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Total source charge and charge screening in Yang-Mills theories

William B. Campbell
Department of Physics and Astronomy, University of Nebraska–Lincoln, Lincoln, Nebraska 68588-0111

Richard E. Norton
*Centre de Physique Théorique de l'Ecole Polytechnique, 91128 Palaiseau CEDEX, France
and Department of Physics, University of California, Los Angeles, 405 Hilgard Avenue, Los Angeles, California 90024*
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New gauge-invariant definitions for the total charge on a static Yang-Mills source are suggested which we argue are better suited for determining when true color screening has occurred. In particular, these new definitions imply that the Abelian Coulomb solution for a simple "electric" dipole source made up of two opposite point charges has zero total source charge and therefore no color screening. With the definition of total source charge previously suggested by other authors, such a source would have a total source charge of $2q$ and therefore a screening charge in the field of $-2q$, where q is the magnitude of the charge of either point charge. Our definitions for more general solutions are not unique because of the path dependence of the parallel transport of charges. Suggestions for removing this ambiguity are offered, but it is not known if a unique, physically meaningful definition of total source charge in fact exists.

I. INTRODUCTION

In discussions of the classical sector of SU(2) Yang-Mills theories gauge-invariant definitions for the total color charge of an external source, the charge in the field and the total charge of the system have been given [1,2]. The total charge on the external source is defined by simply adding the magnitudes of the local charge vectors which make up the source. The total color charge of the system is defined from the $1/r^2$ behavior of the color-electric field for large r. Both of these definitions can be given in a gauge-invariant way. The difference between them is presumed to be the screening charge in the field.

Unfortunately this gauge-invariant definition for the total charge on the external source leads to implausible conclusions for some familiar cases. We discuss below the particular case of a finite dipole source, but it will be evident that the discussion immediately generalizes to any distribution of static charges which allow the Coulomb solution.

Because of these peculiarities we offer in this paper alternative gauge-invariant definitions for the total source charge. For any static system described by the Coulomb solution these definitions will give a unique answer. As we discuss, there is a lack of uniqueness in these definitions when applied to systems whose field configurations have a nonvanishing curvature $\mathbf{F}^{ij}$. Although certain choices from these definitions suggest themselves, we do not know if any choice will be useful or physically relevant, and hence we do not know if the notion of color screening in classical Yang-Mills theory is indeed a well-defined physical concept.

Before elaborating on these remarks, let us review the definitions of the total source charge and total color charge of the system presently employed in the literature. Our notation is standard with $\mathbf{A}^\mu$ the four-potential. We use boldface characters to represent the three isocomponents for potentials and fields. Our metric is such that $g^{00}=+1, g^{11}=g^{22}=g^{33}=-1$. We use units where $c\equiv 1$. In this notation the isovector components of the field tensor are obtained from $\mathbf{A}^\mu$ by

$$\mathbf{F}^{\mu\nu}=\partial^\mu \mathbf{A}^\nu-\partial^\nu \mathbf{A}^\mu+g\,\mathbf{A}^\mu\times\mathbf{A}^\nu\ . \tag{1.1}$$

The field tensor satisfies the field equation

$$D_\mu \mathbf{F}^{\mu\nu}=4\pi\mathbf{j}^\nu\ , \tag{1.2}$$

where

$$D_\mu \mathbf{F}^{\mu\nu}\equiv(\partial_\mu+g\,\mathbf{A}_\mu\times)\mathbf{F}^{\mu\nu}\ ,$$

and $\mathbf{j}^\nu$ is the current density for the external source. It follows that $\mathbf{j}^\nu$ satisfies the equation of continuity

$$D_\mu \mathbf{j}^\mu=0\ . \tag{1.3}$$

Here we will be interested in static sources so that

$$\mathbf{j}^\mu=\boldsymbol{\rho}(\mathbf{r})\delta^\mu{}_0\ . \tag{1.4}$$

It follows from Eq. (1.3) that

$$\boldsymbol{\rho}\times\mathbf{A}^0=0\ , \tag{1.5}$$

and thus $\boldsymbol{\rho}$ and $\mathbf{A}^0$ must be parallel or antiparallel in isospace.

If we restrict ourselves to static solutions and, consistently, to time-independent gauge transformations, then $\mathbf{A}^0$ transforms as an isovector. It follows that one can always transform to a gauge where $\boldsymbol{\rho}$ and $\mathbf{A}^0$ have components along the same fixed direction in isospace, e.g., the third isodirection. In such a gauge one can show that if the third isocomponent of $\boldsymbol{\rho}$ never changes sign (which can always be arranged by a gauge transformation) then the third isocomponent of $\mathbf{A}^0$ must always have the same sign as that of $\boldsymbol{\rho}$ [3].

It follows from this that we can in any gauge define a unit isovector $\mathbf{n}(\mathbf{r})$ which gives the isodirection for both $\boldsymbol{\rho}$ and $\mathbf{A}^0$. We define then $\rho(\mathbf{r})$ and $\Phi(\mathbf{r})$ as the magnitudes of $\boldsymbol{\rho}$ and $\mathbf{A}^0$ so that

$$\boldsymbol{\rho}(r)=\rho(\mathbf{r})\mathbf{n}(\mathbf{r}) \quad \text{and} \quad \mathbf{A}^0=\Phi(\mathbf{r})\mathbf{n}(\mathbf{r}) \ .$$

We point out that our definition of $\mathbf{n}(\mathbf{r})$ is more restrictive than that given by Lai and Oh [2]. They define $\mathbf{n}(\mathbf{r})$ as any unit isovector field. However, in the specific examples they discuss they use the same definition we give above. Their definition is so general that it leads to complete ambiguity in the definitions for the color charges given below. Our definition for $\mathbf{n}(\mathbf{r})$ agrees with that given by Isidro Filho, Kerman, and Trottier [1].

Since $\rho=\mathbf{n}\cdot\boldsymbol{\rho}$ is gauge invariant, a gauge-invariant total source charge can be defined by [2]

$$Q_S \equiv \int \rho(\mathbf{r}) d^3 r \ , \tag{1.6}$$

which is positive definite.

The total color charge for a solution can be defined by introducing the conserved gauge-invariant current density J_T^ν such that [2]

$$J_T^\nu \equiv \frac{1}{4\pi}\partial_\mu(\mathbf{n}\cdot\mathbf{F}^{\mu\nu})=\mathbf{n}\cdot\mathbf{j}^\mu+\frac{1}{4\pi}(D_\mu\mathbf{n})\cdot\mathbf{F}^{\mu\nu} \ , \tag{1.7}$$

where the second term can be interpreted as the current density due to the color in the field. The total color charge Q_T is then given by

$$Q_T \equiv \int J_T^0 d^3 r = \frac{1}{4\pi}\oint_{r\to\infty} \mathbf{n}\cdot\mathbf{F}^{i0} dS_i \ . \tag{1.8}$$

In the gauge where $\boldsymbol{\rho}$ and $\mathbf{A}^0$ point in the third isodirection, Eq. (1.8) is simply Gauss's law for the third isocomponent of $\mathbf{F}^{i0}$. Making reasonable assumptions one can argue that this third component of $\mathbf{F}^{i0}$ is the only one which can behave like $1/r^2$ for large r with the other isocomponents going to zero faster [3]. Thus Eq. (1.8) is a reasonable definition for Q_T. It follows from our earlier discussion about the relative sign of the isocomponents of $\boldsymbol{\rho}$ and $\mathbf{A}^0$ that Q_T must be positive. One can also show that $Q_T \le Q_S$ [3].

II. THE "ELECTRIC" DIPOLE

We consider here the case of the simple finite dipole to illustrate why we feel that alternate definitions to Eq. (1.6) for Q_S are needed. For simplicity we assume that the point sources only have components in the third isodirection and are separated by distance $2a$ along the z axis. The source density $\boldsymbol{\rho}$ is

$$\boldsymbol{\rho}(\mathbf{r})=q[\delta^3(\mathbf{r}-a\hat{\mathbf{z}})-\delta^3(\mathbf{r}+a\hat{\mathbf{z}})]\mathbf{e}_3 \ , \tag{2.1}$$

where q is the magnitude of the charge on each of the two point sources, $\hat{\mathbf{z}}$ is the unit vector in the z direction and $\mathbf{e}_3$ is the unit vector in the third isodirection. The Abelian Coulomb solution for this source is given by

$$\mathbf{A}^0=\left[\frac{q}{|\mathbf{r}-a\hat{\mathbf{z}}|}-\frac{q}{|\mathbf{r}+a\hat{\mathbf{z}}|}\right]\mathbf{e}_3 \ , \tag{2.2}$$

with $\mathbf{A}^i\equiv 0$. The only nonzero components of the field tensor are

$$\mathbf{F}^{i0}=-\partial_i\mathbf{A}^0 \ . \tag{2.3}$$

Using the definition of $\mathbf{n}(\mathbf{r})$ given in Sec. I we have

$$\mathbf{n}(\mathbf{r})\equiv \mathrm{sgn}(z)\mathbf{e}_3 \ , \tag{2.4}$$

with $\mathrm{sgn}(z)$ the sign function which is $+1$ for $z>0$ and -1 for $z<0$. Using this $\mathbf{n}(\mathbf{r})$ we find

$$\rho(\mathbf{r})=q[\delta^3(\mathbf{r}-a\hat{\mathbf{z}})+\delta^3(\mathbf{r}+a\hat{\mathbf{z}})] \ , \tag{2.5}$$

and thus from Eq. (1.6), $Q_S=2q$. It is clear from Eqs. (2.2) and (2.3) that $\mathbf{F}^{i0}\sim 1/r^3$ for large r and thus from Eq. (1.8) that $Q_T=0$. This implies, according to the definitions of Sec. I, that there is color screening and that there must be a color charge of $-2q$ in the field.

Indeed the screening charge can be seen directly by looking at the second term in Eq. (1.7) which gives the current in the field J_F^μ. For the present case where $\mathbf{n}(\mathbf{r})$ is given by Eq. (2.4) we have

$$J_F^\mu=\frac{1}{4\pi}(\partial_i\mathbf{n})\cdot\mathbf{F}^{i0}\delta^\mu{}_0=\frac{1}{2\pi}\delta(z)\mathbf{e}_3\cdot\mathbf{F}^{z0}\delta^\mu{}_0 \ , \tag{2.6}$$

and thus there is only a static field charge. One can show by integrating Eq. (2.6) that the charge in the field is indeed $-2q$. However we can infer that this is correct by a simple electrostatic analogy.

The charge density of Eq. (2.6) can be recognized as being completely equivalent to the electrostatic charge density on a grounded, thin, infinite, conducting sheet located in the xy plane, induced by two positive point charges equidistant on either side of the conducting sheet, equivalent to those of Eq. (2.5). It is well known that such an electrostatic configuration has a field which behaves like a dipole field at large r and that the total charge induced on the conducting sheet is $-2q$.

This dipole case also illustrates why we feel the imprecise definition of $\mathbf{n}(\mathbf{r})$ given by Lai and Oh [2] is not useful. Instead of defining $\mathbf{n}(\mathbf{r})$ as we did in Eq. (2.4) we could, according to Lai and Oh, also define it simply as $\mathbf{n}(\mathbf{r})=\mathbf{e}_3$. In this case we would find

$$\rho(\mathbf{r})=q[\delta^3(\mathbf{r}-a\hat{\mathbf{z}})-\delta^3(\mathbf{r}+a\hat{\mathbf{z}})] \ , \tag{2.7}$$

from which it follows that $Q_S=Q_T=0$ and that the charge in the field is identically zero. Thus for the same solution to the Yang-Mills field equations with the same external source we would draw entirely opposite conclusions about whether there is color screening depending on which $\mathbf{n}(\mathbf{r})$ we use. Clearly this implies that color screening is not well defined by these techniques.

It seems to us that even if we adhere to the definition of $\mathbf{n}(\mathbf{r})$ given in Sec. I and shown explicitly for the dipole in Eq. (2.4) the situation is still not satisfactory. The finite dipole solution is analogous to a simple electric dipole made up to two opposite point charges separated by a finite distance. We should be able to interpret it in an analogous way. In the electrostatic case we would say that the total source charge is zero because the two charges are equal and opposite. In the Abelian Coulomb case the total source charge should also be zero. it seems unreasonable to attribute the fact that the total charge is zero to color screening in the field. We will make this more precise in the next section.

We should point out that the gauge we have used here is *not* the special gauge discussed in Sec. I in which the source density $\rho(\mathbf{r})$ would be lined up in the third isodirection such that $\rho(\mathbf{r})=\rho(\mathbf{r})\mathbf{e}_3$. In that gauge we would have $\mathbf{n}(\mathbf{r})=\mathbf{e}_3$ instead of the form given in Eq. (2.4) and $\mathbf{A}^i$ would not be zero. Of course the results for Q_S and Q_T are independent of the gauge choice.

III. NEW DEFINITIONS FOR TOTAL SOURCE CHARGE

The difficulty with the definition of Q_S given in Sec. I as it applies to examples like that of Sec. II can be eliminated if we make use of the notion of parallel transport. Two isolated isocharges can be compared in a gauge-invariant way only if they are first parallel transported to a common point. In Yang-Mills theories the parallel transport is given in terms of the connection $\mathbf{A}^\mu$. Thus, it would seem natural that a meaningful definition of total source charge would involve the gauge fields $\mathbf{A}^\mu$ as well as the external charge density $\rho(\mathbf{r})$.

That one must consider transport when characterizing an external source is not a new idea. For example, one of us (W.B.C.) pointed out some time ago that one has to be careful in defining a point source to take into account the singularity properties of $\mathbf{A}^\mu$ at the location of the source [4].

In the dipole example of Sec. II one can use transport to make a comparison of the two point sources. If one does, one finds that indeed the two sources are equal and opposite. This conclusion is based on a gauge-invariant procedure which we discuss below. Thus our intuition based on the equivalent electrostatic case is good; the total source charge is zero and there is no color screening by the field. This differs from the conclusions we drew in Sec. II using the definitions of Sec. I.

The situation in Yang-Mills theories is of course like that of general relativity: we do not know the geometry until we have a solution to the field equations. Thus we will assume that the field equations have been solved giving us a static solution for a static external source as defined in Sec. I. Knowing $\mathbf{A}^\mu$ then allows us to compare isovectors at different space-time points using parallel transport. If we have two points infinitesimally separated by a four-displacement dx^μ we require the covariant differential of $\mathbf{q}$ between these points to be zero, i.e.,

$$D\mathbf{q}\equiv(d+g\,\mathbf{A}^\mu dx_\mu\times)\mathbf{q}=0\ ,$$

or

$$d\mathbf{q}=-g(\mathbf{A}^\mu\times\mathbf{q})dx_\mu\ , \tag{3.1}$$

which is consistent with the equation of continuity, Eq. (1.3), for the case where dx^μ is along the world line of $\mathbf{q}$. We note that only the isodirection of $\mathbf{q}$ is affected by this transport, not its magnitude. While Eq. (3.1) could apply to actual physical transport of $\mathbf{q}$ it can also be understood to tell us how to compute the isocomponents of $\mathbf{q}$ relative to the isocoordinates at a nearby space-time point.

We want to compare source charges at the same instant of time in the frame where all the charges are at rest. We take this to mean that dx^μ should be spacelike with $dx^0=0$. We write $dx^i\equiv dr_i$ and Eq. (3.1) becomes

$$d\mathbf{q}=g(\mathbf{A}^i\times\mathbf{q})dr_i\ . \tag{3.2}$$

In order to compare $\mathbf{q}$ with a charge located a finite distance from $\mathbf{q}$, we must integrate Eq. (3.2) along some path connecting the two charges. This integration generally depends on the path and thus our comparison is ambiguous. In order to remove this ambiguity we restrict the path to the straight line connecting the two point charges. Equation (3.2) is actually a differential equation for $\mathbf{q}$ along the path of transport. If we introduce the parameter λ which is the distance along the path then we can rewrite Eq. (3.2) as

$$\frac{d\mathbf{q}}{d\lambda}=g(\mathbf{A}^i\times\mathbf{q})\hat{\lambda}_i\ , \tag{3.3}$$

where $\hat{\boldsymbol{\lambda}}\equiv d\mathbf{r}/d\lambda$ is the unit vector pointing in the direction of transport. We write the solution to Eq. (3.3) as $\mathbf{q}(\lambda)$. When $\lambda=0$ we have the isocomponents of $\mathbf{q}$ relative to its actual location at $\mathbf{r}$. When $\lambda=\lambda_f\equiv|\mathbf{r}'-\mathbf{r}|$ we have the isocomponents of $\mathbf{q}$ relative to the location of the other charge $\mathbf{q}'$, at $\mathbf{r}'$.

Instead of solving Eq. (3.3) for $\mathbf{q}(\lambda_f)$ we can use this equation to solve for the rotation matrix in isospace which relates $\mathbf{q}$ to $\mathbf{q}(\lambda_f)$. The advantage is that this matrix depends only on $\mathbf{r},\mathbf{r}'$, $\mathbf{A}^i$ but not on the particular charge being transported. We will write this matrix as an isodiad. If $\mathbf{R}(\lambda)$ is the rotation which relates $\mathbf{q}$ to $\mathbf{q}(\lambda)$, i.e., $\mathbf{q}(\lambda)=\mathbf{R}(\lambda)\cdot\mathbf{q}$, then it is easy to show using Eq. (3.3) that

$$\frac{d\mathbf{R}}{d\lambda}=g(\mathbf{A}^i\times\mathbf{R})\hat{\lambda}_i\ , \tag{3.4}$$

where it is understood that $\mathbf{R}(0)=\mathbf{I}$, with $\mathbf{I}$ the identity isodiad. Instead of writing the solution we seek as $\mathbf{R}(\lambda_f)$ we use the more symmetric notation $\mathbf{R}(\mathbf{r}',\mathbf{r})$, so that $\mathbf{q}(\lambda_f)=\mathbf{R}(\mathbf{r}',\mathbf{r})\cdot\mathbf{q}$. The advantage of this notation is made clear in the discussion below.

We could define a gauge-invariant total charge for the pair of charges as

$$\sqrt{[\mathbf{q}(\lambda_f)+q']^2}=\sqrt{\mathbf{q}^2+\mathbf{q}'^2+2\mathbf{q}'\cdot\mathbf{R}(\mathbf{r}',\mathbf{r})\cdot\mathbf{q}}\ . \tag{3.5}$$

We note that this definition is completely symmetric between $\mathbf{q}$ and $\mathbf{q}'$. Because the transposed of $\mathbf{R}(\mathbf{r}',\mathbf{r})$ is its inverse, we see that $\mathbf{q}'\cdot\mathbf{R}(\mathbf{r}',\mathbf{r})$ gives us the components of $\mathbf{q}'$ transported to the position of $\mathbf{q}$. Thus Eq. (3.5) can be viewed equivalently from the point of view of either charge. This equation also suggests one way we could define a total source for any distribution of charge.

If we assume that we have found $\mathbf{R}(\mathbf{r}',\mathbf{r})$ for the straight line transport between all possible pairs of source points then we can define

$$Q_S\equiv\sqrt{\int\rho(\mathbf{r}')\cdot\mathbf{R}(\mathbf{r}',\mathbf{r})\cdot\rho(\mathbf{r})d^3r'd^3r}\ , \tag{3.6}$$

which is equivalent to Eq. (3.5) for two point charges. For the gauge used in Sec. II, $\mathbf{A}^i=0$, thus $\mathbf{R}(\mathbf{r}',\mathbf{r})\equiv\mathbf{I}$ and Eq. (3.6) yields $Q_S=0$ for the dipole case, the same result we would get in electrostatics and verifying our earlier assertion.

Another way we could use this equal time transport to define a Q_S is to transport all parts of the source to a common point and then simply add the transported isovector charges. For convenience we will take this common point to be the origin and define the isovector

$$\mathbf{Q}_S \equiv \int \mathbf{R}(0,\mathbf{r})\cdot\boldsymbol{\rho}(\mathbf{r})d^3r \ , \tag{3.7}$$

from which we define the gauge-invariant charge $Q_S\equiv\sqrt{\mathbf{Q}_S\cdot\mathbf{Q}_S}$. This Q_S is also zero for the dipole case of Sec. II, again because $\mathbf{A}^i=0$. This definition for Q_S does depend on the origin and therefore has an additional ambiguity that Eq. (3.6) does not have. However for a given situation there might well be a natural choice for the origin.

The choice of path used in defining $\mathbf{R}(\mathbf{r}',\mathbf{r})$ is irrelevant when the field configurations have zero curvature $\mathbf{F}^{ij}$, as is the case for the Coulomb solutions discussed above. However, when $\mathbf{F}^{ij}\neq 0$ a prescription must be given. Here we have chosen the straight-line path between $\mathbf{r}$ and $\mathbf{r}'$. Other choices could be used provided they are consistent with the condition $\mathbf{R}(\mathbf{r},\mathbf{r})\equiv\mathbf{I}$ and do not depend on gauge. Straight-line paths seem to be a natural choice but one might well ask if there is some other choice which would minimize the value of the Q_S's we have defined here.

As a practical matter either Eq. (3.6) or Eq. (3.7) could be difficult to apply since solving for $\mathbf{R}(\mathbf{r}',\mathbf{r})$ could be difficult. Because of this Eq. (3.7) may be the more useful definition. The transport which gives us $\mathbf{R}(0,\mathbf{r})$ is radially inward along $-\hat{\mathbf{r}}$, with $\hat{\mathbf{r}}$ the radial unit vector, and thus only the $\mathbf{A}^r$ component of $\mathbf{A}^i$ will contribute in Eq. (3.4). One can always transform to a gauge where $\mathbf{A}^r=0$. In this gauge $\mathbf{R}(0,\mathbf{r})=\mathbf{I}$ and $\mathbf{Q}_S$ is simply the integral of $\rho(r)$. Thus if one could work in this gauge from the beginning or at least easily transform to it, $\mathbf{Q}_S$ could be obtained by a straightforward integration.

IV. CONCLUSION

The discussion of the dipole example in Sec. II suggests to us that even when one employs gauge-invariant definitions for total charge Q_T, and total source charge Q_S, the concept of color screening in classical Yang-Mills theory is ambiguous. While the definition of total charge given in Eq. (1.8) is physical and unambiguous, providing one defines $\mathbf{n}(\mathbf{r})$ as in Sec. I, the problem lies in giving a physically sensible definition for the total source charge. As seen from the dipole example, the commonly employed definition of Q_S in Eq. (1.6) is inadequate.

We have not been able to remove the ambiguity in defining Q_S but we have given definitions which offer the hope of being able to distinguish between situations where $Q_T=0$ because the source is a color singlet and those where there is true color screening. The important ingredient in our new definitions is parallel transport which we feel is indispensable in trying to analyze the nature of a source.

We are not the first to suggest that defining color screening can be ambiguous. Hughes [5] pointed out that the total-screening solution of Sikivie and Weiss [6], if looked at in a particular gauge, appears to be a solution for a color-singlet source. Interestingly Lai and Oh [2] have criticized Hughes' conclusion because it appears to be gauge dependent. As we pointed out in Secs. I and II, while the definition of Lai and Oh is gauge invariant, it is nevertheless ambiguous because their definition for $\mathbf{n}(\mathbf{r})$ is ambiguous.

Our conclusion would be that the Sikivie-Weiss total-screening solution is even more complicated than any of these authors has suggested. In fact we would conclude that their solution actually corresponds to a time-dependent external source even though $\rho(\mathbf{r})$ is not time dependent. This is because their $\mathbf{A}^i$ has linear time dependence and thus $\mathbf{R}(\mathbf{r}',\mathbf{r})$ would be time dependent.

Note added in proof. We have recently realized that the integral under the radical in Eq. (3.6) could in some situations be negative, leading to an imaginary Q_S. This can be avoided by using the absolute value of the integral but may indicate that Eq. (3.6) is not a physically useful definition for Q_S.

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