Transverse color field correlations in QCD plasma

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Transverse color field correlations in a quark-gluon plasma are studied in the Coulomb gauge using real-time field theory at finite temperature. A consistent linear response analysis is used to determine the space-time behavior of the correlations. The color correlation function shows a damped oscillatory behavior in space and time indicative of the collective behavior of the plasma. Dispersion relations for transverse collective modes are obtained and compared with earlier results.

I. INTRODUCTION

The understanding of the collective behavior [1–3] of a quark-gluon plasma (QGP) is crucial, as it very clearly differentiates the color deconfined phase (QGP) from the color confined phase of hadronic matter. The collective features would, very likely, also play a significant role in the detection of a QGP in the laboratory and hence in revealing the dynamics of the plasma from observations.

In this article, our main aim is to study color field correlations in a QCD plasma and thereby obtain information about the space-time behavior of the collectivity in the system. Correlations among the longitudinal components of the color electric fields have been studied earlier [4], using gauge-covariant linear response theory, and have given new insight into QGP's. Consequently, in this work we follow the same approach and examine color field correlations in the transverse direction. From these investigations, we also hope to learn about the color-magnetic response of the system. We also determine the dispersion relations for the collective transverse modes of the plasma although they were obtained nearly a decade ago by Weldon and Klimov [3]. The reasons for this reevaluation will become clear later.

The study of collective properties has usually involved the application of perturbative (field-theoretic) methods of finite temperature. This has generally been justified on the basis of the smallness (asymptotic freedom) of the running coupling constant $\alpha_s = g^2/4\pi$ at high temperatures. All the same, it has been known [5,6] for many years that perturbative QCD has infrared problems, when one attempts to carry out calculations beyond leading order. For example, a simple (actually incomplete) linear response analysis [7] gives rise to an incorrect dispersion relation for plasma oscillations. However, consistent gauge-covariant [7] and gauge-invariant [8] formulations of response theory are able to overcome these difficulties. It should also be mentioned that the controversy associated with the sign of the imaginary part of the gluon self-energy has recently been resolved by Pisarski and collaborators [9], by essentially resuming the leading contributions from thermal fluctuations with high ($\sim T$) virtual momenta. It was shown [9] that when such a calculation is carried out, the external perturbation gets damped and moreover this property is gauge invariant. As a result of these developments [7–9], it appears that the perturbative approach if carried out correctly (including all terms of a given order in g^2) ought to produce a reliable description of the collective behavior of a QCD plasma.

In view of these considerations, we proceed with the gauge-covariant formulation of the linear response theory, for the determination of transverse response of QCD plasma. For our purposes (space-time correlations) real-time, finite-temperature field theory [10] (FTFT) is best suited and we carry out the analysis in the Coulomb gauge. Section II of this paper contains the derivation of the basic expression for the correlation function and relations between the various thermal averages. The difference between the simple linear response theory and the modified (gauge covariant) is also discussed. Section III contains the evaluation of the transverse correlation function. The results of our calculations and the discussion of the results are given in Sec. IV.

II. CORRELATION FUNCTION AND LINEAR RESPONSE THEORY

Linear response theory has been discussed by many authors [3,7,11], and hence we very briefly summarize the main features. The response function $\mathcal{X}$ relates the induced field $\delta\mathcal{O}$ in the system to an applied external field $\mathcal{O}_{\rm ext}$. More precisely, for any field $\mathcal{O}$, we have the relation

$$\delta\mathcal{O}(x) = \int d^4x' \mathcal{X}(x-x')\mathcal{O}_{\rm ext}(x') . \tag{1}$$

It can be shown that $\mathcal{X}$ is essentially a retarded commutator between the external Hamiltonian $H_{\rm ext}$ and the field operator $\mathcal{O}$. For a specific choice of $H_{\rm ext} \sim \mathcal{O}\mathcal{O}_{\rm ext}$ it reduces to a retarded commutator of only the field operators $\mathcal{O}$ in Heisenberg representation. Since, in practice, the field-theoretic methods exist to evaluate the time-ordered product of the operators in question, one has to find relations between the retarded and the time-ordered quantities. As we are interested in the response of the plasma due to an external color-electric (-magnetic) field perturbation, we consider the time-ordered product of electric fields $\langle T[E_i^a(x)E_j^b(x')]\rangle$ and the correlation function of electric fields

$$C_{ij}^{ab}(x,x')=\langle E_i^a(x)E_j^b(x')\rangle \ , \tag{2}$$

where

$$E_i^a(x)=\partial_i A_0^a(x)-\partial_0 A_i^a(x)-gf^{abc}A_0^b(x)A_i^c(x) \tag{3}$$

and the angular brackets denote the thermal averages. For a system having translational invariance with respect to space and time, the correlation functions depend only on $(x-x')$. If we denote the Fourier transform of $C_{ij}^{ab}(x-x')$ by $C_{ij}^{ab}(k)$, then it is related to the Fourier transform of the time-ordered product by the fluctuation dissipation theorem [11] according to

$$C_{ij}^{ab}(k)=T_F[\langle E_i^a(x)E_j^b(x')\rangle]$$
$$=\mathrm{Re}T_F\{\langle T[E_i^a(x)E_j^b(x')]\rangle\} \tag{4}$$

where T_F denotes Fourier transform. Using Eqs. (2) and (3) and working in Coulomb gauge with $\partial_i A^i(x)=0$, we get

$$\begin{aligned}\langle T[E_i^a(x)E_j^b(x')]\rangle =& \ \partial_0\partial_0'\langle T[A_i^a(x)A_j^b(x')]\rangle+\partial_i\partial_j'\langle T[A_0^a(x)A_0^b(x')]\rangle+gf^{acd}\partial_0'\langle T[A_0^c(x)A_i^d(x)A_j^b(x')]\rangle\\ &+gf^{bcd}\partial_0\langle T[A_i^a(x)A_0^c(x')A_j^d(x')]\rangle-gf^{acd}\partial_j'\langle T[A_0^c(x)A_i^d(x)A_0^b(x')]\rangle\\ &-gf^{bcd}\partial_i\langle T[A_0^a(x)A_0^c(x')A_j^d(x')]\rangle+g^2f^{acd}f^{beh}\langle T[A_0^c(x)A_i^d(x)A_0^e(x')A_j^h(x')]\rangle\\ &+i\delta(t-t')\left[-\delta^{ab}\left[\delta_{ij}-\frac{\partial_i\partial_j}{\nabla^2}\right]\delta(\mathbf{x}-\mathbf{x}')+g^2N_c\frac{\langle A_i^a(x)A_j^b(x')\rangle}{4\pi|\mathbf{x}-\mathbf{x}'|}\right] \ ,\end{aligned} \tag{5}$$

where N_c is the number of colors. The last two terms in Eq. (5) are obtained by expressing the nondynamical degrees of freedom $A_0(x)$ in terms of dynamical degrees of freedom $A_i(x)$ and we have retained terms only up to order g^2. This is because we intend to evaluate the correlation function to $O(g^2)$. It is important to note the appearance of three-point and four-point functions in Eq. (5) (see Ref. [7]). This is not the case in an Abelian gauge theory (say electron-positron plasma) where, as a consequence, the dispersion relations for collective modes are determined by the two-point functions alone. As will be seen later, the linear response theory with two-point function alone in Eq. (5) yields an incorrect [7] dispersion relation for plasma oscillation in the case of a QCD plasma (non-Abelian gauge theory). It is because of this important fact and for gauge covariance that we consider the correlation function of electric fields and not of color potentials $A_\mu^a(x)$.

Starting from Eq. (5), it is easy to calculate the longitudinal and transverse correlation function using, respectively, the longitudinal and transverse projection operators for the fields. These are given by the expressions

$$E_i^{aL}(x)=\left[\frac{\partial_i\partial_j}{\nabla^2}\right]E_j^a(x) \ , \tag{6}$$

$$E_i^{aT}(x)=\left[\delta_{ij}-\frac{\partial_i\partial_j}{\nabla^2}\right]E_j^a(x) \ . \tag{7}$$

As mentioned earlier, in this paper we consider transverse correlations only. From Eqs. (4) and (5), we obtain the transverse correlation function $\langle E_i^{aT}(x)E_j^{bT}(x')\rangle$ using the definition given in Eq. (7). The last term in Eq. (5) is a delta function in time. This can be neglected [7] when we consider plasma oscillations but it has to be included when we consider the correlation function.

III. EVALUATION OF CORRELATION FUNCTIONS

We now apply real-time perturbative finite-temperature field-theoretic (FTFT) methods (see Ref. [10]) to evaluate the various terms in Eq. (5). It involves the determination of transverse polarization tensor, three- and four-point functions. In the real-time finite-temperature formalism, it is important to recall [10] that due to the effective doubling of the degrees of freedom, the propagators as well as the self-energies have a 2×2 matrix structure. Further in the Feynman rules [10] there are vertices of type 1, type 2 and mixed internal lines of type 1 and type 2, but the external lines are always of type 1.

The full propagator matrix $\mathcal{D}$ satisfies the Dyson-Schwinger equations [10]

$$\mathcal{D}^{(rs)}=D^{(rs)}+(D\pi_T\mathcal{D})^{(rs)} \ , \quad r,s=1,2 \ , \tag{8}$$

where we have omitted the Lorentz and color indices for simplicity. D is 2×2 free propagator and π_T is 2×2 polarization tensor and they are given by [10]

$$\mathcal{D}(k)=U\begin{bmatrix}\dfrac{1}{k^2-G+i\varepsilon} & 0\\ 0 & \dfrac{-1}{k^2-G^*-i\varepsilon}\end{bmatrix}U \ , \tag{9}$$

$$U=\begin{bmatrix}\cosh(\theta_k) & \sinh(\theta_k)\\ \sinh(\theta_k) & \cosh(\theta_k)\end{bmatrix} \ , \tag{10}$$

$$\sinh(\theta_k)=\frac{e^{-\beta|k_0|/2}}{(1-e^{-\beta|k_0|})^{1/2}} \ , \tag{11}$$

$$\cosh(\theta_k)=\frac{1}{(1-e^{-\beta|k_0|})^{1/2}}\ , \tag{12}$$

$$\mathrm{Re}(G)=\mathrm{Re}(\pi_T{}^{(11)})\ , \tag{13}$$

$$\mathrm{Im}(G)=\tanh\left[\frac{\beta|k_0|}{2}\right]\mathrm{Im}(\pi_T{}^{(11)})\ , \tag{14}$$

$$\pi_T^{(22)}=-\pi_T^{(11)*}\ , \tag{15}$$

$$\pi_T^{(12)}=\pi_T^{(21)}=-i\tanh(2\theta_k)\,\mathrm{Im}(\pi_T^{(11)})\ , \tag{16}$$

where $T=1/\beta$ is the temperature. One obtains $D(k)$, the free particle propagator, from $\mathcal{D}(k)$ by setting $G=0$. It is important to keep the matrix structure of π_T and $\mathcal{D}$ in order to eliminate the ill-defined product of delta functions in the Schwinger-Dyson equation given in Eq. (8).

Thus the evaluation of $\mathcal{D}$ involves basically the determination of $\pi_T^{(11)}$. The diagrams that contribute to the polarization tensor are shown in Fig. 1. In Figs. 1(a) and 1(b) the internal lines can be either the transverse (T) gluon or Coulomb (C) gluon or a combination of these; i.e., in the loop we have contribution from (TT), (TC), and (CC). The ghost contribution [Fig. 1(c)] is required to render the theory finite in the vacuum sector ($T=0$ contribution). Restoring the Lorentz indices (i,j) the polarization tensor $\pi_T^{(11)}$ is given as

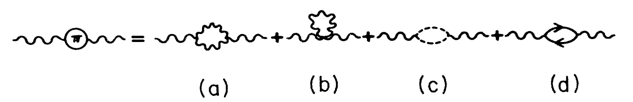

FIG. 1. Feynman diagrams contributing to transverse-polarization tensor. (a) and (b) are the gluon loops and there is contribution from both transverse and Coulomb internal lines. (c) represents the ghost loop contribution and the quark loop contribution is shown in (d).

$$\pi_T=\frac{1}{2}\left[\frac{k_i\pi_{ij}k_j}{\mathbf{k}^2}-\pi_{ii}\right]\ . \tag{17}$$

It turns out that if we consider various terms in Fig. 1 (to order g^2), the evaluation of $\pi_{ij}^{(11)}$ does not involve the vertices of type 2 and $D^{(12)}$ or $D^{(21)}$ propagators. Thus, the calculations simplify. We separate $\pi_T^{(11)}$ into a sum of a vacuum term (temperature independent) $\pi_{T\mathrm{vac}}^{(11)}$ and temperature-dependent term $\pi_{T\beta}^{(11)}$. Therefore we have

$$\pi_T^{(11)}=\pi_{T\mathrm{vac}}^{(11)}+\pi_{T\beta}^{(11)} \tag{18}$$

with

$$\begin{aligned}
\pi_{T\mathrm{vac}}^{(11)}=\frac{g^2N_c}{\pi^2}\Bigg\{&-\frac{(k_0^2-k^2)}{48}\left[-C-\ln\left[\frac{-k_0^2+k^2}{4\pi\Lambda^2}\right]-2\psi(2)+2\psi(4)\right]\\
&+\frac{k_0^2}{12}\left[-C-\ln\left[\frac{k^2}{4\pi\Lambda^2}\right]+\frac{3}{2}-\psi(1)-\psi(\tfrac{3}{2})+2\psi(\tfrac{5}{2})\right]\\
&+\frac{k^2}{140}\left[-\frac{35}{3}C-\frac{35}{3}\ln\left[\frac{k^2}{4\pi\Lambda^2}\right]+\frac{593}{12}-12\psi(\tfrac{3}{2})+5\psi(2)+5\psi(\tfrac{5}{2})-12\psi(3)-14\psi(\tfrac{9}{2})\right]\Bigg\}\\
&+\frac{g^2N_ck^2}{2\pi^2}\int_0^1dx\int_0^1dy_1dy_2dy_3\,\delta(1-y_1-y_2-y_3)y_1^{1/2}\\
&\quad\times\Bigg[\ln|y_2^2+(2xy_1-1)y_2+xy_1(xy_1-1)+xy_1(1-x)k_0^2/\mathbf{k}^2|\\
&\quad\quad-\frac{1}{4}\,\frac{2y_1x(1-x)k_0^2/\mathbf{k}^2-1}{y_2^2+(2xy_1-1)y_2+xy_1(xy_1-1)+y_1x(1-x)k_0^2/\mathbf{k}^2}\Bigg]\\
&-\frac{g^2N_f}{24\pi^2}(k_0^2-k^2)\left[-C-\ln\left[\frac{-k_0^2+k^2}{4\pi\Lambda^2}\right]-3\psi(1)+4\psi(2)-\psi(4)\right]\ ,
\end{aligned} \tag{19}$$

where $C=$ Euler constant $=0.577\ldots$, $\psi(z)=(d/dz)\ln\Gamma(z)$ and N_f is the number of quark flavors.

$$\begin{aligned}\mathrm{Re}(\pi_{T\beta}^{(11)})=\frac{g^2N_c}{8\pi^2}\int_0^\infty dp\, pf_p\Bigg\{&7+\frac{2k_0^2}{k^2}+\frac{7(k_0^2-k^2)(p^2+k^2)}{2p^2k^2}+\frac{(k_0^2-k^2)^2}{2p^2k^2}\\&+\frac{p^4+k^4}{2p^2k^2}-\frac{(k_0^2+p^2)}{pk}\left[\frac{p^2+k^2}{2pk}+\left[1+\frac{(p^2+k^2)^2}{4p^2k^2}\right]\ln\left|\frac{p-k}{p+k}\right|\right]\\&+\frac{1}{16p^3k^3(p+k_0)^3}\Bigg[(k_0^2-k^2)(2p+k_0-k)(2p+k_0+k)\\&\qquad\times[8p^2(p+k_0)^2+4p^2k_0^2+12p^2k^2\\&\qquad\quad+4pk_0^3+12pk^2k_0+k_0^4+k^4+6k_0^2k^2]\\&\qquad\times\ln\left|\frac{(k_0-k)(2p+k_0+k)}{(k_0+k)(2p+k_0-k)}\right|\\&\qquad+2(p^2-k^2)^2(p^4+k^4+6p^2k^2)\ln\left|\frac{p-k}{p+k}\right|\Bigg]\\&+\frac{1}{16p^3k^3(p-k_0)^3}\Bigg[(k_0^2-k^2)(2p-k_0-k)(2p-k_0+k)\\&\qquad\times[8p^2(p-k_0)^2+4p^2k_0^2+12p^2k^2\\&\qquad\quad+4pk_0^3-12pk^2k_0+k_0^4+k^4+6k_0^2k^2]\\&\qquad\times\ln\left|\frac{(k_0+k)(2p-k_0+k)}{(k_0-k)(2p-k_0-k)}\right|\\&\qquad+2(p^2-k^2)^2(p^4+k^4+6p^2k^2)\ln\left|\frac{p-k}{p+k}\right|\Bigg]\Bigg\}\\+\frac{g^2N_f}{\pi^2}\int_0^\infty p\, n_p\Bigg[&p+\frac{p(k_0^2-k^2)}{2k^2}\\&+\frac{(k_0^2-k^2)}{16k^3}[(2p+k_0-k)(2p+k_0+k)+2k^2]\ln\left|\frac{(k_0-k)(2p+k_0+k)}{(k_0+k)(2p+k_0-k)}\right|\\&+\frac{(k_0^2-k^2)}{16k^3}[(2p-k_0-k)(2p-k_0+k)+2k^2]\ln\left|\frac{(k_0+k)(2p-k_0+k)}{(k_0-k)(2p-k_0-k)}\right|\Bigg]\,,\end{aligned}\tag{20}$$

$$\begin{aligned}\mathrm{Im}(\pi_{T\beta}^{(11)})=\frac{g^2N_c}{16\pi}\int_0^\infty dp\, p\Bigg[&(f_p+f_{p+k_0}+2f_pf_{p+k_0})\frac{(k_0^2-k^2)(2p+k_0-k)(2p+k_0+k)}{16p^3k^3(p+k_0)^3}\\&\times[8p^2(p+k_0)^2+4p^2k_0^2+12p^2k^2+4pk_0^3+12pk^2k_0+k_0^4+k^4+6k_0^2k^2]\theta_1\\&+(f_p+f_{p-k_0}+2f_pf_{p-k_0})\frac{(k_0^2-k^2)(2p-k_0-k)(2p-k_0+k)}{16p^3k^3(p-k_0)^3}\\&\times[8p^2(p-k_0)^2+4p^2k_0^2+12p^2k^2-4pk_0^3-12pk^2k_0+k_0^4+k^4+6k_0^2k^2]\theta_2\Bigg]\\+\frac{g^2N_f}{32\pi}\int_0^\infty dp\Bigg\{&(n_p+n_{p+k_0}-2n_pn_{p+k_0})\frac{(k_0^2-k^2)}{k^3}[(2p+k_0-k)(2p+k_0+k)+2k^2]\theta_1\\&+(n_p+n_{p-k_0}-2n_pn_{p-k_0})\frac{(k_0^2-k^2)}{k^3}[(2p-k_0-k)(2p-k_0+k)+2k^2]\theta_2\Bigg\}\,,\end{aligned}\tag{21}$$

$$f_p=\frac{1}{e^{\beta|p|}-1}\ ,\quad n_p=\frac{1}{e^{\beta|p|}+1}\ ,$$

$$\theta_1=1 \quad \text{for } -1\le 2pk_0+k_0^2-k^2\le 1\ , \qquad (22)$$
$$=0 \quad \text{otherwise}\ ,$$

$$\theta_2=1 \quad \text{for } -1\le -2pk_0+k_0^2-k^2\le 1\ , \qquad (23)$$
$$=0 \quad \text{otherwise}\ .$$

In Eqs. (19)–(23) $p=|\mathbf{p}|$, $k=|\mathbf{k}|$, etc.

In order to evaluate the other terms involving three and four field operators (A_μ^a), in Eq. (5) we use Wick's theorem. Since we wish to evaluate all terms to order g^2, the field lines that occur are of type 1 only for four-point function whereas for the three-point function the terms containing the propagator $D^{(12)}$ always occur in product with $D_{00}^{(12)}$, which is zero in Coulomb gauge. Collecting all the terms we finally get, for the Fourier transform of the time-ordered product of color-electric fields,

$$T_F\{\langle T[E_i^{aT}(x)E_j^{bT}(x')]\rangle\}=i\delta^{ab}\left[\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right]\left[-1+k_0^2\mathcal{D}^{(11)}(k)-k_0^2D^{(11)}(k)\Delta+\frac{\Delta}{2}+\frac{\Delta}{2}\right]\ , \qquad (24)$$

where

$$\Delta=\Delta_{\text{vac}}+\Delta_\beta\ , \qquad (25)$$

$$\Delta_{\text{vac}}=\frac{g^2N_c}{3\pi^2}\left[-C-\ln\left[\frac{k^2}{4\pi\Lambda^2}\right]+\frac{3}{2}-\psi(1)-\psi(\tfrac{3}{2})+2\psi(\tfrac{5}{2})\right]\ , \qquad (26)$$

$$\Delta_\beta=-\frac{g^2N_c}{4\pi^2k}\int_0^\infty dp\,pf_p\left[\frac{p^2+k^2}{2pk}+\left[1+\frac{(p^2+k^2)^2}{4p^2k^2}\right]\ln\left|\frac{p-k}{p+k}\right|\right]\ . \qquad (27)$$

Note that the contribution to the second term in Eq. (24) comes from the two-point function and the third and fourth terms are from three- and four-point functions respectively. The last contribution comes from the instantaneous Coulomb term in Eq. (5). The function $\mathcal{D}^{(11)}(k)$ is defined in terms of the Fourier transform $\mathcal{D}_{ij}^{(11)}(k)$ of the two-point function defined in Eq. (8). The relation is

$$\mathcal{D}_{ij}^{ab(11)}(k)=\delta^{ab}\left[\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right]\mathcal{D}^{(11)}(k)\ . \qquad (28)$$

Having obtained the final expressions for various terms in Eq. (5), we need to evaluate the integrals involved numerically in order to obtain dispersion relation for plasma oscillations and also the correlation function. After expanding $\mathcal{D}^{(11)}(k)$ in a series in G we can combine the two-, three-, and four-point contributions along with the polarization tensor G to obtain an effective G^{eff}. It is defined so that the expression

$$i\delta^{ab}\left[\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right]\left[-1+\frac{k_0^2}{k_0^2-\mathbf{k}^2-G^{\text{eff}}}\right] \qquad (29)$$

is equivalent (to order g^2) to the right-hand side of Eq. (24). This gives us

$$G_{\text{plasma}}^{\text{eff}}=G-\frac{(k_0^2-\mathbf{k}^2)}{2k_0^2}(k_0^2+\mathbf{k}^2)\Delta\ . \qquad (30)$$

The instantaneous contribution [last term in Eq. (24)] has been omitted since we are interested here in plasma oscillations. In order to obtain the correct dispersion relation one has to solve the equation

$$k_0^2-\mathbf{k}^2-G_{\text{plasma}}^{\text{eff}}=0\ , \qquad (31)$$

instead of the equation

$$k_0^2-\mathbf{k}^2-G=0\ . \qquad (32)$$

The effective polarization tensor for correlation function can also be defined in a similar way. The correlation function $C_T(k)$ is defined by

$$\begin{aligned}C_{ij}^{abT}(k)&=T_F[C_{ij}^{abT}(x-x')]\\&=T_F[\langle E_i^{aT}(x)E_j^{bT}(x')\rangle]\\&=\text{Re}(T_F\{\langle T[E_i^{aT}(x)E_j^{bT}(x')]\rangle\})\\&=\delta^{ab}\left[\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right]C_T(k)\ ,\end{aligned} \qquad (33)$$

$$C_T(k_0,|\mathbf{k}|)=\frac{k_0^2[\sinh^2(\theta_k)+\cosh^2(\theta_k)]\,\text{Im}(G^{\text{eff}})}{[k_0^2-\mathbf{k}^2-\text{Re}(G^{\text{eff}})]^2+[\text{Im}(G^{\text{eff}})]^2} \qquad (34)$$

with

$$G^{\text{eff}}=G-\frac{(k_0^2-\mathbf{k}^2)\mathbf{k}^2}{k_0^2}\Delta\ . \qquad (35)$$

The correlation function in space-time $C_T(t,|\mathbf{r}|)$ is obtained from Eq. (34) by inverse Fourier transform. Since $G^{\rm eff}$ is a function of $k=|\mathbf{k}|$ and an even function of k_0 and using $\bar{k}_\mu=\beta k_\mu$ and $\bar{x}_\mu=\beta x_\mu$ we can write

$$C_T(\bar{t},\bar{r})=\int_0^\infty d\bar{k}_0\cos(\bar{k}_0\bar{t})\int_0^\infty d\bar{k}\,\sin(\bar{k}\,\bar{r})C_T(\bar{k}_0,\bar{k})/\bar{r} \tag{36}$$

within an unimportant multiplicative factor.

IV. DISCUSSION

As we mentioned earlier, it is important to include the contributions of three- and four-point functions in $G^{\rm eff}$ to get the correct dispersion relation. We have solved Eqs. (31) and (32) numerically for two coupling strengths $\alpha_s=g^2/(4\pi)$. The spectrum of collective plasma oscillation is shown in Fig. 2 and Fig. 3. It is clear from the figures that without these contributions (broken line in Figs. 2 and 3), for small k there is a rise in the frequency which is unphysical. The inclusion of these contributions (solid line in Figs. 2 and 3) makes the dispersion relation behave like $\omega^2=\omega_p^2+6k^2/5$ for small $k=|\mathbf{k}|$, where $\omega_p^2=g^2(N_c+N_f/2)T^2/9$ is the plasma frequency. For large k, the dispersion relation again yields the standard result $\omega^2=3\omega_p^2/2+k^2$ and we find no difference between the results of using the effective polarization tensor and the usual analysis containing a two-point function for reasonably small coupling strengths.

We would like to point out that in Fig. 3, for $k=0$, the frequency is not exactly $\omega=\omega_p=1.1087T$ as it ought to be according to the dispersion relation for small k. This is because when we numerically solve $k_0^2-k^2-\pi_T=0$ for $k=0$, we assume that the coupling α_s is very small and hence a numerical discrepancy shows up for $\alpha_s=0.22$ (Fig. 3) but not for $\alpha_s=0.022$ (Fig. 2).

It is worth pointing out [12] that Klimov and Weldon [3] did obtain the correct (to leading order in g) dispersion relation, in a bare one-loop calculation of the transverse-polarization function π_T, keeping only terms of the form $g^2T^2f(k_0/k)$. These terms, in fact [12], give the entire contribution to the dispersion relation to leading order.

In the present analysis, we have retained all the terms including terms of the form $g^2k^2f(k_0/k)$ in the one-loop diagrams. These terms contribute to the dispersion relation at the same order in g as two-loop contributions of the form $g^4T^2f(k_0/k)$. A proper evaluation of these subleading terms would require a resummation of higher-loop contributions as was carried out in Ref. [9] for the imaginary part of π_T. This has not been done in the present work. A comparison of our dispersion relations (Figs. 2 and 3) with those in Ref. [3] indicates that both the calculations show essentially the same pattern of behavior.

After calculating the correlation function $C_T(k_0,|\mathbf{k}|)$ Eq. (34) numerically, we have evaluated its Fourier transform $C_T(t,|\mathbf{r}|)$ also numerically. These are shown in Fig. 4 and Fig. 5 for two coupling strengths α_s. As in the case of longitudinal correlation [4] functions these also have a damped oscillatory behavior in both space and time. The oscillatory behavior suggests the existence of transverse collective oscillations as well as dynamic (frequency-dependent) screening in the transverse direction. As the coupling strength α_s is increased the oscillations become faster. Unfortunately this analysis does not throw any light onto whether or not there is static magnetic screening. But as far as the space-time behavior is concerned it has all the properties of longitudinal correlations. To obtain the static properties from these correlations is

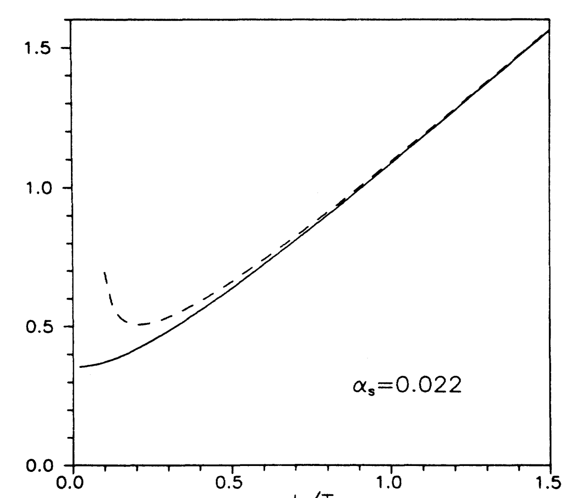

FIG. 2. The plasma dispersion relation is plotted as $k/T=\bar{k}$ vs $\omega/T=\bar{k}_0$. The broken line is the spectrum of collective modes with two-point function alone and the solid line corresponds to the spectrum resulting from the gauge-covariant response analysis. The plasma frequency $\bar{\omega}_p=\omega_p/T=0.3505$ with $N_c=3$ and $N_f=2$.

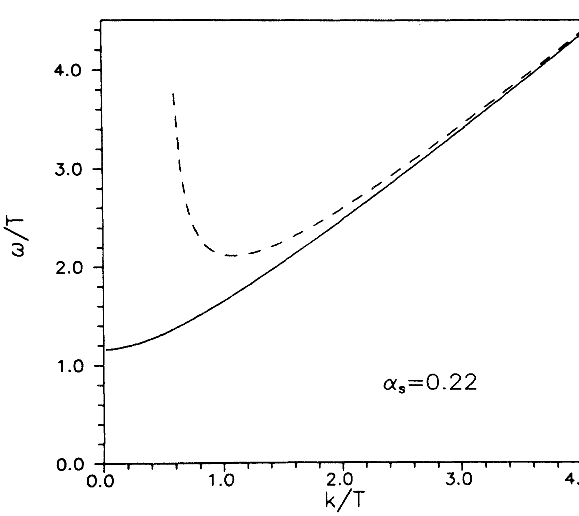

FIG. 3. Same as Fig. 2 with plasma frequency $\bar{\omega}_p=1.1087$.

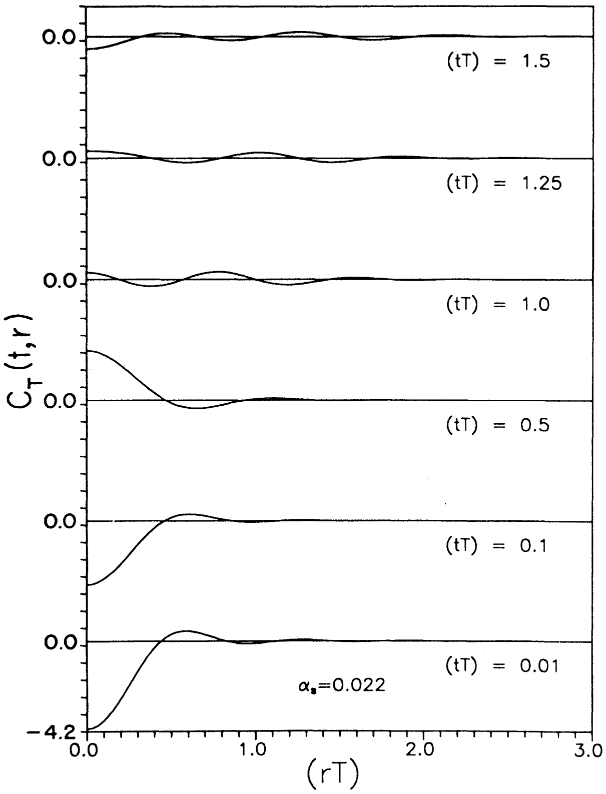

FIG. 4. The transverse correlation function is plotted in arbitrary units, and their dependence on $\bar{r}=rT$ and $\bar{t}=tT$ is shown. The strong coupling strength $\alpha_s=0.022$, $N_c=3$, and $N_f=2$.

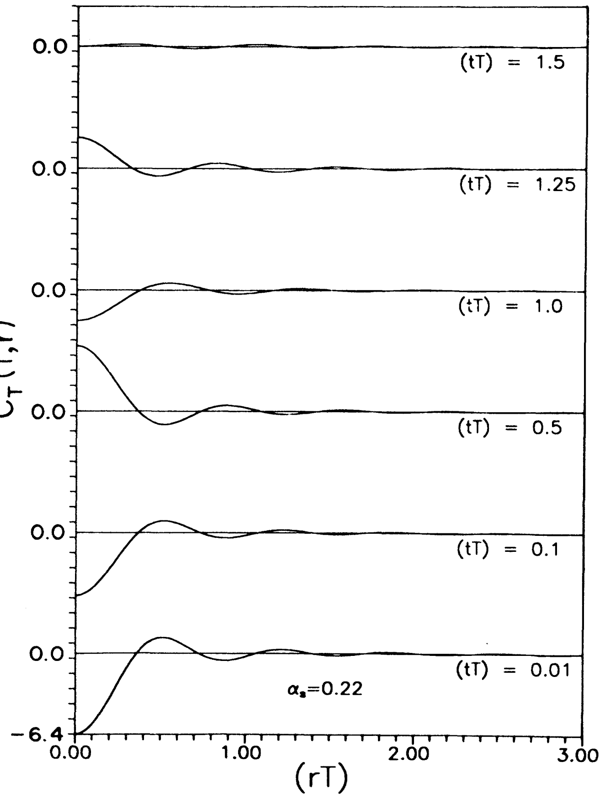

FIG. 5. Same as Fig. 4 with $\alpha_s=0.22$.

beyond this analysis. It is important to analyze the problem of magnetic screening carefully but the frequency-integrated functions such as correlation functions are most probably insensitive to that property.

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[12] We would like to point out that the earlier results [3] on dispersion relations are correct (to leading order) at all temperatures (T), not only at high temperatures.