Renormalization counterterms in linearized lattice QED

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(Received 5 August 1991)

We analyze in weak-coupling perturbation theory the renormalization-counterterm structure of linearized lattice quantum electrodynamics (QED). We find that, although the linearized theory is gauge invariant, counterterms appear that would be forbidden by the Ward identities in the compact formulation of lattice QED. For example, the photon develops a mass, the light-by-light scattering amplitude becomes noncovariant, and the coupling constant is renormalized even in the quenched version of the theory. Gauge invariance does not preclude the appearance of such counterterms in the linearized theory because the formulation is nonlocal.

I. INTRODUCTION

It has been suggested recently [1] that one can formulate quantum electrodynamics (QED) on the lattice by linearizing certain parts of the usual compact lattice action for QED. Specifically, the suggestion is that one linearize, with respect to the transverse component of the gauge field, the electron-photon interaction terms in the lattice action. In this paper, we argue that such a procedure leads, in weak-coupling perturbation theory, to the appearance of ultraviolet counterterms that are not present either in the compact lattice theory or in the continuum theory with a gauge-invariant regulator.

The linearized QED action for the "naive" lattice transcription of the Dirac operator is

$$I=\frac{a^4}{4}\sum_{x,\mu,\nu}[\Theta_{\mu\nu}(x)]^2+a^4\sum_{x,\mu}\bar{\psi}(x)\gamma_\mu\frac{1}{2a}\{[1+iea\theta_\mu^T(x)]U_\mu^L(x)\psi(x+a\hat{\mu})-[1-iea\theta_\mu^T(x-a\hat{\mu})]U_\mu^{L\dagger}(x-a\hat{\mu})\psi(x-a\hat{\mu})\}+a^4\sum_x m\bar{\psi}(x)\psi(x)\ , \tag{1.1}$$

where m is the electron mass; a is the lattice spacing; $\hat{\mu}$ is a unit vector in the μ direction; e is the electron charge; ψ is the electron (Dirac) field; the γ_μ are the anti-Hermitian (Euclidean) Dirac matrices, with $\{\gamma_\mu,\gamma_\nu\}=-2\delta_{\mu\nu}$; θ_μ is the photon field; $U_\mu(x)=\exp[iea\theta_\mu(x)]$; and $\Theta_{\mu\nu}$ is the electromagnetic field strength, which is given by

$$\Theta_{\mu\nu}=\Delta_\mu^+\theta_\nu(x)-\Delta_\nu^+\theta_\mu(x)\ , \tag{1.2}$$

where

$$\Delta_\mu^+f(x)=(1/a)[f(x+a\hat{\mu})-f(x)]\ . \tag{1.3}$$

The superscript T or L indicates that the superscripted quantity is to be evaluated using the transverse or longitudinal component of the photon field, respectively. The transverse and longitudinal components of θ_μ are defined by

$$\begin{aligned}\theta_\mu^T(x)&=\sum_{y,\nu}\mathcal{P}_{\mu\nu}(x-y)\theta_\nu(y)\ ,\\ \theta_\mu^L(x)&=\sum_{y,\nu}[\delta_{xy}\delta_{\mu\nu}-\mathcal{P}_{\mu\nu}(x-y)]\theta_\nu(y)\ ,\end{aligned} \tag{1.4}$$

with

$$\mathcal{P}_{\mu\nu}(x-y)=\int_{-\pi}^{\pi}\frac{d^4l}{(2\pi)^4}\exp[il\cdot(x-y)]\times\exp[i(l_\nu a-l_\mu a)/2]\mathcal{P}_{\mu\nu}(l) \tag{1.5a}$$

and

$$\mathcal{P}_{\mu\nu}(l)=\delta^{\mu\nu}-\frac{\sin(l_\mu a/2)\sin(l_\nu a/2)}{\sum_\alpha\sin^2(l_\alpha a/2)}\ . \tag{1.5b}$$

In order to simplify the analysis, we work throughout in the Landau gauge

$$\sum_\mu \Delta_\mu^- \theta_\mu(x)=0 , \tag{1.6}$$

where

$$\Delta_\mu^- f(x)=(1/a)[f(x)-f(x-a\hat{\mu})] . \tag{1.7}$$

Then θ_μ^L vanishes, and we need consider only the interactions of transverse photons with electrons.

The Feynman rules for weak-coupling perturbation theory for the linearized action (1.1) are given in the appendix, along with the rules for the compact form of the action. Throughout this paper, we use the terms "compact" and "noncompact" to indicate the form of the interaction between the electron and the photon, not the form of the purely photonic part of the action, for which we always choose the expression given in (1.1). (The compact form is obtained by replacing the factors $[1\pm ie\theta_\mu^T]$ in (1.1) by U_μ^T and $(U_\mu^T)^\dagger$, respectively.) In the linearized case, there is a single interaction vertex involving one photon and an electron, whereas in the compact case there are additional multiphoton-electron seagull vertices. This is the only difference between the linearized and compact formulations in perturbation theory, and it is central to our analysis.

One need not formulate the linearized theory in terms of the naive lattice transcription of the Dirac operator. For example, in Ref. [1] staggered fermions were employed. A different choice of lattice Dirac operator, of course, leads to a different expression for a particular Feynman diagram. However, the presence or absence of that Feyman diagram in a given theory is independent of the choice of fermion formulation, once one has specified either the compact or linearized form for the interactions of the electron with the photon field. As we shall see, our conclusions about the counterterm structure of the linearized theory do not depend on the details of the Feynman rules, but only on the presence or absence of certain diagrams. We refer to the case of the naive-fermion formulation in order to have a definite example at hand, but our arguments are more generally valid.

The main point that we wish to demonstrate is the following: although the action (1.1) is gauge invariant and the photon couples to a formally conserved current [1], the Ward identities in the linearized theory do not control the ultraviolet (UV) behavior of the theory in the usual way. We show that new UV counterterms appear: the transverse photon develops a quadratically divergent transverse mass, there are noncovariant renormalizations of the photon's wave function and the light-by-light scattering amplitude, and the vertex renormalization Z_1 is no longer equal to the electron's wave-function renormalization Z_2. As we shall see, these UV counterterms appear, despite the gauge invariance of the theory, because of the nonlocal nature of the transversality projector (1.5a). The appearance of such counterterms is not surprising since nonlocal, gauge-invariant interactions of the form $\sum_\mu(\theta_\mu^T)^2$ or $[\sum_\mu(\theta_\mu^T)^2]^2$ are allowed in the action once θ^T is treated differently from θ^L.

In the remainder of the paper, we explain how the new UV counterterms arise in the linearized theory, and we discuss their consequences. Therefore, we focus on the ultraviolet-divergent subgraphs in the theory, and we ignore contributions that vanish in the continuum limit. In Sec. II we examine the quenched version of the theory (no electron loops); in Sec. III we discuss electron loops with no radiative corrections; in Sec. IV we analyze the case of radiative corrections to electron loops; and in Sec. V we summarize our results and discuss their implications for numerical simulations.

II. THE QUENCHED THEORY

In the quenched theory, that is, the theory with no electron loops, the only divergent subgraphs are the electron self-energy corrections and electron-photon vertex corrections. In the compact version of the quenched lattice theory, the vertex correction receives contributions from continuumlike graphs [for example, Fig. 1(a)], and also from the seagull graphs [for example, Fig. 1(b)] [2]. Then, repeated application of the graphical (perturbative) Ward identities [2] allows one to derive the Ward identity relating the vertex correction to the propagator:

$$\sum_\mu (2/a)\sin(\tfrac{1}{2}l_\mu a)\Gamma_\mu(p,l)=S^{-1}(p+l)-S^{-1}(p) , \tag{2.1}$$

where $\Gamma_\mu(p,l)$ is the complete electron-photon vertex with external legs truncated, $S(p)$ is the complete electron propagator, p is the incoming electron momentum, and l is the incoming photon momentum. Unless otherwise noted, when we speak of truncated Green's functions in the linearized theory, we mean that both the external transverse-photon propagators and the associated transversality projectors have been removed.

In the lattice formulation of QED, unlike the continuum formulation, the seagull contributions [such as the one in Fig. 1(b)] are essential in deriving the Ward identity (2.1). The reason for this is that, on the lattice, the nonseagull vertex depends on the incoming electron momentum, whereas in the continuum the vertex is momentum independent. The seagull contributions compensate for the momentum dependence of the lattice vertex [2].

In the Landau gauge, the linearized version of the lattice theory does not contain seagull graphs. Consequently, the continuumlike Ward identity (2.1) is modified. The Ward identity now contains contributions, corresponding to the missing seagull graphs, in which the

(a) (b)

FIG. 1. One-loop contributions to the electron-photon vertex in compact lattice QED. (a) The continuumlike contribution. (b) One of two lattice seagull contributions. The second seagull contribution is the mirror image of the one shown.

external lattice momentum $(2/a)\sin(\frac{1}{2}l_\mu a)$ is dotted into the seagull vertices. It is easy to see by direct computation in one-loop order that the seagull contributions to the Ward identity are nonvanishing.

All of the seagull contributions that potentially modify the Ward identity (2.1) involve the external photon. Furthermore, since the seagull vertices themselves are suppressed by powers of the lattice spacing a [see (A4) and (A5)], a Feynman graph containing a seagull can give a nonvanishing contribution in the continuum limit $a \to 0$ only if a loop integration containing the seagull diverges. Hence, the seagull contributions to the Ward identity have the structure of an ultraviolet counterterm that renormalizes the electron-photon vertex. In the compact lattice theory (or the continuum theory), one establishes the equality of the vertex renormalization Z_1 and the electron wave-function renormalization Z_2 by considering the Ward identity (2.1) in the limit $l \to 0$. In the linearized theory, owing to the presence of seagull contributions to the Ward identity, Z_1 is no longer equal to Z_2. This implies that, in the linearized theory, even in the absence of dynamical electrons (electron loops), the coupling constant is renormalized by a factor $\tilde{Z}_1 = Z_1/Z_2$. Because the seagull vertices contain at least one explicit power of a, the loop integration corresponding to the factor $\tilde{Z}_1$ must be power divergent in the UV if the contribution is to be nonvanishing. Hence, there is no logarithmic dependence on a: $\tilde{Z}_1$ is a finite renormalization.

Since the Ward identity (2.1) does not hold in the linearized theory, one might wonder whether the electromagnetic current is conserved. In the Landau gauge, the photon propagator is proportional to the transversality projector $\mathcal{P}_{\mu\nu}(l)$ given in (1.5). Of course, since $\sum_\nu \mathcal{P}_{\mu\nu}(l)\mathcal{P}_{\nu\rho}(l) = \mathcal{P}_{\mu\rho}(l)$, one can always associate a transversality projector with the current to which the transverse photon couples. Thus, in this somewhat formal sense, one can define a conserved current. However, owing to the presence of the transversality projector, the conserved current is nonlocal, with infinite range, and the corresponding charge is ill defined. In an arbitrary gauge, there are longitudinal as well as transverse photon modes. The longitudinal modes, however, couple to the electron in the usual compact way, so all the seagulls required for current conservation are present.

Suppose we were to emphasize the conservation of the infinite-range, nonlocal current in the linearized theory by deriving a Ward identity for the vertex function with external legs truncated, but with a transversality projector appended at the photon connection. The result would be (2.1), except that the right-hand side would now vanish. Unfortunately, we learn nothing about Z_1 by taking the limit $l \to 0$ for this Ward identity because $\mathcal{P}_{\mu\nu}(l)$ is ill defined at $l=0$. To put it another way, because $\mathcal{P}_{\mu\nu}(x)$ is nonlocal, with infinite range, once we have appended the transversality projector to a Green's function, we lose information about the short-distance contributions contained therein. Although we have formal current conservation, the fact that it is enforced through an infinite-range, nonlocal mechanism deprives the associated Ward identities of their power to control UV counterterms.

III. ELECTRON LOOPS WITHOUT RADIATIVE CORRECTIONS

Now let us consider the subgraphs involving an electron loop, but with no radiative corrections on the loop. The subgraphs of this type that have a divergent power count are the lowest-order contributions to the vacuum polarization (photon self-energy) and the light-by-light scattering amplitude.

Consider first the vacuum polarization. In the compact lattice theory, the lowest-order vacuum polarization receives two contributions. One is the continuumlike contribution, shown in Fig. 2(a), and the other is the seagull contribution shown in Fig. 2(b). In the compact theory, one can show, through repeated application of the graphical Ward identities [2], that the vacuum polarization is transverse. That is,

$$\sum_\mu (2/a)\sin(\tfrac{1}{2}l_\mu a)\Pi_{\mu\nu}(l) = 0 \;, \qquad (3.1)$$

where $\Pi_{\mu\nu}(l)$ is the vacuum polarization with external legs truncated, and l is the external-photon momentum.

On the other hand, in the linearized theory, the seagull graph of Fig. 2(b) is absent. Consequently, $\Pi_{\mu\nu}(l)$ is no longer transverse: there appears in (3.1) a term which can be expressed as the scalar product of the lattice momentum $(2/a)\sin(\frac{1}{2}l_\mu a)$ with the seagull graph of Fig. 2(b). It is easy to see that this seagull contribution to the Ward identity (3.1) is nonvanishing.

Since, in the linearized case, $\Pi_{\mu\nu}(l)$ is no longer transverse, one cannot argue that $\Pi_{\mu\nu}(0)$ vanishes. Hence, the leading divergence, which according to power counting is quadratic, need not vanish. Indeed, one finds by direct calculation that there is a divergence proportional to $(1/a^2)\delta_{\mu\nu}$, which is precisely the negative of the leading divergence in the missing seagull graph. This divergent contribution corresponds to a counterterm of the form $\sum_{x,\mu}[\theta_\mu^T(x)]^2$, which renormalizes the mass of the transverse photon.

As in the quenched linearized theory, one can exhibit current conservation by associating a transversality projector (1.5b) with each electron-photon vertex. If $\Pi_{\mu\nu}(l)$ were defined in this way, then (3.1) would hold. However, this formal transversality of $\Pi_{\mu\nu}(l)$, so defined, is insufficient to eliminate the quadratic divergence. Obviously, a divergence proportional to $(1/a^2)\delta_{\mu\nu}$ is merely converted into a divergence proportional to $(1/a^2)\mathcal{P}_{\mu\nu}(l)$.

To see why the formal transversality of $\Pi_{\mu\nu}(l)$ fails to

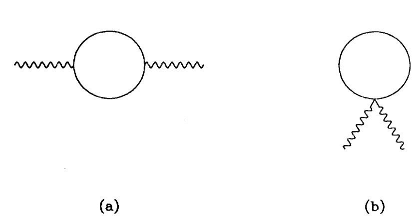

FIG. 2. One-loop contributions to the vacuum polarization in compact lattice QED. (a) The continuumlike contribution. (b) The lattice seagull contribution.

control the quadratic divergence, let us recall the Ward-identity argument for the vanishing of the quadratic divergence in the compact theory [2]. We differentiate (3.1) with respect to l_ρ to obtain

$$\cos(\tfrac{1}{2}l_\rho a)\Pi_{\rho\nu}(l)+\sum_\mu (2/a)\sin(\tfrac{1}{2}l_\mu a)\frac{\partial}{\partial l_\rho}\Pi_{\mu\nu}(l)=0 \ . \quad (3.2)$$

Now, since the electron mass protects electron loops from infrared divergences, $(\partial/\partial l_\rho)\Pi_{\mu\nu}(l)$ is finite in the limit $l\to 0$. Hence, we conclude that

$$\Pi_{\rho\nu}(0)=0 \ . \quad (3.3)$$

That is, the leading divergence in $\Pi_{\rho\nu}(l)$ vanishes. On the other hand, if $\Pi_{\rho\nu}(l)$ is defined to contain transversality projectors, then $(\partial/\partial l_\rho)\Pi_{\mu\nu}(l)$ is no longer finite in the limit $l\to 0$ because the derivative of the transversality projector with respect to l_ρ is singular. In this case, the second term in (3.2) contributes as $l\to 0$, and we can no longer conclude that the leading divergence in $\Pi_{\rho\nu}(l)$ vanishes. Once again, we see that, for Green's functions containing the transversality projector, the infinite-range, nonlocal nature of the projector prevents us from using the Ward identities to constrain local (UV) contributions.

In addition to the leading (quadratically divergent) counterterm, the vacuum polarization generates nonleading counterterms. One of these corresponds to the usual logarithmically divergent counterterm associated with renormalization of the photon's wave function, which, in the Landau gauge, is of the form $\sum_{l,\mu,\nu}(2/a)^2\sin^2(\frac{1}{2}l_\mu a)[\theta_\nu^T(l)]^2$ in momentum space and $\sum_{x,\mu,\nu}[\Delta_\mu^+\theta_\nu^T(x)]^2 \doteq \frac{1}{2}\sum_{x,\mu,\nu}[\Theta_{\mu\nu}^T(x)]^2$ in coordinate space. This counterterm is, of course, covariant in the continuum limit. Because of the absence of seagull graphs in the linearized theory and the consequent lack of transversality of the vacuum polarization, there is also a counterterm that is not covariant in the continuum limit. It is of the form $\sum_{l,\mu}(2/a)^2\sin^2(\frac{1}{2}l_\mu a)[\theta_\mu^T(l)]^2$ in momentum space and $\sum_{x,\mu}[\Delta_\mu^+\theta_\mu^T(x)]^2$ in coordinate space. Power-counting arguments show that this counterterm is finite. In the compact theory, this counterterm is forbidden by the Ward identity (3.1).

In the linearized theory, we also expect the light-by-light scattering amplitude (with external legs truncated) to be nontransverse and to contain a covariant counterterm of the form $\sum_{x,\mu,\nu}[\theta_\mu^T(x)]^2[\theta_\nu^T(x)]^2$ and a noncovariant counterterm of the form $\sum_\mu[\theta_\mu^T(x)]^4$. Power-counting arguments indicate that these counterterms are finite. Nevertheless, even the covariant counterterm must be subtracted in the lattice theory in order to bring the lattice amplitude into agreement with the continuum one.

Fermion-loop amplitudes with more than four external photons are, of course, convergent and hence generate no new counterterms. In the absence of radiative corrections, the superficially divergent fermion-loop amplitudes would, in the compact theory, involve seagull vertices with no more than four photons. Thus, in the absence of radiative corrections, the new counterterms we have discussed in this section could be eliminated by including in the linearized theory all transverse-photon seagull vertices that contain four or fewer photons. The coefficients of the seagull vertices would, of course, be required to be identical to those in the compact theory.

IV. ELECTRON LOOPS WITH RADIATIVE CORRECTIONS

In the quenched version of the linearized theory, we found a new counterterm $\tilde{Z}_1$, which could be canceled by tuning the strength of the electron-photon vertex. In the linearized theory with electron loops but with no radiative corrections, we found several new counterterms that could be eliminated by introducing a finite number of transverse-photon seagull vertices with the same coefficients as in the compact theory. Hence, one might hope that a procedure involving the tuning of $\tilde{Z}_1$ and the inclusion of a finite number of seagulls would be sufficient to control all of the counterterms that arise in the linearized theory with dynamical electrons. However, if we consider radiative corrections to electron loops, then it becomes clear that this is not the case. The difficulty stems from the fact that the linearized theory lacks certain radiative corrections involving seagulls of higher order than e^4. In isolation, such radiative corrections would be of higher order in a, and, hence, negligible in the continuum limit. If the radiative corrections are embedded in a divergent fermion loop, however, the complete graph may be non-negligible. Thus, such a graph can yield a contribution to the new counterterms that appear in the linearized theory.

To illustrate this phenomenon, let us consider an example. The radiative correction of Fig. 3(a), which involves an order e^5 seagull is, by power counting, of order a^1. On the other hand, when it is embedded as a subgraph in a contribution to the vacuum polarization [Fig. 3(b)], the divergence in the electron loop makes the overall ultraviolet behavior order a^{-2}. In the compact theory, such a contribution is canceled by contributions from graphs involving lower-order seagull and nonseagull vertices, since the transversality of the vacuum polarization guarantees that the order-a^{-2} divergence vanishes. In the linearized theory, the truncated vacuum polarization is nontransverse, and so, presumably, contains in order e^8 a divergence of order a^{-2}, which corresponds to the leading divergence of the absent seagull graph [Fig. 3(b)]. Similarly, if we were to embed the radiative correction of Fig. 3(a) in a light-by-light scattering graph, that graph

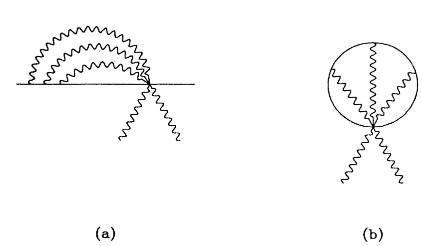

FIG. 3. (a) An order a^1e^8 contribution to the electron–two-photon vertex in compact lattice QED. (b) An order $a^{-2}e^8$ contribution to the vacuum polarization in compact lattice QED.

would generate covariant and noncovariant counterterms that arise in the linearized theory in the light-by-light scattering amplitude in order e^{10}.

More generally, suppose that one includes in the linearized theory all of the seagull vertices from the compact theory through order e^n. Then it is clear that one can construct a radiative correction involving a seagull of order e^{n+1} which, when embedded in a vacuum-polarization or light-by-light-scattering graph, will yield a contribution corresponding to an uncompensated counterterm in the linearized theory. Hence, we see that there is no finite set of seagulls, with coefficients identical to those in the compact theory, whose inclusion in the linearized theory, together with tuning of the electron-photon coupling, would account for all of the new counterterms that arise in the linearized theory. Of course, one might try to introduce seagulls with arbitrary coefficients and tune the coefficients so as to cancel the new counterterms. However, such a procedure offers no advantage over including an explicit term in the bare action for each of the new counterterms.

V. SUMMARY AND DISCUSSION

We have seen that, although it is gauge invariant, the linearized lattice formulation of QED gives rise in weak-coupling perturbation theory to UV counterterms that would be excluded by Ward identities in either the compact lattice formulation or the continuum theory with a gauge-invariant regulator. Specifically, we have found that the vertex renormalization Z_1 is not equal to electron's wave-function renormalization Z_2; the transverse photon develops a quadratically divergent mass; there are subleading noncovariant corrections to the photon propagator; and the light-by-light scattering amplitude develops covariant and noncovariant counterterms. Since θ_μ^T is gauge invariant, counterterms involving θ_μ^T and its derivatives are constrained in form only by the lattice remnant of rotational invariance and by the requirement that the expression be of dimension four or less. All such counterterms appear in the perturbative analysis. Hence, the counterterms that appear in the linearized theory are equivalent to the counterterms that appear if one explicitly breaks the gauge symmetry. That is, there is no advantage in linearizing with respect to θ_μ^T rather than θ_μ: equivalent counterterms appear in the two cases.

The Ward identities for the linearized theory do not preclude the appearance of such counterterms because, owing to the absence of seagull-vertex contributions, the Green's functions with external transverse-photon legs truncated do not, in themselves, respect current conservation. Rather, there is a conserved current in the sense that one can associate a transversality projector (1.5) from a transverse-photon propagator with each photon-electron vertex in a Green's function. Since the transversality projector is an infinite-range, nonlocal object, it is not surprising that the Ward identities for Green's functions with transversality projectors appended yield no constraints on the UV counterterms, which arise from short-distance contributions.

This absence of the usual constraints on renormalization counterterms appears to be a rather general phenomenon that can occur whenever gauge invariance is realized through an infinite-range, nonlocal mechanism. For example, in a theory in which the UV regulator (either continuum or lattice) breaks chiral gauge invariance, one can restore that invariance by introducing a massless auxiliary scalar field [3]. When one carries out the functional integration over the auxiliary field, the resulting effective action contains generalizations of the Wess-Zumino term [4] that restore the gauge symmetry. However, because the auxiliary field is massless, there are infinite-range, nonlocal interactions: transversality projectors of the type (1.5) appear [5]. Consequently, the theory generates ultraviolet counterterms that ordinarily would be forbidden by gauge invariance [5–10].

Another example of this phenomenon can be constructed in the context of continuum QED. Suppose that one adds to the QED action—either by hand or through the effects of a regulator that violates current conservation—a photon mass term $\sum_\mu A_\mu^2$. Such a term is, of course, gauge variant, but one can compensate precisely for the gauge transformations of the photon field by introducing a massless auxiliary Stückelberg field ϕ (Ref. [11]). The photon mass term then takes the form $\sum_\mu (A_\mu - \partial_\mu \phi)^2$ and the action regains a gauge invariance. However, as in the previous example, when one integrates out the massless auxiliary field, one finds an effective action that exhibits infinite-range nonlocality. A term of the form $\sum_\mu (A_\mu^T)^2$ appears; that is, the transverse component of the photon field develops a mass.

In both of the examples cited above, we can understand the failure of gauge invariance to constrain the forms of the counterterms by noting that, in the local action, a transformation of the auxiliary scalar can, by itself, compensate for a gauge transformation of the vector field. Thus, gauge invariance imposes constraints only on the forms of the interactions involving the auxiliary field. But the auxiliary field is unphysical since, by a suitable choice of gauge, it can be decoupled from the theory. This situation is, of course, familiar from standard electroweak theory. There, the transformations of the massless, azimuthal components of the scalar (Higgs) field compensate for transformations of the gauge fields and, hence, allow the appearance of gauge-field mass terms and other potentially nonrenormalizable interactions that would otherwise be forbidden by gauge invariance. The standard-model interactions involving the radial component of the Higgs field then play a crucial role in maintaining the renormalizability of the theory.

The effects discussed in this paper can be understood simply in terms of a toy model. Consider continuum QED with a Pauli-Villars regulator and with an irrelevant operator of the form $\bar\psi[\sum_\mu (A_\mu^T)^2]\psi$ included in the action. This operator is gauge invariant but nonlocal (with infinite range), owing to the explicit appearance of A_μ^T. It corresponds to a two-photon–electron seagull. Although the seagull is of dimension five, it can contribute in leading order in the inverse Pauli-Villars mass through graphs such as the one in Fig. 2(b). Thus, there appear new counterterms of the sort that we have found in linearized QED. Of course, since Lorentz invariance is not broken by the Pauli-Villars regulator, the noncovariant counterterms are absent. As in the case of linearized QED, the irrelevant operator that we introduce is explicitly nonlocal (as opposed to being obtained from a local field theory by integrating out a scalar field). One might be tempted to try to make the theory local by replacing A_μ^T with $(A_\mu - \partial_\mu \phi)$, but, as we have already seen, such a procedure does not yield any additional constraint on the counterterms.

The analysis that we have presented is couched in terms of weak-coupling perturbation theory. Nevertheless, our principal conclusion—that an infinite-range, nonlocal implementation of gauge invariance does not constrain the appearance of UV counterterms—seems to have a generality that extends beyond perturbation theory.

Our discussion in this paper has focused on *infinite*-range, nonlocal implementations of gauge invariance. One can also construct gauge-invariant theories involving *finite*-range, nonlocal mechanisms. This, however, entails the introduction of an additional scale in the action. It is clear that, once a new scale has been introduced, many new counterterms can, in principle, arise.

The appearance of new counterterms in linearized QED has important consequences for numerical simulations. For example, we have found that, even in the quenched approximation, the electric charge is renormalized in linearized QED. Hence, in a quenched numerical simulation, it would be necessary to tune the electron-photon vertex, as one approached the continuum limit, in order to maintain a correspondence with the continuum limit of the compact formulation. In the linearized theory with dynamical fermions, additional new counterterms appear. The set of counterterms in the linearized theory is, in fact, the same as the set that one would obtain by breaking gauge invariance. In the absence of radiative corrections these additional counterterms could be eliminated by including in the linearized theory a finite set of transverse-photon seagull vertices whose coefficients are the same as in the compact theory. However, if one allows for the possibility of radiative corrections to electron loops, then there is no finite set of such seagull vertices which, when combined with tuning of the electron-photon vertex, would eliminate the new counterterms. Thus, in a simulation, one would be faced with the formidable task of tuning all of the new counterterms as well as the charge and the electron mass.

ACKNOWLEDGMENTS

We wish to thank Jim Cline, John Kogut, Paul Mackenzie, Daniel Mustaki, and Robert Perry for enlightening conversations. The work of G.T.B. was supported by the U.S. Department of Energy, Division of High Energy Physics, Contract No. W-31-109-ENG-38. Fermilab is operated by Universities Research Associates, Inc. under contract with the U.S. Department of Energy. The work of J.S. was supported by the U.S. Department of Energy, Division of High Energy Physics, Contract No. DE-AC02ER01545.

APPENDIX: THE FEYNMAN RULES

The Feynman rules for weak-coupling perturbation theory for the action (1.1) are as follows [12]. The photon propagator in Landau gauge is

$$\Delta_{\mu\nu}(l) = \frac{\mathcal{P}_{\mu\nu}(l)}{(2/a)^2 \sin^2(\frac{1}{2} l_\mu a)} , \tag{A1}$$

with $\mathcal{P}_{\mu\nu}(l)$ given in (1.5b); the electron propagator is

$$S_F(p) = \frac{1}{(1/a)\sum_\mu \gamma_\mu \sin(p_\mu a) + m} ; \tag{A2}$$

and the electron-photon vertex is

$$V_\mu^{(1)}(p,l) = -e\gamma_\mu \cos(p_\mu a + \tfrac{1}{2} l_\mu a) , \tag{A3}$$

where p is the incoming electron momentum and l is the incoming photon momentum. As usual, for each closed loop there is an integration $\int_{-\pi/a}^{\pi/a} d^4k/(2\pi)^4$, and for each electron loop there is a factor -1.

In the compact theory there are, in addition to the electron–one-photon vertex (A3), electron-multiphoton seagull vertices. For example, the electron–two-photon seagull vertex is

$$V_{\mu\nu}^{(2)}(p,l_1,l_2) = ae^2 \delta_{\mu\nu} \gamma_\mu \sin(p_\mu a + \tfrac{1}{2} l_{1\mu} a + \tfrac{1}{2} l_{2\mu} a) , \tag{A4}$$

where l_1 and l_2 are the incoming photon momenta. The higher-order seagull vertices can be obtained from the recursion relation [2]

$$V_{\mu_1 \cdots \mu_{n+1}}(p,l_1,\ldots,l_{n+1}) = e\delta_{\mu_n \mu_{n+1}} \frac{V_{\mu_1 \cdots \mu_n}(p+l_{n+1},l_1,\ldots,l_n) - V_{\mu_1 \cdots \mu_n}(p,l_1,\ldots,l_n)}{(2/a)\sin[\frac{1}{2}(l_{n+1})_\mu a]} . \tag{A5}$$

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