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Neutrino mass in grand unified SU(15)

N.G. Deshpande, Palash B. Pal, and H.C. Yang
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403
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We explore various scenarios in which neutrinos can obtain mass in a recently discussed model of grand unification based on the gauge group SU(15). Since SU(15) has only left-handed neutrinos, the mass is of Majorana type which necessarily violates lepton-number symmetry. We show that, by introducing the 15-dimensional Higgs-boson multiplet, one can naturally obtain a small neutrino mass through lepton-number violation which already exists at the unification scale.

Recently, SU(15) as a grand unification group has been shown to be interesting for various reasons [1, 2]. It has been shown that, with a particular chain of symmetry breaking, the unification scale can be very low in this model, in the range of 10^7 to 10^{11} GeV [2, 3]. Baryon-number violation is necessarily present in the model [3,4]. However, despite the low unification scale, the proton lifetime can be comfortably consistent with known experimental bounds [5]. It has also been argued that the model is free from problems with grand unified monopoles [6]. Thus, in these essential aspects, the model is very different from the SU(5) grand unification model. In this article, we want to explore the implication of the model for lepton-number violation, which is another important consequence of grand unification models in general. Since the model does not have any right-handed neutrinos, the question of lepton-number violation is intimately connected with the question of the generation of Majorana mass for neutrinos.

We start with a brief description of the model, which will help us establish the notation. The fifteen known left chiral fermionic fields of the first generation are assigned [2] to the fundamental representation of the gauge group SU(15):

$$\Psi = \left(\nu_e e^- e^+ \; u_r u_b u_y d_r d_b d_y \; \hat{u}_r \hat{u}_b \hat{u}_y \hat{d}_r \hat{d}_b \hat{d}_y\right)_L , \quad (1)$$

where the caret denotes antiparticles. The other generations have similar representations. The subscripts r, b, y represent color indices. The symmetry-breaking chain [2] which occupies most of the recent discussion is

$$\begin{aligned} \mathrm{SU}(15) &\xrightarrow{M_G} \mathrm{SU}(3)_l \times \mathrm{SU}(12)_q \\ &\xrightarrow{M_{12}} \mathrm{SU}(3)_l \times \mathrm{SU}(6)_{qL} \times \mathrm{SU}(6)_{qR} \times \mathrm{U}(1)_B \\ &\xrightarrow{M_6} \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \\ &\xrightarrow{M_W} \mathrm{SU}(3)_c \times \mathrm{U}(1)_Q . \end{aligned} \quad (2)$$

This chain gives the possibility of a low unification scale [2, 3], $\sim 10^{11}$ GeV.

The spontaneous symmetry breaking at the scales M_G, M_{12} and M_6 can be done by using the following Higgs-boson multiplets, respectively:

$$\Phi^{[ijk]} : \mathbf{455}, \quad T^i{}_j : \mathbf{224}, \quad H^{\{ij\}}{}_{\{kl\}} : \mathbf{14175}, \quad (3)$$

where square and curly brackets stand for antisymmetrization and symmetrization of indices respectively. At the weak scale M_W, the symmetry breaking can be done by

$$S^{\{ij\}} : \mathbf{120} \text{ or } A^{[ij]} : \mathbf{105} . \quad (4)$$

They couple to fermion bilinears and give mass to the charged fermions when they develop vacuum expectation values (VEV's). Neutrinos, however, do not obtain any mass through the same VEV because, unlike other fermions, the left-handed neutrino does not have a right-handed partner. Thus, the neutrinos can obtain only a Majorana mass if there is lepton number violation in the model.

To discuss the possibility of lepton number violation, we first have to discuss how we assign lepton number to the particles of the model. Lepton number is not part of gauge symmetry of this model. However, all the particles of Ψ have well-defined conventional assignments of lepton number, viz., $+1$ for the first two components, -1 for the third one, and zero for all the rest. We can use this scheme to assign lepton number to the gauge bosons and the Higgs bosons of the model. In other words, for any multiplet $\phi^{ij\cdots}$, we will count a lepton number $+1$ for each occurence of the index 1 or 2, and -1 for each occurence for the index 3. Multiplets with lower indices are considered to be the complex conjugates, and therefore will have just the opposite lepton number.

Assigned this way, there is no lepton number violation in the gauge couplings, where any given gauge boson couples only to a specific pair of fermions and therefore carries a well-defined lepton number. The Yukawa couplings can at best assign a lepton number to any specific component of a Higgs-boson multiplet. Thus, lepton number violation can occur only through the Higgs potential. In fact, the vacuum structure of the model breaks lepton-number symmetry. This can be seen from the first stage of symmetry breaking in Eq. (2). In the basis introduced in Eq. (1), this breaking is induced when the rank-3 tensor Φ develops the following VEV:

$$\langle \Phi^{123} \rangle \propto \varepsilon^{123}\,, \tag{5}$$

while all other components have a zero VEV. Thus, the component of Φ developing a VEV has the quantum numbers of $\nu_e e^- e^+$, i.e., carries a lepton number of 1 unit. The VEV, therefore, breaks lepton number, irrespective of any other possible source of the violation of the same.

Let us now ask ourselves whether this VEV can induce a Majorana mass for the neutrinos. Two units of lepton-number violation can be induced by two VEV's of Φ. But that is not sufficient to generate a Majorana mass for ν_L. The weak isospin I_{3L} has also to be violated by 1 unit. In this model, this violation can come only through the VEV's of S^{ij} or A^{ij}, the rank-2 Higgs boson fields. In the language of gauge-invariant operators, we are then looking for operators such as

$$\begin{aligned} \mathcal{O}_1 &= \Phi^{ijk}\Phi^{pqr}A_{pj}A_{iq}\Psi_k\Psi_r\,, \\ \mathcal{O}_2 &= \Phi^{ijk}\Phi^{pqr}A_{ij}A_{pq}\Psi_k\Psi_r\,, \end{aligned} \tag{6}$$

where the generation indices for Ψ have been suppressed. In the operator $\mathcal{O}_1$, we can also replace A_{ij} with S_{ij}. However, $\mathcal{O}_2$ will vanish by symmetry under the same replacement. If these operators can be generated in the unbroken theory, one can obtain a Majorana mass of the neutrino once Φ and A (or S) develop a VEV.

It is now easy to see that the operators of Eq. (6) cannot be generated by quantum corrections if the Higgs-boson content of the model consists of the particles shown in Eqs. (3) and (4). This is because, with only these particles, the full Lagrangian of the model can be shown to be invariant under the global symmetry

$$\Phi \rightarrow e^{i\theta}\Phi\,, \tag{7}$$

which forbids the operators of Eq. (6). It must be noted at this point that although the VEV of Φ^{123} breaks this symmetry spontaneously, no Goldstone boson results since the same VEV is breaking the gauge symmetry and therefore the would-be Goldstone bosons are absorbed by gauge bosons. Another way of saying this is that it is possible to assign the global lepton numbers in an alternative way such that Φ does not carry lepton number, so that lepton number is not broken when Φ develops a VEV. Whichever way one prefers to look at it, the physical implication is the same, viz. that the neutrinos are massless in this model.

This conclusion remains unchanged even if one introduces, after Frampton and Kephart [5], a 5-index antisymmetric Higgs multiplet $h^{[ijklm]}$ to break $\mathrm{U}(1)_B$ at the scale M_6. Apart from terms which are invariant under the operation in Eq. (7) and similar independent phase rotation of other Higgs multiplets, there are now nontrivial trilinear terms in the Higgs potential such as

$$\epsilon_{(15)}h^{(5)}h^{(5)}h^{(5)} \quad \text{and} \quad \Phi_{(3)}A_{(2)}h^{(5)}\,, \tag{8}$$

where the numbers in the parentheses stand for the number of upper or lower indices. These terms violate the symmetry of Eq. (7). However, even these terms are invariant under a discrete symmetry

$$\Phi^{(3)} \rightarrow \exp(2\pi i/3)\,\Phi^{(3)}, \quad h^{(5)} \rightarrow \exp(2\pi i/3)\,h^{(5)}\,. \tag{9}$$

This is enough to forbid the opertors in Eq. (6), and consequently neutrino mass, from arising at any order. Thus the field h does not help in generating neutrino mass. Since its only utility in the model is in breaking $\mathrm{U}(1)_B$, which can be broken by a VEV in the rank-3 tensor Φ anyway [3], we will not consider this multiplet in subsequent discussion.

Masslessness of neutrinos is not ruled out unequivocally by terrestrial experiments. There are, however, some claims of evidence of nonzero neutrino masses from β-decay experiments [7], although they remain controversial. On the other hand, there is ample motivation from astrophysics and cosmology for assuming nonzero masses for neutrinos. For example, the solar-neutrino problem [8] can be resolved by assuming small neutrino masses and mixing. In light of these, we try to see whether nonzero neutrino mass can be accommodated in SU(15) model by some simple modification.

Previously, we have seen that there is some symmetry which prohibits neutrino mass. The simplest way to avoid any such symmetry might be to add, to the Higgs boson content of the model, a Higgs multiplet φ transforming like the fundamental representation of the gauge group. In this case, there are the following trilinear couplings which are not invariant under the independent phase rotations of different Higgs multiplets:

$$\mu\Phi^{ijk}A_{ij}\varphi_k + \mu' S^{ij}\varphi_i\varphi_j\,. \tag{10}$$

Although they respect a discrete symmetry

$$\Phi \rightarrow -\Phi, \quad \varphi \rightarrow -\varphi\,, \tag{11}$$

this symmetry does not forbid the operators in Eq. (6). Thus, neutrino mass can be generated once the multiplet φ is introduced in the theory.

The actual magnitude of the mass will depend on the exact mechanism by which I_{3L} is broken in the theory. As we said earlier, it can be broken by VEV's in S^{ij} or A^{ij}. Below, we consider various possibilities, and examine how a neutrino mass is generated in each case. For the sake of simplicity, we ignore generational mixing and consider the mass of ν_e. The masses of neutrinos from other generations can be estimated in a similar fashion.

Case I : Only $\langle A \rangle \neq 0$. In this case, the mass matrices of the charged fermions are antisymmetric at the tree level. For three generations, this yields one zero eigenvalue and two nonzero ones which are equal. This is untenable, so we discard this possibility.

Case II : $\langle S \rangle \neq 0$, *no A present.* In this case, the charged fermions have a symmetric mass matrix, which can explain the mass patterns observed in the real world. A VEV of S^{23}, for example, can give the mass of the electron. The 1-loop diagram of Fig. 1 now shows that the component S^{11} develops a VEV as well, which gives mass to the neutrino. Denoting the strength of the quartic coupling $\Phi^{ijk}\varphi_i S_{jl} T^l{}_k$ by λ, an order of magnitude estimate gives

$$m_\nu \sim \frac{m_e}{M_W}\frac{\lambda^2\mu' v_\Phi^2 v_S^2}{M_G^2 M_{S_{11}}^2}\,. \tag{12}$$

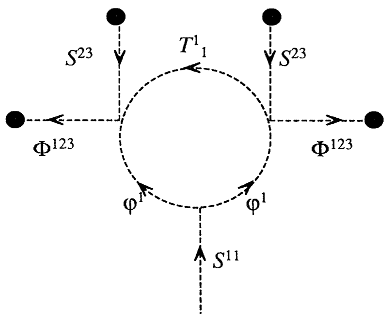

FIG. 1. 1-loop diagram that induces a VEV for S^{11} through the VEV's of S^{23} and Φ.

The factor M_G^2 in the denominator arises out of the loop integration, assuming the masses of all the particles in the loop are of order M_G, the heaviest scale in the model. The symmetry-breaking pattern gives $v_\Phi \simeq M_G$ and $v_S \simeq M_W$, omitting the gauge coupling constants. As for the trilinear coupling μ', we note that the mass of S^{23} coming out of the self-energy graph of Fig. 2 must not exceed the weak scale so that the S^{23} VEV can be of order M_W. This puts a restriction of $\mu' \lesssim M_W$. For the same reason, we can put the constraint $\lambda \lesssim M_W/M_G$. Using the maximum possible value of μ' and λ, we then obtain

$$m_\nu \sim \left(\frac{M_W^4}{M_{S_{11}}^2 M_G^2}\right) m_e \,. \tag{13}$$

Since S^{11} is in the same $SU(3)_l$ submultiplet with S^{23}, and S^{23} mass is $\sim M_W$, we expect $M_{S_{11}} \sim M_6$, so that Eq. (13) gives $m_{\nu_e} \sim (M_W^4/M_6^2 M_G^2) m_e$. If M_6 is close to M_W which would be interesting for experimental purposes, one obtains $m_{\nu_e} \sim 10^{-6}\,\text{eV}$ if $M_G \sim 10^8\,\text{GeV}$. The interesting aspect of this magnitude is that, since the neutrino masses scale like the charged-fermion masses, one expects $m_{\nu_\tau} \sim 10^{-3}\,\text{eV}$, which is in the right range for the solution of the solar-neutrino problem through the resonant neutrino oscillation mechanism.

Case III : $\langle S \rangle \neq 0$, $\langle A \rangle = 0$. Here also, neutrino mass comes through 1-loop diagrams. The diagram coming from Fig. 1 are of course present, but there are additional diagrams shown in Fig. 3. The estimate of this diagram gives

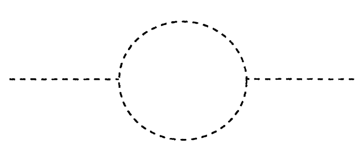

FIG. 2. A typical contribution of the self-energy of S through scalar loops. The induced mass must be $\lesssim M_W$ for the consistency of the theory.

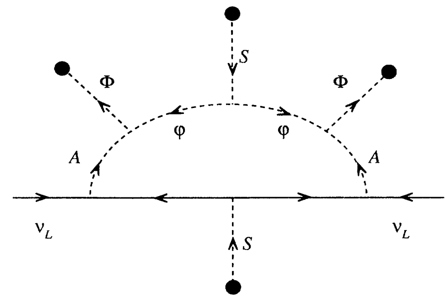

FIG. 3. 1-loop diagram for neutrino mass when both A and S are present in the theory, but A has no VEV.

$$m_\nu \sim \frac{m_e}{M_W}\frac{\mu^2\mu' v_\Phi^2 v_S^2}{M_G^6} \,, \tag{14}$$

where f is the Yukawa coupling of the antisymmetric multiplet A^{ij}, which is not related to the masses of fermions since A does not have any VEV. The natural value for the trilinear coupling μ is $\sim M_G$. Using the estimates of other parameters from the previous case, we obtain

$$m_\nu \sim f^2 (M_W/M_G)^2 m_e \,. \tag{15}$$

If $f \sim 1$, this contribution seems larger than the contribution of Eq. (13), particularly if $M_6 \gg M_W$.

Case IV : $\langle S \rangle \neq 0$, $\langle A \rangle \neq 0$. In this case, there can be tree-level contributions to the VEV of S^{11}, as shown in Fig. 4. This gives rise to neutrino masses at the tree level. A quick estimate gives

$$m_\nu \sim \frac{f\mu^2\mu' v_\Phi^2 v_A^2}{M_{S_{11}}^2 M_\varphi^4} \,. \tag{16}$$

As before, we can use $\mu' \sim M_W$, $M_\varphi \sim M_G$ and $M_{S_{11}} \sim M_6$. In addition, we now need $\mu \lesssim M_W$ in order that the mass of A remains around the weak scale. Then we obtain

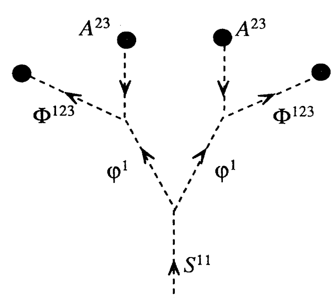

FIG. 4. Tree diagram that induces a VEV for S^{11} when both A and S are present in the theory, and A has VEV's which break $SU(2)_L$ by $\frac{1}{2}$ units.

$$m_\nu \sim \frac{f M_W^5}{M_G^2 M_6^2}, \tag{17}$$

assuming $v_A \sim M_W$. If $f \sim m_e/M_W$, this estimate coincides with the one in Eq. (13). But f can be larger, and thus one can obtain larger values of neutrino mass.

The essential points in the cases II–IV can now be summarized. The component S^{11} of the symmetric multiplet of Higgs bosons couples to the fermion bilinear $\nu_L \nu_L$, so neutrinos obtain a Majorana mass if S^{11} develops a VEV. In cases II and IV, we have shown that in the presence of the 15-dimensional Higgs boson multiplet a consistent solution of the minimization conditions of the Higgs potential demands some nonzero VEV of S^{11} if $\langle \Phi^{123} \rangle \neq 0$ and either S or A has VEV's which break I_{3L} by half units. The latter VEV's are necessary for giving masses to the charged fermions as well as to the W and the Z gauge bosons.

Figure 3 gives an alternative way of giving neutrino masses. In this case, the required $\Delta I_{3L} = 1$ comes not from the VEV of one Higgs particle carrying $I_{3L} = 1$, but rather from two different VEV's each of which contribute $\Delta I_{3L} = \frac{1}{2}$.

To summarize, we have shown that, if one uses only the Higgs-boson multiplets shown in Eqs. (3) and (4), neutrinos remain exactly massless in the SU(15) model. This conclusion is unchanged even if one introduces a 5-index antisymmetric multiplet. However, by the introduction of a 15-dimensional Higgs multiplet φ, one can give masses to the neutrinos. We have discussed how different scenarios of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ breaking can result in different sources of neutrino mass. The masses are naturally small in all scenarios, and they can have the right magnitude to be relevant for the MSW solution of the solar-neutrino problem.

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