Rotational invariance in light-cone quantization

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In the study of the decay of a heavy scalar particle at rest in the Yukawa model at the one- and two-loop levels, it is shown explicitly that naive light-cone quantization leads to a violation of rotational invariance. Noncovariant counterterms are constructed in detail to restore Lorentz covariance. An analysis of surface and zero-mode contributions clarifies the origin of the problem.

I. INTRODUCTION

Light-cone quantization might be a very valuable tool toward a better understanding of the strong interaction. The main advantages of the formalism are the simple vacuum structure, the manifest boost invariance in the z direction, and the Hamiltonian formulation that leads to a very physical approach to field theory.

One of the major disadvantages of the formalism [1] (as for any Hamiltonian form of dynamics) is its nonmanifest Lorentz invariance (here, rotational invariance). Being not manifestly Lorentz covariant one still expects that physical observables (S-matrix elements) exhibit the full Lorentz covariance of the underlying Lagrangian. Since the verification of Lorentz covariance of the S matrix in a noncovariant formalism is in general rather tedious, it has become common practice to simply assume covariance of the S matrix in naive light-cone quantization [2]. This paper deals with the problem of Lorentz covariance (in particular, rotational invariance) in light-cone quantization.

A powerful test of rotational invariance is given by examining the angular distribution of the decay products of a heavy scalar particle at rest, such as

$$\sigma \to f\bar{f} \ . \tag{1.1}$$

Starting out with the light-cone quantized Yukawa model [3]

$$\mathcal{L} = \bar{f}(i\not\partial - m)f + \phi(\Box + \lambda^2)\phi + \gamma \bar{f} f \phi \ , \tag{1.2}$$

we note that any deviation from a uniform $f\bar{f}$ distribution in physical S-matrix elements would indicate a serious violation of rotational invariance.

This paper investigates the decay (1.1) at the one- and two-loop levels. A discussion beyond one loop is important in order to decide whether self-induced inertia terms [4], which naturally arise from normal ordering of the Hamiltonian, could cure the problem. Violations at higher loops would mean, in particular, that any clever arrangement of self-induced inertia terms cannot restore a covariant answer for physical S-matrix elements, since self-induced inertias are of second order in the coupling.

We demonstrate an alternative treatment by adding counterterms to the Lagrangian respecting only those symmetries, which are manifestly preserved on the light cone, i.e., transverse rotations and boosts along the z axis. The goal of this paper is to construct them explicitly and show how rotational invariance can be restored for physical S-matrix elements. To complete the discussion, in Sec. IV we address the question of why light-cone quantization leads to incorrect results, if naively applied.

II. BREAKDOWN OF COVARIANCE AT THE ONE-LOOP LEVEL AND ADDITION OF NONCOVARIANT COUNTERTERMS

We begin our considerations with the decay of a scalar particle into a fermion-antifermion pair $\sigma \to f\bar{f}$ at the tree level. The corresponding matrix element squared is (see Fig. 1)

$$\sum_{s_f, s_{\bar{f}}} |M|^2 = \mathrm{Tr}[(\not p_f + m_v)(-\not q + m_v)] \ . \tag{2.1}$$

Overall light-cone energy conservation constrains the external momenta, leading to

$$\lambda^2 = \frac{m^2 + q_\perp^2}{q^+(1-q^+)} \ . \tag{2.2}$$

Note that, in order to allow for noncovariant counterterms, two different masses have been introduced [5]. A vertex mass m_v appears in the numerator and a kinetic mass m appears in P^- conservation and in all denominators associated with the diagram [6]. Equations (2.1) and (2.2) lead immediately to

$$\sum_{s_f, s_{\bar{f}}} |M|^2 = -2\frac{m_v^2 - m^2}{q^+(1-q^+)} - 2\lambda^2 + 8m_v^2 \ . \tag{2.3}$$

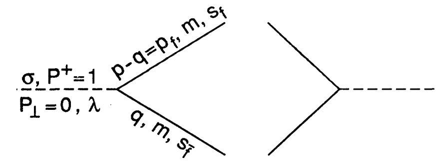

FIG. 1. Tree-level matrix element for the decay $\sigma \to f\bar{f}$. The dashed line represents a heavy boson with mass λ at rest: $p^+ = p^-, p_\perp = 0$. The sum runs over the fermion (mass m) spin labels $s_f, s_{\bar{f}}$.

Obviously rotational invariance is obtained if and only if $m_v = m$; i.e., no problems arise in tree-level physics.

At the one-loop level the set of diagrams in Fig. 2 contributes to the decay. Note that to order γ^4 only interference terms between one-loop and tree-level diagrams contribute. As an example we calculate the contribution from interference between a boson-exchange graph and the tree graph (see Fig. 3) [7]:

$$I_{\text{bos ex}} = \gamma^4 \int_0^1 \frac{dk^+}{(16\pi^3)} d^{2(1-\epsilon)}k_\perp \frac{\theta(1-q^+-k^+)}{(q^++k^+)(1-q^+-k^+)k^+} \times \frac{\mathrm{Tr}[(\not{p}-\not{q}+m)(\not{p}_2+m)(-\not{p}_1+m)(-\not{q}+m)]}{\left[p^- - \frac{m^2+(q_\perp+k_\perp)^2}{q^++k^+} - \frac{m^2+(q_\perp+k_\perp)^2}{1-q^+-k^+}\right]\left[p^- - \frac{m^2+(q_\perp+k_\perp)^2}{1-q^+-k^+} - \frac{\lambda^2+k_\perp^2}{k^+} - \frac{m^2+q_\perp^2}{1-q^+}\right]} . \tag{2.4}$$

Using the Brodsky trick [8] to include instantaneous fermion contributions, performing the trace, combining energy denominators and integrating over $k_\perp$, we obtain

$$I_{\text{bos ex}} = \gamma^4 \int_0^1 \frac{dk^+}{16\pi^3} \frac{\theta(1-q^+-k^+)}{(q^++k^+)(p^+-q^+-k^+)k^+} \int_0^1 d\alpha \frac{1}{\mu^2} \left[\frac{A+B}{2}\frac{\Gamma(\epsilon)}{(M^2)^\epsilon} + \frac{C}{M^2}\right] \tag{2.5}$$

where

$$\mu = -\left[\frac{\alpha}{(k^++q^+)(1-q^+-k^+)} + \frac{(1-\alpha)(1-q^+)}{k^+(1-q^+-k^+)}\right],$$
$$M^2 = \mu^{-1}\left[-q_\perp^2\mu^{-1}\left[\frac{\alpha}{(q^++k^+)(1-q^+-k^+)} + \frac{1-\alpha}{1-q^+-k^+}\right]^2 + \alpha\left[p^- - \frac{m^2+q_\perp^2}{q^+-k^+} - \frac{m^2+q_\perp^2}{1-q^+-k^+}\right] + (1-\alpha)\left[p^- - \frac{m^2+q_\perp^2}{q^+} - \frac{\lambda^2}{k^+} - \frac{m^2+q_\perp^2}{1-k^+-q^+}\right]\right], \tag{2.6}$$
$$A = \frac{2(4m^2q^{+2}-m^2-q_\perp^2)}{(q^++k^+-1)q^+}, \quad B = -\frac{2(4m^2q^{+2}-4m^2q^++m^2+q_\perp^2)}{(1-q^+)q^+} .$$

C acquires terms from zero and linear order in the integration variable $k_\perp$ of the Dirac trace. The linear terms give a contribution after shifting momenta. Since the expression is rather lengthy we do not display it here.

Similar steps must be performed for all the other diagrams of Fig. 2. This involves renormalizing the diagrams using minimal subtraction and performing the integration over k^+ and α numerically. Then rotational invariance can be checked for the total one-loop S-matrix element by computing the diagrams for two different sets of external momenta:

$$\text{set I: } q^+=\tfrac{1}{2},\ q^x=\tfrac{1}{4},\ q^y=0 ,$$
$$\text{set II: } q^+=\tfrac{1}{4},\ q^x=0,\ q^y=0 . \tag{2.7}$$

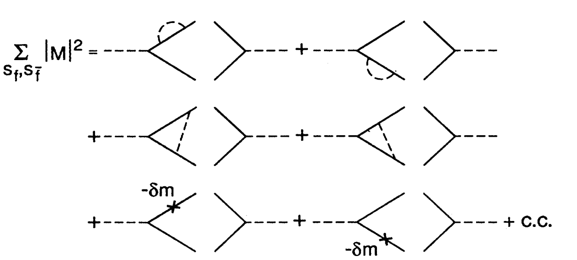

FIG. 2. Fourth-order contributions to $\sigma \to f\bar{f}$. The δm insertion represents the one-loop mass counterterm.

In both cases, we have chosen $\lambda=1$, $m=\sqrt{3/16}$. Since both sets obey Eq. (2.1) and describe a scalar at rest, i.e., $P^+=P^-$ and $P_\perp=0$, the answer is supposed to be the same for both of them, unless rotational invariance is broken.

For the asymmetry r, i.e., the result of the numerical integration for the difference of set I and set II, in terms of

$$a = \frac{8\pi^3}{\gamma^4} \sum_{s_f s_{\bar f}} |M|^2 , \tag{2.8}$$

we find $r=0.02a$. That means rotational invariance is broken for physical S-matrix elements at the one-loop level. In Appendix A we give details of this calculation. In particular it is shown there that the piece which violates rotational invariance comes from the instantane-

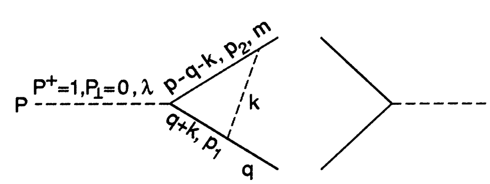

FIG. 3. Typical contribution to the vertex correction of $\sigma \to f\bar{f}$.

ous contribution in the external self-energy diagrams shown in Fig. 4.

In order to keep our discussion as clear as possible, we restrict the number of spatial dimensions to two in what follows. This enables us to disentangle the specific renormalization procedure on the light cone from the ordinary ones, since the Yukawa model is superrenormalizable in 2+1 dimensions.

The remaining goal of this section is to show that the term that violates rotational invariance is of the same form as the first term in the right-hand side (RHS) of Eq. (2.3). Thus, by allowing independent renormalizations for m_v and m one can restore rotational invariance.

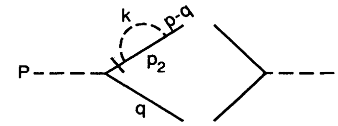

FIG. 4. Instantaneous contributions to the external self-energy.

Using light-cone perturbation theory (LCPT) rules one finds [9], for the graph in Fig. 4,

$$I(q^+,q_\perp)=\int_0^{1-q^+}\frac{dk_\perp dk^+}{16\pi^3}\frac{\mathrm{Tr}[(\not p-\not q+m)(\not p_2+m)\frac{1}{2}\gamma^+(-\not q+m)]}{(1-q^+-k^+)k^+(1-q^+)\left[p^- -\frac{m^2+q_\perp^2}{q^+}-\frac{\lambda^2+k_\perp^2}{k^+}-\frac{m^2+(q_\perp+k_\perp)^2}{1-q^+-k^+}\right]}\ . \quad (2.9)$$

A change of variables $k^+=(1-q^+)x,\hat{k}_\perp=k_\perp+xq_\perp$, combined with use of

$$(p-q)^2=p^-(1-q^+)-(1-q^+)\frac{m^2+q_\perp^2}{q^+}-q_\perp^2=m^2 \quad (2.10)$$

and

$$\frac{\lambda^2+k_\perp^2}{x}+\frac{m^2+(q_\perp+k_\perp)^2}{1-x}=\frac{\lambda^2}{x}+\frac{m^2}{1-x}+\frac{(k_\perp+q_\perp x)^2+xq_\perp^2(1-x)}{x(1-x)} \quad (2.11)$$

yields

$$I=\int_0^1\frac{d\hat{k}_\perp dx}{(16\pi^3)}\frac{\mathrm{Tr}(\dots)}{x(1-x)\left[m^2-\frac{\lambda^2}{x}-\frac{m^2}{1-x}-\frac{\hat{k}_\perp^2}{x(1-x)}\right](1-q^+)}\ . \quad (2.12)$$

To write this in a more compact form, we define the q^+ and $q^\perp$ independent function:

$$f(m,\lambda)=\int_0^1\frac{d\hat{k}_\perp dx}{(16\pi^3)}\frac{2-x}{[x(1-x)m^2-\lambda^2(1-x)-m^2x-\hat{k}_\perp^2]}\ , \quad (2.13)$$

where $\hat{k}_\perp=k_\perp+q_\perp x$. Discarding odd terms in $\hat{k}_\perp$, which do not contribute to the integral, we obtain

$$I=\int_0^1 dx\frac{d\hat{k}_\perp}{(16\pi^3)}\frac{(2-x)m(1-q^+)-(2-x)m^2q^+}{[x(1-x)m^2-\lambda^2(1-x)-m^2x-\hat{k}_\perp^2](1-q^+)} \quad (2.14)$$

$$=\frac{1-2q^+}{1-q^+}f(m,\lambda)=\left[2-\frac{1}{1-q^+}\right]f(m,\lambda)\ . \quad (2.15)$$

A similar calculation for the diagram that correspond to the antifermion self-energy, yields

$$\tilde{I}=\left[2-\frac{1}{q^+}\right]f(m,\lambda)\ , \quad (2.16)$$

which contains the same function $f(m,\lambda)$. The total answer, i.e. the sum of I and $\tilde{I}$, is

$$I_{\mathrm{tot}}=\left[4-\frac{1}{q^+(1-q^+)}\right]f(m,\lambda)\ . \quad (2.17)$$

This result has the remarkable feature that it contains the same q^+ dependence as the term in Eq. (2.3) that violates rotational invariance. Hence the violation of rotational invariance at the one-loop level can be cured by an appropriate renormalization of m and m_v, i.e., by using different bare values for m and m_v in the light-cone Hamiltonian.

III. BREAKDOWN OF COVARIANCE AT THE TWO-LOOP LEVEL

In this section it is shown that violations of rotational invariance in the light-cone formulation are not restricted to the one-loop level. This statement is correct even if the one-loop subdivergences are treated covariantly.

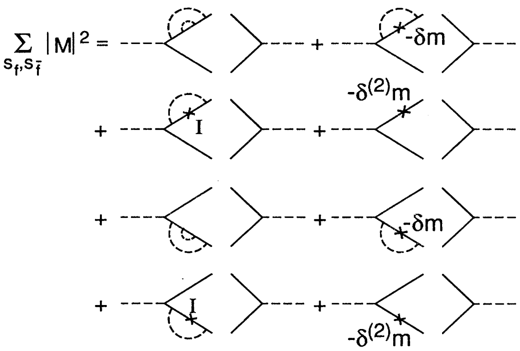

FIG. 5. Two-loop rainbow self-energy contribution to $\sigma \to f\bar{f}$. $\delta m, \delta^{(2)}m$ denote the one- and two-loop self-energy mass corrections, respectively. I corresponds to a counterterm which restores rotational invariance at the one-loop level.

In order to constrain the number of diagrams that contribute to the S matrix, we introduce a second fermion flavor and bosons, which change isospin, into the (2+1)-dimensional Yukawa model. However all couplings at fermion-boson vertices are assigned differently, so that isospin symmetry is broken. The new interaction Lagrangian is

$$\mathcal{L}_{\text{int}} = g_{pn}\bar{p}n\phi_- + g_{pp}\bar{p}p\phi_0 + g_{nn}\bar{n}n\phi_0 + \text{H.c.} \tag{3.1}$$

In this two-flavor model only the rainbow self-energy (Fig. 5) and the ladder vertex correction (Fig. 6) contribute at order $g_{pp}^2 g_{pn}^4$ to the decay $\phi_0 \to \bar{p}p$. All other diagrams contribute with other combinations of coupling constants and must be separately covariant, if covariance is assumed for all values of the couplings.

The rainbow self-energy contribution is shown diagramatically in Fig. 5. The third diagram restores covariance at the one-loop level. Diagrams which contain $\delta m, \delta^{(2)}m$ are one- and two-loop mass counterterms, respectively.

As in Sec. II, we consider the instantaneous contribution to the self-energy diagrams in Fig. 7 separately from the rest. Table I shows the result of the numerical integration for both sets of momenta in (2.7). As in the one-loop case, rotational invariance is violated for the instantaneous contribution to the external self-energy diagrams.

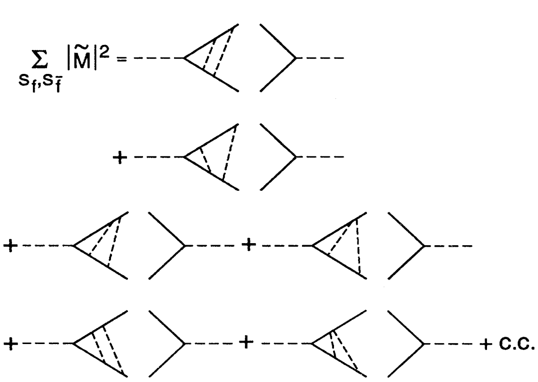

FIG. 6. Two-loop ladder vertex correction to $\sigma \to f\bar{f}$. Six time orderings add up to the covariant answer.

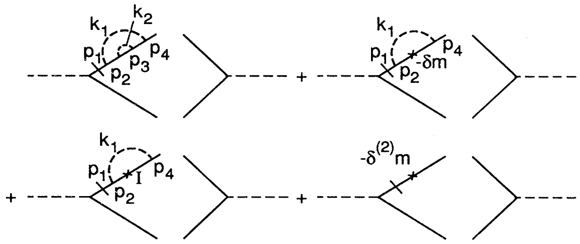

FIG. 7. Instantaneous self-energy correction in two loops. Momentum labels are assigned as indicated.

The ladder vertex contributions yield the six time orderings shown in Fig. 6. The result of the numerical integration is given in Table II.

Thus the ladder diagrams appear to be rotationally invariant by themselves, and a possible cancellation of the noncovariant terms in the self-energy diagram cannot occur. Details of this calculation are given in Appendix B [10].

In the remainder of this section we want to demonstrate that the breakdown of covariance, as in the one-loop case, can be cured by an appropriate renormalization of m_v and m. Since the calculation is similar to that of the one-loop case, we restrict ourselves to an illustration of this procedure.

We start out with the matrix element in Fig. 8 in two loops. In Appendix C it is shown that the two-loop self-energy I_{self} contains a noncovariant piece proportional to $C\gamma^+/p^+$ (see also Ref. [3]), where C is independent of the incoming fermion momentum [11]. Thus, after on-shell renormalization, one finds

$$I_{\text{self}} = \left[\frac{\gamma^+}{p^+} - \frac{\bar{u}\gamma^+ u}{\bar{u}up^+}\right] C + (\not{p} - m) f_1(p^2) + (p^2 - m^2) f_2(p^2) \; . \tag{3.2}$$

The instantaneous self-energy contribution of Fig. 7 becomes

$$\begin{aligned} I &= \text{Tr}\left[(-\not{k} + m)\frac{1}{2}\frac{\gamma^+}{p^+}\frac{1}{m}(\not{p} + m)\right] C\gamma^2 \\ &= 4\gamma^2 C \frac{p^+ - k^+}{p^+} = 4\gamma^2 C \frac{2p^+ - 1}{p^+} \\ &= 4\gamma^2 C \left[2 - \frac{1}{p^+}\right] , \end{aligned} \tag{3.3}$$

where we have set $\not{p} = m$ for the external fermion in Fig.

TABLE I. Self-energy contribution to $\sigma \to f\bar{f}$ in two loops. a_2 describes the contribution from the instantaneous diagrams (Fig. 7), which violate rotational invariance. a_1 is the result of the numerical integration of the residual self-energy diagrams.

Set	a_1	a_2
I	-1.58 ± 0.01	0.015 ± 0.004
II	-1.58 ± 0.01	-0.135 ± 0.002

TABLE II. Result of the numerical integration of the ladder vertex correction to $\sigma \to f\bar{f}$ (Fig. 6). A rotational invariant answer is obtained for both sets.

Set	a_1
I	-2.13 ± 0.01
II	-2.13 ± 0.01

7, and used the following $\gamma^+\gamma^+=0, k^++p^+=1$, and $\bar{u}\gamma^+u=2p^+$.

An analogous calculation for the diagram which corresponds to the antifermion self-energy, yields

$$\tilde{I}=4\gamma^2C\left[2-\frac{1}{1-p^+}\right], \tag{3.4}$$

so that the total contribution becomes

$$I+\tilde{I}=4\gamma^2C\left[4-\frac{1}{p^+(1-p^+)}\right]. \tag{3.5}$$

Again we see that Eq. (3.5) has the same form as the piece that violates rotational invariance in Eq. (2.3), which means that rotational invariance can be restored by tuning the vertex mass and the kinetic mass differently [12].

IV. SURFACE AND ZERO-MODE CONTRIBUTIONS

In the previous sections we have discussed the breakdown of rotational invariance in light-cone quantization and described a way to cure the problem by adding noncovariant counterterms. In order to make the discussion more complete, we will investigate in this section the question of why rotational invariance is broken if light-cone quantization is applied naively. The conclusion will be that naive light-cone quantization omits important surface and zero-mode contributions.

We start our discussion with the $(n+1)$-loop self-energy diagram in Fig. 8 in d dimensions and covariant perturbation theory. Since the theory is based on a manifestly covariant Lagrangian, one expects for the n-loop self-energy I^n_{self} the following structure after mass renormalization:

$$I^n_{\text{self}}=(\not p-\not q-m)f^n_1((p-q)^2) +[(p-q)^2-m^2]f^n_2((p-q)^2), \tag{4.1}$$

where f^n_i must have a spectral representation

$$f^n_i(q^2)=\int_{s_0>0}^{\infty}ds\frac{\rho^n_i(s)}{q^2-s+i\epsilon} \tag{4.2}$$

with no poles for $q^2\leq 0$. We discuss here only the zero-mode effects induced by f^n_1. For f^n_2 the same considerations can be made yielding similar results [13].

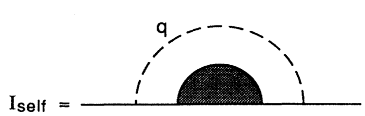

FIG. 8. $(n+1)$-loop rainbow self-energy correction.

One finds, for the f^n_1 contribution to the self-energy in $n+1$ loops,

$$I^{n+1}_{\text{self}}=\int\frac{d^Dq}{(2\pi)^D}\frac{f^n_1((p-q)^2)(\not p-\not q+m)}{[(p-q)^2-m^2+i\epsilon](q^2-\lambda^2+i\epsilon)}. \tag{4.3}$$

Since problems are expected for the γ^+ component only [3], we compute

$$\frac{1}{D}\text{Tr}(\gamma^- I^{n+1}_{\text{self}})=\int\frac{d^Dq}{(2\pi)^D}\left[p^--\frac{q_\perp^2+\lambda^2}{q^+}\right]\times\frac{f^n_1((p-q)^2)}{[(p-q)^2-m^2+i\epsilon](q^2-\lambda^2+i\epsilon)} +\int\frac{d^Dq}{(2\pi)^D}\frac{1}{q^+}\frac{f^n_1((p-q)^2)}{(p-q)^2-m^2+i\epsilon}, \tag{4.4}$$

where

$$q^-=\frac{1}{q^+}[q^2-\lambda^2+(q_\perp^2+\lambda^2)] \tag{4.5}$$

was used.

It should be emphasized that even though light-cone variables have been introduced, only algebraical steps have been performed so far; i.e., no breakdown of covariance can have occurred at this point. The trouble occurs when the integration over q^- is performed, in order to obtain LCPT.

The first integral in Eq. (4.4) poses problems at the one-loop level, i.e., $f\equiv 1$, when trying to perform the q^- integration. This is because the integrand falls off no faster than $1/q^-$ for $p^+-q^+=0$ or $q^+=0$. Whereas the first case should give rise to a contribution of measure 0, we expect nonvanishing contributions from the surface term in the second case, since the denominators are multiplied by a function which diverges for $q^+\to 0$.

What we encounter here is nothing else but the one-loop problem of the self-energy which has been noticed by many authors [14–16].

However, in higher loops we expect no trouble arising from this term. To illustrate this we use the spectral decomposition of Eq. (4.2) and write the first contribution to Eq. (4.4) as

$$I_1=\int_{s_0}^{\infty}ds\int\frac{d^Dq}{(2\pi)^D}\left[p^--\frac{q_\perp^2+\lambda^2}{q^+}\right]\times\frac{\rho_1(s)}{[(p-q)^2-s+i\epsilon][(p-q)^2-m^2+i\epsilon]}\times\frac{1}{q^2-\lambda^2+i\epsilon}. \tag{4.6}$$

If sufficiently regular behavior for $\rho_1(s)$ is assumed, the integrand falls off like $\sim(1/q^-)^2$ or faster, which means that surface terms do not contribute [17–19].

The situation is different for the second integral in Eq. (4.4) however. Performing the q^- integration leads to [20]

$$\frac{1}{2}\int dq^- d^{D-2}q_\perp \frac{1}{q^+}\frac{f_1^n((p-q)^2)}{(p-q)^2-m^2+i\epsilon}$$
$$=\frac{1}{p^+}\delta(p^+-q^+)\int \frac{d^D q f_1^n((p-q)^2)}{(q-p)^2+m^2+i\epsilon}\ . \quad (4.7)$$

This is because for $p^+\neq q^+$ the contour of the left-hand side can be chosen such that its contribution vanishes. The rest follows from

$$\frac{1}{2}\int dq^+\int dq^- d^{D-2}q_\perp \frac{1}{q^+}\frac{f_1^n((p-q)^2)}{(p-q)^2-m^2+i\epsilon}$$
$$=\frac{1}{p^+}\int \frac{d^D p f_1^n((p-q)^2)}{(q-p)^2-m^2+i\epsilon}\ . \quad (4.8)$$

The point is that naive light-cone quantization omits the zero-mode contribution on the right-hand side of Eq. (4.7) and thereby causes a violation of rotational invariance. This also predicts that the piece that violates rotational invariance is always proportional to $1/p^+$, which is in perfect agreement with all our experiences at the one-, two-, and three-loop levels [16]. Since the RHS of Eq. (4.7) does not depend on the outer boson mass, we see that using a heavy Pauli-Villars boson regulator instead of dimensional regularization would have taken care of the problem [21].

To complete this section we want to list again the properties of the diagrams in Fig. 8.

(i) It is very likely that noncovariances appear in any order of perturbation theory.

(ii) The noncovariant piece is always $p_\perp$ and p^- independent and of the form $c(\gamma^+/p^+)$.

(iii) The noncovariant zero-mode contribution is independent of the outer boson mass, which explains why a Pauli-Villars regulator plays an extraordinary role among regulators.

(iv) Dimensional regularization is not sufficient, neither is the so-called "covariant cutoff [22]."

(v) Even supersymmetric theories suffer from this problem (see Appendix D).

At this point we should add a remark to claify some points which we made in a previous paper [3]. In light-cone perturbation theory, quadratic divergences can occur to all orders for many theories (such as QED) if one does not take into account all time orderings. Most of these quadratically divergent terms violate rotational invariance as well. Since QED_{2+1} is superrenormalizable this problem does not occur there for higher orders (i.e., the noncovariant counterterms are UV finite for three or more loops). Nevertheless, even in QED_{2+1} there is a logarithmic UV divergence in the two-loop noncovariant counterterm.

V. SUMMARY AND CONCLUSIONS

We have shown that naive light-cone quantization leads to a violation of rotational invariance in physical S-matrix elements. To do this we investigated the decay of a heavy scalar particle at rest and observed a deviation from a uniform distribution of its decay products. The analysis shows that the effect is not restricted to one loop. Following the general arguments of Sec. IV one expects a violation at any order in perturbation theory.

At the one- and two-loop levels, we explicitly show that the problem can be cured by tuning the vertex mass m_v differently from the kinetic mass m. This procedure corresponds to adding noncovariant counterterms, which preserve only the kinematic light-cone symmetries. That requires an additional renormalization condition, compared to a manifestly covariant theory.

We suggest the decay of a heavy boson at rest because violation of covariance is obvious in this case. Once the additional counterterm is fixed the statement of renormalizability requires that *all* processes can be evaluated to the same order in perturbation theory [23] without encountering any further violations [24]. To complete our discussion, we investigated the question of why light-cone quantization goes wrong if it is not applied carefully enough. We found that nonvanishing surface contributions accompanied by a zero-mode problem at one loop and missing zero-mode contributions at higher-loop orders cause a breakdown of the covariant structure of the theory. At this point it should be mentioned that the same problems are expected to occur in gauge theories (in $A^+=0$ or any other gauge), quantized on the light cone. As far as practical methods are concerned, such as discretized light-cone quantization (DLCQ) [25] or the Tamm-Dancoff-procedure [4], additional violations of rotational invariance are anticipated. This is because one is forced to work with a finite value of a cutoff which by itself breaks Lorentz invariance. In this paper, we have discussed only those violations of rotational invariance which survive the continuum limit.

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APPENDIX A

Using LCPT theory for the self-energy contribution I_{self} (Fig. 9), one finds

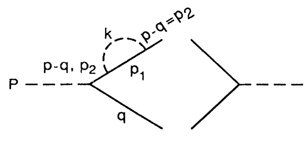

FIG. 9. Self-energy diagram in one loop.

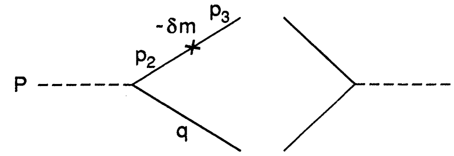

FIG. 10. One-loop mass correction to the self-energy.

$$I_{\text{self}}=\frac{\gamma^4}{16\pi^3}\int dk^+ d^{2(1-\epsilon)}k_\perp \frac{\Theta(1-q^+-k^+)}{(1-q^+)k^+(1-q^+-k^+)} \times \frac{\mathrm{Tr}[(\not p_2+m)(\not p_1+m)(\not p_2+m)(-\not q+m)]}{\left[p^- -\frac{m^2+q_\perp^2}{1-q^+}-\frac{m^2+q_\perp^2}{q^+}\right]\left[p^- -\frac{m^2+(q_\perp+k_\perp)^2}{(1-q^+-k^+)}-\frac{k_\perp^2+\lambda^2}{k^+}-\frac{m^2+q_\perp^2}{q^+}\right]}\,, \tag{A1}$$

where $p_\perp=0$ and $p^+=1$. Note that an off-shell value for p^- has been assigned in order to deal with the double pole. At the end of the calculation, p^- is taken on shell. If one shifts variables to

$$\tilde{k}_\perp=k_\perp+q_\perp\frac{k^+}{1-q^+} \tag{A2}$$

the Dirac trace can be reduced to the simple form $A\tilde{k}_\perp^2+C$, where

$$A=2(4m^2q^+-3m^2+q^{+2}\lambda^2-2q^+\lambda^2+q_\perp^2+\lambda^2)/k_\perp^+ \tag{A3}$$

and C contains terms of zero and linear order in the integration variable $\tilde{k}_\perp$ of the Dirac trace. This is correct only after terms are discarded which do not contribute to the integral. The linear terms give a contribution after shifting momenta. Since the expression is rather lengthy we do not display it here. The $\tilde{k}_\perp$ integration can be trivially performed, yielding

$$I_{\text{self}}=-\frac{\gamma^4}{16\pi^3}\int_0^1 dk^+\frac{\theta(1-q^+-k^+)}{(1-q^+)^2}\frac{1}{p^- -\frac{m^2+q_\perp^2}{1-q^+}-\frac{m^2+q_\perp^2}{q^+}}\left[(1-\epsilon)A\frac{\Gamma(-1+\epsilon)}{(M^2)^{-1+\epsilon}}+C\frac{\Gamma(\epsilon)}{(M^2)^{\epsilon}}\right]\,, \tag{A4}$$

where

$$M^2=-\frac{k^+(1-k^+-q^+)}{1-q^+}\left[q_\perp^2\frac{k^+}{(1-q^+)(1-q^+-k^+)}+p^- -\frac{\lambda^2}{k^+}-\frac{m^2+q_\perp^2}{1-q^+-k^+}-\frac{m^2+q_\perp^2}{q^+}\right]. \tag{A5}$$

$C_{\text{Eul}}=0.577\ldots$ is Euler's constant. The self-energy counterterm that corresponds to the diagram in Fig. 10 is evaluated in a similar fashion. As in the self-energy diagram (see Fig. 9) the instantaneous contribution is included by putting

$$p_2^-=p^- -\frac{m^2+q_\perp^2}{q^+} \tag{A6}$$

on the energy shell. δm is given by

$$\delta m=\frac{1}{2m}\gamma^2\int dk^+d^{(1-\epsilon)}k_\perp\frac{1}{k^+(1-k^+)} \times\frac{\bar{u}(\not p_1+m)u}{p^- -\frac{m^2+k_\perp^2}{1-k^+}-\frac{\lambda^2+k_\perp^2}{k^+}} \tag{A7}$$

with $p_\perp=0, p^+=1$ for the initial fermion. Note that it does not matter whether or not the instantaneous contribution is included, since it is $k_\perp$ independent and therefore gives a vanishing contribution in dimensional regularization.

TABLE III. Total one-loop contribution to $\sigma\to f\bar{f}$.

Set	a_1
I	$0.048\pm0.2\times10^{-4}$
II	$0.285\pm0.6\times10^{-5}$

Performing steps similar to those taken before one finds

$$\delta m=-\frac{\gamma^2}{2m}\int dk^+(1-\epsilon\ln N^2)\left[\frac{1}{\epsilon}-C_{\text{Eul}}\right] \times[-(\lambda^2-m^2)+m^2(1-k^+)+k^+m^2-2m^2]\,, \tag{A8}$$

where

$$N^2=-k^+(1-k^+)\left[p^- -\frac{m^2}{1-k^+}-\frac{\lambda^2}{k^+}\right]. \tag{A9}$$

Table III shows the result for the numerical integration. The result is that rotational invariance is broken at the one-loop level. Numerically we find that the violating piece arises from the instantaneous self-energy contribution.

APPENDIX B

We start out with the two-loop rainbow self-energy diagram (Fig. 5). LCPT yields

$$
\begin{aligned}
I_{\text{rainbow}} = {} & \frac{g_{pp}^4 g_{pp}^2}{(16\pi^3)^2} \int \frac{dk_1^+ dk_2^+ dk_{1\perp} dk_{2\perp}}{p_1^+ p_2^+ p_3^+ p_4^+ k_1^+ k_2^+} \left\{ \frac{1}{P^- - \frac{m^2+p_{1\perp}^2}{p_1^+} - \frac{m^2+q_\perp^2}{q^+}} \right. \\
& \times \frac{\mathrm{Tr}[(\not p_f+m)(\not p_4+m)(\not p_3+m)(\not p_2+m)(\not p_1+m)(-\not q+m)]}{\left[P^- - k_1^- - \frac{m^2+p_{2\perp}^2}{p_2^+} - \frac{m^2+q_\perp^2}{q^+}\right]\left[P^- - k_1^- - k_2^- - \frac{m^2+p_{3\perp}^2}{p_3^+} - \frac{m^2-q_\perp^2}{q^+}\right]\left[P^- - k_1^- - \frac{m^2+p_{4\perp}^2}{p_4^+} - \frac{m^2+q_\perp^2}{q^+}\right]} \\
& \left. + \frac{\mathrm{Tr}[(\not p_f+m)(\not p_4+m)(-\delta m)(\not p_2+m)(\not p_1+m)(-\not q+m)]}{\left[P^- - \frac{m^2+p_{1\perp}^2}{p_2^+} - \frac{m^2+q_\perp^2}{q^+}\right]\left[P^- - k_1^- - \frac{m^2+p_{2\perp}^2}{p_2^+} - \frac{m^2+q_\perp^2}{q^+}\right]\left[P^- - k_1^- - \frac{m^2+p_{4\perp}^2}{p_4^+} - \frac{m^2+q_\perp^2}{q^+}\right]} p_3^+ \right\} \\
& - \frac{g_{pp}^4 g_{pp}^2}{(16\pi^3)^2} \int \frac{dk_1^+ dk_2^+ dk_{1\perp} dl_{2\perp}}{p_1^+ p_2^+ p_3^+ p_4^+ k_1^+ k_2^+} \left\{ \frac{1}{P^- - \frac{m^2+p_{1\perp}^2}{p_1^+} - \frac{m^2+q_\perp^2}{q^2}} \right. \\
& \times \frac{\mathrm{Tr}[(\not p_f+m)(\not p_1+m)(-\not q+m)]\mathrm{Tr}[(\tilde{\not p}_4+m)(\tilde{\not p}_3+m)(\tilde{\not p}_2+m)(1/4m)(\tilde{\not p}_1+m)]}{\left[P_{M,1}^- - k_1^- - \frac{m^2+p_{2\perp}^2}{p_2^+} - \frac{m^2+q_\perp^2}{q^+}\right]\left[P_{M,1}^- - k_1^- - k_2^- - \frac{m^2+p_{3\perp}^2}{p_3^+}\right]\left[P_{M,1}^- - \frac{m^2+p_{4\perp}^2}{p_4^+}\right]} \\
& \left. - \frac{\mathrm{Tr}[(\not p_f+m)(\not p_1+m)(-\not q+m)]\mathrm{Tr}[(\tilde{\not p}_4+m)(-\delta m)(\tilde{\not p}_2+m)(1/4m)(\tilde{\not p}_1+m)]}{\left[P^- - \frac{m^2+p_{1\perp}^2}{p_1^+} - \frac{m^2+q_\perp^2}{q^+}\right]\left[P_{M,1}^- - k_1^- - \frac{m^2+p_{2\perp}^2}{p_2^+}\right]\left[P_{M,1}^- - \frac{m^2+p_{4\perp}^2}{p_4^+} - k_1^-\right]} p_3^+ \right\} .
\end{aligned}
\tag{B1}
$$

The momenta are given by

$$p_1=\left[1-q^+,\lambda^2-\frac{m^2+q_\perp^2}{q^+},-q_\perp\right],$$

$$p_2=p_4=\left[1-q^+-k_1^+,\lambda^2-\frac{m^2+q_\perp^2}{q^+}-k_1^-,-q_\perp-k_{1\perp}\right],$$

$$p_3=\left[1-q^+-k_1^+-k_2^+,\frac{p_2^-}{p_2^+}p_3^+,-q_\perp-k_{1\perp}-k_{2\perp}\right],$$

$$\tilde{p}_2=\tilde{p}_4=(1-q^+-k_1^+,\lambda^2-k_1^-,-q_\perp-k_{1\perp}),$$

$$\tilde{p}_3=\left[1-q^+-k_1^+-k_2^+,\frac{\tilde{p}_2^-}{p_2^+}p_3^+,-q_\perp-k_{1\perp}-k_{2\perp}\right]$$

and $k_1^-=(k_{1,\perp}^2+\lambda^2)/k_1^+, k_2^-=(k_{2,\perp}^2+\lambda^2)/k_2^+, P_{M,1}^-=(p_{1,\perp}^2+\lambda^2)/p_1^+, P^-=\lambda^2$. Note that the third diagram of Fig. 5 which restores covariance at the one-loop level can be taken into account by setting $p_3^-=(p_2^-/p_2^+)p_3^+$ and $\tilde{p}_3^-=(\tilde{p}_2^-/p_2^+)p_3^+$. This rule relates the bad component of the self-energy ($\gamma^+p_1^-$) to the good component ($\gamma^-p_1^+$) and covariance is achieved by construction [26].

The one-loop mass correction δm is given by

$$\delta m=\frac{e^2}{16\pi^3}\int_0^1 dk_2^+ dk_{2\perp}\frac{(1-k_2^+)m^2+m^2}{-k_{2\perp}^2+k_2^+(1-k_2^+)\left[m^2-\frac{m^2}{1-k_2^+}-\frac{\lambda^2}{k_2^+}\right]}.$$

The last two terms of Eq. (B1) correspond to the two-loop mass correction $\delta^{(2)}m$. Note that they are defined quasilocal, i.e., the $\delta^{(2)}m$ subtraction occurs already at the integrands before integration. This makes the expression suitable for numerical integration.

The instantaneous self-energy contribution can be obtained by subtracting a similar expression such as Eq. (B1) from I_{rainbow}, where p_1^- is set on mass shell. The two-loop vertex correction is computed in a similar way.

APPENDIX C

In this section we show by explicit construction that the two-loop rainbow self-energy in naive LCPT contains a noncovariant piece of the form

$$C\frac{\gamma^+}{P^+} \tag{C1}$$

even when all subloops have been rendered covariant. C is independent of the incoming fermion momentum P. Since by assumption the one-loop self-energy $I_{\text{self}}^{(1)}$ (Fig. 9) is covariant, one should be able to express I_{self}^1 in the form (4.1) and (4.2). In this particular example we find

$$\rho_1(s)=\frac{e^2}{8\pi^3}\Omega(D)\int dx(1-x)[\tau(x)]^{D/2-1}\Theta(\tau(x)),$$

$$\rho_2(s)=\frac{e^2}{8\pi^3}\Omega(D)\int dx(2-x)[\tau(x)]^{D/2-1}\Theta(\tau(x))\frac{m}{m^2-s}, \tag{C2}$$

where $\tau(x)=x(1-x)s-[m^2x+\lambda^2(1-x)]$ was introduced. $\Omega(D)$ is the volume of the D-dimensional unit sphere. Thus in a covariant formalism the two-loop rainbow self-energy becomes

$$I^{(2)}=\int\frac{d^Dk}{(2\pi)^D}\frac{\rho_1(s)+(\not{p}-\not{k}+m)\rho_2(s)}{(k^2-\lambda+i\epsilon)(\not{p}-\not{k}-m+i\epsilon)[(p-k)^2-s+i\epsilon]}. \tag{C3}$$

Naive LCPT replaces $I^{(2)}$ with $I_{lc}^{(2)}$, where

$$\begin{aligned}I_{lc}^{(2)}&=\int\frac{ds}{[2(2\pi)^{D-1}]^2}\int\frac{dk^+d^{D-2}k_\perp}{(p^+-k^+)^2k^+}\frac{[\rho_1(s)+(\tilde{\not{p}}_1+m)\rho_2(s)](\tilde{\not{p}}_1+m)}{\left[p^--\frac{(p_\perp-k_\perp)^2+m^2}{p^+-k^+}-\frac{k_\perp^2+\lambda^2}{k^+}\right]\left[p^--\frac{(p_\perp-k_\perp)^2+s}{p^+-k^+}-\frac{k_\perp^2+\lambda^2}{k^+}\right]}\\&=\int\frac{ds}{[2(2\pi)^{D-1}]^2}\int\frac{dk^+d^{D-2}k_\perp}{(p^+-k^+)^2k^+}[\rho_1(s)+(\tilde{\not{p}}_1+m)\rho_2(s)](\tilde{\not{p}}_1+m)\frac{p^+-k^+}{s-m^2}\\&\quad\times\left[\frac{1}{p^--\frac{(p_\perp-k_\perp)^2+m^2}{p^+-k^+}-\frac{k_\perp^2+\lambda^2}{k^+}}-\frac{1}{p^--\frac{(p_\perp-k_\perp)^2+s}{p^+-k^+}-\frac{k_\perp^2+\lambda^2}{k^+}}\right]\end{aligned}$$

and

$$\tilde{p}_1=\left[p^+-k^+,p^--\frac{k_\perp^2+\lambda^2}{k^+}-\frac{m^2+(p_\perp-k_\perp)^2}{p^+-k^+},p_\perp-k_\perp\right].$$

The problem is thus reduced to finding the noncovariant piece of the one-loop self-energy. This has been done [3] and the answer is of the asserted form of Eq. (C1).

APPENDIX D

The two-loop self-energy in the supersymmetric Wess-Zumino model. When dimensional regularization is used in the Yukawa model, there is no need for a one-loop noncovariant counterterm if the boson and fermion masses are equal [27]. This observation could be of crucial importance for the light-cone quantization of supersymmetric field theories. In fact, in Ref. [28] it has been proposed to use the (finite [29]) $N=4$ supersymmetric Yang-Mills theory as a regularized extension of light-cone (3+1)-dimensional QCD (QCD_{3+1}).

Compared to normal theories with similar interactions, supersymmetric theories have a less singular UV behavior. Since part of the problem with the violation of rotational invariance is connected with the loop regularization of light-cone singularities, one might hope that supersymmetry (SUSY) theories are less troubled by noncovariant self-energies. Technically, the improved UV behavior arises from cancellations between various diagrams related by SUSY transformations. Perhaps something similar happens with the noncovariant self-energies in light-cone quantization. As mentioned above this is indeed the case at the one-loop level if one uses dimensional regularization in the transverse coordinates. In order to find out whether such a behavior persists in higher loops, we will investigate the two-loop self-energy of a fermion in the SUSY Wess-Zumino model [30]:

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{2}(\partial_\mu A)^2-\tfrac{1}{2}(\partial_\mu B)^2-\tfrac{1}{2}i\bar{\psi}\gamma^\mu\partial_\mu\psi\\&-\tfrac{1}{2}m^2A^2-\tfrac{1}{2}m^2B^2-\tfrac{1}{2}mi\bar{\psi}\psi-gmA(A^2+B^2)\\&-\tfrac{1}{2}g^2(A^2+B^2)^2-ig\bar{\psi}(A-\gamma_5B)\psi\ ,\end{aligned}\tag{D1}$$

where ψ is a Majorana spinor and A and B are, respectively, a scalar and a pseudoscalar field. The (unsubtracted) one-loop self-energies for bosons and fermions in this model read

$$\tilde{\Sigma}_F=\not{p}_1 f_1(p^2),\quad \tilde{\Sigma}_B=2p^2f_1(p^2)\ ,\tag{D2}$$

where

$$f_1(p^2)=c\int d^{D-2}k_\perp\int_0^1 dx\frac{1-x}{p^2x(1-x)-m^2-k_\perp^2+i\epsilon}\tag{D3}$$

(c is some constant). Performing an on-shell mass subtraction one finds [31]

$$\begin{aligned}\Sigma_F&=(\not{p}-m)f_1(p^2)+(p^2-m^2)\frac{f_2(p^2)}{m}\ ,\\ \Sigma_B&=2[(p^2-m^2)f_1(p^2)+(p^2-m^2)f_2(p^2)]\ ,\end{aligned}\tag{D4}$$

where

$$f_2(p^2)=m^2\frac{f_1(p^2)-f_1(m^2)}{p^2-m^2}\ .\tag{D5}$$

Inserting these one-loop corrections into the one-loop self-energy yields the nested (rainbow-type) contributions to the fermion self-energy at $O(g^4)$ [32]:

$$\begin{aligned}\Sigma^{1a}(p^\mu)&=\tilde{c}\int d^Dk\frac{1}{k^2-m^2+i\epsilon}\frac{(\not{p}-\not{k})f_1((p-k)^2)}{(p-k)^2-m^2+i\epsilon}\ ,\\ \Sigma^{1b}(p^\mu)&=\tilde{c}\int d^Dk\frac{1}{k^2-m^2+i\epsilon}\frac{2(\not{p}-\not{k})f_1(k^2)}{(p-k)^2-m^2+i\epsilon}\ ,\\ \Sigma^{2a}(p^\mu)&=\tilde{c}\int d^Dk\frac{1}{k^2-m^2+i\epsilon}\frac{2(\not{p}-\not{k})f_2((p-k)^2)}{(p-k)^2-m^2+i\epsilon}\ ,\\ \Sigma^{2b}(p^\mu)&=\tilde{c}\int d^Dk\frac{1}{k^2-m^2+i\epsilon}\frac{2(\not{p}-\not{k})f_2(k^2)}{(p-k)^2-m^2+i\epsilon}\ ,\end{aligned}\tag{D6}$$

where $\tilde{c}$ is some constant.

Σ^a and Σ^b correspond to insertions of $\Sigma^{1\text{ loop}}$ into the fermion and boson line, respectively.

Following Sec. IV we substitute in the numerator of the γ^+ component:

$$\begin{aligned}&\Sigma^{1a},\Sigma^{2a}:\ -k^-\mapsto-\frac{k^2-m^2}{k^+}-\frac{k_\perp^2+m^2}{k^+}\ ,\\ &\Sigma^{1b},\Sigma^{2b}:\ p^--k^-\mapsto\frac{(p-k)^2-m^2}{p^+-k^+}+\frac{(p_\perp-k_\perp)^2+m^2}{p^+-k^+}\ .\end{aligned}\tag{D7}$$

As we have shown there naive light-cone quantization (NLCQ) simply neglects the first term thus omitting

$$\begin{aligned}\Delta\Sigma^{1a}&=-\tilde{c}\gamma^+\int\frac{d^Dk}{k^+}\frac{f_1((p-k)^2)}{(p-k)^2-m^2}\\&=-\frac{\tilde{c}}{p^+}\int d^Dk\frac{f_1(k^2)}{k^2-m^2}\ ,\\ \Delta\Sigma^{1b}&=2\frac{\tilde{c}}{p^+}\gamma^+\int d^Dk\frac{f_1(k^2)}{k^2-m^2}\ ,\\ \Delta\Sigma^{2a}&=-2\frac{\tilde{c}}{p^+}\gamma^+\int d^Dk\frac{f_2(k^2)}{k^2-m^2}\ ,\\ \Delta\Sigma^{2b}&=2\frac{\tilde{c}}{p^+}\gamma^+\int d^Dk\frac{f_2(k^2)}{k^2-m^2}\ .\end{aligned}\tag{D8}$$

One can easily verify that the $\Delta\Sigma$ terms arising from f_2 insertions cancel whereas this does not happen for f_1. Thus NLCQ falls short of the correct result by an amount

$$\Delta\Sigma_{\rm NLCQ}=\frac{\tilde{c}}{p^+}\gamma^+\int d^Dk\frac{f_1(k^2)}{k^2-m^2}\neq0\ .\tag{D9}$$

In the beginning of this appendix we raised the hope that SUSY theories are free of the zero-mode problem. Unfortunately this turned out to be false as Eq. (D9) shows. This means that if one wants to use SUSY theories as a regulator for other theories one still has to preregulate them in such a way that there are no noncovariant terms or use some other technique (e.g., noncovariant counterterms) to compensate for $\Delta\Sigma$. This might limit the practical use of SUSY regulators in light-cone quantization considerably.

[1] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[2] For example, only certain components of tensors are calculated while other components are constructed on the basis of general Lorentz-covariance arguments.
[3] M. Burkardt and A. Langnau, Phys. Rev. D **44**, 1187 (1991).
[4] R. J. Perry, A. Harindranath, and K. G. Wilson, Phys. Rev. Lett. **65**, 2959 (1990); R. J. Perry and A. Harindranath, Phys. Rev. D **43**, 4051 (1991); A. Harindranath and R. J. Perry, *ibid.* **43**, 492 (1991); **43**, 3580(E) (1991).
[5] In the Hamiltonian formulation the kinetic mass can be identified with the mass term that appears in the kinetic energy term for the fermions and the vertex mass is the mass that appears in the helicity-flip fermion-boson vertices.
[6] In this simple example there is no energy denominator.
[7] J. Collins, *Renormalization* (Cambridge University Press, Cambridge, England, 1984), p. 66.
[8] S. J. Brodsky, R. Roskies, and R. Suaya, Phys. Rev. D **8**, 4574 (1973).
[9] Invited papers, in *Quarks and Leptons,* Proceedings of the Fourth South African Summer School, Stellenbosch, South Africa, 1985, edited by C. A. Engelbrecht, Lecture Notes in Physics Vol. 248 (Springer-Verlag, New York, 1986).
[10] In the case of the ladder diagram, rotational invariance is preserved even for the corresponding off-shell Green's function. The numbers displayed in Table II have been obtained by setting the fermion mass off shell and equal to the boson mass in the energy denominators. Numerical difficulties occur in the on-shell case at the point $k_\perp \simeq 0, k^+ \simeq 0$ since one energy denominator approaches 0 in this case. We solved this problem by integrating the difference of set I and set II directly rather than computing the difference of the integrated values for both sets.
[11] Provided all noncovariant pieces have been removed from subloops.
[12] It should be noted that this calculation remains correct at higher loops, provided the noncovariant piece has again a structure such as γ^+/p^+.
[13] In principle, a cancellation between noncovariant terms from f_1^n and f_2^n is conceivable. However, in practice we have not encountered an example where this actually happens. In particular, even in the SUSY Wess-Zumino model, such terms do not cancel, as the explicit calculation in Appendix D demonstrates.
[14] D. Mustaki, S. Pinsky, J. Shigemitsu, and K. Wilson, Phys. Rev. D **43**, 3411 (1991).
[15] A. Tang, Ph.D. thesis, SLAC Report No. 351, 1990.
[16] A. Langnau, Ph.D. thesis (in preparation).
[17] G. McCartor, Z. Phys. C **41**, 271 (1988); F. Lenz, in *Hadrons and Hadronic Matter,* Proceedings, Cargese, France 1989, edited by D. Vautherin, F. Lenz, and J. W. Negele (Plenum, New York, 1990); F. Lenz *et al.*, Ann. Phys. (N.Y.) **208**, 1 (1991).
[18] It is not clear whether the zero-mode terms, which appear in (4.7), are related to those zero modes which are necessary to ensure a proper description of the vacuum structure [17].
[19] Again one has to be careful about $q^+=0$ and $q^+=p^+$. It turns out that the additional term $1/[(p-q)^2-s+i\epsilon]$ ensures sufficiently regular behavior for $q^+=0, q^-\rightarrow\infty$ such that no contribution arises from this point.
[20] A similar trick has been used in T.-M. Yan, Phys. Rev. D **7**, 1780 (1973), as well as in Ref. [8].
[21] C. Bouchiat, P. Fayet, and N. Sourlas, Lett. Nuovo Cimento **4**, 9 (1972); S.-J. Chang and T.-M. Yan, Phys. Rev. D **7**, 1147 (1973).
[22] The so-called covariant cutoff regularizes a theory by truncating the Hilbert space with respect to a given total invariant mass [15].
[23] Of course by order we mean order in the coupling constant.
[24] If there are no elementary scalars in the theory one can compare two scattering experiments in the c.m. frame, which are related by a nontrivial rotation, such as, e.g., the total Compton cross section in QED for incoming particles along the z direction, in comparison to the process where the incoming particles fly along the x direction. In $1+1$ dimensions the breakdown of parity invariance in naive light-cone quantization is conceivable. It would be of interest to check this in a nontrivial example. However, one has to be careful in selecting a process which is really sensitive to parity violation because quite often C parity or isospin symmetry (which are manifest in light-cone quantization) guarantee parity invariance for certain amplitudes.
[25] H. C. Pauli and S. J. Brodsky, Phys. Rev. D **32**, 1993 (1987); **32**, 2001 (1987); T. Eller, H.-C. Pauli, and S. J. Brodsky, *ibid.* **35**, 1493 (1987); K. Hornbostel, Ph.D. thesis, SLAC Report No. 0333, 1989; M. Burkardt, Nucl. Phys. **A504**, 762 (1989).
[26] This method works only if the tensor structure is simple enough, so that different components of the tensor can be related easily. For the vertex correction in QED in the Feynman gauge this is not the case. A more general subtraction scheme will be presented elsewhere.
[27] This can be directly read off from Eq. (2.9) in Ref. [3].
[28] S. J. Brodsky, invited lectures presented at Field Theory, the 30th Schladming Winter School in Particle Physics, Schladming, Austria, 1991 (unpublished).
[29] S. Mandelstam, Nucl. Phys. **B213**, 149 (1983); L. Brink, O. Lindgren, and B. E. W. Nilsson, Phys. Lett. **123B**, 323 (1983); M. A. Namazie, A. Salam, and J. Strathdee, Phys. Rev. D **28**, 1481 (1983).
[30] J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).
[31] It should be emphasized that bosons and fermions acquire the same value for the self-mass corrections. Thus, if we would not perform a mass renormalization but rather allow the masses to change via the one-loop correction, this would not induce new noncovariant effects. Since the masses of bosons and fermions would remain equal, the one-loop cancellation of noncovariant terms still applies.
[32] Of course there are more diagrams contributing to the $O(g^4)$ self-energy of the fermion, such as, e.g., two overlapping loops. However, those kinds of graphs do not give rise to noncovariant self-energies even in non-SUSY theories and can thus be omitted in the discussion here.