Electroweak corrections in the nonminimal technicolor model

Robert N. Cahn
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

Mahiko Suzuki
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
and Physics Department, University of California, Berkeley, California 94720
(Received 29 March 1991)

The electroweak radiative corrections induced by a one-family technicolor model are examined using the dispersion relation technique proposed by Peskin and Takeuchi. Large uncertainties in Δr remain because its value is sensitive to the masses of the technipions and technirho.

Technicolor schemes for breaking electroweak symmetry [1,2] remain attractive despite the absence of fully realistic models. Direct searches for technicolored particles are under way at existing high-energy hadron colliders and the extent of these searches will be increased enormously at the CERN Large Hadron Collider (LHC) and the Superconducting Super Collider (SSC) in Texas. Indirect searches are possible by looking for deviations from the predictions of the standard model in the fundamental quantities such as the W-Z mass difference. The influence of technicolor on electroweak corrections was investigated as early as 1983 by Renken and Peskin [3], and subsequently by Lynn, Peskin, and Stuart [4]. Improved measurements of the W and Z masses have renewed interest in these effects and they have been reinvestigated by Golden and Randall [5], by Peskin and Takeuchi [6], and by Holdom and Terning [7]. These authors draw generally negative conclusions because they predict that the "one-family" model of technicolor [8] would change the W-Z mass difference by about 500 MeV relative to the prediction of the standard model. The good agreement at the level of about 300 MeV with the predicted value is some circumstantial evidence against the model. We find, however, that the mass shift is subject to large uncertainties.

The one-family model is comprised of four weak isodoublets of technifermions, three techniquark doublets (U,D) carrying $SU(3)_{color}$ and one technilepton doublet (N,E) that is colorless. The technifermions all carry technicolor, which we indicate by $SU(N_{TC})$. The eight technifermions give a theory that is invariant under global $SU(8)\times SU(8)$, except for gauge interactions and masses. The spontaneous symmetry breaking that occurs when the technifermion condensate forms breaks the global symmetry to the diagonal SU(8). This generates 63 Goldstone bosons, three of which are absorbed by the W and Z. The analogue F_π of the usual pion-decay constant $f_\pi=93$ MeV is related to $v=2^{-1/4}G_F^{-1/2}=246$ GeV, which in the conventional model is the vacuum expectation value of the Higgs field, and N_d, the number of technidoublets in the model, by

$$v^2=N_dF_\pi^2 \ . \tag{1}$$

For the one-family model, $N_d=4$ so $F_\pi=123$ GeV.

The relation between m_W and m_Z is given in terms of Δr, which is determined by radiative corrections:

$$m_W^2=\frac{1}{2}\left[1+\left[1-\frac{4\pi\alpha(1+\Delta r)}{\sqrt{2}m_Z^2G_F}\right]^{1/2}\right]m_Z^2 \ . \tag{2}$$

A shift $\delta\Delta r$ produces a change in the W mass with the Z mass fixed of

$$\Delta m_W=-17\ \text{GeV}\ \delta\Delta r \ . \tag{3}$$

The value of Δr is determined by the vacuum-polarization functions of the W, Z, and photon. If we assume that isospin is not broken (as it would be if there were large mass splittings inside isodoublets), and if the masses of the W and Z are dropped, the expression for Δr may be written [6]

$$\Delta r=-\frac{g^2}{2\pi}\int\frac{ds}{s^2}[\mathrm{Im}\Pi_{VV}(s)-\mathrm{Im}\Pi_{AA}(s)] \ . \tag{4}$$

Traditional current-algebra and vector-dominance techniques provide a framework in which we can estimate the contributions to Eq. (4). These principles are not sufficient to determine the result and we shall therefore investigate several alternative assumptions about the dynamics.

Among the technivector particles is one that we call the ρ_{TC} and whose composition is

$$\rho_\mu=\frac{1}{\sqrt{N_d}}\sum_i\rho_\mu^i \ . \tag{5}$$

The sum extends over $i=1,\ldots,N_d$, that is, over the techniquark and technilepton doublets, and ρ_μ^i is the ρ-like meson made from technifermions of the ith doublet. Analogous to the technirho, we define a pure-isospin technipion multiplet:

$$\pi=\frac{1}{\sqrt{N_d}}\sum_i\pi_i \ . \tag{6}$$

where π_i is the triplet of "pions" made from quarks or leptons of the ith doublet.

The axial-vector and vector currents are

$$\mathbf{V}_\mu=\sum_i \bar{Q}_i \gamma_\mu \frac{\tau}{2} Q_i \ , \tag{7}$$

$$\mathbf{A}_\mu=\sum_i \bar{Q}_i \gamma_\mu \gamma_5 \frac{\tau}{2} Q_i \ . \tag{8}$$

The isospin associated with $\mathbf{V}_\mu$ is the sum of the isospins from each generation.

We assume a strong form of vector dominance whereby the couplings of the vector current are determined by the couplings of the technirho. The coupling of the vector current to the technirho itself is designated $m_{\rho\mathrm{TC}}^2/f_{\rho\mathrm{TC}}$. The coupling of the technirho to technipion pairs at zero-momentum transfer is taken to be $g_{\rho\mathrm{TC}}$. This coupling is the same for all technipion pairs since the technirho couples to the technipions according to their isospin, which is in all cases equal to one. Thus the coupling of the vector current to the pairs is, at $q^2=0$, $g_{\rho\mathrm{TC}}/f_{\rho\mathrm{TC}}$, so vector-dominance requires $g_{\rho\mathrm{TC}}=f_{\rho\mathrm{TC}}$.

The partial width of the technirho into a single technipion-technipion channel is

$$\Gamma=\frac{g_{\rho\mathrm{TC}}^2 m_{\rho\mathrm{TC}}}{48\pi}\left[1-\frac{4m_{\pi\mathrm{TC}}^2}{m_{\rho\mathrm{TC}}^2}\right]^{3/2} , \tag{9}$$

where we have implicitly assumed that $g_{\rho\mathrm{TC}}$ varies little between $q=0$ and $q^2=m_{\rho\mathrm{TC}}^2$. Since there are N_d^2 technipion triplets (ignoring the losses to the W and Z),

$$\Gamma=N_d^2\frac{g_{\rho\mathrm{TC}}^2 m_{\rho\mathrm{TC}}}{48\pi}\left[1-\frac{4m_{\pi\mathrm{TC}}^2}{m_{\rho\mathrm{TC}}^2}\right]^{3/2} , \tag{10}$$

assuming, for simplicity, degeneracy of all the technipions. The imaginary part of the technirho-technirho vacuum polarization is, analogously,

$$\mathrm{Im}\Pi_{\rho\rho}(s)=-\frac{N_d^2 g_{\rho\mathrm{TC}}^2 s}{48\pi}\left[1-\frac{4m_{\pi\mathrm{TC}}^2}{s}\right]^{3/2} . \tag{11}$$

On the other hand, we express the vector-current spectral function as

$$\Pi_{VV}(s)=\left[\frac{m_{\rho\mathrm{TC}}^2}{f_{\rho\mathrm{TC}}}\right]^2\frac{1}{s-m_{\rho\mathrm{TC}}^2-i\,\mathrm{Im}\Pi_{\rho\rho}(s)} , \tag{12}$$

so

$$\mathrm{Im}\Pi_{VV}(s)=\left[\frac{m_{\rho\mathrm{TC}}^2}{f_{\rho\mathrm{TC}}}\right]^2\frac{\mathrm{Im}\Pi_{\rho\rho}(s)}{(s-m_{\rho\mathrm{TC}}^2)^2+[\mathrm{Im}\Pi_{\rho\rho}(s)]^2} . \tag{13}$$

This expression is heuristic. In particular, we should regard $m_{\rho\mathrm{TC}}$ to be a function of s.

The behavior of the vector contribution to Δr near threshold is obtained directly from Eq. (13) by noting that there $\mathrm{Im}\Pi_{\rho\rho} \ll m_{\rho\mathrm{TC}}^2$ so

$$\mathrm{Im}\Pi_{VV}(s)=\frac{1}{f_{\rho\mathrm{TC}}^2}\mathrm{Im}\Pi_{\rho\rho}(s) \tag{14}$$

$$=-\frac{sN_d^2}{48\pi}\left[1-\frac{4m_{\pi\mathrm{TC}}^2}{s}\right]^{3/2} , \tag{15}$$

where we used the vector-dominance relation $f_{\rho\mathrm{TC}}=g_{\rho\mathrm{TC}}$. Were we to use this value from a threshold $4m_{\pi\mathrm{TC}}^2$ to a cutoff Λ^2, ignoring the threshold suppression and the contribution from the axial-vector current, we would have, from Eq. (4),

$$(\Delta r)_{\mathrm{nonresonant}}=\frac{g^2N_d^2}{96\pi^2}\ln\frac{\Lambda^2}{4m_{\pi\mathrm{TC}}^2} , \tag{16}$$

the result of Golden and Randall [5].

The value of Δr derived from these formulas is uncertain because of ambiguities in the parameters. The formula of Golden and Randall depends on the threshold specified by $m_{\pi\mathrm{TC}}$ and the cutoff Λ. If we take $\Lambda=4\pi v/\sqrt{N_d}=1.5$ TeV and $N_d=4$, $(\Delta r)_{\mathrm{nonresonant}}=0.030$, giving a shift of -520 MeV, while taking $\Lambda=4\pi v/N_d=0.75$ TeV gives $(\Delta r)_{\mathrm{nonresonant}}=0.020$. These results and those from other approximations are summarized in Table I.

This calculation is open to certain criticisms. Ignoring the threshold factor $(1-4m_{\pi\mathrm{TC}}^2/s)^{3/2}$ increases the resulting shift significantly. Furthermore, this approach ignores the dominant dynamics, that of the technirho. Even if one declines to introduce a technirho explicitly, it is inconsistent to assume that the amplitude determined by chiral symmetry persists away from threshold. This can be expressed in terms of the Omnes function

$$F(s)=\exp\left[\frac{s}{\pi}P\int ds'\frac{\delta(s')}{s'(s'-s)}\right] , \tag{17}$$

TABLE I. Values of Δr resulting from the technicolor corrections in a variety of models. The factor $\frac{3}{4}$ results from subtracting $\frac{1}{4}$ of the vector contribution to represent the axial contribution in Eq. (4).

Model	Δr
Nonresonant, $\Lambda=1.5$ TeV, $m_{\pi\mathrm{TC}}=91$ GeV	0.030
Nonresonant, $\Lambda=0.75$ TeV, $m_{\pi\mathrm{TC}}=91$ GeV	0.020
Narrow resonance, $m_{\rho\mathrm{TC}}=1$ TeV	$0.026\times\frac{3}{4}=0.019$
Resonance, $m_{\rho\mathrm{TC}}=1$ TeV, $m_{\pi\mathrm{TC}}=100$ GeV	$0.030\times\frac{3}{4}=0.022$
Resonance, $m_{\rho\mathrm{TC}}=1$ TeV, $m_{\pi\mathrm{TC}}=200$ GeV	$0.020\times\frac{3}{4}=0.015$

which gives the energy dependence of the form factor required by a given phase shift, $\delta(s)$. Of course we do not know the technipi-technipi phase shift in the technirho channel, so we cannot evaluate this reliably. The tree-level calculation from a chiral Lagrangian gives, for the p-wave channel for $N_d=4$,

$$a_{l=1}^{\text{Born}}=e^{i\delta_{l=1}}\sin\delta_{l=1}=\frac{(s-4m_{\pi\text{TC}}^2)^{3/2}}{3\pi v^2\sqrt{s}} . \tag{18}$$

Unitarizing this amplitude necessarily introduces ambiguity. The choice

$$\tan\delta_{l=1}=a_{l=1}^{\text{Born}} \tag{19}$$

gives a phase shift that goes asymptotically to $\pi/2$, a pathological behavior. From Eq. (17) we see that the form factor behavior asymptotically is

$$F(s\rightarrow\infty)\propto s^{-\delta(\infty)/\pi} . \tag{20}$$

An ordinary resonance, for which $\delta(\infty)=\pi$, will give a form factor falling as $1/s$, while the peculiar one resulting from Eq. (19) will fall as $s^{-1/2}$. In any event, the rising, positive phase shift in the p-wave channel will inevitably produce a form factor that rises above unity near threshold and then falls far above threshold, whether or not a resonance is introduced explicitly. The important form-factor effects and the threshold factor make estimates based on Eq. (16) unreliable.

The simplest way to introduce the technirho is to treat it as a narrow resonance and assume it saturates the contribution we have from Eq. (13). Then we find

$$\text{Im}\Pi_{VV}(s)=-\pi\left[\frac{m_{\rho\text{TC}}^2}{f_{\rho\text{TC}}}\right]^2\delta(s-m_{\rho\text{TC}}^2) , \tag{21}$$

and a contribution to Δr:

$$(\Delta r)_{\text{resonant}}^{V}=\frac{g^2}{2f_{\rho\text{TC}}^2} . \tag{22}$$

Before proceeding further, we invoke the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [9,10], which follows from current algebra and vector dominance. In the usual QCD context it reads

$$f_\rho^2=\frac{m_\rho^2}{2f_\pi^2} . \tag{23}$$

Our vector and axial-vector currents, Eqs. (7) and (8), obey the usual current-algebra commutation relations. Our PCAC (partial conservation of axial-vector current) statement is

$$\partial\mathbf{A}=\sqrt{N_d}F_\pi m_{\pi\text{TC}}^2\pi , \tag{24}$$

but on the other hand the connection between F_π and v is precisely $N_dF_\pi^2=v^2$, so we see that the appropriate form of the KSRF relation is

$$f_{\rho\text{TC}}^2=\frac{m_{\rho\text{TC}}^2}{2v^2} . \tag{25}$$

As in the instance of the usual KSRF relation, the mass of the ρ is not determined. For the vector contribution to Δr we have

$$(\Delta r)_{\text{resonant}}^{V}=\frac{g^2v^2}{m_{\rho\text{TC}}^2}=0.026\left[\frac{1\text{ TeV}}{m_{\rho\text{TC}}}\right]^2 . \tag{26}$$

The value of $f_{\rho\text{TC}}$ is given by the KSRF relation as $f_{\rho\text{TC}}^2=m_{\rho\text{TC}}^2/(2v^2)$, but still the value of $m_{\rho\text{TC}}$ is unspecified. Analysis of the leading planar diagrams in a $1/N_{\text{TC}}$ expansion gives the scaling rule $m_{\rho\text{TC}}^2/v^2\propto 1/N_{\text{TC}}$ [11]. Combining this with the relation $v^2=N_dF_\pi^2$ implies that at fixed F_π, $m_{\rho\text{TC}}^2/v^2\propto 1/N_{\text{TC}}N_d$. If this relation is true even for QCD with N_{TC} replaced by 3, then

$$m_{\rho\text{TC}}^2/v^2=(3/N_{\text{TC}})(1/N_d)(m_\rho^2/f_\pi^2) ,$$

or

$$m_{\rho\text{TC}}=(3/N_{\text{TC}})^{1/2}(4/N_d)^{1/2}\times 1\text{ TeV} .$$

The narrow resonance approximation differs from the nonresonant approximation by receiving its dominant contribution from a different region, and by having a different dependence on N_d. With the hypothesized $m_{\rho\text{TC}}^2\propto 1/N_d$, we have $(\Delta r)_{\text{resonant}}\propto N_d$, rather than N_d^2, as in Eq. (16).

The full expression for $\text{Im}\Pi_{VV}$ is obtained from Eqs. (11) and (13) with the result

$$\text{Im}\Pi_{VV}(s)=-\frac{sN_d^2}{48\pi}\mathcal{R} , \tag{27}$$

where

$$\mathcal{R}=\frac{(1-4m_{\pi\text{TC}}^2/s)^{3/2}}{\left[1-\frac{s}{m_{\rho\text{TC}}^2}\right]^2+\left[\frac{N_d^2sf_{\rho\text{TC}}^2}{48\pi m_{\rho\text{TC}}^2}\right]^2\left[1-\frac{4m_{\pi\text{TC}}^2}{s}\right]^3} . \tag{28}$$

If the full expression given in Eq. (28) is used and the result integrated numerically, we find for $N_d=4$ a vector contribution $\Delta r_{\text{resonant}}=0.030$ if we use $m_{\pi\text{TC}}=100$ GeV. If instead we use $m_{\pi\text{TC}}=200$ GeV we find $\Delta r_{\text{resonant}}=0.020$. Of course we do not really know what to choose for $m_{\pi\text{TC}}$. The integrand from the vector spectral function is shown in Fig. 1.

Rather than attempt to model the axial-vector-current spectral function, we rely on the Weinberg sum rules [12] and the KSRF relation. The Weinberg sum rules in this context are

$$\frac{1}{\pi}\int_0^\infty\frac{ds}{s}[\text{Im}\Pi_{VV}(s)-\text{Im}\Pi_{AA}(s)]=v^2 , \tag{29}$$

$$\frac{1}{\pi}\int_0^\infty ds[\text{Im}\Pi_{VV}(s)-\text{Im}\Pi_{AA}(s)]=0 . \tag{30}$$

Assuming these are saturated by a single technirho and techni-$a1$ with

$$\frac{1}{\pi}\text{Im}\Pi_{VV}(s)=g_V^2\delta(s-m_V^2) , \tag{31}$$

$$\frac{1}{\pi}\text{Im}\Pi_{AA}(s)=g_A^2\delta(s-m_A^2) , \tag{32}$$

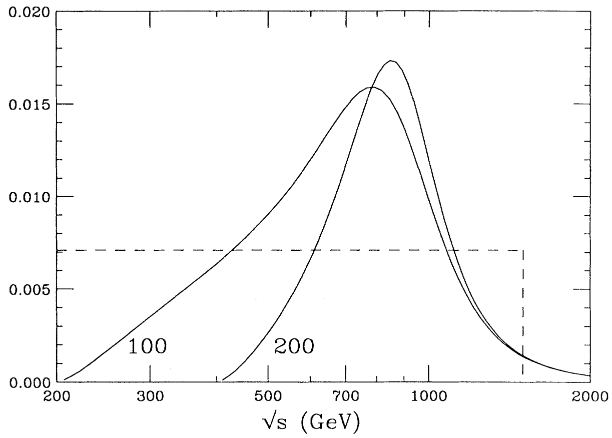

FIG. 1. The vector portion of the integrand in the expression for Δr. The quantity plotted is $-g^2 \mathrm{Im}\Pi_{VV}(s)/(2\pi s)$, so the contribution to Eq. (4) is obtained by integrating with the differential $2d\ln\sqrt{s}$. For the values $m_{\pi\mathrm{TC}}=100$ and 200 GeV the results of Eqs. (27) and (28) are shown as solid curves. The dashed curve shows the result from the nonresonant approximation, Eq. (15), without any threshold suppression, cut off at $\Lambda=1.5$ GeV.

we have

$$g_V=g_A\ ,\tag{33}$$

$$\frac{g_V^2}{m_V^2}-\frac{g_A^2}{m_A^2}=v^2\ .\tag{34}$$

The correspondence with the previous notation is $g_V=m_{\rho\mathrm{TC}}^2/f_{\rho\mathrm{TC}}$. Thus the KSRF relation reads

$$g_V^2=2v^2m_{\rho\mathrm{TC}}^2\ ,\tag{35}$$

from which follows

$$m_A^2=2m_V^2\ .\tag{36}$$

Consequently, the contribution of the axial spectral function is $-\frac{1}{4}$ times the contribution from the vector spectral function. Taking this into account we find

$$(\Delta r)_{\mathrm{resonant}}=\frac{3}{4}\frac{g^2v^2}{m_{\rho\mathrm{TC}}^2}=0.019\left[\frac{1\ \mathrm{TeV}}{m_{\rho\mathrm{TC}}}\right]^2\ .\tag{37}$$

The factor $\frac{3}{4}$ is to be included also for the calculations in which the width of the technirho is retained. Table I shows that the resonance models give Δr in the range 0.0155–0.022 in the one family ($N_d=4$) model. This would produce a shift in the W mass of -250 MeV to -375 MeV, which is not contradicted by the data.

We see that using the conventional scaling for the technirho mass $m_{\rho\mathrm{TC}}=m_\rho[v/(f_\pi\sqrt{N_d})]\approx 1$ TeV gives a smaller predicted shift of the W mass than does the nonresonant approximation, Eq. (10), which is not in contradiction with the data. On the other hand, it is clear from the narrow resonance approximation, Eq. (26), that a decrease in the mass of the technirho will increase the contribution to Δr.

There is a reason for expecting that the technirho may have a mass less than that predicted by the standard scaling arguments. In the one-family technicolor model, the forces in certain channels are enhanced by the large multiplicity of pions [13]. Among these channels are the singlet, in which there may be a scalar bound state, and the antisymmetric adjoint. The latter representation contains the technirho. The increased attractive force could well decrease the technirho mass. If we take $m_{\rho\mathrm{TC}}=750$ GeV and $m_{\pi\mathrm{TC}}=100$ and 200 GeV, we find for Δr the values 0.034 and 0.027, respectively. This demonstrates how sensitive the result is to the values of both the parameters $m_{\rho\mathrm{TC}}$ and $m_{\pi\mathrm{TC}}$.

The strong force in the technirho channel due to the chirally invariant technipion interaction might lead to two separate resonances in this channel, one primarily a bound state of technipions, the other a bound state of technifermions. In the narrow-resonance approximation there are six parameters g_{V1}, g_{V2}, g_A, m_{V1}, m_{V2}, and m_A. The Weinberg sum rules generalize to

$$g_{V1}^2+g_{V2}^2=g_A^2\ ,\tag{38}$$

$$\frac{g_{V1}^2}{m_{V1}^2}+\frac{g_{V2}^2}{m_{V2}^2}-\frac{g_A^2}{m_A^2}=v^2\ ,\tag{39}$$

while the KSRF relation becomes

$$\frac{g_{V1}^2}{m_{V1}^2}+\frac{g_{V2}^2}{m_{V2}^2}=2v^2\ .\tag{40}$$

From this it follows that

$$\frac{g_A^2}{m_A^2}=v^2\ ,\tag{41}$$

just as in the case with a single resonance.

The Weinberg sum rules and the KSRF relation reduce the number of free parameters to three. One way to reduce this number is to require that unlike the technirho's mass, the techni-$a1$'s mass scales naively, so $m_{a1}=1400$ GeV. It is easy to show then that the minimal value of Δr is obtained if the lower resonance decouples, $g_{V1}=0$. This just reduces the problem to that with a single narrow vector resonance, with the previous result

$$\Delta r=\frac{3g^2v^2}{2m_A^2}=0.019\ ,\tag{42}$$

where the numerical value follows from our choice for m_A.

In summary, while the dispersion relation approach of Peskin and Takeuchi offers a convenient means of estimating the electroweak radiative corrections induced by the technicolor sector, the uncertainties inherent in the strong dynamics make it impossible to make a reliable calculation of the shift in Δr since its value depends critically on the masses of the technipion and technirho, neither of which can be estimated with accuracy. The best that can be said at present is that the contribution to Δr is probably at least 0.015.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and by NSF Grant PHY-85-15857.

[1] S. Weinberg, Phys. Rev. D **19**, 1277 (1979).
[2] L. Susskind, Phys. Rev. D **20**, 2619 (1979).
[3] R. Renken and M. Peskin, Nucl. Phys. **B211**, 93 (1983).
[4] See B. W. Lynn, M. E. Peskin, and R. Stuart, in *Physics at LEP,* LEP Jamboree, Geneva, Switzerland, 1985, edited by J. Ellis and R. Peccei (CERN Report No. 86-02, Geneva, 1986), Vol. 1, p. 151, Fig. 20.
[5] M. Golden and L. Randall, Fermilab Report No. 90/83-T (unpublished).
[6] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990).
[7] B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990).
[8] S. Dimopoulos, Nucl. Phys. **B168**, 69 (1980).
[9] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16**, 255 (1966).
[10] Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).
[11] G. 't Hooft, Nucl. Phys. **B72**, 461 (1972).
[12] S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967).
[13] R. N. Cahn and M. Suzuki, Phys. Rev. Lett. **67**, 169 (1991).