Pion interferometry and intermittency in heavy-ion collisions

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We study the expectations of the conventional theory of soft hadronic interactions, based on the Reggeon calculus and on the generalized optical theorem, for the Bose-Einstein correlations between identical pions. The attention is mainly focused on heavy-ion collisions, where the presence of large nuclear scales significantly improves the predictive power of the theory. We find that the interferometry image of the "radiating source" is that of a pancake, with a nuclear transverse size and a hadronic longitudinal size. The implications of our results for the studies of intermittency are pointed out. A tentative extension of this discussion to hadron-hadron collisions is also presented.

I. INTRODUCTION

We shall discuss the theory of Bose-Einstein correlations among pions produced in collisions of very energetic heavy ions. Some consequences of our results for the recent studies of intermittency will also be presented. We shall use the general framework of the Reggeon theory [1], but we shall adopt a rather pedagogical presentation to make the paper readable for nonexperts. This work is a direct continuation of Refs. [2,3], where some partial results have already been obtained.

The interest of heavy-ion collisions resides in the possibility of creating hadronic systems with high matter density. In this unusual regime one expects new phenomena to occur and to yield precious information on the nonperturbative QCD dynamics. Unfortunately, these dense systems must be ephemeral and the observation of the new physics very indirect and, consequently, rather difficult. This is well illustrated by the case of the J/ψ suppression. The idea is beautiful [4], and it has received experimental support [5]. Hence evidence for quark-gluon plasma formation has rapidly been claimed. However, it was found later that similar effects are expected from conventional dynamics. After a long debate the interpretation of the data remains uncertain [6].

It is obvious that in order to extract from data a signal of new physics one must know what is the background, by which we mean the expectation of the conventional picture of hadronic interactions. A reader may object at this point that there is nothing like a conventional picture of soft hadronic interactions, that there is only a variety of models, which are all equally founded (or rather unfounded, since we do not have any, even approximate, control of nonperturbative QCD, apart from lattice calculations, which are not suitable for high-energy production processes). We strongly disagree with this opinion. The Reggeon theory, together with the (generalized) optical theorem [7] and supplemented by a few empirical rules derived from phenomenology, adequately summarizes the bulk of knowledge accumulated during the last 30 years on soft hadronic interactions. This is an effective theory, since it does not refer to the microscopic degrees of freedom, but it is self-consistent and incorporates the constraints of quantum mechanics (such as unitarity or quantum interference). It is unable to answer all relevant questions, but the answers it does give are certainly not *ad hoc* and should be taken seriously.

In Sec. II we present a heuristic model which at high energy is equivalent to the Reggeon theory as long as one neglects Reggeon interactions and quantum interference. We then give some plausibility arguments which tend to explain the modifications expected from quantum interference and derived later on. The full understanding of these results, obtained in Sec. III and only summarized at the end of Sec. II, requires some familiarity with the basic concepts of the Reggeon theory. Section III can be omitted in first reading, especially by readers who are not interested in technicalities. In Sec. IV we compare our results with data and discuss their implications for the behavior of scaled multiplicity moments used in the studies of intermittency in heavy-ion and hadron-hadron collisions. A final summary and conclusions can be found in Sec. V.

II. HEURISTIC REVIEW AND A FIRST SUMMARY

At high enough (although not asymptotic) energies, the Reggeon theory leads to a simple picture which enables one to reproduce the salient features of multiparticle production in soft hadronic interactions. We present this picture below using a language which should be easily understandable for nonexperts. Explicit reference to the underlying theory will be avoided. A more theoretical discussion is postponed to Sec. III. We shall come back to phenomenology in Sec. IV.

Let $N(q)$ denote the observed density of produced pions. Here q denotes the transverse-momentum vector q_T and the rapidity y of a secondary, $q=(q_T,y)$. The one-particle inclusive spectrum is obtained by taking the average of $N(q)$:

$$\left(\frac{1}{\sigma_{\rm in}}\right)\frac{d\sigma}{dy\, d^2q_T}\equiv I_1(q)=\langle N(q)\rangle \ . \qquad (1)$$

The physical picture mentioned at the beginning of this section consists of regarding $N(q)$, in the so-called central rapidity region, as a superposition of a fluctuating number of "elementary" densities (representing radiation from individual "chains," or "strings" as one often writes in the literature). This reflects the composite nature of colliding hadronic systems, which implies the occurrence of subcollisions involving different constituents. What we call an "elementary density" is the density corresponding to a subcollision. The number n of subcollisions is distributed according to some probability $P(n)$.

A crucial ingredient of the whole picture is that particles belonging to distinct strings do not interact with each other. Within each string there are only short-range correlations in rapidity. The strings are simple universal building blocks. The choice of projectile and target only influences the probability of creating a given number of strings, viz., $P(n)$.

More precisely, we write

$$N(q)=\sum_{\alpha=1}^{n} N_\alpha(q) . \tag{2}$$

Here $N_\alpha(q)$, $\alpha=1,\ldots,n$, are the elementary densities mentioned above. We assume that the densities $N_\alpha(q_1)$ and $N_\beta(q_2)$ are, for all q_1 and q_2, statistically independent, provided $\alpha\neq\beta$. Furthermore, for all α, $N_\alpha(q_1)$ and $N_\alpha(q_2)$ are statistically independent, provided the rapidity distance $|y_1-y_2|$ is much larger than some energy-independent, universal correlation length $\Lambda_{\rm SR}$.

Assuming that all elementary densities have the same average, i.e., $\langle N_\alpha(q)\rangle=i_1(q)$ for any α, we obtain for the one-particle inclusive spectrum defined in Eq. (1)

$$I_1(q)=\sum_{n\geq 1} P(n) n i_1(q)=\langle n\rangle i_1(q) . \tag{3}$$

Likewise, taking the average of the product $N(q_1)N(q_2)$ (for $q_1\neq q_2$; see Ref. [8]), one further obtains

$$I_2(q_1,q_2)\equiv\langle N(q_1)N(q_2)\rangle =\langle n(n-1)\rangle i_1(q_1)i_1(q_2)+\langle n\rangle i_2(q_1,q_2) , \tag{4}$$

where

$$i_2(q_1,q_2)\equiv\langle N_\alpha(q_1)N_\alpha(q_2)\rangle =i_1(q_1)i_1(q_2)+c_{\rm SR}(q_1,q_2) , \tag{5}$$

and $c_{\rm SR}(q_1,q_2)$ rapidly vanishes when $|y_1-y_2|\gg\Lambda_{\rm SR}$. The first term on the right-hand side of (4) results from the possibility of choosing the two observed particles in two distinct strings. The combinatorial significance of the factor $\langle n(n-1)\rangle$ is obvious. Equation (4) can also be rewritten

$$I_2(q_1,q_2)=I_1(q_1)I_1(q_2)\langle n^2\rangle/\langle n\rangle^2 +\langle n\rangle c_{\rm SR}(q_1,q_2) . \tag{6}$$

The deviation of the right-hand side of (6) from a simple product of one-particle average densities is the observable correlation function. A long-range rapidity correlation emerges because $\langle n^2\rangle\neq\langle n\rangle^2$ and the short-range one of Eq. (5) is multiplied by the factor $\langle n\rangle$. The physical significance of all this is discussed at length in Ref. [8].

Until now, our discussion has been classical, in the sense that we have neglected quantum interference. A truly convincing discussion of the latter requires dealing with amplitudes, not probabilities, and will be carried out in Sec. III. However, even without going to a more rigorous framework one can easily guess what kind of corrections to the probabilistic model presented above will be generated by quantum statistics.

The two-particle Bose-Einstein correlation vanishes rapidly when the relative momentum of the two particles becomes large. The interference between particles belonging to the same string produces an extra short-range correlation which adds to $c_{\rm SR}(q_1,q_2)$. Only the sum of the two is observable. However, one also expects interference between particles emitted from different strings. The corresponding contribution to the observable inclusive spectrum is again proportional to the combinatorial factor $\langle n(n-1)\rangle$. Indeed, for identical pions, we shall find, instead of (6),

$$I_2(q_1,q_2)=I_1(q_1)I_1(q_2) \times\{\langle n^2\rangle/\langle n\rangle^2 +[\langle n(n-1)\rangle/\langle n\rangle^2]C_{\rm BE}(q_1,q_2)\} +\langle n\rangle c_{\rm SR}(q_1,q_2) , \tag{7}$$

where $C_{\rm BE}(q_1,q_2)$ vanishes when $|y_1-y_2|\gg\Lambda_{\rm BE}$ and $C_{\rm BE}(q,q)=1$.

A partial result for $C_{\rm BE}(q_1,q_2)$ has been obtained in Ref. [2] for the case $y_1=y_2$. Our study of the general case yields, in first approximation, the following result at high enough energy and for $A,B\gg 1$:

$$C_{\rm BE}(q_1,q_2)=c_{\rm BE}(q_1,q_2)\exp[-\Delta_T^2 R_{AB}^2/3] , \quad \Delta=q_1-q_2 , \tag{8}$$

where $c_{\rm BE}(q,q)=1$ and R_{AB} is a nuclear scale depending on R_A and R_B, the root-mean-square radii of the colliding nuclei. We also find that the function $c_{\rm BE}(q_1,q_2)$, which contains the whole dependence on $|y_1-y_2|$ generated by quantum interference, is actually independent of nuclear parameters. In particular, the correlation length $\Lambda_{\rm BE}$ is universal.

Assuming Gaussian forms of nuclear densities, we find the analytic result

$$R_{AB}^2=R_A^2R_B^2/(R_A^2+R_B^2) . \tag{9a}$$

When $A\ll B$, it is the radius of the smaller nucleus which controls the correlation:

$$R_{AB}=R_A, \quad A\ll B . \tag{9b}$$

When precise predictions are needed, it is easy to repeat the calculations with a more realistic input, having recourse to a numerical computation. Using Saxon-Woods densities, one then finds that (9b) is actually a better approximation to R_{AB} than (9a) for O-Au or S-Ag collisions, just to give two representative examples.

Corrections to Eq. (8) have been estimated and will be presented in Sec. III C. At this point it suffices to say that the corrections are not important enough to change our conclusions.

The regime relevant for heavy-ion collisions is $\langle n \rangle \gg 1$. For a collision of two heavy ions with atomic mass numbers A and B, one finds from (3) that

$$\langle n \rangle_{AB} = \langle n \rangle_{NN} I_1^{AB}(q) / I_1^{NN}(q) , \tag{10}$$

so that, up to a factor of order unity, $\langle n \rangle_{AB}$ is given by the ratio of the heights of rapidity plateaus in A-B and N-N collisions, which is known to be large.

It is clear from Eq. (7) that for $\langle n \rangle \gg 1$ the last term, viz., $\langle n \rangle c_{\rm SR}(q_1,q_2)$, becomes small compared to the other terms on the right-hand side [remember Eq. (3)]. Thus, using (8), we rewrite (7) as

$$I_2(q_1,q_2) = I_1(q_1) I_1(q_2) (\langle n^2 \rangle / \langle n \rangle^2) \times [1 + c_{\rm BE}(q_1,q_2) \exp(-\Delta_T^2 R_{AB}^2/3)] . \tag{11}$$

Integrating out the transverse momenta and remembering that hadronic scales involved in $c_{\rm BE}$ are small compared to R_{AB}, we get, for identical pions and $\langle n \rangle \gg 1$,

$$I_2(y_1,y_2) = I_1(y_1) I_1(y_2) (\langle n^2 \rangle / \langle n \rangle^2) \times [1 + 3 c_{\rm BE}(y_1 - y_2) / 2 \langle q_T^2 \rangle R_{AB}^2] . \tag{12}$$

The shape of the rapidity correlation induced by quantum interference is independent of nuclear parameters, but its strength is inversely proportional to R_{AB}^2.

The aim of the next section is to derive the results summarized above. A reader who is not conversant with the basic ideas of the Reggeon calculus can skip this section [especially Sec. III C, where we calculate corrections to (8)] and go over directly to Sec. IV.

III. BOSE-EINSTEIN CORRELATION IN REGGEON THEORY

A. General framework

In Reggeon theory the hadronic diffraction is described in first (Born) approximation by the exchange of a quasiparticle called the Pomeron. The latter is strongly coupled to hadron sources, and therefore the description of elastic hadron-hadron scattering requires considering multi-Pomeron exchanges. If mutual interactions between these Pomerons are neglected, one is led to the picture of Sec. II. Indeed, the cross sections for producing multiparticle final states are obtained by taking the discontinuities of the elastic amplitude, in other words by "cutting" through the exchanged Pomerons. One cut Pomeron yields what has been called in the last section an "elementary" density.

The strong coupling of Pomerons to hadron sources inevitably generates an interaction between the Pomerons themselves. The simplest such interaction and the only one which has been experimentally observed is the triple-Pomeron coupling. However, the coupling strength is experimentally found to be small ($r = 0.05 \pm 0.01$ GeV^{-1}; cf. Ref. [9]), and this interaction is of little importance for the bulk of multiparticle production at accessible energies [the dimensionless parameter controlling the perturbative expansion in r is proportional to $(r^2/\alpha')\log(s)$]. Triple couplings involving Pomerons and other Reggeons have been measured too. The indirect interaction between Pomerons induced by these couplings is also of little relevance at present energies. The magnitude of direct higher-order Pomeron couplings is, strictly speaking, unknown. However, essentially all existing data on hadron diffraction and soft production processes have been successfully described neglecting these interactions. In the context of multiparticle production, phenomenology uses the "dual parton model" [10] (DPM) or the conceptually equivalent "quark-gluon string model" [11] (to quote the most familiar examples), which have both been abstracted from the Reegeon theory along the lines discussed in Sec. II and which are commonly adopted by experiments to estimate the soft background.

The neglect of Pomeron interactions can be rationalized using topological expansion arguments, first put forward by 't Hooft [12] and further developed by Veneziano [13]. In a gauge theory with SU(N) symmetry, the Feynman diagrams can be classified according to their topology and one finds a common suppression factor $1/N^2$ for each new handle. There is a correspondence between this topological expansion and the loop expansion of the Reggeon theory: A new loop can be regarded as a new handle, and therefore loops are suppressed in the large-N limit, which is usually a good guide in strong interaction physics. Actually, the scenario is more subtle, since phenomenology favors the "supercritical" Pomeron, with intercept above unity: $\epsilon \equiv \alpha(0) - 1 \approx 0.1$ or so. A Pomeron line propagating from the bottom to the top of a Reggeon diagram yields a factor s^ϵ. Thus, when the energy becomes large enough to have $s^\epsilon/N^2 \approx 1$, such an extra Pomeron line is not supressed. In particular, multi-Pomeron exchanges between the target and projectile cannot be neglected. However, generic Pomeron interactions produce loops extending over a fraction of the total rapidity interval and are suppressed. Of course, at truly asymptotic energies these topological arguments become irrelevant when $\epsilon > 0$.

The inclusive k-particle cross section can be found using the generalized optical theorem and is given by a discontinuity of the appropriate $k \to k$ forward scattering amplitude. Considering the absorption corrections to this amplitude in the framework of the Reggeon theory, Abramovski, Gribov, and Kancheli [14] (AGK) have discovered a remarkable cancellation: Neglecting the interaction between Pomerons, one finds that the single-particle inclusive cross section is given by the unique diagram shown in Fig. 1, while only the two diagrams of Fig. 2 contribute to the two-particle cross section. It is obvious from Fig. 1 that the A dependence of the density of particles produced in hadron-nucleus collisions in the central rapidity region is $I_1(y) \sim A \sigma_{\rm in}^{hN} / \sigma_{\rm in}^{hA}$, without any shadowing. The fact that data show little shadowing is actually a strong argument in favor of the neglect of Pomeron interactions at present energies.

One can show [8] that, apart from factors corresponding to the insertions on Pomeron lines, the magnitude of the discontinuity represented by the diagram of Fig. 1 or 2(a) [Fig. 2(b)] can be interpreted as $\langle n \rangle$ [$\langle n(n-1) \rangle$],

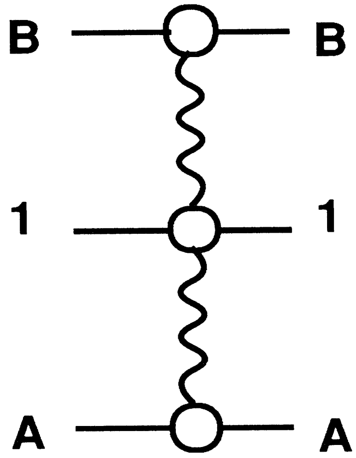

FIG. 1. Optical theorem states that the inclusive cross section for $A+B\to\pi(q_1)+$anything is equal to a discontinuity of the forward amplitude for $A+\pi+B\to A+\pi+B$. At high energy this $3\to3$ amplitude is dominated by Pomeron exchanges (wiggly lines). The contributions of all diagrams, other than the one shown in the figure, cancel when interactions between Pomerons are neglected (AKG cancellation).

where n denotes the number of cut Pomerons. In the discussion of heavy-ion collisions, we are therefore primarily interested in the diagram of Fig. 2(b). If the two observed particles are identical pions, the $4\to4$ amplitude should be symmetrized and a new contribution represented in Fig. 2(c) appears. In this case, when $\Delta=q_1-q_2\neq0$, there are insertions of four-momenta $\pm\Delta$ into the Pomeron lines. In the next subsection we shall calculate the one-loop amplitude of Fig. 2(c) for $\Delta\neq0$, assuming that only

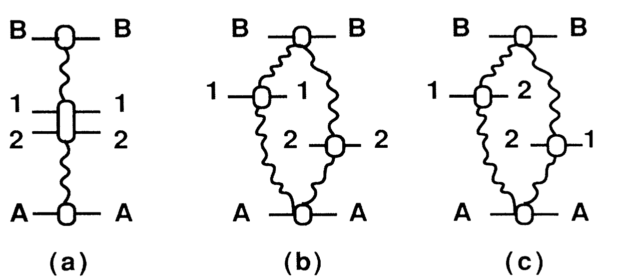

FIG. 2. In analogy to Fig. 1, we show here diagrams representing $(4\to4)$ forward amplitudes whose discontinuities add up to give the full two-body inclusive cross section for $A+B\to\pi(q_1)+\pi(q_2)+$anything (neglecting interactions between Pomerons): (a) generates short-range correlations only, (b) is responsible for long-range correlations, and the presence of (c) is due to Bose-Einstein interference. For heavy-ion collisions (a) can be neglected (see text).

low-mass excitations at the top and bottom vertices are relevant. High-mass excitations will be considered in Sec. III C.

B. Calculation to one-loop order

The Reggeon theory is from the outset formulated in a kinematical regime where energies are so large that one can consider all momentum transfers as transverse. In our case there are four-momentum insertions in the Pomeron lines, and we must check whether the Reggeon theory formulas do apply without any modification.

Let us fix our notations. The four-momentum of the incident nucleus A (B) will be denoted p_A (p_B). The corresponding masses are m_A and m_B. Let the masses of the intermediate states produced at the top and the bottom vertices of Fig. 2(c) be M_A and M_B, respectively. Consider the "process" where two nuclei A and B interact by the exchange of a Pomeron, get converted into two excited systems on the mass shell and produce in the central rapidity region a "pion pair" with four-momentum Δ (our rapidities will always be defined independently of m_A and m_B; hence the "central region" refers to the rapidity location half-way between a nucleon in A and a nucleon in B). Studying the kinematics of this process, one finds after some algebra that the invariant momentum transfers carried by the Pomeron, before and after the insertion of Δ, read

$$k^2=-k_T^2+(\Delta^+/p_A^+)(M_A^2-m_A^2)-(\Delta^+/p_A^+)^2M_A^2-|k^2|_{\min} \tag{13a}$$

and

$$(k+\Delta)^2=-(k_T+\Delta_T)^2+(\Delta^-/p_B^-)(M_B^2-m_B^2)-(\Delta^-/p_B^-)^2M_B^2-|(k+\Delta)^2|_{\min}\ , \tag{13b}$$

where

$$|k^2|_{\min}=|(k+\Delta)^2|_{\min}=(M_A^2-m_A^2)(M_B^2-m_B^2)/(p_A^+p_B^-+p_A^-p_B^+) \tag{13c}$$

is the familiar minimum momentum transfer ($p^{\pm}$ are the light-cone variables $p^0\pm p^3$). We conclude that the invariant momentum transfers, at high enough energy, depend on the transverse components of Δ only.

This simple kinematic observation has an important consequence. The only dependence on $\Delta^{\pm}$ which survives at very high energies comes from the pion-Pomeron vertices, which are universal, in the sense that they do not depend on what happens at the top and bottom vertices of the diagram. This is why the shape of rapidity correlations induced by Bose-Einstein interference is independent of nuclear parameters.

The cross section corresponding to the diagram of Fig. 2(c) is a familiar one-particle-exchange cross section modified by the insertion of the two pion-Pomeron vertices $\psi(q_1,q_2,k)$ and $\psi(q_2,q_1k)$:

$$\sigma(2c;\Delta)=(16/s_0^2)\int d\nu_A\, d\nu_B\, d^2k_T\, P^2(y_0-y_A,k_T)P^2[y_B-y_0,k_T+\Delta_T] \times N_A(k_T,\nu_A)N_B[k_T+\Delta_T,\nu_B]\psi(q_1,q_2,k_T)\psi(q_2,q_1,k_T)\ . \tag{14}$$

Here

$$P(y,k_T)=\exp\{y[\alpha(-k_T^2)-1]\} \tag{15}$$

is the Pomeron propagator and the invariants $N_{A(B)}$ are discontinuities of the Pomeron-A (B) forward elastic amplitudes. The variable $\nu_{A(B)}=k\cdot p_{A(B)}/m_{A(B)}$ is the energy transfer carried by the Pomeron in the rest frame of A (B), the rapidity $y_0=(y_1+y_2)/2$, and the parameter s_0 is a hadronic scale of the order of 1 GeV^2. We have neglected the dependence of ψ on $k_{0,L}$. Indeed, energy-momentum conservation and Lorentz invariance imply that in two dimensions a four-point function can depend on the (transverse) masses and on a single other invariant only. However, according to Eqs. (13), the transverse masses of the Pomerons are negligible, Note that the dependence of ψ on k_T involves a hadronic scale which is small compared to the nuclear one controlling the loop integration and can actually be also neglected, even at energies where the Regge slope $\alpha' y$ is not large. The presence of the nuclear scale improves the predictive power of the Reggeon theory: The detailed structure of hadronic vertices, which are almost unconstrained by the theory, becomes irrelevant. The overall normalization of the right-hand side in (14) is at this point arbitrary and will be fixed later on.

The precise meaning of the neglect of high-mass excitations of our intermediate systems is that the nuclei can possibly be broken, but without production of other particles than liberated nucleons. Hence the nuclear vertices can be treated nonrelativistically, provided one works in the rest frame of the target nucleus. The function $N(k^2,\nu)$ can be regarded as the cross section of a process where some probe is scattered off the nucleus with momentum transfer k_T, the energy transfer $k_0=\nu$ being small compared to the target mass. Let $U(x)$ be the effective "potential" of probe-nucleus interaction. Hence the cross section is given by the familiar formula

$$N(k_T,\nu)=\sum_f \left|\left\langle f\left|\int d^3x\, e^{ik_T\cdot x}U(x)\right|i\right\rangle\right|^2\delta(E_f+\nu-E_i)\ , \tag{16}$$

where $E_{f,i}$ are the energies of the final and initial nuclear states and $U(x)$ is the sum of terms describing the interactions with individual constituent nucleons located at x_j:

$$U(x)=\sum_{j=1}^{A} V(x-x_j)\ . \tag{17}$$

Using the closure property of the final states,

$$\sum_f |f\rangle\langle f|=1\ , \tag{18}$$

we find

$$\int d\nu_A\, N_A(k_T,\nu_A) = v^2(k_T)\left[A+\sum_{m\neq n}\langle i|e^{ik_T\cdot(x_m-x_n)}|i\rangle\right]\ , \tag{19}$$

where $v(k_T)$ is the Fourier transform of $V(x)$. We repeat these known arguments [15] for the sake of completeness.

Introducing the nuclear ground-state wave function $\Psi(x_1,\ldots,x_a)$, one, of course, has

$$\langle i|e^{ik_T\cdot(x_m-x_n)}|i\rangle = \int\prod_j d^3x_j|\Psi(x_1,\ldots,x_A)|^2e^{ik_T\cdot(x_m-x_n)}\ . \tag{20}$$

If one represents the nucleus as a gas of independent particles,

$$|\Psi(x_1,\ldots,x_A)|^2=\prod_j\rho(x_j)\ , \tag{21}$$

one gets

$$\sum_{m\neq n}\langle i|e^{ik_T\cdot(x_m-x_n)}|i\rangle = A(A-1)F^2(k_T)\ , \tag{22}$$

with

$$F(k_T)=\int d^3x\,\rho(x)e^{ik_T\cdot x}\ . \tag{23}$$

In the Gaussian approximation one writes

$$\rho(x)=(3/2\pi R_A^2)^{3/2}\exp(-3x^2/2R_A^2)\ , \tag{24}$$

where $R_A=\langle x^2\rangle^{1/2}$ is the root-mean-square radius of the nucleus. Neglecting the longitudinal component of k, we finally write

$$F(k_T)=\exp(-R_A^2k_T^2/6)\ . \tag{25}$$

The corrections to the right-hand side of (22) come from correlations between the constituent nucleons. However, these terms are of order $O(A)$. Since we are interested in the leading behavior as A increases, we drop all these terms. In other words, we concentrate our attention on the coherent interaction of the Pomeron with the nucleus in the diagrams of Figs. 2(b) and 2(c). Note that this does not mean that nuclei act coherently in the multiparticle production process. The optical theorem and AGK cutting rules enable us to reduce the discussion of the incoherent multiparticle process to the elastic-scattering problem involving coherent interactions of Pomerons with nuclei.

It is time now to fix our normalizations. We do this by noting that the first term in the square brackets in (19) corresponds to incoherent interaction of individual nucleons of A. Let us normalize the elastic scattering amplitude so that the optical theorem reads

$\sigma_{\rm tot}=8\pi\,{\rm Im}f(0)$. The dimensionless Pomeron-nucleon vertex $\beta(k_T)$ is defined by the one-Pomeron-exchange amplitude

$$f_1(k_T)=is_0^{-1}\beta^2(k_T)P(y,k_T)\ . \tag{26}$$

In the eikonal model

$$f(k_T)=(i/4\pi)\int d^2b\ e^{ik_Tb}(1-e^{i\chi})\ , \tag{27}$$

the amplitude $f_1(k_T)$ is obtained by linearizing with respect to χ and the forward two-Pomeron-exchange amplitude reads

$$f_2(0)=-(i/2\pi)\int d^2k_T\, f_1(k_T)f_1(-k_T)\ . \tag{28}$$

Comparing (28) with (14) (at $\Delta=0$) and (19) and remembering the factor 8π from the optical theorem, the fact that there are two ways of inserting the pion-Pomeron vertices ψ, and finally the factor -2 from the AGK cutting rules, one concludes that $v(k_T)=\beta(k_T)$. It is known that the eikonal model is a realistic approximation to NN scattering, and we use it here without hesitation.

With the conventions adopted above, the single-pion inclusive spectrum as given by the diagram of Fig. 1 is

$$I_1(q)=8\pi s_0^{-1}AB\beta^2(0)P(y,0)\psi(q,q,0)/\sigma_{AB}\ . \tag{29}$$

Integrating in (14) over k_T using (29), and setting

$$\psi(q_1,q_2,0)=\psi(q_1,q_1,0)f(q_1,q_2)\ , \tag{30}$$

so that $f(q,q)=1$, one obtains the contribution to $I_2(q_1,q_2)$ from Fig. 2(c):

$$\delta I_2(q_1,q_2)=[\langle n(n-1)\rangle/\langle n\rangle^2]\times I_1(q_1)I_1(q_2)c_{\rm BE}(q_1,q_2)\times\exp(-\Delta_T^2R_{AB}^2/3)\ , \tag{31}$$

where

$$\langle n(n-1)\rangle/\langle n\rangle^2=3\sigma_{AB}/4\pi(R_A^2+R_B^2) \tag{32}$$

and

$$c_{\rm BE}(q_1,q_2)=f(q_1,q_2)f(q_2,q_1)\ , \tag{33}$$

and, of course, $c_{\rm BE}(q,q)=1$. Note that the last, exponential factor in (31) results from calculating the following convolution integral:

$$\int d^2k_T\,F_A^2(k_T)F_B^2(k_T+\Delta_T)\sim\exp(-\Delta_T^2R_{AB}^2/3)\ . \tag{34}$$

In the Gaussian approximation for the form factors, one has a strict equality, apart from a normalization factor, and R_{AB} is given by (9a). When realistic shapes of form factors are used, one has to calculate the convolution numerically. For not too large $|\Delta_T|$, the result can be well represented by a Gaussian as in (34) but R_{AB} is usually somewhat larger. The results given in Sec. II follow from Eq. (31).

We have neglected hadronic radii as compared to the nuclear ones for the sake of simplicity. This is certainly legitimate for A,B large enough, but in applications one

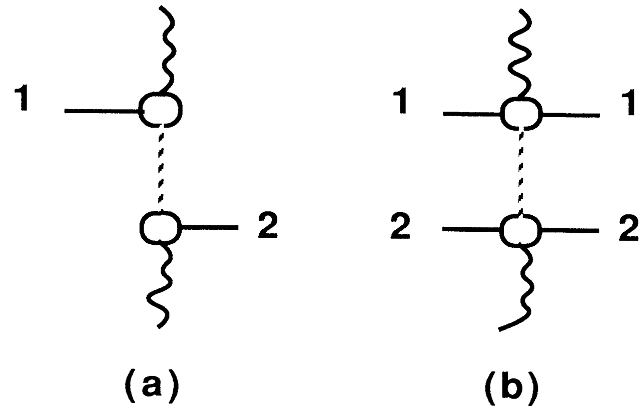

FIG. 3. (a) Diagram represents a Pomeron-$\pi(q_1)$-Pomeron-$\pi(q_2)$ vertex appearing in Fig. 2(c) when the two pions are widely separated in rapidity; the exchanged object (dotted line) has unnatural spin parity 0^-, 1^+, etc., and its Regge intercept is close to zero. (b) The standard diagram representing the contribution of the central six-point vertex of Fig. 2(a) to the dynamical two-body correlation function, when the two pions are widely separated in rapidity; the exchanged object (dotted line) has Regge intercept close to $\frac{1}{2}$.

may add to R_A^2 (viz., R_B^2) the slopes of hadronic vertices including the Regge slope $\alpha' y$.

Let us now discuss what can be said about the function $c_{\rm BE}(q_1,q_2)$. For $|y_1-y_2|\gg1$, the amplitude $\psi(q_1,q_2,0)$ is represented by the Reggeon diagram of Fig. 3(a), where the exchanged object has negative G parity, the spin-parity assignment 0^-, 1^+, etc., and consequently a Regge intercept close to that of the pion. Hence,

$$\psi(q_1,q_2,0)\sim e^{\epsilon_\pi|y_1-y_2|},\quad |y_1-y_2|\gg1\ , \tag{35}$$

where $\epsilon_\pi\equiv\alpha_\pi(0)-1$. Since $\alpha_\pi(0)\approx0$ and two factors ψ enter into the definition of $c_{\rm BE}$ [cf. (30) and (33)], we expect that

$$c_{\rm BE}(q_1,q_2)\sim\exp(-|y_1-y_2|/\Lambda_{\rm BE})\ ,\quad |y_1-y_2|\gg1\ , \tag{36}$$

with $\Lambda_{\rm BE}\approx0.5$. This argument is analogous to the standard one, which uses the diagram of Fig. 3(b) to find the behavior of the dynamical short-range correlation

$$c_{\rm SR}(q_1,q_2)\sim\exp(-|y_1-y_2|/\Lambda_{\rm SR})\ ,\quad |y_1-y_2|\gg1\ , \tag{37}$$

with $\Lambda_{\rm SR}\approx2$. It is interesting that, at least at large rapidity separations, $c_{\rm BE}(q_1,q_2)$ is much steeper than $c_{\rm SR}(q_1,q_2)$.

C. High-mass excitations

We shall now consider the case where, for example, the intermediate mass $M_A\gg m_A$ [still being much smaller than the total c.m. system energy, since there must be enough rapidity space for the two Pomerons in Figs. 2(b) and 2(c)]. The Reggeon diagram of Fig. 4(a) yields a con-

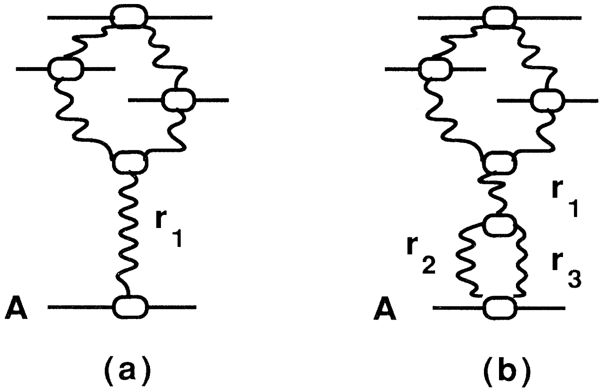

FIG. 4. In Sec. III B it is assumed that only low-mass excitations occur in the bottom and top vertices of the diagrams shown in Figs. 2(b) and 2(c). The lower halves of Figs. 4(a) and 4(b) are nothing but high-energy models for the vertex function appearing at the bottom of the diagrams of Figs. 2(b) and 2(c) when high-mass intermediate states are produced in the Pomeron-nucleus collision. As discussed in Sec. III C, only the diagram of Fig. 4(b) is relevant when the mass number A is large (the Reggeons r_j are not necessarily Pomerons). As shown in Sec. III C, taking into account high-mass excitations corrects the results of Sec. III B by a few percent only.

tribution proportional to A instead of A^2, and we therefore focus our attention on the diagram of Fig. 4(b) (where the Reggeons $r_{1,2,3}$ are not necessarily Pomerons).

We assume again that the interaction of the bottom Reggeons with the nucleus can be treated nonrelativistically in the nucleus rest frame. Thus the energy p_0 transferred by the Reggeon in this frame is very small. However, the longitudinal momentum transfer p_L is not necessarily small, because the mass M of the produced particles is not necessarily small compared to M_A. Let us denote by (y_1,y_2) the rapidity interval where the secondaries corresponding to the cut Reggeon r_1 are produced. We have

$$M_A^2 \approx 2k\cdot p_A = 2Am_N k_0 \approx As_0 e^{y_2} \tag{38}$$

and

$$M^2 \approx 2k_L p_L \approx 2k_0 p_L \approx s_0 e^{y_2 - y_1} \tag{39}$$

[note that $k_0^2 - k_L^2 = O(1/s)$ according to (13a) and therefore $k_0 \approx k_L$; furthermore, p_0 is small by assumption]. From the above relations one easily finds

$$e^{y_1} \approx m_N / p_L \ . \tag{40}$$

To ensure a coherent interaction of the Reggeons $r_{2,3}$ with the nucleus (in order to get a contribution proportional to A^2), p_L must be small:

$$p_L R_A / \sqrt{3} \leq 1 \tag{41}$$

or, using (40),

$$y_1 \geq y_A = \tfrac{1}{2}\ln(s_0 R_A^2/3) \ . \tag{42}$$

This constraint being found, the calculation of $\sigma(4b:\Lambda)$ is a standard exercise in Reggeon theory [a sharp cutoff as in (42) is sufficient for our purposes; a more precise consideration of the damping of the longitudinal-momentum transfer by the nuclear form factor gives similar results]. One finds, using the conventions of Ref. [9] for triple-Reggeon couplings,

$$\sigma(4b;\Delta) = \sigma(2c;\Delta=0)(3/2R_{AB}^2)\Gamma\Theta \times \exp(-\Delta_T^2 R_0^2/3) \ , \tag{43}$$

where

$$\Gamma = [(r_{PP1} r_{123} \beta_2 \beta_3)/\beta^2]\mathrm{Im}(i\eta_2\eta_3) \tag{44}$$

(the bottom loop is cut in all possible manners) and

$$\Theta = \int_{y_0 > y_2 > y_1 > y_A} \exp[\epsilon_1(y_2 - y_1) + (\epsilon_2 + \epsilon_3) y_1] dy_1\, dy_2 \ . \tag{45}$$

Here $\epsilon_i = 1 - \alpha_i(0)$ and $\beta_i \equiv \beta_i(0)$ is the coupling to the nucleon of the Reggeon r_i. The signature factor is denoted η_i, and r_{ijk} is the coupling of the three Reggeons $r_i - r_j - r_k$. It is appropriate to set the Pomeron intercept to unity, since all rapidity intervals are small for collision energies of current interest. Obviously, the dependence of $\sigma(4b;\Delta)$ on Δ_T results from the integration in the upper loop. We have already mentioned the fact that this Δ_T dependence is controlled by the scale of the vertex function which falls the least rapidly with k_T. In our case it is the PPr_1 vertex, which is known to be even softer than a generic Regge residue, so that R_0 is relatively small, $R_0 \approx 1\ \mathrm{GeV}^{-1}$, say

We see from (43) that at $\Delta = 0$, $\sigma(4b;\Delta)$, compared to $\sigma(2c;\Delta)$, is supressed by the factor R_{AB}^{-2}. Hence we shall neglect in the following the correction to $\sigma(2b)$ from the diagram of Fig. 4(b). However, the suppression factor disappears once the cross sections are integrated over Δ. Let us put together the contributions from diagrams of Figs. 2(b), 2(c), and 4(b) and integrate the resulting $I_2(q_1,q_2)$ with respect to the transverse momenta q_{T1} and q_{T2}. The result of the integration of $\sigma(4b;\Delta)$ is somewhat uncertain, since all factors in the integrand depend on hadronic scales and the function $c_{\mathrm{BE}}(q_1,q_2)$ is unknown. We shall neglect the dependence of $\exp[-\Delta_T^2 R_0^2/3]$ and $c_{\mathrm{BE}}(q_1,q_2)$ on the transverse momenta, as compared to that of the factor $I_1(q_1)I_1(q_2)$, overestimating in this way the correction due to $\sigma(4b;\Delta)$. The result of the integration is

$$I_2(y_1,y_2) = I_1(y_1)I_1(y_2)[\langle n(n-1)\rangle / \langle n\rangle^2] \times \{1 + [3c_{\mathrm{BE}}(y_1 - y_2)/2\langle q_T^2\rangle R_{AB}^2](1+\delta)\} \ , \tag{46}$$

which is essentially the same as (12), except for the correction δ given by

$$\delta \approx \langle q_T^2 \rangle \Gamma\Theta \ . \tag{47}$$

Before estimating δ, from the triple-Regge data, let us observe that $\delta=0$ if the Reggeons r_1, r_2, and r_3 are either P' or ω, with intercept $\alpha(0)=0.5$. Indeed, from parity conservation r_1 must be P' and $r_2=r_2$. But then the signature factor $\text{Im}(i\eta_2\eta_3)$ vanishes. In triple-Regge phenomenology [9], one writes the cross section for $a+b\rightarrow c+X$ in the form

$$\frac{d\sigma}{dt\,dx}=\sum_{ijk} G_{ijk}(t)(1-x)^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)}(s/s_0)^{\alpha_k(0)-1} . \tag{48}$$

Consider the case $r_1=r_2=r_3=P$ as an example. With the conventions of Ref. [9] chosen in this paper,

$$G_{PPP}(0)=4\pi r\beta^3 . \tag{49}$$

Using $\beta^2=4\ \text{GeV}^{-2}$, $G_{PPP}(0)=2.65\ \text{mb/GeV}^2$ from Ref. [16] (this yields $r=0.066\ \text{GeV}^{-1}$, slightly larger than $0.05\ \text{GeV}^{-1}$ quoted in Sec. III), $y_0=4$ (corresponding to the laboratory energy per nucleon of 1000 GeV), and $R_A=3$ fm, we find $\delta=-1.8\times10^{-2}$. The correction is smaller at lower energies.

Similar estimate can be obtained for other choices of r_i, $i=1,2,3$. The full estimate for δ at 1000 GeV and for $R_A=3$ fm is about -3%. This correction should be multiplied by 2 if the high-mass excitation of the other nucleus is also taken into account and if $A=B$.

The main lesson from the exercise carried out in this subsection is that the corrections coming from high-mass excitations are of the order of a few percent and are negative. These corrections are small and will be neglected in phenomenological applications.

IV. PHENOMENOLOGICAL APPLICATIONS

A. Interferometry parameters

Let us now discuss our expectations for the interferometry parameters. For $\langle n\rangle \gg 1$ we get, from Eq. (7),

$$K\int d^2\Sigma\, I_2^{\pm\pm}(q_1,q_2)/\int d^2\Sigma\, I_1^{\pm}(q_1)I_1^{\pm}(q_2)=1+F_{\text{BE}}(q_1-q_2) , \tag{50}$$

where $\Sigma=q_{1T}+q_{2T}$ and

$$F_{\text{BE}}(q_1-q_2)=\int d^2\Sigma\, I_1^{\pm}(q_1)I_1^{\pm}(q_2)C_{\text{BE}}(q_1,q_2)/\int d^2\Sigma\, I_1^{\pm}(q_1)I_1^{\pm}(q_2) . \tag{51}$$

As in experimental papers, the normalization K has been chosen so that the right-hand side is 1 when the two particles are far away from each other in phase space. Neglecting corrections calculated in Sec. III C, one has $C_{\text{BE}}(q,q)=1$. We have found that these corrections are negative, but very small, for heavy-ion collisions. The calculation of the corresponding corrections for high-multiplicity NN collisions would lead us beyond the scope of this paper, but we do not expect them to be large. Neglecting these corrections altogether, one has $F_{\text{BE}}(0)=1$. In the interferometry language this means that the chaoticity parameter λ equals unity. This is compatible with NA35 data [17,18] on central nucleus-nucleus collisions, but is at variance with the UA1 high-multiplicity data [19]. In this last case we do expect λ to be less than unity, because $\langle n\rangle$ is not large enough to neglect $O(1/\langle n\rangle)$ corrections to the right-hand side of (51). The formalism presented in Sec. IV C enables one to estimate $\langle n\rangle$. One finds that the highest rapidity densities observed by UA1 correspond to $\langle n\rangle=8$–9. Hence we would not be surprised if λ were less than unity by 10%–20%. However, the lowest observed value of the chaoticity parameter is $\lambda=0.15$.

We would like to conclude that UA1 data point toward some new dynamics. However, we are rather reluctant to do so. The measurement of λ is a delicate thing. There are notorious ambiguities associated with the definition of the "reference sample" [20], not to mention all other purely experimental difficulties. Also, the determination of λ does depend on the assumed shape of the BE correlation. Furthermore, every projection operation tends to decrease the observed λ. One such projection is explicit in (51). Another one results from putting together all pairs with $\delta q_L \le 0.2$ GeV/c [19], which corresponds to a rapidity resolution δy of the order of unity. Now the equality $C_{\text{BE}}(q,q)=1$ is consistent with $C_{\text{BE}}(q_1,q_2)$, depending not only on Δ_T and y_1-y_2, but also on $\Delta_T u(\Sigma)$ and $(y_1-y_2)w(\Sigma)$, where $u(\Sigma)$ and $w(\Sigma)$ are unknown. The integration over rapidities yields an effective λ proportional to $\Lambda_{\text{BE}}/\delta y$. The Regge arguments of Sec. III B suggest that Λ_{BE} is significantly smaller than unity. Furthermore, the integration in (51) is sensitive to the unknown function $w(\Sigma)$. These two projections could perhaps explain the smallness of the effective λ.

Note also that a decrease of λ with multiplicity reported by UA1, considered as a genuine dynamical effect, appears rather strange. Indeed, for all other quantities (intermittency slope, interferometry radius, etc.), the multiplicity dependence observed in proton-antiproton, when extrapolated to high multiplicities, yields estimates qualitatively consistent with nuclear data. Here the value of λ drops from $\lambda=0.4$–0.5 at low multiplicities to $\lambda=0.15$ at large multiplicities [18], while it is found to be close to 1 in O-Au collisions [17].

The parametrizations used by experimenters to fit their data are not really natural from our point of view. The parametrization using the single invariant $Q^2=(q_1-q_2)^2$ seems too restrictive. Independently of any model, only the invariance with respect to longitudinal boosts is expected in the central rapidity region. The parametrization of Ref. [18] using Δ_T and Δ_L (but transformed to the center of longitudinal-momentum frame of each pair) unnecessarily mixes the transverse and longitudinal degrees of freedom. We propose to write $F_{BE}(q_1-q_2)$ as a product of factors depending on y_1-y_2 and $q_{1T}-q_{2T}$, respectively, for example,

$$F_{BE}(q_1-q_2)=\exp(-|y_1-y_2|/\Lambda-R_T^2\Delta_T^2/2) . \quad (52)$$

For small rapidity separations this form is, of course, not a theoretical prediction, but just a simple guess. The prediction is that Λ is universal, i.e., independent of A and B, while $R_T=(\frac{2}{3})^{1/2}R_{AB}$, where R_{AB} is given by (9) for nucleus-nucleus collisions. As already mentioned, this estimate has been obtained neglecting the contributions to R_T from hadronic vertices and, strictly speaking, holds only when R_{AB} is large enough. A more realistic estimate for $A \ll B$ (the case of main physical interest) is easily found:

$$R_T=[2R_A^2/3+R_N^2/3+(R_T^{NN})^2]^{1/2}, \quad A \ll B , \quad (53)$$

where R_N is the nucleon radius and R_T^{NN} is the interferometry radius in a high-multiplicity NN collision. R_T^{NN} is expected to be a "hadronic scale," thus of the order of 1 fm or so; we are unable to predict anything more precise. The increase of R_T as one goes from hadron-hadron to nucleus-nucleus collisions is an unescapable consequence of the theory.

The value of R_T at midrapidities reported by the NA35 Collaboration for O-Au (8.1 ± 1.6 fm [17] is 3 times larger than expected from our conventional theory. This appears very exciting. Unfortunately, the preliminary result [18] of the same group for S+Ag is $R_T\approx4.6\pm0.7$ fm, much closer to our expectation, which is 2.4 fm when the asymptotic estimate is employed and becomes 2.9 fm when (53) and $R_T^{NN}\approx1.4$ fm (from Ref. [19]) are used.

NA35 data also indicate that the rapidity correlation is affected by the sizes of the colliding nuclei, at variance with the expectation of the theory considered in this paper. However, it is not quite clear what the result would be if the experimenters used the parametrization we propose. It is important to check this point. Every disagreement with the conventional theory is potentially very interesting.

At this point let us mention the suggestion [21] that large interferometry radii can be due to the production of resonances. This is a possibility. However, the rate of production of resonances in the central region mildly depends on the nature of the targets. Therefore, it is difficult to see how resonance production could lead to "abnormally" large interferometry radii in nuclear collisions and to radii of the order of 1 fm in hadron-hadron collisions. Note, however, that the momentum resolution in most studies of BE correlations in hadron-hadron collisions is considerably worse than that in NA35. Resonance production produces a sharp spike in the interferometry signal at very small values of relative momentum. While this spike could not possibly have been seen with the UA1 setup, it can perhaps contribute to the large radii observed by NA35. The spike in question is independent of the atomic mass numbers of the colliding objects and will not be further discussed in this paper, where we focus our attention on nuclear effects. Let us only emphasize that high-resolution measurements of the shape of the interferometry signal in NN collisions would be very helpful in interpreting the heavy-ion data.

B. Intermittency slopes in nucleus-nucleus collisions

The studies of intermittency [22], more precisely of the variation of the scaled multiplicity moments with the decreasing size δ of the rapidity (or pseudo rapidity) window, are a source of precious information on the short-range structure of rapidity correlations. In this section we discuss the consequences of Eq. (12) for the behavior of the moment $F_2=\langle N(N-1)\rangle/\langle N\rangle^2$, for the simplest case of a central nucleus-nucleus collision, where the condition $\langle n\rangle \gg 1$ is met and (12) is expected to be a very good approximation. Strictly speaking, our derivation holds for an average collision. However, the only thing which changes when only central collisions are considered is the probability distribution of n, so that the estimate given by (32) no longer holds. Integrating (12) with respect to rapidities in the range $-\delta/2<y_1$, $y_2<\delta/2$, one gets

$$F_2(\delta)=(\langle n^2\rangle/\langle n\rangle^2) \times[1+3G(\delta/\Lambda_{BE})/4\langle q_T^2\rangle R_{AB}^2] . \quad (54)$$

We have multiplied the second term in square brackets by an extra factor of $\frac{1}{2}$ because we consider here arbitrary charged pions, while the BE correlation is only present for identical ones. Assuming, for simplicity, an exponential form of c_{BE}, i.e.,

$$c_{BE}(y_1,y_2)=\exp(-|y_1-y_2|/\Lambda_{BE}) , \quad (55)$$

we have

$$G(x)=2(x-1+e^{-x})/x^2 . \quad (56)$$

As shown in Sec. III B, one expects $\Lambda_{BE}=0.5$ at large distances. Nothing guarantees that the exponential shape (55) with $\Lambda_{BE}=0.5$ is also adequate at small $|y_1-y_2|$. We shall use it below to get a tentative estimate, also quoting the result obtained setting $\Lambda_{BE}=1$.

Using $\langle q_T^2\rangle=0.16$ $(\text{GeV}/c)^2$ together with the values of R_{AB} given by (9b), we obtain the effective intermittency slope ϕ_2, defined as $\Delta\ln F_2/\Delta\ln(1/\delta)$ for a given range of δ, always chosen to be the same as that used by experimenters in their fits. From now on, if not stated otherwise, ϕ_2 will denote the slope parameter for arbitrary charged pions. We calculate the rms radii from the interpolating formula [23]

$$R_A=0.82A^{1/3}+0.58 \text{ fm} . \quad (57)$$

For O-Au collisions we find, for the range $2>\delta>0.1$ (cf. Ref. [24]),

$$\phi_2=0.005 \quad \text{for } \Lambda_{BE}=0.5 , \qquad \phi_2=0.003 \quad \text{for } \Lambda_{BE}=1 . \quad (58)$$

When A and B are changed, ϕ_2 is predicted to scale with R_{AB}^2. In particular, for S-initiated collisions, the above values of ϕ_2 are scaled down by $R_{\text{O-Au}}^2/R_{\text{S-Au}}^2\approx0.7$.

The above estimates of ϕ_2 are reduced when hadronic contributions to R_{AB} are not neglected. This is a 30% reduction if R_T^{NN} is 1.4 fm, as claimed by UA1 in Ref. [19].

Note that (54) is obtained from (7) when $\langle n \rangle$ is so large that the last term, of order $\langle n \rangle$, can be neglected compared to the first two terms, which are of order $\langle n \rangle^2$. Once one is in the large $\langle n \rangle$ regime, changing the degree of centrality only affects the factor $\langle n^2 \rangle / \langle n \rangle^2$ in (54), which is irrelevant for the intermittency slope. This observation is pertinent for central and semicentral O-Au and S-Au collisions. On the other hand, for O(S)-Em collisions, we expect some decrease of ϕ_2 with increasing $\langle n \rangle$ (also as one goes from average to central ones, even for S-Au).

The above tentative estimates of the intermittency slopes are in the same ballpark as the data of the EMU01 Collaboration [24]. However, these data seem to indicate a decrease of ϕ_2 with increasing centrality and also an increase of ϕ_2 with increasing projectile size. These features, if confirmed with better statistics, would point toward some collective effect of a new type. However, in view of the poor accuracy of the present data, such conclusion is premature.

As a final remark, we would like to emphasize again the point already made in Ref. [3]: In heavy-ion collisions the short-range correlations are dominated by the BE effect and therefore we predict $\phi_2^{\pm\pm} = 2\phi_2$; the intermittency slopes should be twice larger when only identical instead of arbitrary charged pions are observed.

C. Tentative discussion of intermittency slopes in hadron-hadron collisions

We now turn to hadron-hadron collisions. Of course, this extension of our discussion involves extra uncertainties. Furthermore, our prediction is that the chaoticity parameter λ is not far from unity when very large multiplicity events are selected. If the value $\lambda = 0.15$ of UA1 Collaboration is taken literally, our theory simply does not apply. In Sec. IV A we expressed already our doubts concerning the interpretation of this small λ. In this section we shall assume that the small effective value of λ indeed results from an experimental bias, and we shall proceed disregarding this result. Thus the following discussion is necessarily very tentative.

Let us start with central proton-antiproton collisions, i.e., with high-multiplicity selection, where the $\langle n \rangle \gg 1$ regime is approached. Results, both for BE interferometry and intermittency slopes, are available from the UA1 Collaboration [19] for values of $I_1(y)$ up to 10. At the highest rapidity densities, UA1 finds

$$F_{\rm BE}(\Delta_T) \sim \exp(-R_T^2 \Delta_T^2 / 2), \quad R_T \approx 1.4 \text{ fm} . \tag{59}$$

We can use Eq. (54) after replacing $R_{AB}^2/3$ by the hadronic parameter $R_T^2/2$. Here we have to rely exclusively on data since the theory is unable to make any quantitative prediction about $C_{\rm BE}(q_1, q_2)$. Using (59) and (54), we get for the intermittency slope in the range $1 > \delta > 0.1$ and, for $\langle n \rangle \gg 1$,

$$\begin{aligned} \phi_2 &= 0.009 \quad \text{for } \Lambda_{\rm BE} = 0.5 , \\ \phi_2 &= 0.006 \quad \text{for } \Lambda_{\rm BE} = 1 . \end{aligned} \tag{60}$$

The UA1 value in the same δ range is $\phi_2 \approx 0.007$. Strictly speaking, the $\langle n \rangle \gg 1$ regime is not really reached in the UA1 data. The rapidity densities they observe are large in hadronic standards, but still much less than those produced in heavy-ion collisions. Thus the neglect of terms of order $\langle n \rangle$ in (7) is a rather crude aproximation and the slope given by (60) is presumably underestimated.

Consider next the fully inclusive data, i.e., without high-multiplicity selection, for which the dynamical short-range correlation is important. We have to include an extra effect, which has been neglected in Sec. II for the sake of simplicity. Phenomenology indicates that in hadron-hadron collisions the total available energy is unequally partitioned among the different strings. In the dual parton model (DPM) one string (involving valence constituents) takes a large fraction of the available energy and the rest is equally shared by the remaining ones (involving sea or gluon constituents). This finite-energy effect should be taken into account when $\langle n \rangle$ is not large. Equation (2) then becomes

$$N(q) = N_v(q) + \sum_{\alpha=1}^{n-1} N_{s\alpha}(q) , \tag{61}$$

and, consequently,

$$I_1(q) = i_{1v}(q) + \langle n-1 \rangle i_{1s}(q) . \tag{62}$$

For identical pions simple algebra leads to

$$I_2(q_1,q_2) = I_1(q_1) I_1(q_2) + (C_{\rm LR} + B C_{\rm BE} + C_{\rm SR})(q_1,q_2) , \tag{63}$$

where

$$C_{\rm LR}(q_1,q_2) = (\langle n^2 \rangle - \langle n \rangle^2) i_{1s}(q_1) i_{1s}(q_2) \tag{64}$$

is the long-range correlation,

$$C_{\rm SR}(q_1,q_2) = c_{\rm SR}^v(q_1,q_2) + \langle n-1 \rangle c_{\rm SR}^s(q_1,q_2) \tag{65}$$

is the short-range correlation, and

$$\begin{aligned} B(q_1,q_2) = {} & \langle (n-1)(n-2) \rangle i_{1s}(q_1) i_{1s}(q_2) \\ & + \langle n-1 \rangle [i_{1v}(q_1) i_{1s}(q_2) + q_2 \leftrightarrow q_1] . \end{aligned} \tag{66}$$

Clearly, for $N_v(q) \equiv N_s(q)$, Eq. (63) reduces to Eq. (7) of Sec. II.

Integrating over transverse momenta and over rapidities in the range $-\delta/2 < y_1, y_2 < \delta/2$, and using (59), we get, for arbitrary charged pions,

$$F_2(\delta) = 1 + g_{\rm LR} + g_{\rm BE} G(\delta/\Lambda_{\rm BE}) + g_{\rm SR} G(\delta/\Lambda_{\rm SR}) , \tag{67}$$

where

$$g_{\rm LR} = (\langle n^2 \rangle - \langle n \rangle^2)(h_s/H)^2 , \tag{68a}$$

$$g_{\rm BE} = \frac{\langle (n-1)(n-2) \rangle h_s^2 + 2 \langle n-1 \rangle h_s h_v}{2H^2 \langle q_T^2 \rangle R_T^2} , \tag{68b}$$

$$g_{\rm SR} = c_{\rm SR}/H^2 . \tag{68c}$$

Equation (67) reduces to Eq. (54) when $\langle n \rangle \gg 1$. Here c_{SR} is the strength of the dynamical short-range correlation and Λ_{SR} is the corresponding correlation length, H and $h_{s(v)}$ are the heights of the observed and $s(v)$-string rapidity plateaus, respectively.

In the DPM one has at collider energies $h_v = 2.1$, $h_s = 0.9$, and $\langle n \rangle = 2$, which yield the (pseudo)rapidity plateau height $H = 3$. The variance of n can be directly related to the experimentally measured values of the long-range forward-backward correlation and of the multiplicity moment C_{2F} in the forward or backward rapidity interval under consideration. Indeed, one has

$$g_{LR} = b_{FB}(C_{2F} - 1) \ . \tag{69}$$

Since we are interested in the central rapidity region, we use the experimental values [25] of b_{FB} and C_{2F} in the pseudorapidity intervals $1 \leq |\eta| \leq 2$, which are close to $\eta = 0$ and are separated by a rapidity cut of two units that removes the short-range correlation. Using $b_{FB} = 0.412(11)$ and $C_{2F} = 1.85(1)$, from the UA5 Collaboration, we get $g_{LR} = 0.35$.

Let us tentatively set $\Lambda_{BE} = 0.5$ and $\Lambda_{SR} = 2$, the values expected from Regge arguments at large rapidity separations ($\Lambda_{SR} = 2$ is actually a more realistic effective correlation length than $\Lambda_{SR} = 1.1$ used in Ref. [3]). From (68b), with $R_T = 1.4$ fm and $\langle q_T^2 \rangle = 0.16$ $(\mathrm{GeV}/c)^2$, we get $g_{BE} = 0.045$. The constant g_{SR} is determined from the UA1 experimental result [26] $F_2(1) = 1.617(13)$. We get $g_{SR} = 0.283$. As a cross-check we first compute from Eq. (63) the value of $R(0,0) = C(0,0)/H^2$, where $C(y_1, y_2)$ is the full correlation function. We get $R(0,0) = 0.68$ to be compared to the UA5 value [25] 0.68(2). The contribution of the BE correlation is $\delta R(0,0) = g_{BE}$ and is therefore small. Computing the intermittency slope ϕ_2 in the interval $1 > \delta > 0.1$, we find $\phi_2 = 0.014$ to be compared to the UA1 value [26] $\phi_2 = 0.011(2)$.

We turn next to like-charged pions, in order to calculate the slope ϕ_2^{--}. We again use (67), but with g_{BE} multiplied by 2. We keep of course, the same g_{LR} as before, and we determine g_{SR} from the experimental value [27] $F_2^{--}(1) = 1.542(14)$. We get $g_{SR} = 0.165$. We finally obtain $\phi_2^{--} = 0.015$, to be compared to the UA1 result $\phi_2^{--} = 0.011(3)$.

Had we chosen the value $\Lambda_{BE} = 1$, instead of $\Lambda_{BE} = 0.5$, the above estimates would become $\phi_2 = 0.012$ and $\phi_2^{--} = 0.011$

As a final comment, let us mention that, in our approach, we do not have enough information to consider the finite statistics distortion [28] of the above estimates. The intermittency slopes corresponding to an infinite statistics can be calculated from the two-particle inclusive spectrum, as we have done. The finite statistics distortion depends on the multiplicity distributions in rapidity windows of varying size. The knowledge of these multiplicity distributions is equivalent to the knowledge of correlation functions of all orders, i.e., to the knowledge of an infinity of parameters. The theory we have considered has no power to predict these parameters. *A priori*, it can accommodate a wide variety of patterns, and it seems virtually impossible to estimate the finite statistics distortion without adopting an *ad hoc* model.

V. SUMMARY AND CONCLUSIONS

In this work we have studied the expectations of the conventional picture of soft hadronic interactions for the Bose-Einstein correlations between like-charged pions. We used the framework of the Reggeon theory combined with the generalized optical theorem. Our strategy, motivated by the topological $1/N$ expansion, has been to neglect interactions between Pomerons (except in Sec. III C, where it has been checked explicitly that the triple-Pomeron interaction is indeed irrelevant for our problem) or, using a more pictorial language, between the radiating "strings." We focused our attention on nucleus-nucleus collisions. There is no point to insist on the intrinsic interest of these collisions; the subject is among the most topical ones. However, it is perhaps not commonly realized that the appearance of nuclear (hence large) scales considerably improves the predictive power of the conventional theory. The most unambiguous results in Reggeon theory have been obtained in the asymptotic limit where all hadronic parameters are small compared to the Regge slope $\alpha' y$. Here a similar role is played by the nuclear radii. One should remark, that such a large scale is absent in high-multiplicity hadron-hadron collisions. Thus, although the latter share some features in common with nuclear collisions, predictions concerning them are less reliable.

Our results for pion interferometry in nuclear collisions are easy to summarize: (1) The correlations between transverse momenta are controlled by the size of the smaller of the colliding nuclei; (2) the rapidity correlations are universal, in the sense that they are insensitive to nuclear parameters; (3) the chaoticity parameter λ is close to unity (actually smaller than 1 by a few percent). Thus the interferometry image of the "source" is that of a pancake, with a nuclear transverse size and a hadronic longitudinal size.

The interferometry data have been discussed in Sec. IV A. The value of the chaoticity parameter is compatible with $\lambda = 1$ in collisions of nuclei. The transverse size of the source is larger than in *NN* collisions, actually larger than we expect. However, the large difference observed at *NN* midrapidities between radii for O- and S-initiated reactions cast some doubt on the significance of the disagreement (especially that the transverse radius observed using S as the projectile is not very different from our expectation). Moreover, it seems that the longitudinal size of the source is larger than that observed in *NN*. If confirmed, this result would require some unconventional dynamics. In this context we propose a new parametrization of the Bose-Einstein correlation, which is more natural from the theoretical point of view [see Eq. (52)].

In the theory we have explored, there is an inescapable relation between pion interferometry and intermittency in collisions involving heavy nuclei. Indeed, in this case, the only relevant short-range rapidity correlations are those generated by quantum interference. And the so-called in-

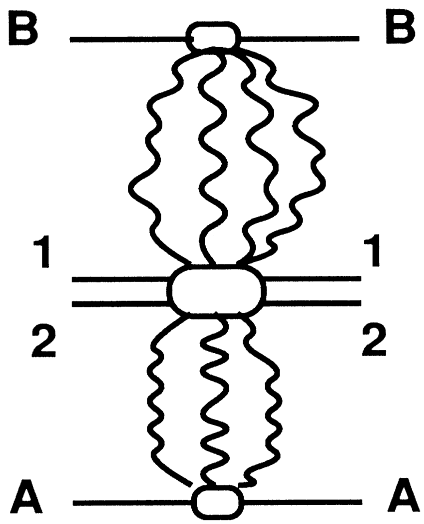

FIG. 5. Diagram represents a contribution to the two-body inclusive spectrum, which involves Pomeron interactions and survives the AKG cancellation. Processes of this kind could perhaps be introduced to mimic unconventional dynamics. Such diagrams are neglected in this work because of the absence of evidence for interactions between Pomerons, which are strong enough to be relevant for our problem at presently accessible energies (see Sec. III A).

termittency, in hadron production, is a manifestation of short-distance structure of rapidity correlations. This point has been discussed in Sec. IV B. Our estimated intermittency slopes are in the same ballpark as the observed ones, but certain trends of the data seem to be at variance with our expectations (e.g., a decrease of the slopes with the increasing degree of centrality of collisions). For completeness, we also discuss in Sec. IV C the intermittency slopes observed in collider data.

The experimental situation can be summarized by saying that there are several indications of a failure of the conventional dynamics, but more work is necessary to convert these indications into a clear evidence. Let us be optimistic, and let us suppose that this evidence will soon be available. Would it be possible to "upgrade" the conventional approach, so as to embrace the new information? This would require introducing interactions between Pomerons such as in Fig. 5. The problem is then to get large interferometry radii. In the diagram of Fig. 2(c), in a sense, the secondary particles receive information about the size of the projectiles from the loop integration. As shown in Ref. [2], this mechanism no longer works in the diagram of Fig. 5. However, it might be that the secondaries receive information about the size of the projectiles from the dependence of the central vertex on the number of coupled Pomerons. It seems possible to introduce such interactions [29], but it is not clear whether this can be done without generating unwanted effects (such as important shadowing in particle production off nuclei). Furthermore, such an approach seems unavoidably *ad hoc*. Reggeon theory can accommodate new interactions, but its predictive power is then rapidly lost (at least at finite energy). We are therefore inclined to think that a proper understanding of deviations from the background behavior described in this paper will be impossible in a purely S-matrix theory and will require a new insight into the early stage of the development of the collision process. We would be happy if our conclusions were perceived as an encouragement by experimenters working in this field.

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