# Energy dependence of color transparency

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A quantum-mechanical approach is used to study high-momentum-transfer reactions in which a nucleon is knocked out of a target nucleus. It is found that the nuclear interactions of the wave packet produced in such a hard interaction may cancel, so that the nuclear medium is transparent. Expressions for the effective wave-packet–nucleon interaction and the corresponding distorted waves are presented. An analysis of existing $(p,pp)$ data at beam momenta from about 5 to 12 GeV/$c$ suggests that only the nucleon and Roper resonance components of the wave packet contribute to the scattering amplitude. The $(p,pp)$ data and our theory imply that color-transparency effects in the $(e,e'p)$ reaction may be significant at relatively low-momentum transfers of $Q^2 \approx 3-6$ GeV$^2$.

## I. INTRODUCTION

This paper is concerned with high-momentum-transfer (greater than about 1–2 GeV/$c$) nuclear processes which proceed by the emission of a single fast nucleon from the nucleus. Examples are the semiexclusive $(e,e'p)$ and $(p,pp)$ reactions occurring on nuclear targets. We consider only those experiments in which the detected protons have enough energy to ensure that no pions are produced.

The semiexclusive experiments under consideration here are special. For such cases [1,2], the nucleus will be anomalously transparent to nucleons. The arguments are based on three main points.

(i) To obtain an appreciable amplitude for a high-momentum-transfer reaction on a nucleon leading to a nucleon, the colored constituents must be close together, i.e., small objects are produced in high-momentum-transfer exclusive processes.

(ii) If the constituents are close together, their color electric dipole moment is small and the soft interactions with the medium are suppressed; i.e., small objects interact weakly.

(iii) If the particle stays small it can escape from the nucleus without further interaction.

The particle suffers no distortion when all three conditions are satisfied. Any of the conditions may be considered controversial. The medium is thus "transparent," leading to the name "color transparency." Color transparency is expected to hold even when the projectile-nucleus optical potential is not small.

Color transparency has generated much interest from theorists [3–8] and experimentalists [9,10]. This is because of the arguments [1,2] that the existence of color transparency is a testable prediction of quantum chromodynamics (QCD). There is also some hope that calculations of proton-proton scattering [11] using perturbative QCD will provide the guidance necessary to implement the qualitative ideas regarding color transparency.

The main purpose of the present paper is to study how color transparency arises as a function of momentum transfer (or energy if the scattering angle is fixed). The aim is to help experimentalists to find color transparency by using experiments over a range of energy. At the lowest energies there is no transparency (the nucleus acts as a black disk) and standard nuclear-physics techniques are valid. At the highest energies available perturbative QCD applies (one hopes) and color transparency will be observed. A successful treatment of the energy dependence of color transparency therefore requires a correct treatment of both relatively low- and high-energy physics. Thus, it seems that an approach using both hadronic and quark degrees of freedom is inevitable.

Our procedure uses the ideas [12] of soft (dipole) gluon exchange and color neutrality to model the interaction between the quarks in the fast ejected particle (ejectile) and the bound slowly moving nucleons. One obtains the ejectile-nucleon cross section $\tilde{\sigma}$ of Eq. (21), for example. This is an operator depending on the transverse separation $b$ of the quarks in the ejectile. The evaluation of this operator in a hadronic basis, combined with the eikonal propagation of the ejectile through the nucleus is used to construct the ejectile-nucleon effective cross section, Eq. (33). Previous authors [3,13,14] postulated reasonable forms of the effective cross section. Here we provide an underlying derivation. It turns out that our Eq. (33) is not very different than previous hypotheses, even though the input $\tilde{\sigma}$ is different. Furthermore, our underlying quantum-mechanical framework will eventually allow improvements of the present qualitative, simplified treatments of $\tilde{\sigma}$ and the baryon wave functions.

We previously developed [8] an approach to color transparency based on a quantum-mechanical treatment of the wave function of the ejectile in the nucleus. The action of a high-momentum-transfer operator on the struck nucleon creates an object which has a small spatial extent. This small object can be described as a coherent sum over nucleon excited states, $m$, i.e., as a wave packet. In that case, the small size is described as a destructive interference between the different states [8] and depends on the relative phases of the states of the wave packet. If the important states in the wave packet have similar energies the important relative phases vary slowly with time and the wave packet remains spatially small. Alternately, we say that assuming all the energies are equal allows one to employ closure (completeness) to do the sum over states, $m$; this is the so-called closure approximation.

Our argument seems to rely on the use of hadronic rather than quark degrees of freedom. The hadronic basis is a convenient tool to evaluate matrix elements; moreover these matrix elements may be measured in independent experiments. We intend to use and stimulate new measurements of the matrix elements relevant for color transparency in future work.

Even though we stress use of the hadronic basis, the color transparency phenomena is certainly a consequence of the quark-gluon nature of QCD. Indeed, one way to argue for the validity of our closure approximation is to use perturbative QCD factorization theorems [15,16]. Such theorems allow the separate treatment of the hard and soft interactions. In perturbative QCD we expect that the minimum possible number of hard interactions occur. Hard final-state interactions would increase the number of hard interactions from that minimum; hence the final-state interactions between the ejectile and the nuclear medium are of a soft, low-momentum-transfer nature. The soft interactions do not have very large matrix elements between baryons of low and high mass, according to the quark model. This is why the closure approximation is expected to be valid. This is explained more in Sec. III.

Note further that the closure approximation is not expected to be valid for those inclusive processes in which hadrons with a very broad range of internal energies are produced. Thus most inclusive processes are not expected to exhibit color transparency.

We now come to an outline of the paper. In Sec. II we review and restate the idea that an object produced in a high-momentum-transfer reaction is small. The physics of the closure approximation is discussed in Sec. III. Our quantum-mechanical approach to the propagation of the ejectile through the nucleus is presented in Sec. IV. The ejectile-nucleon cross section is introduced and used to evaluate the ejectile-nucleus interaction $U$ in Sec. V. The closure approximation is used to compute amplitudes in Sec. VI. In our present model, the approach to color transparency is controlled by a $1/Q^2$ dependence [Eq. (28)]. In Sec. VII the expression for the effective interaction between the ejectile and the nucleus is derived and the eikonal approximation for the baryon propagation discussed. In Sec. VIII we present our numerical results and compare with the $(p,pp)$ data of Carroll *et al.* [9]. These data are used to predict color transparency effects in the $(e,e'p)$ reaction. The work of Ref. [3] was the first to treat the color transparency of the $(p,pp)$, $(e,e'p)$ and $(\pi,\pi N)$ reactions in a unified numerical fashion. Our numerical results are similar to those of Ref. [3]. A discussion and summary are presented in Sec. IX.

## II. HARD INTERACTIONS PRODUCE A SMALL OBJECT

Consider a process in which a photon interacts with a free nucleon and transfers a large momentum $Q$ to it; see, e.g., Fig. 1. The full field-theory calculations are more complicated and generally require treatment of gluon radiation or Sudakov effects [17]. Here we follow Mueller [13]. The nucleon Breit frame is used, since in this frame the photon transfers no energy. If the photon is absorbed by one of the partons (quarks) the momentum $Q/3$ must then be transferred to each of the other quarks in order for the momentum to be divided equally between the (three) partons. The gluons that transfer the momentum are off shell by $Q/3$, so by the uncertainty principle they exist for only a time $3/Q$ and travel a distance of $c$ times this. Thus the partons must all be within a distance $3c/Q$. Hence the object is small if $Q$ is large.

The above simplified reasoning may not be applicable for the experiments which do not have asymptotically large momentum transfer. At moderate values of $Q^2$, effects of nucleonic pointlike configurations [14] (PLC) which involve quarks existing in the same location may be responsible for the form factors. In either case, small sizes are relevant.

It is worthwhile to obtain a simple formula that schematically portrays the physics of the diagram of Fig. 1. We consider, for simplicity, a system of two quarks [18] and obtain one essential feature: the form factor is determined by the properties of the wave function at near the origin. This is used in our calculations below.

We model perturbative QCD calculations of form factors, such as that of Brodsky and Lepage [16]. In that work, the high-momentum components of the meson wave function are dominated by the effects of a one-gluon-exchange interaction, $V_{\text{eff}}$. Thus our focus is on the gluon propagator, whose inverse is $q_\perp^2+q_\parallel^2-q_0^2$. Since there is no net energy transfer in the Breit frame, the average value of the change in the quark energy is zero. Hence, $q_0\approx 0$. The longitudinal momentum of the quark can be written as $xQ/2$, and for each quark the average value of the change in this quantity is $Q/2$. Thus

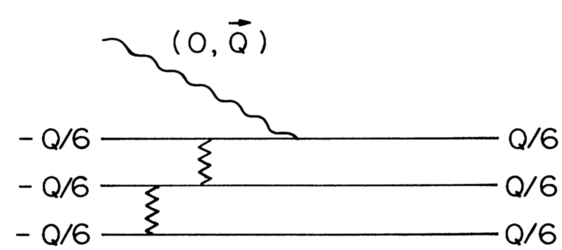


FIG. 1. An example of a high $Q^2$ process, $(\gamma^* p,p)$ in the Breit frame. The quark lines are labeled by the momentum components parallel to the photon momentum **Q**.

$q_{\parallel} \approx Q/2$. Proper form-factor calculations are most easily carried out in momentum space. Here we are concerned with the sizes of fast moving objects and therefore work in coordinate space. Thus we introduce the variable $\mathbf{b}$ which is the transverse separation between the quarks, and is conjugate to the transverse-momentum variable $\mathbf{k}_{\perp}$. Since $q_{\parallel}$ and $q_0$ have been fixed, the only remaining dependence is on $\mathbf{q}_{\perp}$. Thus, the coordinate-space interaction $V_{\text{eff}}$ looks like

$$V_{\text{eff}}(b) = \int \frac{d^2 q_{\perp}}{(2\pi)^2} \frac{e^{i\mathbf{q}_{\perp}\cdot\mathbf{b}}}{q_{\perp}^2 + (Q/2)^2} = \frac{1}{2\pi} K_0\left[\frac{Qb}{2}\right] \sim \frac{1}{2\pi}\left[\frac{\pi}{2}\right]^{1/2} \frac{\exp(-Qb/2)}{\sqrt{Qb/2}} . \quad (1)$$

Equation (1) tells us that the longitudinal-momentum transfer $Q/2$ acts as a mass for the transverse propagation. Thus the transverse separation varies like $1/Q$ and the object is small.

We have handled the $x$ dependence by an averaging procedure. This could be dangerous since the explicit integration may include regions of small $x$ which may have an unusually large contribution. We assume that Sudakov effects [17] suppress such contributions. Thus we do not treat possible end-point singularities that could cause perturbative QCD to be inapplicable [19]. The assumption that this $x$ dependence is not essential is also made in some recent work [7]. Our main point here is to stress the importance of the region with $b \approx 1/Q$. This is obtainable even without details of the proper integration over $x$.

In calculating the amplitude for a system with $N_p - 1$ quarks, $N_p - 1$ such factors appear and reproduce the well-known [20] scaling $Q^{-2(N_p-1)}$ expected from quark counting rules.

Immediately after the collision, the transverse size of the small object is of order $2/Q$. Such an object is not a nucleon; we shall find it convenient below to express it as a sum of baryon resonances. There is a component that is a nucleon: the overlap between the small object and the nucleon is the nucleon form factor.

We now illustrate these ideas by making a schematic calculation of an elastic form factor. For simplicity, we again consider a system of two quarks that is struck by a spacelike photon of four-momentum $-Q^2$.

In perturbative QCD the central idea is that the one-gluon-exchange interaction is the origin of the high-momentum components [16]. Then the form factor can be approximated [21] as a matrix element of $V_{\text{eff}}$ between unperturbed confined wave functions. Thus Eq. (1) shows that the form factor at high $Q^2$ is determined by the probability for quarks to be close together.

To proceed we need a model of the unperturbed wave functions. Keeping just the transverse part of the wave functions, the ground-state (nucleon, $N$) and first-excited-state ($N_R^*$) wave functions are given by

$$\langle b|N\rangle = \frac{1}{b_H\sqrt{\pi}} e^{-b^2/2b_H^2} \quad (2)$$

and

$$\langle b|N_R^*\rangle = (1 - b^2/b_H^2)\langle b|N\rangle , \quad (3)$$

where $b$ is the transverse coordinate. The parameter $b_H$ is the rms size of the hadron which is taken as 1 fm. For the sake of simplicity, we used the (two-dimensional) harmonic oscillator. The state $|N_R^*\rangle$ is the first excited state of this model. It is a radial excitation, so we call it the Roper resonance and denote it with an $R$. A calculation using realistic properties of observed resonances will be a subject of a future investigation. For now, we just use the idea that excited states exist.

In general, the form factor $F(Q^2)$ is given by an overlap

$$F(Q^2) = F_{N,N}(Q^2) = \langle N|T_H|N\rangle , \quad (4)$$

and we take the hard-scattering operator $T_H$ to be

$$T_H \approx V_{\text{eff}} . \quad (5)$$

We next use Eq. (1) so that

$$F(Q^2) \propto \int d^2b\, e^{-b^2/b_H^2} K_0(Qb/2) \sim \frac{1}{Q^2/4}\left[1 - \frac{15}{Q^2 b_H^2}\right] = \frac{1}{Q^2/4}\left[1 - \frac{0.6\ \text{GeV}^2\,\text{fm}^2}{Q^2 b_H^2}\right] . \quad (6)$$

The integration is carried out by using the asymptotic form of $K_0$ shown in Eq. (1). Since $Q \gg (1/b_H)$ it is safe to approximate the Gaussian by a power-series expansion. The expression is completed by converting the units of $Q$ to GeV and $b$ to fm.

If $Q^2$ is large enough, one again sees the expected quark counting rules [20]. Furthermore the requirement that the $Q^{-4}$ term vanishes,

$$Q^2 \gg (0.6\ \text{GeV}^2\,\text{fm}^2)/b_H^2 , \quad (7)$$

provides a crude necessary condition for the validity of the quark counting rules. For a system of $N_p$ partons the form factor has a leading behavior of $Q^{-2(N_p-1)}$ with a $Q^{-2}$ correction term. For $N_p$ partons the numerical factor in Eq. (7) varies as $N_p - 1$. In the following we assume that the validity of Eq. (7) is sufficient for perturbative QCD to be valid.

We shall also need to know an expression for the inelastic form factor defined by

$$F_{N_R^*,N}(Q^2)=\langle N_R^*|T_H|N\rangle \ . \tag{8}$$

Using the wave functions of Eqs. (2) and (3) and the operator $V_{\rm eff}$ of Eqs. (1) and (5) leads to

$$F_{N_R^*,N}(Q^2)\propto \int d^2b\, e^{-b^2/b_H^2}(1-b^2/b_H^2)K_0(Qb/2)\sim\frac{1}{Q^2/4}\left[1-\frac{0.6\ \text{GeV}^2\,\text{fm}^2}{Q^2b_H^2}\right] . \tag{9}$$

Thus, the elastic and inelastic form factors are equal

$$F_{N_R^*,N}(Q^2)\approx F(Q^2) \ . \tag{10}$$

This is caused by the equality of the model wave functions [Eqs. (2) and (3)] at $b=0$.

The reader who has experience in computing form factors using perturbative QCD might be shocked by the appearance of Eqs. (6) and (9). But no substantive difference is intended. The standard procedure [16] is to integrate over the transverse-momentum variables first, obtaining a final expression that is an integral over the longitudinal coordinates. Our procedure is to integrate (via using an average value) over the longitudinal variables first [22], obtaining a final expression that is an integral over transverse spatial coordinates. This is done to emphasize the influence of the transverse size which is relevant for color transparency effects.

## III. EXPANSION TIME

Consider the time required for a small baryon to expand to its normal size. In the rest frame of the ejected particle this time $t_i$ is expected to be about equal to the time it takes a quark to orbit the hadron [1]. For light quarks this is of order $\pi/\Lambda_{\rm QCD}$. The ejected particle has a large velocity in the laboratory frame, so time dilation increases the expansion time to $(E/M)t_i$. Here $M$ is the mass of the nucleon and $E$ its energy in the laboratory frame. The exact value of $t_i$ is not very important, because of the large $\gamma=E/M$ time dilation factor. For large $E$ the expansion time is long enough for the ejected particle to leave the nucleus before expanding. The argument also works in the rest frame of the hadron. In that case the nucleus is Lorentz contracted, so its small longitudinal size allows the fast nucleon to escape before it becomes large.

The arguments just presented contain a hidden assumption. To see this consider an alternate argument. In the nuclear rest frame the object produced in the hard interaction has a transverse size the order of $1/Q$ [23]. By the uncertainty principle, the partons have a spread in transverse momentum of order $Q$ which is about the same as the longitudinal momentum. Thus if the object has moved forward 1 fm, it will have a radius of about 1 fm. In this case, the object will expand and therefore interact long before it can leave the nucleus. This second argument depends only on the uncertainty principle. The first argument breaks down because the nucleon mass $M$ is used to compute $\gamma$. Immediately after the collision we are not dealing with a nucleon. Instead the ejected object can be regarded as a coherent sum of baryon states [8]. Thus the mass that enters in computing $\gamma$ should be not $M$, but rather the mass of intermediate state which is of order $Q$. Thus, it appears there is no time dilation and no color transparency.

The color transparency phenomena can be regained if only the low-lying states of the coherent superposition are relevant for experiments that detect a final proton. In that case, all of the masses $M^*$ are of the order of the nucleon mass $M$ and each is much less that $Q$. Thus the object does escape before expanding. For $\gamma\gg 1$, the condition on the masses $M^*$ can be restated in terms of the Mandelstam energy $s$ as $M^{*2}/s\ll 1$. This is well satisfied for excited states produced diffractively in $pp\to pX$ reactions [24].

Why should only low-lying states $(M^{*2}/s\ll 1)$ be relevant? Certainly the initial hard interaction produces all states with an energy up to $\approx Q$. Consider a component $|m\rangle$ of the ejectile wave packet. This component is relevant if final-state interactions with the nuclear medium, $U$, cause a transition to the proton $|N\rangle$, i.e., $\langle m|U|N\rangle\neq 0$. If $U$ is a soft (low momentum transfer) operator, the matrix element $\langle m|U|N\rangle$ becomes very small for very-high-energy states $|m\rangle$. This is expected in quark models, since quark wave functions of high-lying states have nodes which suppress matrix elements of (smooth) soft low-momentum-transfer operators such as $U$ [25].

To complete the argument that the expansion time is long, we need to show that $U$ really is a soft operator. This follows from the ideas about perturbative QCD (PQCD). In this theory high-momentum transfer occurs via the fewest possible numbers of hard interactions. Consider for example, the proton electromagnetic form factor. In that case at least two hard interactions are required to get the three quarks moving at high speeds in the same direction. In PQCD multiple-scattering terms with two hard and any number of soft interactions are favored over three hard interactions. This is because, the third hard interaction would introduce a factor of order $\Lambda_{\rm QCD}^2/Q^2$. Thus any additional interactions with the nucleus must be low-momentum-transfer ones [26].

Note the importance of the exclusive nature of the process. If we were concerned with inclusive processes and the final state were not a proton but a rapidly moving set of highly excited hadronic states, the relevant intermediate states would also be highly excited. In that case, the closure approximation would not hold, the expansion would be rapid and color transparency would not be observed.

The crucial role [8] of the interaction with the nuclear

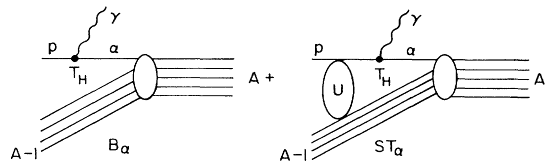


FIG. 2. The high $Q^2$ process, $(\gamma^* p,p)$ for a nucleon in a shell model orbit $\alpha$. The Born $B_\alpha$ and first scattering $ST_\alpha$ terms are shown.

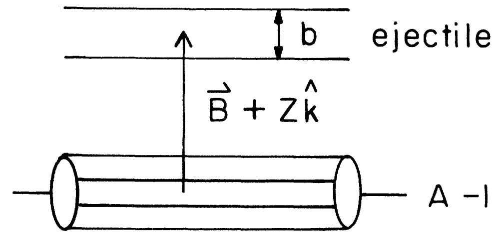


FIG. 3. Coordinates describing the interaction between the ejectile and the nucleus.

medium is stressed in the present work. However, our argument also rests firmly on specific notions regarding quark-gluon dynamics.

## IV. FORMALISM

In this section we present a quantum-mechanical approach that can be used to compute the cross section for ejectile-nucleon interactions. This changes as the ejectile expands during its motion through the nucleus.

To be definite, consider a high-momentum-transfer process in which a photon of three-momentum $\mathbf{q}$ is absorbed and a nucleon of momentum $\mathbf{p}$ leaves the nucleus. As usual, $q^2=-Q^2$. A special case is that of quasielastic kinematics. Then, the detector is set so that $\mathbf{p}=\mathbf{q}$. In that case, in the absence of final state interactions, the struck nucleon is at rest before the collision. Our previous work [8] employed such kinematics. The present equations are more general. For the moment, we consider knockout from only a single shell model orbital, denoted by $\alpha$. The necessary incoherent sums over all the occupied orbitals are done below.

Let the amplitude be defined as $\mathcal{M}_\alpha$, the two lowest-order terms are shown in Fig. 2. In the first, or Born term $B_\alpha$ the proton escapes without interaction. If full color transparency is obtained, this is the only term to survive. The first correction to the Born term is the scattering term or second term denoted by $ST_\alpha$. To the stated order we have

$$\mathcal{M}_\alpha = B_\alpha + ST_\alpha \ . \tag{11}$$

The Born term is given by

$$B_\alpha = \langle \mathbf{p} | T_H(Q) | \alpha \rangle = F(Q^2) \langle \mathbf{p}-\mathbf{q} | \alpha \rangle \ , \tag{12}$$

using the notation of Sec. II. The born term has a form resulting from the use of the nonrelativistic separation of internal and center-of-mass coordinates. The form factor $F(Q^2)$ includes the integral over internal coordinates, the factor $\langle \mathbf{p}-\mathbf{q}|\alpha\rangle$ is the Fourier transform of the bound-state wave function. We shall be concerned with situations such that $\mathbf{p}\approx\mathbf{q}$, so Eq. (12) is reasonable. Specific effects of spin are ignored here and throughout this work.

The second term is denoted by

$$ST_\alpha = \langle \mathbf{p} | U G T_H(Q) | \alpha \rangle \ , \tag{13}$$

where $G$ is the Green's operator for the emerging small object (ejectile). This object is not a nucleon, but rather a wave packet of which the nucleon is just one component. The wave packet is in the relative coordinates and corresponds to the sum over baryon resonances discussed in Sec. II. It is formed by the action of the hard absorption operator, $T_H$ on the bound nucleon.

The operator $U$ represents the interaction between the ejected baryon and the nuclear medium. The nuclear interaction can change the momentum of the ejectile and also excite or de-excite the internal degrees of freedom. That is, $U$ acts in the product space of the internal (quark) motion and the ejectile center-of-mass motion, see Fig. 3. Its matrix elements are written as

$$U_{N,m}(\mathbf{B},Z) = \int d^2b \langle N | b \rangle U(\mathbf{B},Z;b) \langle b | m \rangle \ , \tag{14}$$

where the internal baryon states are denoted by $|m\rangle$, with the nucleon as $|N\rangle$. Note that the $|m\rangle$ are eigenstates of the internal Hamiltonian and can only be changed through interactions with an external potential. $\mathbf{B}$ and $Z$ are the transverse and longitudinal distances between the ejectile and the nuclear center and $b$ denotes schematically the transverse separations between the quarks in the fast ejectile. Our present numerical calculations employ a simplification of using only two quarks in the baryon. Once again, we neglect the dependence on the internal longitudinal coordinates. The operator $U$ is of central importance in our procedure, and the next section is devoted to explaining and hypothesizing a form for this operator.

Inelastic form factors enter into Eq. (13) and are defined as in Eq. (8):

$$F_{mN}(Q^2) = \langle m | T_H(Q) | N \rangle \ . \tag{15}$$

Here the index $m$ denotes a nucleon $N$ or any excited nucleon $N^*$ or $\Delta$.

The explicit representation of the $ST_\alpha$ term of Eqs. (11) and (13) is

$$ST_\alpha = \sum_{m=N,N^*,\Delta} \int d^2B \, dZ \, dZ' \frac{e^{-ipZ}}{(2\pi)^{3/2}} U_{Nm}(\mathbf{B},Z) G_m(Z,Z') e^{iqZ'} \langle \mathbf{B},Z' | \alpha \rangle F_{mN}(Q^2) \ , \tag{16}$$

where $p=|\mathbf{p}|$ and

$$q=|\mathbf{q}|=(Q^2+\omega^2)^{1/2} . \tag{17}$$

The eikonal Green's function for the emerging baryon in a state $m$ is

$$G_m(Z,Z')=\theta(Z-Z')\frac{e^{ip_m(Z-Z')}}{2ip_m} , \tag{18}$$

with

$$\mathbf{p}_m^2=\mathbf{p}^2+M^2-M_m^2 . \tag{19}$$

Here $M$ is the nucleon mass. Equation (16) is our method of computing the quantum diffusion effects of Refs. [3,13,14].

Our intent is to evaluate the full scattering term. However, it is useful to compare that term with one in which the excited states are ignored. This is the distorted wave (DW) approximation [or distorted wave Born approximation (DWBA)] under which

$$ST_{\alpha,\mathrm{DW}}\equiv \int d^2B\, dZ\, dZ'\, e^{-ipZ}U_{NN}(B,Z)G_N(Z,Z') \times e^{iqZ'}\langle \mathbf{B},Z'|\alpha\rangle F(Q^2) . \tag{20}$$

Numerical comparisons between the amplitudes $ST_\alpha$ and $ST_{\alpha,\mathrm{DW}}$ are provided in Sec. VII.

## V. THE EJECTILE NUCLEAR INTERACTION $U$

To proceed it is necessary to devise a model for the interaction $U$. This is essentially the product of the ejectile-nucleon total cross section by the probability to find a nucleon (nuclear density).

The first step is to model the ejectile-nucleon cross section (imaginary part of the forward ejectile-nucleon scattering amplitude) $\tilde{\sigma}$ as [12]

$$\tilde{\sigma}(b)=\sigma\frac{b^2}{b_H^2} . \tag{21}$$

Note that $b$ is the quantum-mechanical position operator, representing the transverse separation of the quarks in the baryon. The usual nucleon-nucleon cross section is $\sigma$, and $b_H$ is the nucleon rms radius. The matrix element of $\tilde{\sigma}$ in the normal proton then yields the expected value, $\sigma$. For a small ejectile, the factor $(b/b_H)^2$ leads to a cross section smaller than $\sigma$. This describes the effects of color neutrality. Moreover, the operator $b$ has off-diagonal matrix elements that lead to excitation of nucleon resonances.

The main point of Eq. (21) is that the ejectile-nucleon cross section vanishes for small values of $b$, but takes on a standard size for larger values. The form (21) is not meant to be detailed representation of the nucleon-nucleon interaction. However, it is useful for our first evaluations of color transparency effects.

The combination of the quantum-mechanical $b$ dependence of Eq. (21) along with the eikonal propagation of the ejectile Eq. (16) leads to an effective ejectile-nucleon interaction that depends on the distance $Z-Z'$.

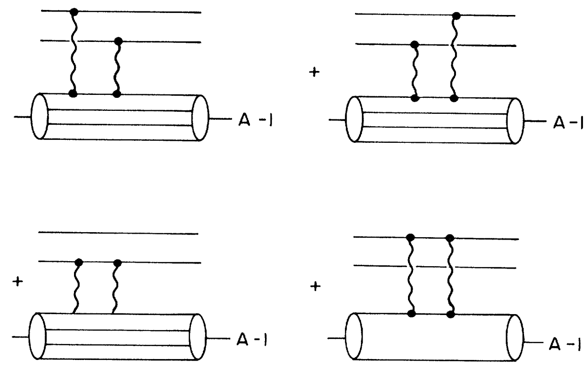


FIG. 4. Soft-gluon exchanges leading to an effective ejectile-nucleon cross section varying as $b^2$.

A crude phenomenological justification for the expression (21), see Fig. 4, is that the interaction is generated by the exchange of soft gluons provided by the nuclear medium; these are "dipole" or $E1$ gluons. Two such exchanges (each providing a factor of $b$) are needed so that the ejectile remains a color singlet. Povh and Hufner [27] found that hadron-proton total cross sections and forward angular distributions (slope parameters) are closely related for c.m. energies $\sqrt{s}\geq 15$ GeV. This indicates that hadron-proton cross sections vary as the square of the hadron radius, and provides some support for our chosen form.

Kopeliovich, Litov, and Nemchik [28] have investigated the quadratic $b$ dependence of the hadron-proton interaction. They consider the form of Eq. (21), another obtained by computing two-gluon-exchange diagrams, and a third form that is essentially linear in $b/b_H$.

In our calculations below, we mainly use the quadratic form, but check that the linear and other forms produce similar numerical results.

The ejectile-nucleon interaction can be used to construct the ejectile-nucleus interaction, $U$. We find

$$U(\mathbf{B},Z;b)=-i2E\frac{\sigma}{2}\rho(B,Z)\frac{b^2}{b_H^2} . \tag{22}$$

The factor $-i$ enters since $U$ depends on the ejectile-nucleon transition matrix. The factors $2E$ and $\frac{1}{2}$ are standard kinematical factors.

## VI. CLOSURE LIMIT

In this section we compute the quantity $ST_\alpha$ of Eq. (13) in the limit where closure is valid. This is relevant since color transparency occurs if $ST_\alpha$ vanishes.

Suppose that $p_m$ are approximately the same for all states $m$ contributing significantly to the sum appearing in Eq. (16), one may use closure to perform the sum over $m$. (This is the "closure approximation.") The necessary condition for the validity of closure [8] is discussed below.

When closure is valid, $p_m\approx p$ and Eq. (16) can be rewritten as

$$ST_\alpha = \sum_{m=N,N^*\Delta} \int d^2B\, dZ\, dZ' \frac{1}{(2\pi)^{3/2}} U_{Nm}(\mathbf{B},Z) \frac{\theta(Z-Z')}{2ip} \langle \mathbf{B},Z'|\alpha\rangle F_{mN}(Q^2) . \quad (23)$$

Notice that the exponentials have vanished and that the propagator has reduced to a step function divided by $2ip$. The only $m$ dependencies are in $U_{Nm}(\mathbf{B},Z)$ and $F_{mN}(Q^2)$. Let us then define the quantity $S$ as the sum of the $m$-dependent terms. Hence,

$$S = \sum_m U_{Nm}(\mathbf{B},Z) F_{mN}(Q^2) , \quad (24)$$

which has the explicit representation [using Eqs. (14) and (4)]

$$S = \int \langle N|b\rangle U(\mathbf{B},Z;b) K_0(Qb/2) \langle b|N\rangle d^2b . \quad (25)$$

In writing the above equation we have ignored the (constant) proportionality factors that relate $T_H$ and the function $K_0$. This is done throughout the remainder of the paper. These factors are common to all the cross sections and divide out in the ratios we plot.

The basic idea is that the function $K_0$ is non-negligible only for small values of the separation $b$ and vanishes for larger $b$, while the opposite is true for the function $U$. $U$ is a long-range soft operator because all of the hard interactions are contained in the operator $T_H$ (here represented by the function $K_0$).

The use of Eq. (22) for $U$ allows one to write $S$ as

$$S = -i2E\frac{\sigma}{2}\rho(B,Z) \int d^2b\, e^{-b^2/b_H^2} \frac{b^2}{b_H^2} K_0(Qb/2) . \quad (26)$$

The factor $b^2/b_H^2$ provides the suppression that leads to color transparency. Without it the above integral would yield the nucleon elastic form factor times the standard value of $U$, the so-called optical potential.

Note that we will ultimately evaluate Eq. (13) by using an intermediate complete set of baryon states. Thus $b^2/b_H^2$ is to be treated as a quantum-mechanical operator.

Evaluation of the integral of Eq. (26) leads to

$$S \approx -i2E\frac{\sigma}{2}\rho(B,Z) F(Q^2) \frac{0.3\ \text{GeV}^2\,\text{fm}^2}{Q^2 b_H^2} . \quad (27)$$

To be explicit, we compare the closure result with the standard DW expression (obtained by setting $b^2/b_H^2$ to unity). The scattering terms are related by

$$ST_{\alpha,\text{clos}} = \frac{0.3\ \text{GeV}^2\,\text{fm}^2}{Q^2 b_H^2} ST_{\alpha,\text{DW}} . \quad (28)$$

If the condition Eq. (9) that perturbative QCD applies, then $S$ essentially vanishes. In the closure limit, the scattering term $ST_\alpha$ is proportional to $S$ and it too vanishes.

Next we turn to the question of the condition necessary for the closure approximation to be accurate. To do this, examine the quantity $p_m$ for large values of the energy $E$ of the outgoing nucleon. One can use closure or completeness to do the sum on $m$ in Eq. (23) if $p_m$ is approximately independent of $m$. The relevant idea is that $U$ is a soft interaction that does not connect the nucleon ground state to excited states, $m$, of high mass. The difference between $p_m$ and $p$ the momentum of the outgoing proton is $\Delta p_m = p_m - p$ with

$$\Delta p_m = \sqrt{E^2 - M_m^2} - \sqrt{E^2 - M^2} \approx -(M_m^2 - M^2)/(2E) . \quad (29)$$

Since $\Delta p_m$ decreases as $E$ increases, $\Delta p_m$ vanishes. Then closure will hold, and color transparency will be obtained for large enough values of $E$.

To see how large is "large enough," examine the $m$ dependence of the eikonal Green's function of Eq. (18). There is a factor of $1/p_m$ times an eikonal phase factor. The term $1/p_m$ is essentially the same as $1/p$ for the energies and masses of interest here. However, the requirement that there is no $p_m$ dependence introduced by eikonal phase factor, $e^{ip_m(Z-Z')}$, provides a much more stringent criterion. This is because this variation is governed by the nuclear rms radius $R_A$, and $R_A \gg 1/p$. Nuclear form factors vary as $1 - q^2R_A^2/6$, so one requires

$$|\Delta p_m| R_A/\sqrt{6} = \frac{R_A(M_m^2 - M^2)}{2\sqrt{6}E} \ll 1 . \quad (30)$$

The above criteria, derived in a time-independent fashion, is compared with the time-dependent approach of [1] in Ref. [8]. It is shown there that the requirements for the validity of closure are essentially the same as those for the escape time ($\gamma t_i$) to be sufficiently small.

## VII. EVALUATIONS

The next step is to make an explicit evaluation of the scattering term. All of the inputs here are specified.

The oscillator model [Eqs. (2) and (3)] that represents the baryon wave functions is of special use here because the interaction $U$ is proportional to $b^2$. Since

$$b^2|N\rangle = b_H^2(|N\rangle - |N_R^*\rangle) , \quad (31)$$

only the ground $N$ state and first excited $N^*$ state (i.e., the Roper resonance) enter into computing the scattering term Eq. (16) $ST_\alpha$. Equation (31) is an exact consequence of our model for the baryon wave functions, Eqs. (2) and (3). Other choices of wave functions (i.e., hydrogenic) lead to a more complicated superposition, but the coefficients of the ground and first excited states are by far the largest [30].

Using the baryon wave functions and elastic and inelastic form factors of Sec. II along with Eq. (31) in Eq. (16) leads to the result

$$ST_\alpha = -F(Q^2)\int d^2B\, dZ\, \rho(B,Z)\frac{e^{-ipZ}}{(2\pi)^{3/2}}\int_{-\infty}^{Z} dZ' e^{-ip(Z'-Z)}\frac{\sigma_{\text{eff}}}{2}(Z,Z')e^{iqZ'}\langle \mathbf{B},Z'|\alpha\rangle \ , \tag{32}$$

where the notation is simplified by defining an effective cross section $\sigma_{\text{eff}}$. This term is

$$\sigma_{\text{eff}}(Z,Z') \equiv \sigma\left[1-\frac{p}{p_1}e^{i(p-p_1)(Z'-Z)}\right] . \tag{33}$$

The notation $p_1$ is an abbreviation for the momentum of the outgoing $N_R^*$ so that $p_1 = p_{N_R^*}$. An overall factor of $E/p$ is ignored on the right-hand side of Eq. (32), but is included in the numerical work. Equation (10), which incorporates the similarity of the nucleon and first excited states for $b=0$, has been used in arriving at Eq. (32).

It is useful to examine the effective cross section of Eq. (33). The corrections to closure appear in the presence of the factor $p/p_1$ and in the exponential. The quantity $p/p_1-1$ is very small for the energies and masses relevant here. Thus it is safe to replace $p/p_1$ by unity and the approximation

$$\sigma_{\text{eff}}(Z,Z') \approx \sigma(1-e^{i(p-p_1)(Z'-Z)}) \tag{34}$$

is valid. The exponent contains the distance $Z'-Z$ which can be as large as the nuclear diameter. According to Eq. (29) (with $E \approx p$), the exponent varies as the nuclear radius divided by the momentum $p$. At values of $p$ large compared to the reciprocal of the nuclear diameter the argument of the expotential function in Eq. (34) approaches zero, the effective ejectile-nucleon cross section $\sigma_{\text{eff}}$ vanishes, and color transparency is obtained.

The next step is to include terms beyond the first order in $U$. It is then worthwhile to define a bit of formalism. Start with the general expression for the amplitude to knock out a bound nucleon of state $\alpha$ from the nucleus. This is the knockout amplitude:

$$\mathcal{M}_\alpha = F(Q^2)\,^{(-)}\langle \psi_p|J_q|\alpha\rangle \ , \tag{35}$$

where

$$\langle \mathbf{r}|J_q|\mathbf{r}'\rangle \equiv \delta(\mathbf{r}-\mathbf{r}')e^{i\mathbf{q}\cdot\mathbf{r}} \ . \tag{36}$$

Here $^{(-)}\langle \psi_p|$ is the adjoint of the scattering wave function with incoming boundary conditions. The comparison of Eqs. (32) and (35) shows that to the present order the scattering wave function is given by

$$^{(-)}\langle \psi_p|\mathbf{r}\rangle = \frac{e^{-i\mathbf{p}\cdot\mathbf{r}}}{(2\pi)^{3/2}}\left[1-\int_Z^{\infty} dZ'\frac{\sigma_{\text{eff}}}{2}(Z',Z)\rho(B,Z')\right] . \tag{37}$$

The above is only the first Born approximation; however, its form is suggestive of the first term of the optical potential approximation to the full eikonal multiple-scattering series. Thus one sums an important series by exponentiating the right-hand side. Then

$$^{(-)}\langle \psi_p|\mathbf{r}\rangle = \frac{e^{-i\mathbf{p}\cdot\mathbf{r}}}{(2\pi)^{3/2}}\exp\left[-\int_Z^{\infty} dZ'\frac{\sigma_{\text{eff}}(Z',Z)}{2}\rho(B,Z')\right] . \tag{38}$$

It is also useful to display the wave function for outgoing boundary conditions:

$$\langle \mathbf{r}|\psi_p\rangle^{(+)} = \frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{(2\pi)^{3/2}}\exp\left[-\int_{-\infty}^{Z} dZ'\frac{\sigma_{\text{eff}}(Z,Z')}{2}\rho(B,Z')\right] . \tag{39}$$

If the standard DW approximation [recall Eq. (20)] were used, the nucleon inelastic excitations would be ignored and the wave function would be

$$\langle \mathbf{r}|\psi_p\rangle_{\text{DW}}^{(+)} = \frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{(2\pi)^{3/2}}\exp[-\chi(B,Z)] \ , \tag{40}$$

with

$$\chi(B,Z) = \frac{\sigma}{2}\int_{-\infty}^{Z} dZ'\rho(B,Z') \ . \tag{41}$$

These equations for the wave functions are the central formal results of this paper. The effects of perturbative QCD can be included by using a wave function $\psi_p$ of a standard form except for the use of the effective ejectile-nucleon cross section $\sigma_{\text{eff}}$, Eqs. (33) and (34). The utility is that these scattering wave functions can be used to compute any reaction involving outgoing protons. Consider the $(p,pp)$ reaction. Then the amplitude can be written as

$$\mathcal{M}_\alpha(p_1,p_2;p) \equiv \,^{(-)}\langle \psi_{p_1}\psi_{p_2}|t|\alpha\psi_p\rangle^{(+)} \ . \tag{42}$$

Here the interaction between the incident and bound nucleon is represented by the operator $t$.

We are now prepared to calculate the cross sections for the $(\gamma^*,p)$ and $(p,pp)$ reactions in which a nucleon is knocked out of an orbital $\alpha$. Most experiments measure a process in which a nucleon can be knocked out of any (partially) occupied orbital. Then the relevant quantity is an incoherent sum such as

$$d\sigma \propto \sum_\alpha |\mathcal{M}_\alpha|^2 \ . \tag{43}$$

Note that this is the correct quantum-mechanical expression. The semiclassical treatment of Farrar *et al.* involves ratios of cross sections obtained by integrating the product of the absolute squares of the bound and scattering wave functions. Our numerical evaluations of Eq. (43) allow us to determine that the semiclassical approximation is accurate to a few percent in the $(e,e'p)$ reaction, but only to about 25% in the $(p,pp)$ reaction. In both cases the effect is energy independent for the energies relevant here. The quantum-mechanical calculation is needed at lower energies.

In the following sections we shall be concerned with the ratios of full and distorted wave cross sections to the Born cross section. The calculations are performed by obtaining orbitals $\alpha$ from the harmonic-oscillator shell model with oscillator frequency $\omega=41$ MeV$/A^{1/3}$.

## VIII. NUMERICAL RESULTS

The ratios of cross sections obtained with our formalism are presented in this section. The first ratio of interest is obtained by dividing the cross section obtained from our wave-packet (full coupled-channel) formalism by that from the Born approximation. A ratio of unity indicates that the medium is transparent. Another ratio is obtained by comparing the cross section obtained in the standard DWA (just the nucleon in the intermediate state) with that of the Born approximation. The curves obtained with the DWA are called "standard" in the figures. Our theory is compared with the results of the $(p,pp)$ experiment of Carroll *et al.* in Fig. 5. Ratios of $d\sigma/dt(\approx 90°)$ in the c.m. obtained with Eq. (43) are plotted. The proton-proton transition matrix $t$ is assumed to provide a constant that divides out of the ratios. The cross section $\sigma$ is taken as 40 mb. This represents the total $pp$ cross section including elastic scattering. Events resulting from elastic $pp$ scattering are considered to be removed from the beam due to nuclear inelasticities. The imaginary part of the $pp$ transition matrix includes processes in which the struck bound proton is knocked out of the nucleus. The exact number of such processes counted in an experiment will depend strongly on the exact conditions of the experiment especially energy resolution. Since the results are largely determined by first-order perturbation theory the effects of the interaction will scale linearly with the value of $\sigma$ used.

For the $(p,pp)$ calculations, $\rho(R)$ of Eq. (22) is taken as a square well. The numerical results are not very sensitive to this choice (a few percent for $^{27}$Al).

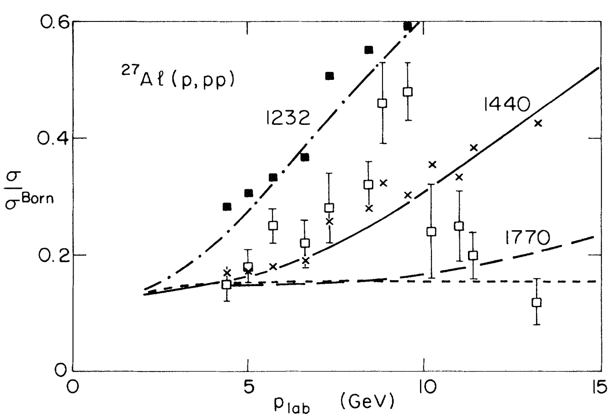


FIG. 5. Various models of the $^{27}$Al$(p,pp)$ reaction; see text for explanation: Energy dependence of ratios of $d\sigma/dt$. The numerators are calculations using the color transparency or DW (Glauber) calculations defined in the text. The denominator is the Born approximation. The $pp$ scattering angle is $\approx 90°$ in the c.m. The open boxes are the experimental data of Carroll *et al.* [9].

In our model, the ejectile is a superposition of many states, but only two contribute to the $(p,pp)$ reaction. One is a nucleon. The other is an excited state, with a mass that we treat as the only free parameter. This second state could represent the $\Delta$, the actual Roper resonance, or the average of the lowest $p$-wave $N^*$ resonances. Thus in Fig. 5 we show the results with three different values of its rest mass. The lines give the result when we have used $Q=p_{\rm lab}$ for the incident momentum and $Q/2$ (there are two exiting nucleons) for the exit momentum. This assumes that the struck nucleon has no momentum. The crosses (mass 1440-MeV intermediate state) and filled boxes (mass 1232 intermediate state) are obtained using the experimentally obtained values of momentum of the struck nucleon. The experiment is shown by the open boxes. By far the best agreement in the region below 10 GeV comes when 1440 MeV, the Roper mass, is used. Thus if this experiment is really seeing color transparency the condition that the mass difference between the Roper and the nucleon be small compared to the energy of the outgoing proton is sufficient for closure. For a $\Delta$ intermediate state the rise due to the onset of color transparency occurs at too low an energy, while for an intermediate state of mass 1770 MeV the rise occurs at energies greater than that of the energy of the experiment. None of the curves we show in Fig. 5 display any sign of the effect decreasing drastically with increasing momentum as the data does.

A downturn in the ratio of cross sections could be produced if a higher-energy state contributes with the opposite sign. We explore this possibility in Fig. 6 which shows results when two excited baryon states are included. One is again the Roper but with the coupling increased to 1.2930 times the original matrix element of the operator $U$. The other has mass 2186.5 MeV and coupling $-4.36$. These parameters are the result of a fit. This does give a reasonable description of the data, but this is only a curiosity since we do not have a model which will give these values of the parameters.

The use of an interaction $U$ that varies as some power of $b$ other that two or the use of another set of baryon

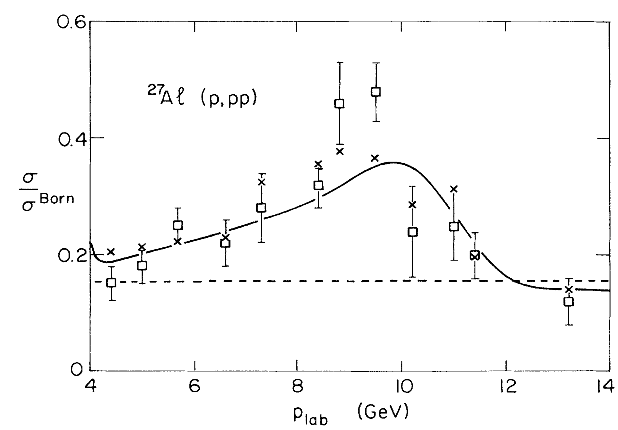


FIG. 6. Effect of an arbitrary third resonance on the $^{27}$Al$(p,pp)$ reaction.

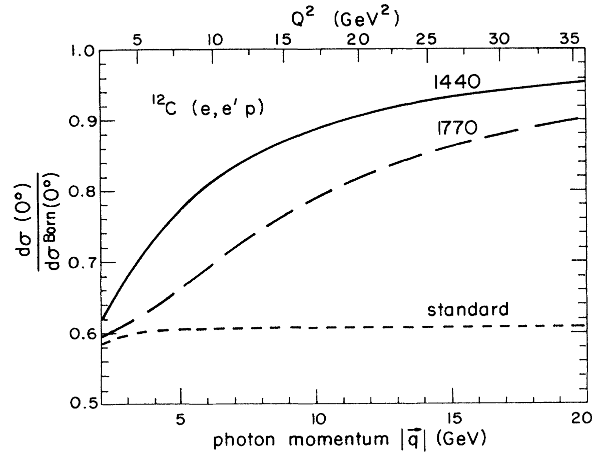


FIG. 7. The $(e,e'p)$ reaction for a $^{12}C$ target: Energy dependence of ratios of forward ($\mathbf{p}=\mathbf{q}$) values of $d\sigma/dt$. The numerators are calculations using color transparency or the DW (Glauber) calculation defined in the text. The denominator is the Born approximation to the forward value of $d\sigma/dt$. The numbers refer to the values of $M_1$ in MeV. The notation "standard" refers to the DW or Glauber calculation.

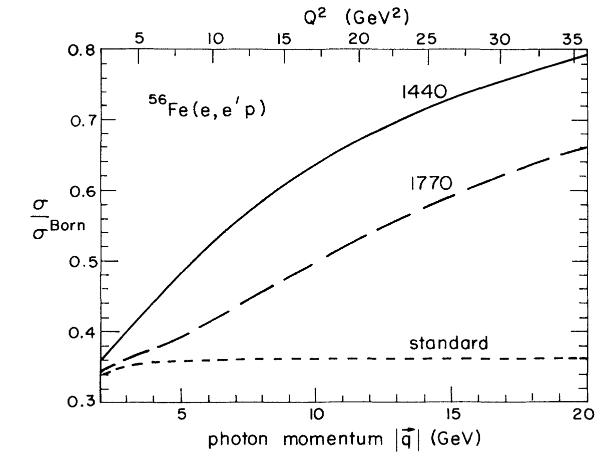


FIG. 9. The $(e,e'p)$ reaction for a $^{56}Fe$ target: Ratios of cross sections to Born approximation cross sections.

wave functions would lead to coupling to more states. A brief investigation using an interaction $U$ that varies as $b$ and the harmonic-oscillator basis states reveals that the closure approximation would be valid at some perhaps slightly higher energy and a smooth rise towards color transparency would still be obtained. Modifying $U$ by including a factor of $\ln(b)$ is computed to have little effect at the energies of interest.

We next explore the use of the $(e,e'p)$ reaction which should be simpler to understand than the $(p,pp)$ reaction [31]. Indeed, the central feature of the Ralston-Pire explanation [6] of the $(p,pp)$ energy dependence involves the specific oscillatory $s$-dependence of the $pp$ interaction at high energies. Such an $s$ dependence has not been observed in studies of the interaction between a proton and a virtual photon. The Brodsky–de Teramond [5] explanation of the $(p,pp)$ reaction involves a proton-proton resonance at energies near the threshold for $c\bar{c}$ production. One can perform the $(e,e'p)$ reaction for high-momentum-transfer kinematics that avoid this resonance.

Thus we present results for a $(e,e'p)$ reaction on the nuclear targets $^{12}C$ (Figs. 7 and 8), $^{56}Fe$ (Fig. 9), and $^{197}Au$ (Fig. 10) as a function of $Q=\mathbf{q}$. The quantity $\rho(R)$ used to obtain $U$ of Eq. (22) is taken to be of the standard Wood-Saxon (Fermi) form. The proposed experimental [10] (quasi-elastic) kinematics are used, so that $Q$ is the magnitude of the momentum of the outgoing proton which is the same as that $\mathbf{q}$ of the incident virtual photon. The photon energy $\omega$ is then given by $\omega=(Q^2+M^2)^{1/2}$. Except for Fig. 7 the ratios shown are those of angular distributions integrated over the direction of the outgoing proton.

In Fig. 7 ratios of forward ($\mathbf{p}=\mathbf{q}$) values of $d\sigma/dt$ are presented. A comparison of Figs. 7 and 8 shows that color transparency effects for fixed angle $d\sigma/dt$ and the integrated angular distributions are different. In general, the effects of color transparency occur at relatively low values of $Q$. This is because of the assumption that the $(p,pp)$ data of Carroll *et al.* displays the effects of color transparency. In that case, it is reasonable to use the relatively low mass of 1440 MeV to compute the momentum

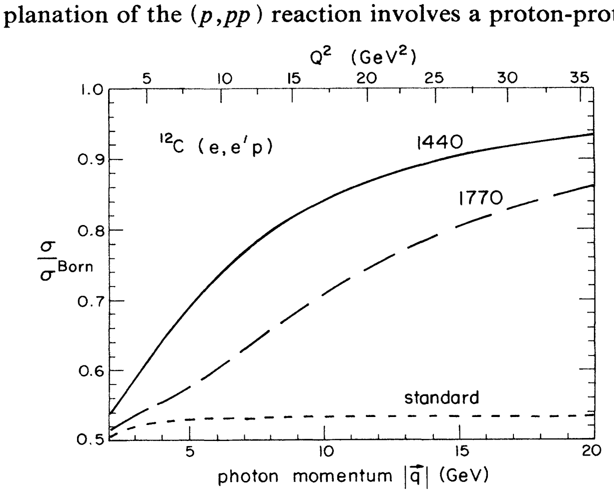


FIG. 8. The $(e,e'p)$ reaction for a $^{12}C$ target: Ratios of cross sections to Born approximation cross sections.

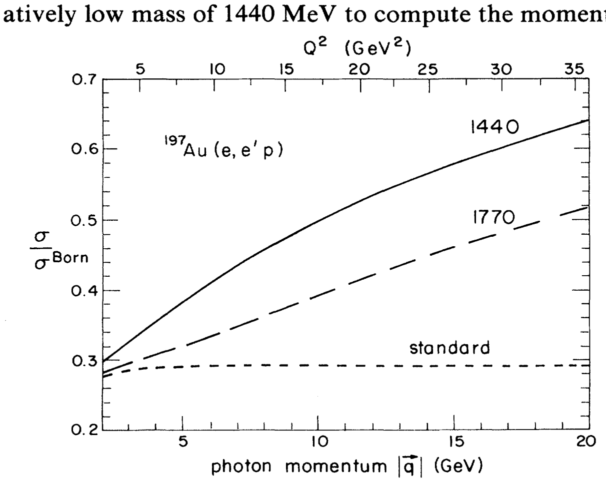


FIG. 10. The $(e,e'p)$ reaction for a $^{197}Au$ target: Ratios of cross sections to Born approximation cross sections.

$p_1$. Note also that the effects of color transparency are more likely to be observed using a lighter target. This, along with the energy dependence needed for complete color transparency, is qualitatively described by the criterion of Eq. (30). The relevant experiment will be done at the Nuclear Physics at SLAC (NPAS) facility in the near future [10].

## IX. DISCUSSION AND SUMMARY

We briefly review the assumptions, procedures and findings of the quantum-mechanical approach to color transparency presented here. First, we agree with the standard analysis [1,2] that perturbative QCD (PQCD) does indeed predict that the nuclear medium becomes transparent at some high-momentum transfer $Q^2$. The required value of $Q^2$ is presently unknown, and cannot be computed reliably. Thus, data must be used to determine the conditions necessary for the validity of PQCD. The interpretation of the data will not be simple, since the many nonperturbative effects disappear at different rates with increasing values of $Q^2$. Hence it is important to use as many different reactions as possible. The $(p,pp)$ and $(e,e'p)$ processes are studied here, but other good possibilities include $(\pi,\pi p)$ and $(K,Kp)$.

It is also possible that color transparency will be observed for $Q^2$ too small for PQCD to be valid [14]. In this case as well more experiments are needed.

Our approach is completely quantum mechanical and avoids the use of semiclassical approximations. Hadronic degrees of freedom are found to be useful in our calculations, but the underlying dynamics are those of the quark model and perturbative QCD.

The arguments for color transparency are based on three main points.

(1) Small objects (ejectiles) are produced in high-momentum-transfer processes. This seems solidly based on the uncertainty principle and relativity. Small objects are important at high-momentum transfer, even if PQCD is not strictly valid [14].

(2) Small objects interact weakly. This seems true for field theories such as QED or QCD which have a concept of neutrality. However, it is difficult to know how weak. We have used the Gunion-Soper [13] effective ejectile-nucleon interaction that varies as $b^2/b_H^2$, where $b$ is the separation between the quarks in the ejectile and $b_H$ is the standard hadronic size. Other forms yield similar results. The present calculation is qualitative, but our quantum-mechanical framework can also be used for more detailed calculations as information about the inputs is improved.

(3) If the particle stays small it can escape from the nucleus without further interaction. This point is the focus of our investigation. The use of the form $b^2/b_H^2$ leads to the effective ejectile-nucleon interaction of, e.g., Eq. (33). This interaction is small if the closure approximation defined in the Introduction is valid. When the closure approximation is accurate the effective-ejectile nucleon interaction vanishes.

The (third) condition that the ejectile escape the nucleus before expanding has been shown to be equivalent to the closure approximation [8]. Identifying the expansion time condition or equivalently the phase coherence condition [8,32] with the closure approximation makes it easier, at least in principle, to assess the validity of the approximations. Here we have used calculations made without the closure approximation to show how the closure approximation becomes more accurate as the energy and momentum transfer increases. Our work stresses the crucial role of the final (or initial) state interaction allowing the closure approximation ever to be valid. In addition we find, based on the comparison with the data of Carroll *et al.* [9], that color transparency can be understood as an interference effect between nucleon and Roper intermediate states in a coupled channel calculation. Our computations are consistent with the data below 10 GeV, but at higher energies additional physics is needed.

Moreover, we showed that a partial validity of the closure approximation, leads to partial transparency. As a result, significant enhancements over standard distorted wave approximations are obtained at fairly modest values of $Q^2$. Thus future experiments with diverse beams could verify the existence of color transparency and elucidate perturbative QCD.

## ACKNOWLEDGMENTS

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[21] QCD calculations that reproduce the observed proton form factor require the use of asymmetric nonperturbative wave functions, e.g., V. L. Chernyak and A. R. Zhitnitskii, Phys. Rep. **112**, 173 (1984). If we used those wave functions, we would integrate over $x$ to obtain an effective operator, such as $V_{\text{eff}}$ of Eq. (1). At high-momentum transfer, we expect that the $V_{\text{eff}}$ would be significant only at small values of $b$, even though the detailed form of $V_{\text{eff}}$ is not known. This is because, the momentum carried by the "slow" quark(s) is not negligible, even though one quark carries most of the momentum. Thus gluon exchange(s) requiring small interquark separations are necessary. Moreover, we expect that the appearance of one "fast quark" requires small interquark separations [22]. Another issue is that strictly speaking there should be an extra propagator in our equations for form factors. However, we borrow a result from the relativistic calculations that factors of $1/\gamma Q$ arising from fermion propagators are cancelled by factors of $\gamma Q$ occurring at photon-quark vertices.

[22] One might wonder whether Feynman's parton mechanism [R.P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, MA, 1972)] is included in this approach. Feynman's mechanism is that a single "fast" quark changes its momentum as a result of being hit by the photon. In Fig. 1, each quark acquires a longitudinal momentum of $Q/3$, so that the top quark line may have a momentum $-\frac{5}{6}Q$ before being struck by the photon. Thus for high $Q$, Fig. 1 includes aspects of the Feynman mechanism.

[23] A more complete treatment [9] of $pp$ scattering incorporating gluon radiative corrections (including Sudakov effects) shows that the $1/Q^2$ factors of quark counting rules can instead be $1/Q^{1.4}$. For high $Q$, this corresponds to a small size.

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[25] In the plane-wave approximation, the inelastic scattering amplitude vanishes at $\mathbf{q}=0$ because of orthogonality. The effects of initial- and final-state absorptive interaction effects cause a nonzero forward amplitude. However, the presence of nodes in high-lying states significantly inhibits transitions between low- and high-lying states.

[26] We take the hard exchanges to occur between quarks in a single escaping proton. Processes involving a hard gluon from another nucleon are suppressed by an additional form factor, since the residual nucleus is not to be highly excited in transparency searches.

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[28] B. Z. Kopeliovich, L. B. Litov, and J. Nemchik, Dubna Report No. E2-90-344, 1990 (unpublished).

[29] Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).

[30] Eq. (31) demonstrates that the use of Eq. (21) along with the model baryon wave functions of Eqs. (2) and (3) is a significant simplification. For example, $pp$ interactions can lead only to the production of the lowest excited state. However, there is some evidence showing that only a few low-lying excited states are important. Consider the low-$t$ diffractive production $pp\to pX$ process. See, e.g., Fig. 2 of Ref. [24]. At the highest energies displayed, $s=500$ GeV$^2$, the cross section has a peak at $M_X$ equal to the mass of the Roper resonance. The resonances of higher mass (1.6 GeV) give a contribution lower by a factor of about two and then fall off like $1/M_X^2$ as $M_X$ increases. The energy of $s=13$ GeV$^2$ is more relevant for us here. In that case, there are only three peaks at masses 1232, 1440, and 1700 MeV corresponding to the Roper, $\Delta$, and lowest odd-parity resonances. Thus the idea that only a few states are relevant is supported by some data. The use of only one excited state is a simplification, but seems sufficiently realistic to provide significant guidance in this first calculation. Models of $U$ that lead to excitation of many states are considered in the section on numerical results.

[31] The electromagnetic form factor is proven to be perturbatively dominated even without performing Sudakov resummations [17]; while such summations [9] are necessary to show that proton-proton elastic scattering at high momentum transfer is dominated by short-transverse-distance effects.

[32] G. Farrar, Phys. Lett. **56B**, 185 (1975).