ARTICLES

Formation of topological defects

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We consider the formation of point and line topological defects (monopoles and strings) from a general point of view by allowing the probability of formation of a defect to vary. To investigate the statistical properties of the defects at formation we give qualitative arguments that are independent of any particular model in which such defects occur. These arguments are substantiated by numerical results in the case of strings and for monopoles in two dimensions. We find that the network of strings at formation undergoes a transition at a certain critical density below which there are no infinite strings and the closed-string (loop) distribution is exponentially suppressed at large lengths. The results are contrasted with the results of statistical arguments applied to a box of strings in dynamical equilibrium. We argue that if point defects were to form with smaller probability, the distance between monopoles and antimonopoles would decrease while the monopole-to-monopole distance would increase. We find that monopoles are always paired with antimonopoles but the pairing becomes clean only when the number density of defects is small. A similar reasoning would also apply to other defects.

I. INTRODUCTION

After a phase transition, it is possible that the system that has undergone the phase transition is left with defects. These defects are regions that are unable to make the transition into the new phase. Line defects and points defects have been observed in the laboratory in nematic liquid crystals [1]. They are also studied theoretically in the context of field theories in the early Universe under the name of cosmic strings [2] and magnetic monopoles. Clearly, the formation of defects can occur in a very wide variety of circumstances. For this reason, in this paper, we will look at the statistical properties of the defects at formation from a general point of view, without any specific model of formation in mind.

A central idea in the present paper is the "probability of defect formation." By this phrase we mean the likelihood of having a defect in a certain *isolated* region of space. The probability could depend on combinatorics, external magnetic fields, existing anisotropies, and other factors. In the case of U(1) strings, for example, the original simulation [3] gave a probability of $\frac{2}{9}$ for a string to pass through an isolated plaquette in the cubic lattice. This number is just due to certain combinatorial factors and depends on the algorithm and the structure of the lattice used in the simulation.

The formation of U(1) strings has been studied by several workers by performing simulations on cubic and tetrahedral lattices. There has also been some effort to analytically understand the statistical properties of the network of strings at formation [4]. All the different studies found that the network of strings could be divided into two distinct classes—the closed strings (loops) and the infinite strings. However, the fraction of string in infinite strings varied from roughly 66% to 80% and it is apparent that the amount of infinite string depends on the algorithm used in the simulation. In this paper, when we look at the formation of strings from a general viewpoint, we will be able to see how the infinite-string density can vary depending on the probability of string formation in the algorithm used in the simulation. The simulation described in detail in Sec. IV, studies the formation of Z_2 strings for which the long-string density has been estimated at about 94% [5,6].

Strings and monopoles in particle physics can arise from very complicated phase transitions in which a very large symmetry group is broken down to some other smaller symmetry group. Although it is a very difficult task to actually find the probability of defect formation in a given model, it seems reasonable to expect that this probability will be different for different symmetry-breaking patterns. Hence, the fraction of infinite strings and the distribution of loops formed during a phase transition will depend on the actual symmetry breaking.

From another angle, the presence of infinite strings in a network of strings can be viewed as a percolation process. The percolation of domains in a system where there are only two states has been well studied by mathematicians and physicists [7]. It has been found that if one of the two states is laid down on a cubic lattice with a probability p, then the percolation probability is 0.37. That is, if $p > 0.37$, then infinite clusters of the state will be found. On the other had, if $p < 0.37$, then the state will only occur in small isolated clusters. If we think of the boundary of the two states as a wall, this says that there are infinite domain walls only if $p > 0.37$ for both the states. For subpercolation probabilities, the domain walls occur as isolated, closed walls. Is there a corresponding effect for strings? What is the critical probability at which strings will percolate? We can also ask about the effect of varying the formation probability of monopoles. There is no percolation here since there are no "infinite" monopoles, but the question is still meaningful.

Defects have been observed and studied in the laboratory in various condensed-matter systems. In such a controlled environment, it may be possible to vary the conditions of the experiment so as to vary the probability of defect formation. For example, if the probability is sensitive to the presence of magnetic fields, a suitable field may be applied externally. (This amounts to having an explicitly broken symmetry and to controlling the strength of the term in the Lagrangian that violates the symmetry.) If this can be done, the results of this paper may even be tested in the laboratory.

The study in this paper also makes connection with the work of Mitchell and Turok [8] on the statistical mechanics of a network of strings that is in dynamical equilibrium. They find that, as the density of strings is lowered, the network of strings goes through a phase transition. At high densities, most of the string density is in infinite strings. At densities below some critical density, the infinite strings go away and only a distribution of loops remains. This qualitative description will also be seen in our work which deals with the statistical properties of the network at formation and not with a network in dynamical equilibrium. However, the detailed statistical features are different. For example, the loop size distribution obtained by us in the low-density regime does not agree with that predicted for a string network in equilibrium in Ref. [8]. This indicates that the statistical results of a network in equilibrium do not apply to the network at formation.

We shall begin by first finding the probability of string formation in Sec. II in certain hypothetical models. This calculation will indicate the kind of calculation that needs to be done in realistic particle-physics theories to estimate the probability of string formation. Section II will also orient the reader with the general strategy of algorithms used for simulating defect formation. In Sec. III we give qualitative arguments describing the effect of a change in the probability of string and monopole formation. These arguments can easily be extended to other defects. In Sec. IV we describe our numerical results for the simulation of strings and in Sec. V we give the results for the formation of monopoles in two dimensions. We conclude in Sec. VI.

II. PROBABILITY OF STRING FORMATION

Here we will estimate the probability of string formation in the context of particle-physics models in which a Higgs field (the order parameter) acquires a vacuum expectation value (VEV) at the phase transition. Strings are formed if the vacuum manifold of the theory is not simply connected. The vacuum manifold M is the coset space G/H where G is the symmetry group before and H the symmetry group after the phase transition. A property of the manifold M is that it is a homogeneous space—it does not contain any preferred points. For example, in the simplest case of U(1) strings, $G=\mathrm{U}(1)$, $H=1$, and $M=S^1$. That is, the vacuum manifold is a circle and is not simply connected.

To find the probability of string formation in some tractable case, we will consider the symmetry breaking $\mathrm{O}(n+1)\rightarrow\mathrm{O}(n)\times Z_2$. This means that M is S^n/Z_2, where S^n is an n-dimensional sphere and the Z_2 tells us that antipodal points on the sphere must be identified. When the Higgs field acquires a VEV, each point in space is assigned a point on M, that is, a pair of points (a point and its antipode) on S^n. Now when a closed circuit is traversed in physical space, a closed curve in M must also be traversed if the mapping is to be single valued. However, there are two ways in which we can get a closed curve in M. To see this, imagine that we start at point A in physical space at which the VEV corresponds to the north and south poles of S^n. When we transverse the closed circuit in physical space and return to point A, we could either have started from the north pole of S^n and returned to the north pole, or else have returned to the south pole. The path running from the north to the south pole is incontractable. If this path is traversed, then there is a string passing through the closed circuit in physical space. Hence to find the probability of a string passing through a closed circuit in physical space, we need to find the ratio of paths on S^n that start at the north pole and end up at the south pole to the number of paths that start at the north pole and end up at the north pole. This has to be done under an additional constraint which comes from realizing that, at the phase transition, the VEV of the Higgs field is uncorrelated beyond some distance ξ. Therefore the pairs of points on S^n assigned to two points in physical space that are separated by distances larger than ξ will be completely uncorrelated. Furthermore, and this is only an assumption, if we know the points on M at two points A and B in physical space that are separated by a distance of about ξ, then the path traced out on M as we go from A to B is the shortest path. This is usually justified by saying that the shortest path gives the configuration of minimum gradient energy.

A concrete way to visualize what is happening is to think in terms of a simulation. Suppose we take a triangular plaquette with vertices A, B, and C in some bigger lattice and wish to determine if there is a string passing through the plaquette in the model where M is S^n/Z_2. If we assume that the distances between each pair of vertices is larger than ξ, the VEV of the Higgs field will take on completely uncorrelated values at the vertices. In the simulation this means that we should randomly assign pairs of points on S^n to each of the vertices. Without loss of generality we can take the pair to be the north (N) and south (S) pole at the vertex A. Next, let the points at B be any other antipodal pair of points P and P′ on S^n (Fig. 1). Assume that P is in the northern hemisphere. Then if we start from the north pole at A, the path traversed in going along the side AB of the triangular plaquette is the arc of the great circle from N to P. Next let the points at the vertex C be Q and Q′. So we now need to connect P to Q or Q′, depending on which is closer. Suppose Q is closer. Then P is connected to Q and the shortest path from P to Q is traversed when we go from B to C. Finally, in going from C to A on the triangle, we will either connect Q to N or Q to S, depending on which is closer. If Q gets connected to S, a string passes through the plaquette, whereas if Q gets connected to N, there is no string.

A little reflection shows that for Q to get connected to

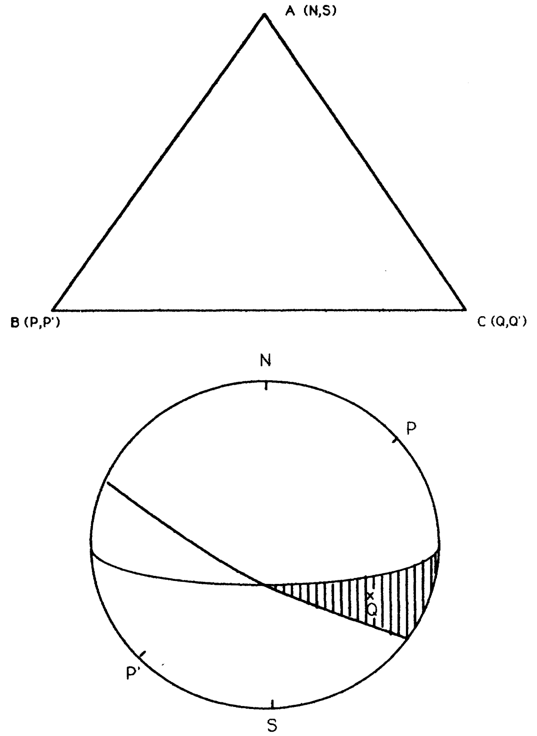

FIG. 1. The triangular plaquette in physical space and the manifold S^2/Z_2. The vertices A, B, C, have been assigned the points (N,S), (P,P′) and (Q,Q′) on S^2, respectively. (Q′ is the antipode of Q and has not been shown in the figure.) If Q lies in the shaded region on S^2, then there must be a string passing through the plaquette.

P and also to S, it must lie in the intersection of two hemispheres: the first hemisphere is the one with P as the pole and the second hemisphere is the one that has S as the pole. Hence the probability for a string to form is given by the ratio of the volume in the intersection of the two hemispheres to the volume of the sphere (see Fig. 1). This ratio is θ/π, where θ is the angular distance between N and P. This gives the probability that a string will form when the points P and P′ are assigned to the vertex B. To get the probability that a string will pass through the plaquette ABC, we need to integrate over all positions of P. Then the result is

$$p_s=\frac{2}{\pi}\frac{\int_0^{\pi/2}d\theta\,\theta\sin^{n-1}\theta}{\int_0^{\pi}d\theta\sin^{n-1}\theta}\ . \tag{1}$$

The integrations can be done but are not needed. It is illustrative, however, to see that for $n=1$ the result is $\frac{1}{4}$ for $n=2$ it is $\frac{1}{\pi}$ and for $n\to\infty$ it is $\frac{1}{2}$. The probability of string formation *grows* with increasing n, and ranges from $\frac{1}{4}$ to $\frac{1}{2}$. It is amusing to note that, at least in these models, the simplest manifold ($n=1$) gives the smallest probability of string formation.

The result of Eq. (1) is not difficult to understand. As the dimension of the sphere increases, more and more volume is packed near the equator. Therefore it becomes increasingly likely for both P and Q to be situated at the equator. Then, in the final step of going from the vertex C to the vertex A, the choice of connecting Q to N or to S must be made randomly. This means that the probability of getting a string tends to half as the dimension of the sphere tends to infinity.

It is not clear if there are particle physics models where the probability of string formation lies outside the range (0.25, 0.50). Perhaps it is possible to use the homogeneity of the manifold M to give some proof that this result is always true.

To find the probability of monopole formation is even more complicated and we shall not attempt it here. However, we would like to mention the case of monopoles when M is S^2/Z_2. In this case, the probability of monopoles is greatly suppressed [9] because even though opposite points on the S^2 are identified, the mapping from a sphere in physical space onto the manifold has to wrap around the whole S^2 for the Higgs field to be single valued everywhere. So, effectively, getting a monopole in this case is like getting a charge-two monopole when M is simply S^2. But there are other complications in this model as it also contains strings that carry magnetic charge. In other words, a loop of string may shrink and form a monopole.

III. QUALITATIVE ARGUMENTS

In this section we will give some qualitative arguments about what to expect as the formation probability of strings and monopoles varies.

First, consider the statistical mechanics of a box of strings in dynamical equilibrium [8]. The density of strings in this box is treated as a free parameter [10]. As we decrease the density of strings, it is expected that only the infinite-string density will decrease and the loop density will remain constant. At high densities, the size distribution of loops is

$$dn(l)=\alpha\frac{dl}{l^{5/2}}\ , \tag{2}$$

where $dn(l)$ gives the number density of loops having length between l and $l+dl$ and α is a constant. This is a scale-invariant distribution of loops [11]. At a certain critical density, the network will undergo a phase transition in which the infinite strings will suddenly disappear and only loops will remain. At low densities, the statistical mechanics arguments predict a loop distribution

$$dn(l)=a\frac{e^{-bl}}{l^c}dl\ , \tag{3}$$

where a,b, and $c=\frac{5}{2}$ are constants.

We shall find that the qualitative features of the statistical arguments will apply in our case. Indeed, the distribution in Eq. (2) has already been seen [3] and a (relatively slow) transition in the properties of the network will also be evident in our results. However, our results give $c=2$ instead of $\frac{5}{2}$ at low densities and we find that, on decreasing the string density by reducing the probability of string formation, not only does the infinite string density decrease but the loop density also increases. This is not what the statistical arguments predict for a box of strings in equilibrium.

There is another approach to the problem that allows us to anticipate, from a "microscopic" point of view, the changes in the network as we reduce the probability of string formation. For this, consider a simulation of string formation on a cubic lattice (Fig. 2). The probability of a string passing through face 1 of the cell is p_s and depends only on the values of the field on the plaquette of face 1. Hence, the probability of a string passing through the opposite face, face 2, is completely independent of what is happening on face 1 and is also p_s. Therefore, the probability that the string bends after entering the cell through face 1 is $1-p_s$. Now, if we reduce p_s, the bending probability increases and the chances of the string closing up to form a loop also increases. This tells us that by reducing the probability of string formation, or equivalently, by decreasing the string density, we can decrease the infinite string density and increase the loop density.

A simple argument allows us to anticipate the changes in the distribution of monopoles as we reduce their probability of formation. Once again, consider a cubic lattice. Suppose that the probability for a monopole to form in one of the cells is $p_m \ll 1$. Now consider a $3\times3\times3$ sublattice. The probability of having one or more monopole charge in this sub-lattice is at most $3^3 p_m$ (to lowest order in p_m) and only depends on the VEV of the Higgs field on the surface of the sublattice. Therefore the probability of having zero charge in the sublattice is greater than $1-27p_m$ and is independent of whether or not there is a monopole present in the central cell of the sublattice. As p_m gets smaller, the probability of having zero charge increases. Hence, given a monopole at the center of the sublattice, it is quite certain that there will also be antimonopole within the sublattice. This means that, in the limit $p_m \rightarrow 0$, an antimonopole will always neighbor a monopole and the average distance between the neighboring monopole and antimonopole will be one lattice spacing in a simulation. Therefore we expect that a reduction of p_m will bring monopoles and antimonopoles closer. But, at the same time, since the number density of monopoles is expected to behave like $1/p_m$, a reduction in p_m will increase the monopole-monopole distance. Therefore a decrease in p_m increases the monopole-antimonopole positional correlation but decreases the monopole-monopole positional correlation.

We shall verify this prediction in the case of monopoles in two dimensions by performing a simulation.

IV. STRING SIMULATION

To simulate the formation of strings, we shall use a very simple algorithm that allows for a continuous variation of the probability of string formation. On the links of the cells of a cubic lattice, we shall assign $+1$ with probability $1-p$ and -1 with probability p. If we now take a plaquette and find the product of these link phases, it can be either $+1$ with probabilty $(1-p)^4+6p^2(1-p)^2+p^4$ or -1 with probabilty $4p(1-p)[(1-p)^2+p^2]$. We shall say that a string passes through the plaquette if the product is -1. It can be verified that this algorithm satisfies the requirement that strings do not have ends and can only occur as loops on a periodic lattice. Furthermore, these strings do not have a direction associated with them and come under the category of Z_2 strings [5,6].

When $p=0.5$ the probability of string formation p_s is also 0.5. As p gets smaller, p_s gets smaller too. Hence, we control the probability of string formation by controlling the parameter p. In the simulations we have used a lattice with periodic boundaries. To decide which string is infinite and which is not, we have used the criterion that if the length of the loop is larger $2L^2$, where L is the lattice size, it should be classified as an infinite string [10]. We have checked that the density in infinite strings obtained is not very sensitive to the exact definition we use.

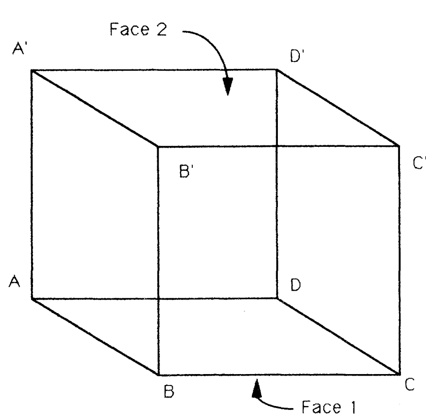

FIG. 2. A cell of a cubic lattice. The presence of a string through face 1 ($ABCD$) of the cell makes no difference as to whether a string passes through face 2 ($A'B'C'D'$) since the criterion for string formation only depends on the vertices and edges of each face.

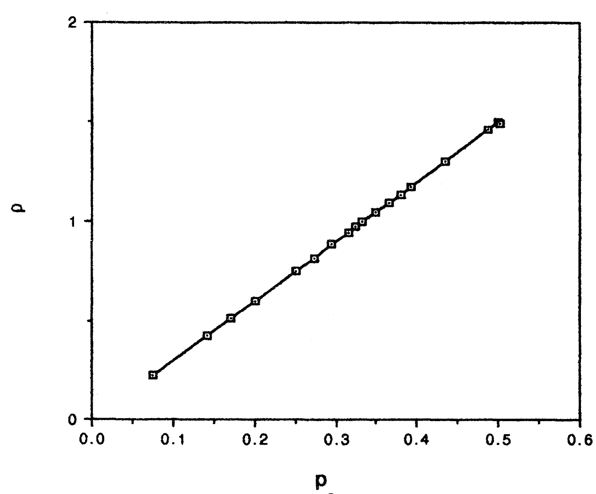

FIG. 3. A plot of the string density ρ vs string probability p_s. The best-fit line gives $\rho=3p_s$ as may be expected for a three-dimensional cubic lattice.

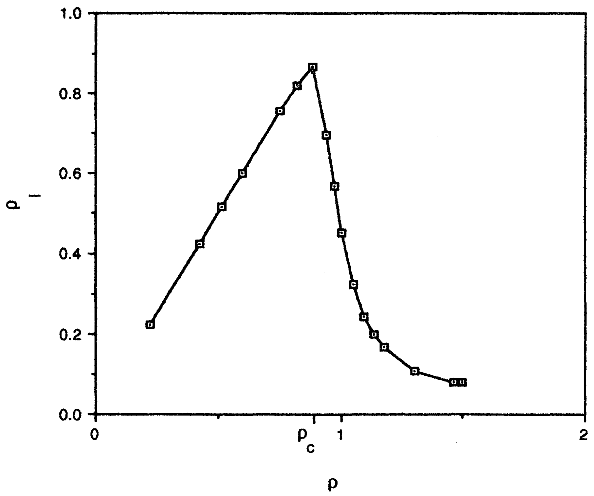

FIG. 4. A plot of the string density in loops ρ_l vs the total string density ρ. For $\rho < \rho_c \sim 0.88$ there are no infinite strings and the graph is just $\rho = \rho_l$. For $\rho > \rho_c$ the density in loops decreases sharply while the density in infinite strings increases.

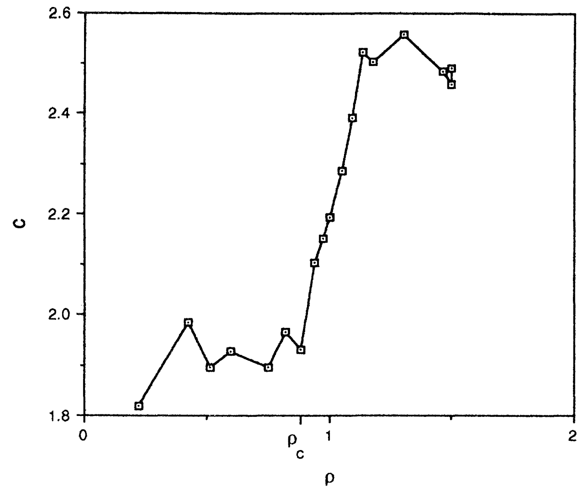

FIG. 6. A plot of the parameter c in Eq. (3) vs the density in strings. For $\rho < \rho_c$ the value of c is consistent with 2.0 while for large ρ, the value is 2.5.

Only when we get very close to the transition density does it becomes difficult to define an infinite string in a stable manner.

All the simulations were done on a 70^3 lattice and the data were accumulated over six runs (10 in the case of $p = 0.02$). When the distribution of loops became sparse, that is, the number of loops of a certain size became less than about 5, the data were truncated. For the smallest few values of p, the number of different loop sizes is not large and we retained all the data.

In Fig. 3 we show the relation between the probability of string formation p_s and the density of strings ρ. The line in Fig. 3 is best fitted by $\rho = 3p_s$. The factor of 3 follows because each cubic cell can have at most three string lengths, that is, six half lengths, associated with it. In what follows, we will study the properties of the network as a function of ρ since the density of strings is a physical property of the network.

In Fig. 4 we plot the density in loops $\rho_l = \rho - \rho_\infty$ vs ρ. The transition in the network properties occurs at the density $\rho_c \sim 0.88$, or at $p_s \sim 0.29$ and $p \sim 0.10$. Below ρ_c there are no infinite strings and all the string is in loops. Note that ρ_l is not constant for densities above ρ_c. This may be understood by saying that as the density of strings is lowered, the infinite strings "break up" into small loops. In this way, a decrease in the infinite string density is accompanied by an increase in the loop density. This is in contrast with the result in Refs. [8,10] where a

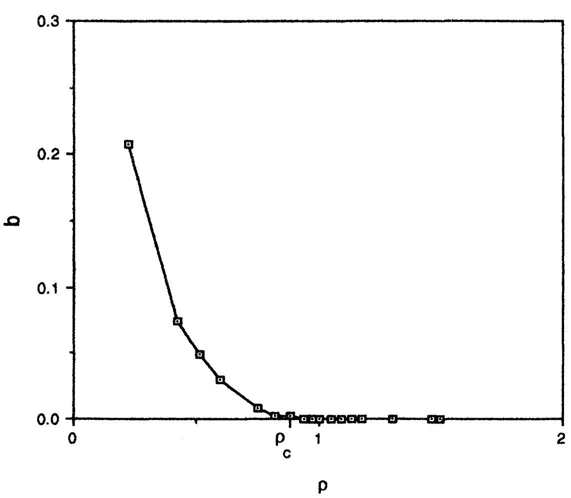

FIG. 5. A plot of the parameter b in Eq. (3) vs the density in strings.

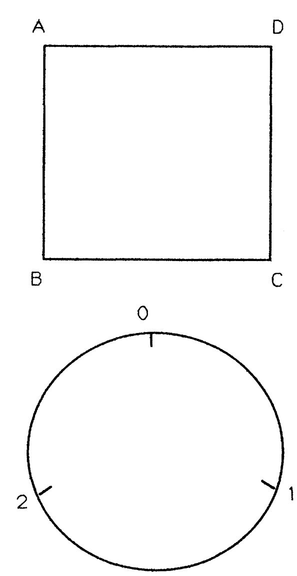

FIG. 7. The square plaquette in physical space and the circular coset space. The coset space is approximated by three points labeled 0, 1, and 2. To each of the vertices A, B, C, and D, a point on the coset space has to be assigned randomly.

network of strings in dynamic equilibrium is studied. In such a system, the infinite string density decreases with a decrease in the total string density but the loop density remains constant. In other words, the infinite strings disappear without breaking up into loops.

The loop distribution obtained for any value of p is well fitted by the functional form of Eq. (3). The parameters a,b, and c can be found from the simulations. The parameter a is just an overall normalization and depends on the total string density and we shall ignore it. The parameter b is zero above the critical density ρ_c and grows as we get to lower densities. A plot of b vs ρ is shown in Fig. 5. In Fig. 6, we show the plot of c vs ρ: the figure shows a remarkable change from a value consistent with 2.0 to a value of 2.5 as we increase ρ through the transition density. In a system of strings at equilibrium, statistical arguments tell us that the value of c would remain at 2.5 whether the strings were at high or low densities [8].

We have performed the string simulations with two other algorithms with similar results. One of the algorithms is a slight modification of the U(1) string algorithm of Ref. [3]. This simulation is also interesting because U(1) strings have an associated direction whereas the strings discussed in the preceding paragraphs do not have an associated direction. As we shall be using the U(1) algorithm to simulate the formation of two-dimensional monopoles, we now describe it.

The plaquette in physical space is taken to be a square while the coset space is a circle which is discretized and taken to be a triangle (see Fig. 7). We assign one of the phases 0, 1, and 2 to each of the vertices A,B,C, and D. If on traversing the plaquette $ABCDA$ we find that we wind around the triangle in coset space, we say that a string must pass through the plaquette $ABCD$. This is just the old algorithm of Ref. [3]. The modification we make to this algorithm is that we assign the phases 0, 1, and 2 to the vertices of the plaquettes not with equal probabilities but with different probabilities. We varied the probability of assigning the phase 2 to any vertex. As every phase must necessarily occur at the vertices of a plaquette to get a string, a change in the probability of phase 2 also changes the probability of getting a string and hence the density of strings. It should be noted that this algorithm simulates the formation of U(1) strings when there is an explicit violation of the U(1) symmetry present in the Lagrangian.

V. MONOPOLES IN TWO DIMENSIONS

The U(1) string simulation immediately allows us to study the formation of monopoles in two dimensions. All that we need to do is to take a slice of the U(1) string simulation. This will give us points on the plane which correspond to monopoles or antimonopoles depending on whether the point is at the intersection of a string (winding number $+1$ in the coset space) with the plane or at the intersection of an antistring (winding number -1) with the plane. We then find the average separation (d) between neighboring monopoles and the average distance $(\bar{d})$ between neighboring monopoles and antimonopoles. A plot of d and $\bar{d}$ versus the number density n of monopoles is shown in Fig. 8. It is apparent that the monopole-monopole distances grows with decreasing density $(d \approx 0.55 n^{-1/2})$ while the monopole-antimonopole distance decreases (slightly) with decreasing density. This confirms the qualitative arguments of Sec. III.

It should be pointed out that the distance between monopoles and antimonopoles is always about one correlation distance, even when the number density of monopoles is high. So we can always imagine the monopoles and antimonopoles as occurring in pairs with a separation of about the correlation distance. When the number density is large, a given pair is close to other pairs and so the pairing is not "clean." As we decrease the formation probability, the number density of monopoles decreases. This is accompanied by a slight decrease in the distance between the members of a pair and a relatively large increase in the distance between pairs. In this way, the pairing becomes clean and the correlation between monopoles and antimonopoles grows.

We show a picture of monopoles in two dimensions in Fig. 9. The positional correlation of monopoles and antimonopoles is obvious. We also notice a few clusters consisting of two pairs. This is because, in the U(1) algorithm that we have used, while it is extremely likely for an antimonopole to be in the cell neighboring a monopole, it is also likely for a second monopole to be in the cell neighboring both the first monopole cell and the antimonopole cell—that is, in the cell diagonally across from the first monopole cell. The presence of this "second-order" correlation between monopoles results in a few of the monopole neighbors being separated by a distance of $\sqrt{2}$ and is seen in Fig. 9 as a cluster of four defects.

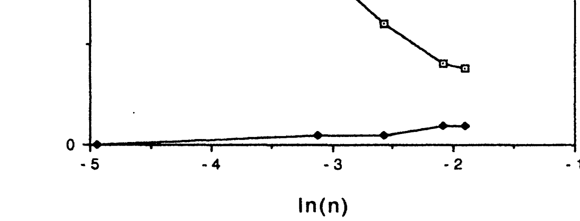

FIG. 8. A logarithmic plot of the average distance between neighboring monopoles d (squares on the plot) and the average distance between neighboring monopoles and antimonopoles $\bar{d}$ (filled circles) vs the monopole number density n. A decrease in the defect density results in an increase in d $(d \approx 0.55 n^{-1/2})$ and a slight decrease in $\bar{d}$. Also note that $\bar{d} \sim 1$ at all densities, that is, the monopoles and antimonopoles are always separated by distances of about one correlation length.

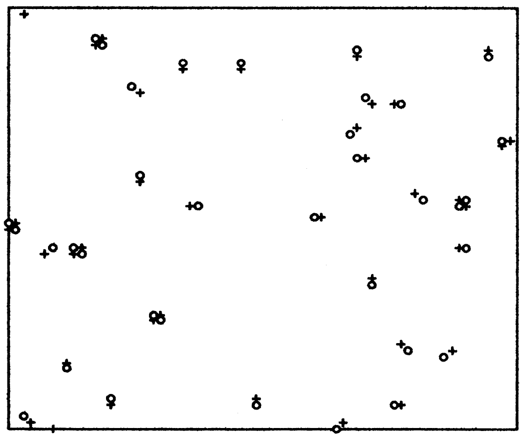

FIG. 9. A picture of the two-dimensional monopole distribution on a 70^2 lattice when the defect density is ~ 0.02. The positional correlation between monopoles (+) and antimonopoles (○) is clear.

We have not done a monopole simulation in three dimensions since we believe that these results give ample indication of what to expect. Furthermore, it would only be useful to do the three-dimensional simulation once we have a physical situation that prescribes a definite algorithm for the formation of monopoles with a variable probability of formation. As mentioned at the end of Sec. IV, the simulation leading to Fig. 9 corresponds to the situation where there is an explicit violation of symmetry in the Lagrangian. For example, in the case of cosmological monopoles, this could occur due to the presence of a primordial magnetic field.

VI. SUMMARY

We have studied the formation of strings and monopoles from a more general point of view where their probability of formation can vary. The network of strings undergoes a transition as we go from high string density to low string density. We have found the loop distribution in both regimes. While the loop distribution in the high-density regime is the same as that obtained by existing statistical arguments for a box of strings in dynamical equilibrium, the distribution at low densities is different. Another new feature is the simultaneous decrease in infinite-string density and increase in loop density as we lower the string density in the high-density regime. An interpretation of this phenomenon is that as the string density is lowered, the infinite strings "break up" into many small loops. In this way, not only do the infinite strings go away but the loop density also increases. This is to be contrasted with the statistical arguments that tell us that as the string density is decreased, the infinite strings just go away without "breaking up" into loops. In this way, a decrease in infinite-string density is not accompanied by an increase in loop density. These features emphasize the fact that a string network at formation is very different from a string network in dynamical equilibrium.

For monopoles, we found that the monopoles and antimonopoles get highly correlated as we decrease their number density. Such a correlation would, in fact, be observed for any defect occurring at low densities. This has been confirmed in the case of textures by numerical simulations done in Ref. [12].

We also find that monopole formation in two dimensions can be viewed as the formation of monopole-antimonopole pairs whose separation is always about one correlation length. At high number densities, these pairs are close to each other and the pairing is not clean. At lower densities, however, the distance between pairs becomes large and it is easy to identify the antimonopole corresponding to any given monopole. This would have some consequences for the cosmological evolution of monopoles since such a correlation would significantly enhance the annihilation of monopole pairs. A more extensive study of this question is presently under way.

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