Effect of neutrino heating in the early Universe on neutrino decoupling temperatures and nucleosynthesis

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The neutrino energy-loss rates during pair annihilation in the early Universe are calculated using the formulation of Herrera and Hacyan and assuming neutrinos to be massless. The work is extended to include the neutrinos of the muon and tauon families, and a method to calculate the evolution of neutrino temperatures is suggested. It is found that neutrinos of the electron family decouple at 1.5×10^{10} K, and those of the other families at 2.5×10^{10} K. The former component is heated by about 0.36%, resulting in a *reduction* of the primordial abundance by a mass fraction of helium (Y_p) in the standard hot big-bang model by about 0.003.

I. INTRODUCTION

In the standard hot big-bang model of the early Universe, it is generally assumed that neutrinos decoupled from the primordial soup well before the freezing out of the neutron-to-proton ratio. However, Dicus *et al.*[1] showed that during $e^{\pm}$ pair annihilation a fraction of the entropy goes into forming pairs of neutrinos and thereby heating the decoupled neutrino gas by about 0.3%. Recently, Herrera and Hacyan[2] have computed the relaxation time of neutrinos in the early Universe. Using a method described in an earlier paper,[3] they have formulated the problem of the elastic electron-neutrino scattering and inelastic neutrino pair formation from $e^{\pm}$ pair annihilation. Although they must have computed the individual relaxation times for $e^-\nu_e$, $e^-\bar{\nu}_e$, $e^-e^+\to\nu_e\bar{\nu}_e$ processes, in their paper,[2] they have presented the results in a graphical form only for the combined relaxation time.

In this work we recompute the neutrino energy-loss rates and relaxation times separately for each component of the binary mixture and extend the computation to include the neutrinos of the muon and tauon families. It is important to know the neutrino energy-loss rates in order to compute the full effect of neutrino heating on the element synthesis in the hot big bang. Dicus *et al.* considered the effect of neutrino heating only on the weak-interaction rates through the implied changes in the neutrino temperature. However, the energy loss due to neutrinos is also going to affect the temperature evolution of the matter segment.[4] We have used the value of the Weinberg mixing angle, given by $\sin^2\theta_W=0.23$, instead of Herrera and Hacyan's adopted value 0.25 and, finally, computed both the effects of neutrino energy loss, as mentioned above.

In Sec. II we merely present the formulas which are taken from Herrera and Hacyan[2] for $\nu_e\bar{\nu}_e$ processes and include the modifications needed for the other families of neutrinos. In Sec. III we present a method of calculating neutrino heating and in Sec. IV the results of our calculation and the conclusion.

II. FORMULATION OF HERRERA AND HACYAN FOR NEUTRINO RELAXATION

Following the method of Herrera and Hacyan,[2] we consider a system composed of four types of particles, of which particle indices 1 and 2 are electrons and positrons at a temperature $T_1=T_2=T$, and the indices 3 and 4 are neutrinos and antineutrinos of any one family at a slightly different temperature $T_3=T_4=T+\delta T$, with $\delta T/T\ll1$. In thermodynamic equilibrium the Lorentz-scalar distribution function f_i $(i=1,\ldots,4)$ for the ith constituent in nondegenerate conditions must follow the usual Fermi-Dirac distribution, given by

$$f_i=\{\exp[\beta_i(p_i^\mu U_\mu)+1]\}^{-1},$$

where $\beta_i=(k_BT_i)^{-1}$, k_B is Boltzmann's constant, and U^μ the hydrodynamic four-velocity. The distribution function f_i satisfies the relativistically invariant Boltzmann equation

$$p_i^\mu f_{i,\mu}=\sum_j C_{ij},$$

where p_i^μ is the four-momentum of particles i and C_{ij} are the collision terms.[5] The energy-momentum tensor of the ith-type particles, $T_i^{\mu\nu}$, is given by

$$T_i^{\mu\nu}=\frac{cg_i}{h^3}\int f_ip_i^\mu p_i^\nu d\Gamma_i .$$

Herrera and Hacyan defined the relaxation time τ of the above mixture as

$$\tau^{-1}=\sum_j\tau_j^{-1},$$

where τ_j's are the relaxation times due to the different microscopic interactions involving electrons and are given by $\tau_j^{-1}=I_j/C_T$, I_j being the rate of energy transfer per unit volume per unit temperature difference (δT) due to the binary interactions of electrons with particles j, and the total heat capacity C_T being given by

$$C_T^{-1}=C_e^{-1}+C_\nu^{-1} .$$

C_e and C_ν are the specific heat of electrons and neutrinos at a constant volume per unit volume of the mixture, respectively. The I_j's are given by

$$I_{\nu}=\frac{c}{32\pi^6 k_B T^2\hbar^6}\int d\Gamma_p d\Gamma_q d\Gamma_{p'} d\Gamma_{q'}(p\cdot U)(p+q)^2[(q'-q)\cdot U]\delta^{(4)}(p+q-p'-q')\sigma_{e\nu} fN\hat{f}'\hat{N}' ,$$

$$I_{\bar{\nu}}=\frac{c}{32\pi^6 k_E T^2\hbar^6}\int d\Gamma_p d\Gamma_{\bar{q}} d\Gamma_{p'} d\Gamma_{\bar{q}'}(p\cdot U)(p+\bar{q})^2[(\bar{q}'-\bar{q})\cdot U]\delta^{(4)}(p+\bar{q}-p'-\bar{q}')\sigma_{e\nu} f\bar{N}\hat{f}'\hat{\bar{N}}' ,$$

$$I_{e^+}=\frac{c}{16\pi^6 k_B T^2\hbar^6}\int d\Gamma_p d\Gamma_{\bar{p}} d\Gamma_q d\Gamma_{\bar{q}}(p\cdot U)(p+\bar{p})^2[(q+\bar{q})\cdot U]\delta^{(4)}(p+\bar{p}-q-\bar{q})\sigma_{ee^+} f\bar{f}\hat{N}\hat{\bar{N}} ,$$

where $p,q,\bar{p},\bar{q}$ are the four-momenta of electrons and neutrinos and their respective antiparticles, N and $\bar{N}$ being the distribution function of neutrinos and antineutrinos, respectively; the quantities with carets are the available phase-space factor, that is, $\hat{f}=1-f$ and $\hat{N}=1-N$.

The differential cross section for the process $e+\nu \rightleftharpoons e+\nu$ is given by

$$\sigma_{e\nu}=4\sigma_0\frac{(s-m_e^2c^2)^2}{m_e^2c^2s}(A_1+A_2\cos\theta+A_3\cos^2\theta) , \tag{1}$$

where $s=p_\mu p^\mu$, p^μ is the total four-momentum of the collision, m_e the rest mass of the electron, $\sigma_0\equiv(G_F m_e/8\pi\hbar^2)^2$, G_F being the Fermi coupling constant for weak interaction, θ the scattering angle in the center-of-momentum frame, and

$$A_1=(C_V+C_A)^2+(C_V-C_A)^2\left[\frac{s+m_e^2c^2}{2s}\right]^2-(C_V^2-C_A^2)\frac{m_e^2c^2}{s} , \tag{2a}$$

$$A_2=(C_V-C_A)^2\left[\frac{s^2-m_e^4c^4}{2s^2}\right]+(C_V^2-C_A^2)\frac{m_e^2c^2}{s} , \tag{2b}$$

and

$$A_3=(C_V-C_A)^2\left[\frac{s-m_e^2c^2}{2s}\right]^2 , \tag{2c}$$

with C_V and C_A to be defined later. The differential cross section for the process $e+\bar{\nu}\rightleftharpoons e+\bar{\nu}$ is given by the same formula (1), but in Eq. (2) C_A has to be replaced by $-C_A$, and this rule is valid for all the three families of neutrinos, namely, for the processes $e+\nu_{e,\mu,\tau}\rightleftharpoons e+\nu_{e,\mu,\tau}$ and $e+\bar{\nu}_{e,\mu,\tau}\rightleftharpoons e+\bar{\nu}_{e,\mu,\tau}$. For the different families of neutrinos, the expressions for C_V and C_A are different. It was originally 't Hooft[6] who had shown that for muonic and tauonic types of neutrino interaction which mediate only via neutral currents or by exchange of Z^0 bosons, $C_V=-0.5+2\sin^2\theta_W$ and $C_A=-0.5$, θ_W being Weinberg's mixing angle. For electron types of neutrino interaction, both charged- and neutral-current processes are responsible and effectively both C_V and C_A change to C_V+1 and C_A+1, respectively, that is, $C_V=0.5+2\sin^2\theta_W$ and $C_A=0.5$. The most up-to-date value of $\sin^2\theta_W$ is found to range from 0.21 to 0.23, and we have taken $\sin^2\theta_W=0.23$.

So far as the pair annihilation processes, namely, $e^++e^-\rightleftharpoons\nu_e+\bar{\nu}_e$, $e^++e^-\rightleftharpoons\nu_\mu+\bar{\nu}_\mu$, and $e^++e^-\rightleftharpoons\nu_\tau+\bar{\nu}_\tau$, with all the neutrinos and their antiparticles being assumed to be essentially massless, the cross sections are again identical for the muon and tauon types of neutrino, but differ for the electron-type neutrinos because of the additional charged-current channel for the latter. The differential cross section for each of the three processes is given by

$$\sigma_{ee^+}=\sigma_0\frac{s(s)^{1/2}}{m_e^2c^2(s-4m_e^2c^2)^{1/2}}(C_1+C_2\cos\theta+C_3\cos^2\theta) ,$$

where

$$C_1=(C_V^2+C_A^2)+(C_V^2-C_A^2)\frac{4m_e^2c^2}{s} ,$$

$$C_2=4C_VC_A\left[1-\frac{4m_e^2c^2}{s}\right]^{1/2} ,$$

$$C_3=(C_V^2+C_A^2)\left[1-\frac{4m_e^2c^2}{s}\right] ,$$

and $C_V=0.5+2\sin^2\theta_W$ and $C_A=0.5$ for $e^\pm\rightarrow\nu_e\bar{\nu}_e$, but $C_V=-0.5+2\sin^2\theta_W$ and $C_A=-0.5$ for $e^\pm\rightarrow\nu_\mu\bar{\nu}_\mu$ $(\nu_\tau\bar{\nu}_\tau)$.

We use Herrera and Hacyan's final expressions for the neutrino power-loss rates per unit volume per unit temperature difference to be given by

$$I_{\nu,\bar{\nu}}=\pi^3 c\sigma_0 k_B\left[\frac{m_e c}{h}\right]^6 z^2\int_0^\infty dw\,\sinh^4 w\int_1^\infty dv\frac{(v^2-1)^6}{v^3}H_\nu(a_1,a_2,a_3)\ ,$$

$$I_{e^+}=1024\pi^3 c\sigma_0 k_B\left[\frac{m_e c}{h}\right]^6 z^2\int_0^\infty dw\,\sinh^2 w\cosh^2 w\int_1^\infty dv\, v^8(v^2-1)^{1/2}H_{e^+}(c_1,c_2,c_3)\ , \tag{3}$$

where $z=m_ec^2/k_BT$ and the integrals $H_{\nu,\bar{\nu}}$ and H_{e^+} are given by

$$H_{\nu,\bar{\nu}}=\int_{-1}^{1}\frac{x(x-x')[2A_1+2A_2xx'+A_3(3x^2x'^2-x^2-x'^2+1)]dx\,dx'}{[\cosh a_1+\cosh(a_2-a_3x)][\cosh a_1+\cosh(a_2-a_3x')]}\ ,$$

$$H_{e^+}=\int_{-1}^{1}\frac{[(2C_1+C_3)+C_3(3x^2x'^2-x^2-x'^2)]dx\,dx'}{(\cosh c_1+\cosh c_2x)(\cosh c_1+\cosh c_3x')}\ ,$$

and the constants are given by

$$a_1=\tfrac{1}{2}zv\cosh w\ ,$$

$$a_2=\frac{a_1}{v^2}\ ,$$

$$a_3=(a_1-a_2)\tanh w\ ,$$

$$c_1=zv\cosh w\ ,$$

$$c_2=z^2(v^2-1)^{1/2}\sinh w\ ,$$

$$c_3=c_1\tanh w\ .$$

The dummy variables w and v in these integrals are related to s by the equations

$$v^2=s\quad\text{and}\quad w=\operatorname{arctanh}\beta_e\ ,$$

except that $v^2=4s$ for all the cases of pair annihilations, β_e being the speed of electron in the center-of-momentum frame in units of c.

In the above formulation the chemical potential of the species are neglected, which is a valid assumption for the scenario of the early Universe prior to the onset of primordial nucleosynthesis. The specific heats of the electrons and neutrinos are given by

$$C_e=4\pi\left[\frac{m_ec}{h}\right]^3 k_Bz^2\int_0^\infty dw\,\sinh^2 w\cosh^3 w\times[\cosh(z\cosh w)+1]^{-1}\ ,$$

$$C_\nu=84\pi\left[\frac{m_ec}{h}\right]^3 k_B\zeta(4)z^{-3}\ ,$$

where $\zeta(4)=\pi^4/90$.

III. METHOD OF CALCULATING THE NEUTRINO HEATING

In order to calculate the amount of neutrino heating at the time of decoupling of the neutrinos, one usually follows the method of Lee and Weinberg.[7] The method used by Dicus *et al.*[1] varies little from the above method and has been followed by Herrera and Hacyan, and they obtained the freezing ratio of photon to neutrino temperature about 0.2% smaller than the standard value $(\frac{11}{4})^{1/3}=1.401$. We think that the Lee-Weinberg[7] (LW) equation for the evolution of the number density of massless neutrinos does not account for the total heating effect of the neutrino gas because the elastic processes which also contribute to heating kinematically the neutrino gas conserve the numbers and therefore we do not use the LW equation. The fraction of the energy of the primordial soup containing $e^\pm$ and photons is exchanged with neutrinos through both elastic and inelastic processes. Therefore, a formulation in terms of the energy-loss integrals I_ν, $I_{\bar{\nu}}$, etc., is called for and, perhaps, ideally suited for studying the effect of neutrino heating.

Since matter and radiation interact very strongly via the electromagnetic interaction, they maintain thermal equilibrium down to a temperature of the order of 4500 K. So, basically, $T_e=T_\gamma=T$ is maintained for $T>4500$ K. Throughout, the standard procedure of the calculation of the temperature evolution of the Universe before, during, and after the nucleosynthesis does not include neutrinos as a thermodynamically participating species, but their interactions in terms of energy gain or loss from the matter-radiation sector are included as an extraneous term in the expression for the temperature evolution. In fact, Wagoner's numerical code for the big-bang nucleosynthesis[4] includes such a term as for the energy losses due to production of neutrino pairs during the latter's decoupling from matter and radiation. An expression for the neutrino loss at temperatures greater than 10^{10} K and at baryonic densities less than 0.1 g/cm^3 is not yet available. Beaudet, Petrosian, and Salpeter[8] derived fitting formulas for neutrino power losses which were valid at relatively lower temperatures and at much higher densities. They also did not include the neutral-current processes in their formulation. Schinder *et al.*[9] and Itoh *et al.*[10] have computed the neutrino-loss terms using the standard theory of electroweak interaction. Even in their calculations the temperature and density ranges do not extend to those pertaining to the era of neutrino decoupling in the early Universe. Instead, we find that the integral formulations for neutrino loss processes of Herrera and Hacyan are exact and simple enough to be used for computing the neutrino loss terms referred to in Wagoner's numerical code, which can now be written as

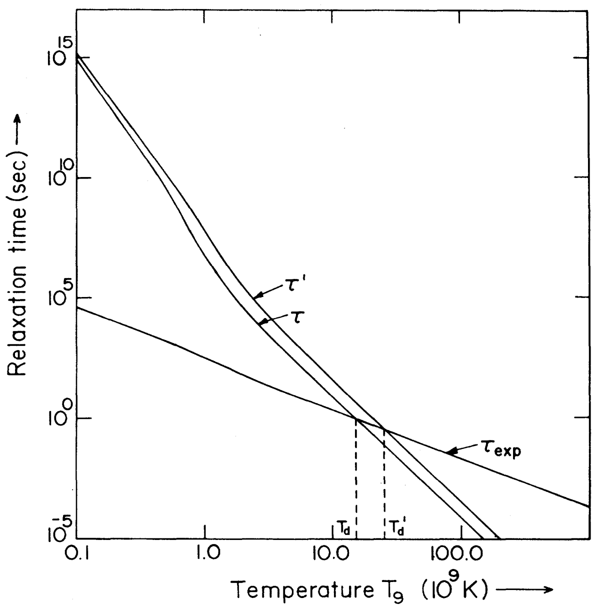

FIG. 1. Relaxation times for neutrinos of electron (τ) and muon/tauon (τ') families and the Hubble rate of expansion τ_{exp} as functions of temperature (T_9). Neutrinos of the muon/tauon family decouple at about 2.5×10^{10} K, whereas those of the electron family at about 1.5×10^{10} K.

$$\frac{du}{dt}\equiv\frac{du_\nu}{dt}=\sum_i I_i(T-T_\nu)\ , \tag{4}$$

provided the difference $(T-T_\nu)\ll T$. This assumption remains perfectly valid during the era of neutrino decoupling. Here the summation extends over all families of neutrinos, each having three terms, two of which are for scattering and one for pair annihilation.

In the high-temperature limit $z\ll1$, $I_i\propto z^{-8}$. This is consistent with the expression of Beaudet *et al.*[8] for $du/dt=Q\propto z^{-9}$ except that the net rate of energy flow must be proportional to the temperature difference in a binary mixture rather than the temperature itself. Since

$$I_i=\frac{C_T}{\tau_i}\quad\text{and}\quad\frac{du_\nu}{dt}=4C_\nu\frac{dT_\nu}{dt}\ ,$$

from Eq. (2) we can write, for the evolution of neutrino temperature T_ν,

$$\frac{dT_\nu}{dt}=\frac{C_T}{C_\nu}\frac{T-T_\nu}{\tau}-HT_\nu\ , \tag{5}$$

where T is the temperature of the electron gas, H the Hubble constant, and τ the total relaxation time for a given family of neutrinos. The second term comes from the usual expansion of the Universe, the temperature falling in proportion to the rate of expansion. The temperature evolution for ν_e's and ν_μ's (or ν_τ's) are slightly different because of the small difference between τ and τ', where τ is the total relaxation time for neutrinos of electron type and τ' for the other two families.

TABLE I. Neutrino energy-loss rates, specific heats, and relaxation times.

Temp. (10^9 K)	Neutrino energy-loss rates (W m^{-3} K^{-1}) due to $\nu_e\bar\nu_e$			due to $\nu_\mu\bar\nu_\mu$ (or $\nu_\tau\bar\nu_\tau$)			Total	Specific heats in (J m^{-3} K^{-1})		Relaxation times in sec for neutrinos of electron type	muon type
T_9	I_{ν_e}	$I_{\bar\nu_e}$	I_{e^+}	I_{ν_μ}	$I_{\bar\nu_\mu}$	I'_{e^+}	$\sum I_i$	C_ν	C_e	τ	τ'
100.0	0.158(+22)	0.427(+21)	0.112(+23)	0.240(+21)	0.192(+21)	0.239(+22)	0.188(+23)	0.132(+19)	0.265(+19)	0.671(−04)	0.312(−03)
50.0	0.618(+19)	0.167(+19)	0.443(+20)	0.936(+18)	0.748(+18)	0.950(+19)	0.745(+20)	0.166(+18)	0.331(+18)	0.211(−02)	0.986(−02)
30.0	0.103(+18)	0.279(+17)	0.745(+18)	0.157(+17)	0.125(+17)	0.159(+18)	0.125(+19)	0.358(+17)	0.714(+17)	0.272(−01)	0.127(+00)
20.0	0.400(+16)	0.108(+16)	0.290(+17)	0.608(+15)	0.487(+15)	0.617(+16)	0.486(+17)	0.106(+17)	0.211(+17)	0.207(+00)	0.103(+01)
10.0	0.150(+14)	0.404(+13)	0.112(+15)	0.230(+13)	0.185(+13)	0.231(+14)	0.185(+15)	0.132(+16)	0.261(+16)	0.672(+01)	0.322(+02)
5.0	0.501(+11)	0.135(+11)	0.408(+12)	0.797(+10)	0.644(+10)	0.764(+11)	0.653(+12)	0.166(+15)	0.313(+15)	0.119(+04)	0.351(+03)
3.0	0.600(+09)	0.163(+09)	0.558(+10)	0.102(+09)	0.833(+08)	0.849(+09)	0.841(+10)	0.358(+14)	0.601(+14)	0.217(+05)	0.564(+04)
2.0	0.133(+08)	0.368(+07)	0.141(+09)	0.243(+07)	0.203(+07)	0.158(+08)	0.198(+09)	0.106(+14)	0.138(+14)	0.296(+06)	0.672(+05)
1.0	0.577(+04)	0.176(+04)	0.503(+05)	0.128(+04)	0.111(+04)	0.267(+04)	0.679(+05)	0.132(+13)	0.459(+12)	0.674(+08)	0.229(+08)
0.5	0.120(+00)	0.467(−01)	0.965(−01)	0.346(−01)	0.316(−01)	0.236(−02)	0.400(+00)	0.166(+12)	0.113(+10)	0.426(+10)	0.164(+11)
0.3	0.118(−05)	0.567(−06)	0.626(−08)	0.407(−06)	0.382(−06)	0.904(−10)	0.300(−06)	0.358(+11)	0.453(+06)	0.258(+12)	0.574(+12)
0.2	0.326(−11)	0.184(−11)	0.971(−17)	0.127(−11)	0.121(−11)	0.965(−19)	0.101(−10)	0.106(+11)	0.260(+02)	0.510(+13)	0.105(+14)
0.1	0.276(−26)	0.196(−26)	0.773(−43)	0.127(−26)	0.123(−26)	0.444(−45)	0.971(−26)	0.132(+10)	0.450(−11)	0.953(+15)	0.180(+16)

We have inserted these new equations in Wagoner's code and evolved it numerically to determine the time evolution of T_{ν_e} and T_{ν_μ}.

IV. RESULTS AND DISCUSSIONS

The integrations in Eq. (3) are carried out using the 32-point Gauss-Legendre quadrature method with 10–20 logarithmic subintervals for covering the range of the variables v and w. An upper limit to $w=8$ has been sufficient, and its lower limit has suitably been chosen depending on the value of the temperatures. Using the Cyber 170 main frame computer, the evaluation of the integrals down to an accuracy of 1 part in 10^4 took about 2 h for each run of temperature and reaction types. The results of the computation for all the I_{ν_e}, $I_{\bar{\nu}_e}$, I_{e^+}, I_{ν_μ}, $I_{\bar{\nu}_\mu}$, I'_{e^+}, $\sum I_i$ $(=I_{\nu_e}+I_{\bar{\nu}_e}+I_{e^+}+2I_{\nu_\mu}+2I_{\bar{\nu}_\mu}+2I'_{e^+})$, C_e, C_ν, τ, and τ' are presented in Table I for temperatures in units of 10^9 K, $T_9=100$, 50, 30, 20, 10, 5, 3, 2, 1, 0.5, 0.3, 0.2, and 0.1. The total relaxation time τ of the electron-type neutrinos, electrons, and their antiparticles is found to follow the expected power law $\tau=91.5z^5$ sec. This compares very well with the result of Herrera and Hacyan, namely, $\tau=87.1z^5$ sec, granting that they assumed $\sin^2\theta_W=0.25$ instead of our 0.23. This power law deviates from its z^5 law at the onset of the pair annihilation and seems to go smoothly over to another power law of the form $z^{7.5}$ at $T_9<0.4$ (as seen in Fig. 1). For the muonic (or tauonic) type of neutrinos, the total relaxation time $\tau'=426z^5$ sec for $T_9>30$.

Defining the decoupling temperature for the neutrinos T_d to be the one at which the relaxation time for the neutrino interactions becomes equal to Hubble expansion time of the Universe τ_{exp} given by

$$\tau_{\text{exp}}=5.69z^2\,\text{sec} \quad (\text{for } z\leq 1)\ .$$

The value of T_d for the electron type of neutrinos turns out to be 1.50×10^{10} K, and that for the ν_μ and ν_τ, $T'_d=2.50\times10^{10}$ K. However, our result for T_d differs from the Herrera-Hacyan calculated value of $T_d=2.1\times10^{10}$ K for the electron type of neutrinos simply because there was an error in their expression for the expansion time scale $\tau_{\text{exp}}=2.08z^2$ sec. The final freezing value for the ratio T_γ/T_{ν_e} turns out to be 1.396, which is about 0.36% lower than the canonical value 1.401, compared to the Herrera-Hacyan value 0.18%. The heating of the neutrinos of the other families is about 0.1% only. In the process a total of about 1.1% of the entropy of matter and radiation is transferred to the neutrinos.

We also have computed the primordial yield by mass fraction (Y_p) of ^{4}He which is found to be lower by 0.003 because of the effect of neutrino heating during the pair annihilation.

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