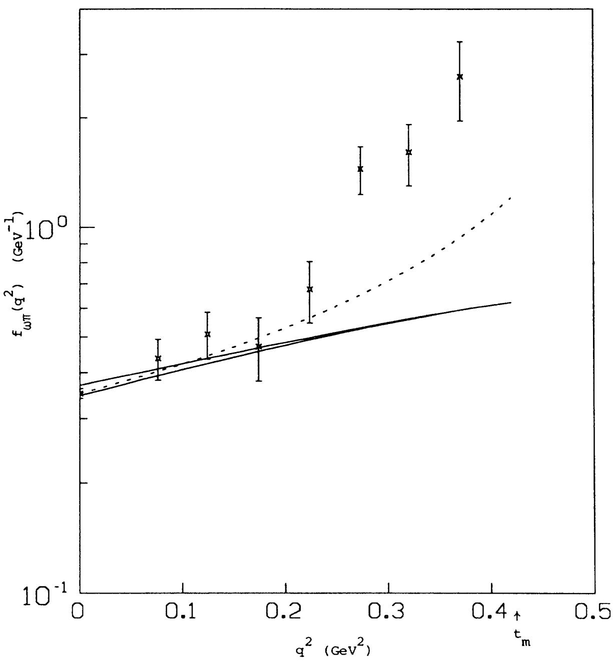

FIG. 3. The quark-model calculation of the $\omega\pi^0$ transition form factor $f_{\omega\pi}(q^2)$ compared to experiment [22]. The top curve results from using the nonrelativistic form of the recoil momentum in the momentum wave function; the bottom curve uses the relativistic form. The dashed line is the prediction of the vector-dominance model with a pole at $m_\rho = 770$ MeV and is normalized at $q^2 = 0$ using [23] $\Gamma(\omega \to \pi^0\gamma) = 717$ keV.

coil compared to the data. This comes from the factor $2\sqrt{m_\pi E_\pi}/(E_\pi + m_\pi)$ in Eq. (13) which goes down too fast with q^2. An obvious way of bringing the two curves into line with the data is to fix this factor at zero recoil, i.e., set $2\sqrt{m_\pi E_\pi}/(E_\pi + m_\pi) = 1$. The results are shown in Fig. 2(b). In this figure, $f_\pi(q^2)$ from the nonrelativistic form of $|\mathbf{k}|$ in (17) agrees very well with experiment over a wide range of q^2, whereas the one using the relativistic form in (16) is still suppressed and goes below experiment at large recoil. Thus, there is some preference for the use of the nonrelativistic form of the recoil momentum in the momentum wave function here.

After fixing the above normalization factor of $f_\pi(q^2)$ at zero recoil there is no longer any need to introduce a correction factor $1/\kappa$ to the momentum wave function [18] in order to bring the calculated result into line with data. The role of the correction factor $1/\kappa$ seems to be taken over by the recoil dependence of spinor normalizations; the calculation is perhaps cleaner than previously expected.

In Fig. 3, we show the results of our calculated $\omega\pi^0$ transition form factor $f_{\omega\pi}(q^2)$ in Eq. (15) using the relativistic and the nonrelativistic forms of $|\mathbf{k}|$, Eqs. (16) and (17), in the momentum wave function. Also shown in Fig. 3 are the data [22] for $f_{\omega\pi}(q^2)$ together with that from the $\omega \to \pi^0\gamma$ width of [23] 717 ± 43 keV at $q^2 = 0$. The two forms for $f_{\omega\pi}(q^2)$ both agree very well with experiment over a wide range of q^2 but fail after $q^2 > 0.2$ GeV^2. Using the relativistic form of $|\mathbf{k}|$ gives a $\omega \to \pi^0\gamma$ width of about 695 keV while the nonrelativistic one gives a value of about 792 keV; both results, however, are within two standard deviations from experiment. According to Fig. 3, there is definitely no preference between the use of the nonrelativistic and the relativistic form of the recoil momentum in the momentum wave function. However, Fig. 3 shows clearly the need to maintain the fully relativistic meson and spinor normalizations in the quark-model calculations. Any attempt to fix these normalization factors at zero recoil will make $f_{\omega\pi}(0)$ far too high. It would be useful to have another experiment to look at this process since the quark model seems to fail at the high q^2 end of the range.

IV. EXCLUSIVE HEAVY-MESON PROCESSES

We see from the agreement with the experimental results of the pion form factor $f_\pi(q^2)$ that there may be a slight preference for the use of the nonrelativistic form of the recoil momentum $|\mathbf{k}|$ in the momentum wave function even in the large recoil region. The pion form factor is a tricky calculation for the quark model though, due to the binding energy problem. In the $\omega\pi^0$ transition form factor $f_{\omega\pi}(q^2)$ it is not possible with the present data to resolve this question. In all cases, the nonrelativistic form of $|\mathbf{k}|$ gives less suppression to the form factors in the region of large recoil.

For the pion form factor the full use of the relativistic normalization does away with the need to introduce an extra parameter to bring the curve into line with the data. This parameter seems to play an important role [18] in the semileptonic decays $B \to X_u$ transitions.

Are there any lessons from the light mesons that could be applied to the exclusive B system? At first glance it might not seem so since the range of available energy is very much larger. However in the kinematics it is often the relative size of the squares of the meson masses which matters and even for the ω and π the heavier one is sufficiently dominant.

One way to try to settle this issue is to look for Dalitz pairs in the process $B \to K^*e^+e^-$ and track the data across the Dalitz plot from the region of low recoil momentum to the highest at $q^2 = 0$. The full matrix element for $B \to K^*e^+e^-$ with QCD corrections is given by [9, 24, 25]

$$M_{B\to K^*e^+e^-} = \frac{4G_F}{\sqrt{2}}\frac{\alpha}{4\pi}V_{ts}^*V_{tb}\left(\langle K^*|\bar{s}\gamma_\mu b_L|B\rangle(A_1\bar{e}\gamma^\mu e + A_3\bar{e}\gamma^\mu e_L) - \frac{2m_b}{q^2}A_2\langle K^*|\bar{s}i\sigma_{\mu\nu}q^\nu b_R|B\rangle\bar{e}\gamma^\mu e\right). \tag{18}$$

The differential width of $B \to K^*e^+e^-$ is calculated, ignoring the electron mass, to be

$$\frac{d\Gamma(B\to K^*e^+e^-)}{dq^2} = \frac{\alpha^2 G_F^2}{96\pi^5}|V_{ts}^*V_{tb}|^2|\mathbf{k}|^3\left((|A_1+\tfrac{1}{2}A_3|^2+\tfrac{1}{4}|A_3|^2)\Lambda_T - 4m_b\mathrm{Re}[(A_1+\tfrac{1}{2}A_3)A_2^*]\Lambda_{Tf}+\frac{4m_b^2}{q^2}|A_2|^2\Lambda_f\right), \tag{19}$$

where

$$\Lambda_T = T_1^2q^2+T_2^2\frac{(m_B^2-m_{K^*}^2)^2}{2m_{K^*}^2}\left(1+\frac{3q^2m_{K^*}^2}{m_B^2|\mathbf{k}|^2}\right)+T_3^2\frac{2m_B^2|\mathbf{k}|^2}{m_{K^*}^2}+T_2T_3\frac{m_B^2-m_{K^*}^2}{m_{K^*}^2}(m_B^2-m_{K^*}^2-q^2),$$

$$\Lambda_{Tf} = T_1f_1+T_2f_2\frac{m_B^2-m_{K^*}^2}{2m_{K^*}^2}\left(1+\frac{3(m_B^2-m_{K^*}^2)m_{K^*}^2}{|\mathbf{k}|^2m_B^2}\right)-T_3f_3\frac{2m_B^2|\mathbf{k}|^2}{m_{K^*}^2(m_B^2-m_{K^*}^2)}$$
$$-T_2f_3\frac{m_B^2-m_{K^*}^2-q^2}{2m_{K^*}^2}+T_3f_2\frac{m_B^2+3m_{K^*}^2-q^2}{2m_{K^*}^2},$$

$$\Lambda_f = f_1^2+\frac{1}{2}f_2^2\left(\frac{q^2-4m_{K^*}^2}{m_{K^*}^2}+\frac{3(m_B^2-m_{K^*}^2)^2}{m_B^2|\mathbf{k}|^2}\right)+f_3^2\frac{2q^2m_B^2|\mathbf{k}|^2}{m_{K^*}^2(m_B^2-m_{K^*}^2)^2}-f_2f_3\frac{q^2(m_B^2+3m_{K^*}^2-q^2)}{m_{K^*}^2(m_B^2-m_{K^*}^2)}.$$

Here, f_1, f_2, f_3, T_1, T_2, and T_3 are form factors introduced in the Appendix. The quantities A_1, A_2, and A_3 are coefficients responsible [9,24] for the QCD corrections to the $b\to s$ process.

There are contributions from the diagrams involving $b\to s\gamma$ and also from diagrams involving Z and W exchanges. In Fig. 4 we show how use of the different forms of exponents give possibly measurable effects as we go from the region of zero recoil (large q^2) to the large recoil at $q^2=0$ appropriate for the exclusive decay $B\to K^*\gamma$. Although the branching ratio is small—these are rare decays—there is a significant difference as we get below $(q/m_B)^2=0.1$.

In Fig. 5 we show the expected behavior of the parts of the differential branching ratio coming from the γ, Z, and W contributions. Note that in Fig. 5 the total contribution does not equal the sum of the γ and $Z+W$ contributions because of various interference effects among the QCD correction coefficients.

In recent papers [26] it has been pointed out that heavy-quark symmetries could relate the data on the semi-leptonic decays $D\to Ke^+\nu$ and $D\to K^*e^+\nu$ and provide information relevant to the exclusive decay $B\to K^*\gamma$. However, there is the problem that the s quark may not be sufficiently heavy to apply these sym-

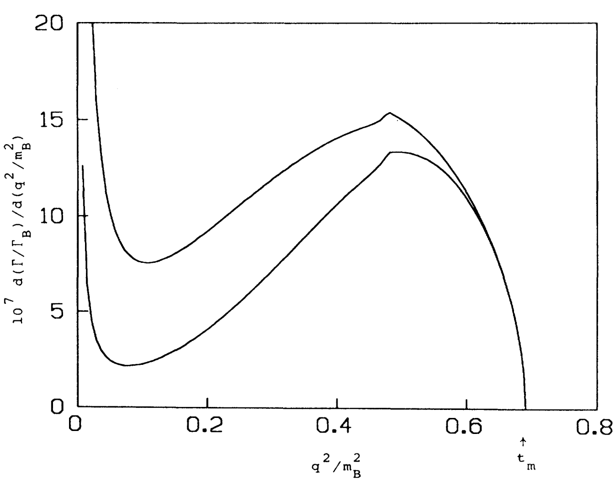

FIG. 4. Differential branching ratios for $B\to K^*e^+e^-$. The top curve uses the nonrelativistic form of the recoil momentum in the momentum wave function; the bottom curve uses the relativistic form. The parameters used are $m_t=120$ GeV, scale $\mu=5$ GeV and $\Lambda_{\rm QCD}=150$ MeV.

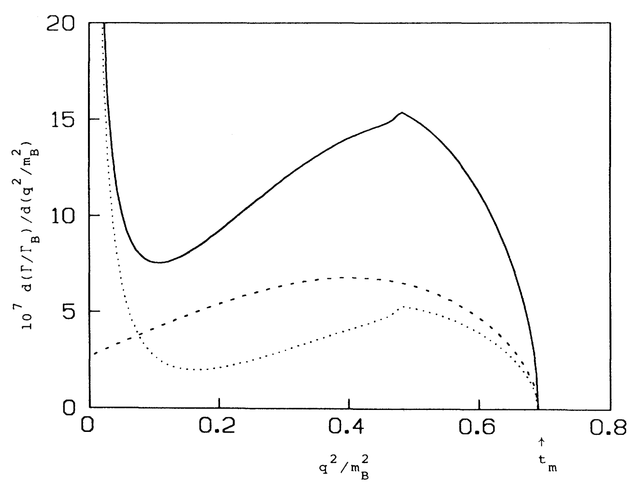

FIG. 5. Differential branching ratios of $B\to K^*e^+e^-$ with the nonrelativistic form of the recoil momentum. The solid line corresponds to the total contributions from all of the γ, Z, and W diagrams. The dotted line corresponds to the contribution from the γ only and the dashed line comes from the Z and W. Note that the sum of the dotted and dashed lines does not equal to the solid line because of various interference effects among the QCD-correction coefficients.

metries to the K^*. There is also the important problem of continuation of these symmetries from zero-recoil momentum across the Dalitz plot to the largest recoil $q^2 = 0$. It has been suggested [27] that the relations among the operator matrix elements might be valid over the full kinematic ranges even for transitions of the type $b \to s$. From the very different type of behavior shown in Fig. 5 this might seem very unlikely. Now, although the heavy-quark symmetries are derived in the large-mass limit, many of the relations have been known to hold in at least an approximate sense in models such as the quark model. Here we will use our quark model to evaluate the extent to which the heavy-quark symmetries hold even in the presence of mass corrections. The symmetries relate the form factors in the following ways [26]:

$$\begin{aligned} T_1 &= \frac{-2(m_B^2 - m_{K^*}^2)}{(m_B + m_{K^*})^2 - q^2} T_2 = 2T_3 = -2T_4 \\ &= \frac{-1}{m_B + m_{K^*}} f_1 = \frac{2(m_B + m_{K^*})}{(m_B + m_{K^*})^2 - q^2} f_2 \\ &= \frac{2}{m_B - m_{K^*}} f_3. \end{aligned} \tag{20}$$

Figure 6 shows the plot of individual terms in Eq. (20) times $\sqrt{4m_B m_{K^*}}$ using the form factors as calculated in the Appendix. The equalities hold very well, with less than about a 15% discrepancy, across the whole kinematic region to $q^2 = 0$. The small discrepancy is caused by the ambiguity in choosing between m_{K^*} and m_s in the symmetry relations [26] where the derivation assumes the heavy-quark mass constitutes most of the meson mass. Note that the choice between relativistic and nonrelativistic kinematic structures in the momentum wave function affects the form factors in the same way; it does not spoil the agreement in Fig. 6.

Using the heavy-quark symmetry relations Eq. (20), we can write the differential width $d\Gamma(B \to K^* e^+ e^-)/dq^2$ in term of one form factor, say $T_1(q^2)$. We see from Fig. 6 that T_1 decreases from a value of about 0.23 GeV $^{-1}$ at zero recoil to about 0.08 GeV $^{-1}$ at the maximum recoil region while the differential width has components which vary dramatically. This difference between the γ and $Z + W$ contributions near $q^2 = 0$ in Fig. 5 is caused by the kinematic factors in Eq. (19). The fact that the heavy-quark symmetries hold across the full kinematic range would seem to bear out the conclusion [27] that possible symmetry-breaking high-p_T tails do not affect the behavior of the form factors. Kinematic effects in these processes, however, are extremely important.

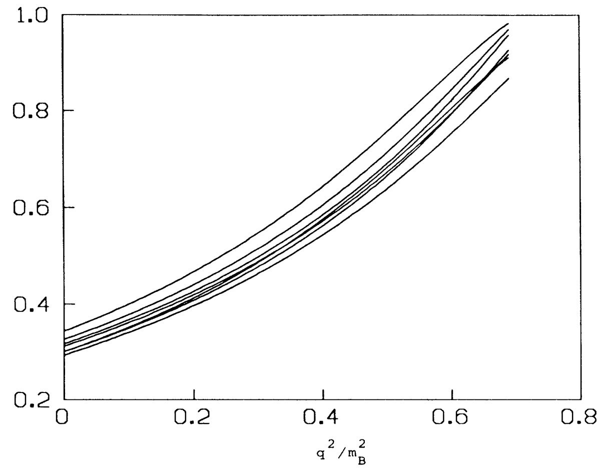

FIG. 6. Plot of individual terms in the symmetry relations Eq. (20) using the nonrelativistic kinematics in the momentum wave functions. From the top curve to the bottom at $q^2 = t_m$ we show $\sqrt{4m_B m_{K^*}}$ times T_1, $-2T_4$, $2T_3$, $2(m_B + m_{K^*})f_2/[(m_B + m_{K^*})^2 - q^2]$, $-2(m_B^2 - m_{K^*}^2)T_2/[(m_B + m_{K^*})^2 - q^2]$, $-f_1/(m_B + m_{K^*})$, $2f_3/(m_B - m_{K^*})$.

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APPENDIX

The hadronic matrix elements relevant to the transition $B \to K^*$ are given by

$$\begin{aligned} \langle K^*(k)|\bar{s} i\sigma_{\mu\nu} q^\nu b_R|B(k')\rangle = {} & f_1(q^2) i\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu} k'^\lambda k^\sigma + [(m_B^2 - m_{K^*}^2)\epsilon_\mu^* - \epsilon^* \cdot q(k'+k)_\mu] f_2(q^2) \\ & + \epsilon^* \cdot q\left((k'-k)_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(k'+k)_\mu\right) f_3(q^2), \end{aligned} \tag{A1}$$

$$\langle K^*(k)|\bar{s}\gamma_\mu b_L|B(k')\rangle = T_1(q^2) i\varepsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu} k'^\lambda k^\sigma + (m_B^2 - m_{K^*}^2) T_2(q^2)\epsilon_\mu^* + T_3(q^2)\epsilon^* \cdot q(k'+k)_\mu + T_4(q^2)\epsilon^* \cdot q(k'-k)_\mu. \tag{A2}$$

In the nonrelativistic quark model, the above form factors are calculated to be

$$f_1(q^2) = -\sqrt{\frac{E_{K^*}}{m_B}} \int d\mathbf{p}\, \phi_B \phi_{K^*} \Omega_s \Omega_b \left[1 + \frac{E_\gamma}{\alpha_s} + \frac{E_\gamma}{|\mathbf{k}|^2}\left(\frac{1}{\alpha_s} + \frac{1}{\alpha_b}\right)\mathbf{k}\cdot\mathbf{p} + \frac{\mathbf{k}\cdot\mathbf{p} + p_z^2}{\alpha_s \alpha_b}\right], \tag{A3}$$

$$f_2(q^2) = \frac{\sqrt{m_B E_{K^*}}}{m_B^2 - m_{K^*}^2} \int d\mathbf{p}\, \phi_B \phi_{K^*} \Omega_s \Omega_b \left[E_\gamma + \frac{|\mathbf{k}|^2}{\alpha_s} + \left(\frac{1}{\alpha_s} + \frac{1}{\alpha_b}\right) \mathbf{k}\cdot\mathbf{p} + \frac{E_\gamma(\mathbf{k}\cdot\mathbf{p} + p_z^2)}{\alpha_s \alpha_b} \right], \tag{A4}$$

$$\begin{aligned} f_3(q^2) = \frac{1}{2m_B}\sqrt{\frac{E_{K^*}}{m_B}} \int d\mathbf{p}\, \phi_B \phi_{K^*} \Omega_s \Omega_b \Bigg[& -m_B + \left(\frac{m_B^2 - m_{K^*}^2}{E_{K^*} + m_{K^*}}\right) + \frac{(m_{K^*}^2 + m_B E_{K^*})}{\alpha_s} \\ & + \frac{m_{K^*}^2 + m_B E_{K^*}}{|\mathbf{k}|^2} \left(\frac{1}{\alpha_s} + \frac{1}{\alpha_b}\right) \mathbf{k}\cdot\mathbf{p} \\ & + \left[\left(\frac{m_B^2 - m_{K^*}^2}{E_{K^*} - m_{K^*}}\right) - m_B\right] \frac{(\mathbf{k}\cdot\mathbf{p} + p_z^2)}{\alpha_s\alpha_b} - \frac{m_{K^*}(m_B^2 - m_{K^*}^2)}{|\mathbf{k}|^2} \frac{(p_x^2 + p_y^2)}{\alpha_s\alpha_b} \Bigg], \end{aligned} \tag{A5}$$

$$T_1(q^2) = \sqrt{\frac{E_{K^*}}{m_B}} \int d\mathbf{p}\, \phi_B \phi_{K^*} \Omega_s \Omega_b \left[\frac{1}{\alpha_s} + \frac{1}{|\mathbf{k}|^2}\left(\frac{1}{\alpha_s} - \frac{1}{\alpha_b}\right) \mathbf{k}\cdot\mathbf{p} \right], \tag{A6}$$

$$T_2(q^2) = \frac{\sqrt{m_B E_{K^*}}}{m_B^2 - m_{K^*}^2} \int d\mathbf{p}\, \phi_B \phi_{K^*} \Omega_s \Omega_b \left[-1 + \frac{\mathbf{k}\cdot\mathbf{p} + p_z^2}{\alpha_s\alpha_b} \right], \tag{A7}$$

$$\begin{aligned} T_3(q^2) = \frac{1}{2m_B}\sqrt{\frac{E_{K^*}}{m_B}} \int d\mathbf{p}\, \phi_B \phi_{K^*} \Omega_s \Omega_b \Bigg[& \left(\frac{m_{K^*} + m_B}{E_{K^*} + m_{K^*}}\right) - \frac{m_{K^*}}{|\mathbf{k}|^2}\left(\frac{1}{\alpha_s} + \frac{1}{\alpha_b}\right) \mathbf{k}\cdot\mathbf{p} - \frac{m_{K^*}}{\alpha_s} \\ & + \left(\frac{m_{K^*} - m_B}{E_{K^*} - m_{K^*}}\right) \frac{(\mathbf{k}\cdot\mathbf{p} + p_z^2)}{\alpha_s\alpha_b} - \frac{m_{K^*}(E_{K^*} - m_B)}{|\mathbf{k}|^2} \frac{(p_x^2 + p_y^2)}{\alpha_s\alpha_b} \Bigg], \end{aligned} \tag{A8}$$

$$\begin{aligned} T_4(q^2) = \frac{1}{2m_B}\sqrt{\frac{E_{K^*}}{m_B}} \int d\mathbf{p}\, \phi_B \phi_{K^*} \Omega_s \Omega_b \Bigg[& \left(\frac{m_{K^*} - m_B}{E_{K^*} + m_{K^*}}\right) - \frac{m_{K^*}}{|\mathbf{k}|^2}\left(\frac{1}{\alpha_s} + \frac{1}{\alpha_b}\right) \mathbf{k}\cdot\mathbf{p} - \frac{m_{K^*}}{\alpha_s} \\ & + \left(\frac{m_{K^*} + m_B}{E_{K^*} - m_{K^*}}\right) \frac{(\mathbf{k}\cdot\mathbf{p} + p_z^2)}{\alpha_s\alpha_b} - \frac{m_{K^*}(E_{K^*} + m_B)}{|\mathbf{k}|^2} \frac{(p_x^2 + p_y^2)}{\alpha_s\alpha_b} \Bigg], \end{aligned} \tag{A9}$$

where the energy and momentum terms are given by

$$E_{K^*} = \frac{m_B^2 + m_{K^*}^2 - q^2}{2m_B}, \quad |\mathbf{k}| = \sqrt{E_{K^*}^2 - m_{K^*}^2}, \quad E_\gamma = \frac{m_B^2 - m_{K^*}^2 + q^2}{2m_B},$$

$$E_b = \sqrt{\mathbf{p}^{\,2} + m_b^2}, \quad E_s = \sqrt{(\mathbf{k} + \mathbf{p})^2 + m_s^2},$$

$$\alpha_s = E_s + m_s, \quad \alpha_b = E_b + m_b, \quad \Omega_s = \sqrt{\frac{E_s + m_s}{2E_s}}, \quad \Omega_b = \sqrt{\frac{E_b + m_b}{2E_b}},$$

$$\phi_B = \phi_B(\mathbf{p}), \quad \phi_{K^*} = \phi_{K^*}\left(\mathbf{p} + \frac{m_d}{m_s + m_d}\mathbf{k}\right).$$

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