PHYSICAL REVIEW D VOLUME 44, NUMBER 11 1 DECEMBER 1991

When do neutrinos really oscillate? Quantum mechanics of neutrino oscillations

C. Giunti
Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218

C. W. Kim
*Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218
and Center for Theoretical Physics, Seoul National University, Seoul 151, Korea*

U. W. Lee
Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218
(Received 22 July 1991)

The quantum mechanics of neutrino oscillations is reexamined by studying the propagation of a flavor neutrino described by a superposition of mass-eigenstate wave packets, *without making the usual relativistic assumption*. The space-dependent oscillation probability is derived by averaging over the propagation time. The time average leads to interesting factors in the oscillation probability, from which the coherence length and the bound for the size of the wave packets are derived. The coherence length is the distance beyond which neutrinos cease to oscillate, although a flavor change may still take place. It is also shown that if one of the mass eigenstates is nonrelativistic, it may dominate the constant flavor-changing probability.

I. INTRODUCTION

If neutrinos are massive particles and mixed, a flavor neutrino is created by a weak-interaction process as a coherent superposition of mass eigenstates. The neutrino oscillations are due to the interference among the different mass eigenstates that propagate with different phase velocities. In the standard treatment of neutrino oscillations [1,2] the following assumptions are made. (a) A neutrino propagates with a definite momentum which is common for all the constituent mass eigenstates. (b) The different mass eigenstates have different energies; these energies are well defined and are given by the energy-momentum dispersion relations; the neutrino wave function is given by a superposition of plane waves, each one corresponding to a mass eigenstate. (c) The neutrino is relativistic; i.e., its momentum is much larger than all the mass eigenvalues.

The standard approach is very useful because of its simplicity and for its physical insight, but it is not satisfactory for a *complete* understanding of the physics involved in the neutrino oscillations, in particular when neutrinos are nonrelativistic, which may be the case in some oscillation experiments because of very poor experimental upper limits on the neutrino masses, especially for ν_μ and ν_τ. Since neutrino oscillation appears to be the most promising phenomenon to probe basic properties of neutrinos, such as masses and mixing angles, a complete understanding of the physics involved is necessary in order to infer any meaningful conclusion from oscillation experiments. A complete treatment of neutrino oscillations must address the following additional issues. (i) A necessary condition for neutrino oscillations to occur is that the neutrino source and detector are localized within a region much smaller than the oscillation length; then the neutrino momentum has at least the corresponding spread given by the uncertainty principle [3]. (ii) The energy-momentum conservation in the process in which the neutrino is created implies that the different mass-eigenstate components have different momenta as well as different energies [4]. (iii) The different mass eigenstates must be produced and detected coherently [5]; this is possible only if the other particles associated with the production and detection processes have energy-momentum spreads larger than the energy-momentum differences of the mass eigenstates. (iv) The wave function of a propagating neutrino must be a superposition of the wave functions of the mass eigenstates with proper coefficients given by the amplitudes of the processes in which mass-eigenstate neutrinos are produced.

The localization of the neutrino source and the spread of the neutrino momentum imply that the propagating flavor neutrino is described not by a superposition of plane waves, but by a superposition of localized wave packets. Therefore, a wave-packet treatment is necessary for a correct quantum-mechanical description of neutrino oscillations. In Ref. [3] neutrino oscillations have been extensively discussed by using wave packets and the standard assumptions (a), (b), and (c) mentioned above, confirming the results obtained in the standard treatment without wave packets.

In this paper we will discuss the quantum mechanics of neutrino oscillations by studying the propagation of a flavor neutrino described by a *superposition of wave packets*, each corresponding to a mass eigenstate, by *implementing the additional issues mentioned above*. This is a first step towards a complete understanding of neutrino oscillations. Since we do not calculate the amplitudes of the interaction process in which the neutrino is produced and detected, the wave packets for the mass eigenstates will be assumed to have a Gaussian form with size σ_x and the coefficients of their superposition will be assumed to be given by the elements of the mixing matrix $\mathcal{U}$ which connects the weak and mass-eigenstate bases of the neutrino fields. This simple quantum-mechanical approach is sufficient for understanding the oscillations of relativistic neutrinos. In fact, for relativistic neutrinos, the amplitudes of the production and detection processes can be approximated to lowest order by the amplitude for massless neutrinos. Hence, as usual, the observable cross section can be factorized as a product of the oscillation probability and the massless cross section. The oscillation probability depends only on the elements of the mixing matrix $\mathcal{U}$ and on the space-time-dependent quantum-mechanical interference between the mass eigenstates occurring as a consequence of the neutrino propagation. The same is not true for nonrelativistic neutrinos, since in this case the production and detection amplitudes depend on the mass eigenvalues and the oscillation probability cannot be factored out.

One of the predictions of the wave-packet treatment is the existence of a coherence length for neutrino oscillations: since the mass-eigenstate wave packets propagate with different velocities, they overlap and interfere only for a finite distance. The coherence length has never been derived in the literature, although its existence and order of magnitude for relativistic neutrinos has been suggested from physical arguments [6]. In this paper we derive the coherence length by averaging over time the space-time-dependent oscillation probability. The average over time is appropriate because, in practical experiments, the propagation time between the neutrino source and the detector is not measured, whereas the distance is known. From the time average, it can also be seen naturally that in order for the mass eigenstates to interfere coherently, the size σ_x of the wave packets must be smaller than the oscillation length L_{ab}^{osc}, as required by (i) (throughout this paper the Greek indices α, β refer to flavor neutrinos, whereas the Latin indices a, b refer to the mass eigenstates). As argued in Ref. [3], the size σ_x of the the wave packets is given by the dimension of the region within which the production process is effectively localized. For example, for solar neutrinos $\sigma_x \lesssim 10^{-6}$ cm [6] and for supernova neutrinos $\sigma_x \sim 10^{-14}$ cm [7].

In our approach, the average values of the energy and momentum of each mass-eigenstate wave packet are different and they are given by energy-momentum conservation in the production process. Hence in this paper we address issues (i) and (ii) above in a natural way. Issue (iii) is addressed by assuming that the momentum spread $\sigma_p \sim 1/\sigma_x$ of the neutrino wave packets is of the same order as the spread of the wave packets of the other particles associated with the production and detection processes. Since $L_{ab}^{\mathrm{osc}} \sim 1/(|\langle E_a \rangle - \langle E_b \rangle|)$, the condition $L_{ab}^{\mathrm{osc}} \gg \sigma_x$ implies that the energy difference $|\langle E_a \rangle - \langle E_b \rangle|$ is much smaller than the momentum spread σ_p, as required by (iii). A complete implementation of the issues (iii) and (iv) in the framework of quantum field theory, including the effect of the production and detection processes and a rigorous treatment of the nonrelativistic case, will be discussed elsewhere [8].

II. NEUTRINO OSCILLATIONS

In order to construct the wave packets for the mass eigenstates, we make the following assumptions: the problem is one dimensional; i.e., we neglect the momentum spread orthogonal to the direction of propagation x; the mass-eigenstate wave packets have a Gaussian form with the same width σ_p in momentum space; the Gaussian mass-eigenstate wave packets in momentum space are sharply peaked around the mean value momenta $\langle p_a \rangle$ determined by the kinematics of the production process (energy-momentum conservation).

The normalized mass-eigenstate wave packets in momentum and coordinate spaces, respectively, are given by

$$\psi_a(p) = (\sqrt{2\pi}\sigma_p)^{1/2} \exp\left[-\frac{(p-\langle p_a \rangle)^2}{4\sigma_p^2}\right],$$
$$\psi_a(x,t) = (\sqrt{2\pi}\sigma_x)^{-1/2} \exp\left[i(\langle p_a \rangle x - \langle E_a \rangle t) - \frac{(x - v_a t)^2}{4\sigma_x^2}\right], \tag{1}$$

where the energy $\langle E_a \rangle$ and the group velocity v_a are given by

$$\langle E_a \rangle = \sqrt{\langle p_a \rangle^2 + m_a^2}, \quad v_a = \frac{\langle p_a \rangle}{\langle E_a \rangle} \tag{2}$$

and the widths σ_x and σ_p are related by $\sigma_x \sigma_p = \frac{1}{2}$.

Let us consider a neutrino created at coordinates $x=0, t=0$ by a weak process as a flavor neutrino ν_α. The quantum-mechanical probability to find the neutrino in the flavor state ν_β detected at a distance $x=X$ and time $t=T$ is given by (here we assume $|\nu_\alpha\rangle = \sum_a \mathcal{U}_{\alpha a}^* |\nu_a\rangle$)

$$P_{\alpha\to\beta}(X,T) = \left|\sum_a \mathcal{U}_{\beta a} \psi_a(X,T) \mathcal{U}_{\alpha a}^*\right|^2 = \frac{1}{\sqrt{2\pi}\sigma_x} \sum_{a,b} \mathcal{U}_{\beta a} \mathcal{U}_{\alpha a}^* \mathcal{U}_{\beta b}^* \mathcal{U}_{\alpha b} \exp[i(\langle p_a \rangle - \langle p_b \rangle)X - i(\langle E_a \rangle - \langle E_b \rangle)T] \times \exp\left[-\frac{(X-v_a T)^2}{4\sigma_x^2} - \frac{(X-v_b T)^2}{4\sigma_x^2}\right]. \tag{3}$$

In practical experiments, the distance X from the neutrino source is known, whereas the time of propagation T is not measured. Hence the probability at the distance X is given by the time average of the probability given in Eq. (3). In the standard treatment for relativistic neutrinos, the time average is accomplished by taking $T=X$. In our approach, this corresponds to approximating the time integration by taking the dominant contribution in the stationary point of the exponent, which is given by

$$T=\frac{v_a+v_b}{v_a^2+v_b^2}X \ . \tag{4}$$

However, without making any approximation, we can easily perform the time integration by completing the squares in the exponent. The final result is given by

$$P_{\alpha\to\beta}(X)=\left[\sum_{a'}\frac{|\mathcal{U}_{\alpha a'}|^2}{|v_{a'}|}\right]^{-1}\sum_{a,b}\mathcal{U}_{\beta a}\mathcal{U}^*_{\alpha a}\mathcal{U}^*_{\beta b}\mathcal{U}_{\alpha b}\exp\left\{i\left[(\langle p_a\rangle-\langle p_b\rangle)-(\langle E_a\rangle-\langle E_b\rangle)\left[\frac{v_a+v_b}{v_a^2+v_b^2}\right]\right]X\right\}$$
$$\times\left[\frac{2}{v_a^2+v_b^2}\right]^{1/2}\exp\left[-\frac{X^2}{4\sigma_x^2}\frac{(v_a-v_b)^2}{v_a^2+v_b^2}-\frac{(\langle E_a\rangle-\langle E_b\rangle)^2}{4\sigma_p^2(v_a^2+v_b^2)}\right] \ . \tag{5}$$

Given a neutrino created in the flavor state ν_α at $x=0$ and detected at the distance $x=X$ from the neutrino source, Eq. (5) gives the quantum-mechanical probability to find it int he flavor state ν_β. The probability given in Eq. (5) contains a double sum over the contributions of the mass eigenstates. In addition to the elements of the mixing matrix $\mathcal{U}$, each term of the sum contains factors (1) and (2) (see below), which can also be obtained by the approximation given in Eq. (4), and factors (3) and (4) (see below) which are due to the time integration.

(1) Phase factor

$$\exp\left[-i(\langle E_a\rangle-\langle E_b\rangle)\left[\frac{v_a+v_b}{v_a^2+v_b^2}-\frac{\langle p_a\rangle-\langle p_b\rangle}{\langle E_a\rangle-\langle E_b\rangle}\right]X\right] \ .$$

This gives the neutrino oscillation as a function of the distance X from the neutrino source. The oscillation lengths are given by

$$L_{ab}^{\rm osc}=\frac{2\pi}{|\langle E_a\rangle-\langle E_b\rangle|}\left[\frac{v_a+v_b}{v_a^2+v_b^2}-\frac{\langle p_a\rangle-\langle p_b\rangle}{\langle E_a\rangle-\langle E_b\rangle}\right]^{-1} \ . \tag{6}$$

The quantity in large parentheses becomes unity for extremely relativistic neutrinos, leading to the usual well-known result.

(2) Damping factor

$$\exp\left[-\frac{X^2}{4\sigma_x^2}\frac{(v_a-v_b)^2}{v_a^2+v_b^2}\right] \ .$$

This measures the coherence of the contributions of the wave packets of the different mass eigenstates. The coherence length for $a\neq b$ is given by

$$L_{ab}^{\rm coh}\sim\sigma_x\left[\frac{v_a^2+v_b^2}{(v_a-v_b)^2}\right]^{1/2} , \tag{7}$$

to be compared with the usual (intuitive) expression $L_{ab}^{\rm coh}\sim\sigma_x/|v_a-v_b|$ [6]. The two expressions coincide only for extremely relativistic neutrinos. Neutrino oscillations occur if the coherence length is much larger than the size σ_x of the wave packets. This is the case if $|v_a-v_b|\ll 1$, i.e., if the two mass eigenstates ν_a and ν_b are almost degenerate.

(3) Exponential factor

$$\exp\left[-\frac{(\langle E_a\rangle-\langle E_b\rangle)^2}{4\sigma_p^2(v_a^2+v_b^2)}\right] \ .$$

This is due to the time integration and guarantees energy conservation within the uncertainly given by the width σ_p of the neutrino wave packets. This factor does *not* depend on the distance X and its presence means that, if $(\langle E_a\rangle-\langle E_b\rangle)\gtrsim\sigma_p\sqrt{v_a^2+v_b^2}$, the time integration suppresses the interference of the different mass eigenstates.

(4) Other factors

$$\left[\sum_{a'}\frac{|\mathcal{U}_{\alpha a'}|^2}{|v_{a'}|}\right]^{-1} , \quad \left[\frac{2}{v_a^2+v_b^2}\right]^{1/2} .$$

The first is a normalization factor that has been put by hand. The second factor is due to the time integration and takes into account the fact that the time-averaged probability to find a mass-eigenstate neutrino at the distance X is inversely proportional to its velocity. This factor is practically unity for relativistic neutrinos, but deviates from unity if a mass eigenstate is nonrelativistic, with velocity $|v_a|\ll 1$.

From Eq. (6), one can see that the oscillation length $L_{ab}^{\rm osc}$ can be macroscopic if the mass eigenstates ν_a and ν_b are almost degenerate, regardless of whether or not they are relativistic. We consider this case and investigate when the mass eigenstates ν_a and ν_b contribute coherently to the oscillation probability given in Eq. (5), *without making the usual relativistic assumption.* We define

$$\Delta m_{ab}^2 \equiv m_a^2 - m_b^2 \;, \quad \overline{m}\,_{ab}^2 \equiv \frac{m_a^2 + m_b^2}{2} \;,$$
$$\langle \overline{p}_{ab} \rangle \equiv \frac{\langle p_a \rangle + \langle p_b \rangle}{2} \;, \quad \langle \overline{E}_{ab} \rangle \equiv \sqrt{\langle \overline{p}_{ab} \rangle^2 + \overline{m}\,_{ab}^2} \;. \tag{8}$$

To lowest order in Δm_{ab}^2 one has

$$\langle p_a \rangle^2 - \langle p_b \rangle^2 \simeq \xi_{ab} \Delta m_{ab}^2 \;, \tag{9}$$

where ξ_{ab} is a dimensionless quantity that depends on the production process and can be calculated from energy-momentum conservation. For example, for the pion decay $\pi \to \mu + \nu$, one obtains, in the rest frame of the pion,

$$\xi_{ab} = -\frac{1}{2}\left[1 + \frac{m_\mu^2}{m_\pi^2}\right] + \frac{m_a^2 + m_b^2}{4 m_\pi^2} \;, \tag{10}$$

and one can see that the value of ξ_{ab} is of order of unity. To lowest order in Δm_{ab}^2 the oscillation and the coherence lengths are given by

$$L_{ab}^{\text{osc}} \simeq 2\pi \frac{2\langle \overline{p}_{ab} \rangle}{|\Delta m_{ab}^2|} \;,$$
$$L_{ab}^{\text{coh}} \sim \sigma_x \frac{\langle \overline{p}_{ab} \rangle^2 \langle \overline{E}_{ab} \rangle^2}{|(\langle \overline{p}_{ab} \rangle^2 - \xi_{ab} \overline{m}\,_{ab}^2) \Delta m_{ab}^2|} \;. \tag{11}$$

The different mass eigenstates contribute coherently to the oscillation probability if the energy-conservation factor (3) mentioned above can be approximated by unity, i.e., for $\Delta m_{ab}^2 \lesssim \sigma_p \langle \overline{p}_{ab} \rangle / (1 + \xi_{ab})$. This condition can be written as $L_{ab}^{\text{osc}} \gtrsim \sigma_x$, i.e., the neutrino source must be localized within a region much smaller than the oscillation length, as required by (i). The maximum number of oscillations is given by

$$N_{\text{osc}} = \frac{L_{ab}^{\text{coh}}}{L_{ab}^{\text{osc}}} \sim \frac{\langle \overline{p}_{ab} \rangle \langle \overline{E}_{ab} \rangle^2}{\sigma_p |(\langle \overline{p}_{ab} \rangle^2 - \xi_{ab} \overline{m}\,_{ab}^2)|} \;. \tag{12}$$

However, in practical experiments, the maximum number of observable oscillations is smaller than the theoretical value given in Eq. (12), since the probability must be further averaged over the dimensions of the source and the detector, and over the energy spectrum of the source and the energy resolution of the detector [9,10]. It is to be emphasized here again that the results given in Eqs. (11) and (12) are valid for nonrelativistic as well as relativistic neutrinos.

If the mass eigenstates are relativistic, to lowest order in the relativistic approximation, $\xi_{ab} \to \xi$, where ξ is independent from the mass eigenvalues, and $\langle \overline{p}_{ab} \rangle \to \langle p_0 \rangle$, where $\langle p_0 \rangle$ is the mean value momentum in the limit $m_a = 0$. The time-averaged oscillation probability given in Eq. (5) becomes

$$P_{\alpha \to \beta}(X) \simeq \sum_{a,b} \mathcal{U}_{\beta a} \mathcal{U}_{\alpha a}^* \mathcal{U}_{\beta b}^* \mathcal{U}_{\alpha b} \exp\left[-i \frac{m_a^2 - m_b^2}{2\langle p_0 \rangle} X\right] \exp\left[-\frac{X^2}{8\sigma_x^2}\left[\frac{m_a^2 - m_b^2}{2\langle p_0 \rangle^2}\right]^2 - (1+\xi)^2 \frac{(m_a^2 - m_b^2)^2}{32 \sigma_p^2 \langle p_0 \rangle^2}\right] \;. \tag{13}$$

Apart from the last energy conservation term, the oscillation probability given in Eq. (13) is the same as that obtained from the approximation given in Eq. (4), i.e., by taking $T = X$ as commonly done. However, due to the wave-packet treatment, the exponent in Eq. (13) is different from the usual one which contains only the phase proportional to X. In particular, the term proportional to X^2, which measures the coherence of the mass eigenstates, gives the coherence length.

So far, we have neglected the spreading of the wave packets during their propagation, which becomes significant for a propagation time longer than the spreading times $T_a^{\text{spr}} \sim \sigma_x^2 \langle E_a \rangle^3 / m_a^2$. For nonrelativistic neutrinos, $v_a T_a^{\text{spr}}$ can be shorter than the coherence length given in Eq. (11) and the spreading cannot be neglected. By taking the spreading into account, the mass-eigenstate wave packets in coordinate space are given by

$$\psi_a(x,t) = \left[\sqrt{2\pi}\sigma_x \left[1 + i \frac{s_a}{2\sigma_x^2} t\right]\right]^{-1/2} \times \exp\left[i(\langle p_a \rangle x - \langle E_a \rangle t) - \frac{(x - v_a t)^2}{4\sigma_x^2 + 2 i s_a t}\right] \;, \tag{14}$$

where $s_a = m_a^2 / \langle E_a \rangle^3$. In this case the time average of the space-time-dependent oscillation probability cannot be calculated exactly. However, one can approximate the time integration by taking the dominant contribution in the stationary point of the exponent of the space-time-dependent probability. The oscillation length is found to be the same as that given in Eq. (11), whereas the corresponding coherence length is given by

$$L_{ab}^{\text{coh}} \sim \sigma_x \frac{\langle \overline{p}_{ab} \rangle^2 \langle \overline{E}_{ab} \rangle^2}{\sqrt{(\langle \overline{p}_{ab} \rangle^2 - \xi_{ab} \overline{m}\,_{ab}^2)^2 (\Delta m_{ab}^2)^2 - 32 \sigma_p^2 (\overline{m}\,_{ab}^2)^2 \langle \overline{p}_{ab} \rangle^2}} \;. \tag{15}$$

The coherence length given in Eq. (15) is longer than that given in Eq. (11) because the spreading of the mass-eigenstate wave packets increases their overlap. One can see that the spreading effect is negligible for

$$\Delta m_{ab}^2 \gg \sigma_p \overline{m}\,{}_{ab}^2 \langle \overline{p}_{ab} \rangle / | \langle \overline{p}_{ab} \rangle^2 - \xi_{ab} \overline{m}\,{}_{ab}^2 | \ .$$

This condition is satisfied by relativistic neutrinos for reasonable values of the parameters involved (the separation between the mass-eigenstate wave packets increases faster than their spreading [11]). For nonrelativistic neutrinos, it is possible that

$$\Delta m_{ab}^2 \lesssim \sigma_p \overline{m}\,{}_{ab}^2 \langle \overline{p}_{ab} \rangle / | \langle \overline{p}_{ab} \rangle^2 - \xi_{ab} \overline{m}\,{}_{ab}^2 | \ ,$$

so that the spreading may be important. In this case, the coherence length becomes infinite, because the spreading of the wave packets increases faster than their separation.

Finally, let us consider a flavor neutrino for which the mass eigenstates are far from degenerate and at least one mass eigenstate is nonrelativistic. If ν_a is one of the nonrelativistic mass eigenstates, $(v_a - v_b)^2 \sim v_a^2 + v_b^2$ and the corresponding coherence length becomes very short $L_{ab}^{\rm coh} \sim \sigma_x$. In this case, the neutrino oscillations due to the interference between the mass eigenstates ν_a and ν_b do not take place and one can measure only a *constant flavor-changing probability.* If all the mass eigenstates (except possibly one) are nonrelativistic, the neutrino oscillations do not occur and the constant flavor-changing probability is given by

$$P_{\alpha \to \beta} \simeq \left[\sum_{a'} \frac{|\mathcal{U}_{\alpha a'}|^2}{|v_{a'}|} \right]^{-1} \sum_a |\mathcal{U}_{\beta a}|^2 |\mathcal{U}_{\alpha a}|^2 |v_a|^{-1} \ . \qquad (16)$$

This probability is different from the usual constant flavor-changing probability for relativistic neutrinos,

$$P_{\alpha \to \beta} \simeq \sum_a |\mathcal{U}_{\beta a}|^2 |\mathcal{U}_{\alpha a}|^2 \ , \qquad (17)$$

since the former contains the factors $|v_a|^{-1}$. For nonrelativistic mass eigenstates, the factor $|v_a|^{-1}$ gives a significant correction to the probability and may not be negligible. For example, let us consider the two-generation case in which ν_1 is relativistic and ν_2 is extremely nonrelativistic. From Eq. (16), if the order of magnitude of all the elements of the mixing matrix $\mathcal{U}$ is about the same, the constant flavor-changing probability is given by

$$P_{\alpha \to \beta} \simeq |\mathcal{U}_{\beta 2}|^2 \qquad (18)$$

instead of the usual probability given in Eq. (17). Notice that the probability given in Eq. (18) does not depend on the initial neutrino flavor since the contribution of the nonrelativistic mass eigenstate is dominant (as long as the elements of the mixing matrix are of the same order of magnitude).

III. CONCLUSIONS

We have calculated the quantum-mechanical probability $P_{\alpha \to \beta}(X)$ to find a neutrino flavor ν_β at the distance X from a source of neutrinos with initial flavor ν_α by studying the neutrino propagation as a superposition of mass-eigenstate wave packets. We have also derived the coherence length for nonrelativistic as well as relativistic neutrinos. The coherence length can be quite short, depending on the neutrino mass difference and on whether or not the mass eigenstates are relativistic. Beyond the coherence length oscillations are suppressed and one can observe only a constant flavor-changing probability.

In our calculation, we have not considered the corrections to the mixing of the neutrino states due to the production and detection processes. For relativistic neutrinos, the effects of the production and detection processes are irrelevant, because they can be approximated by the massless interaction amplitudes and can be factorized outside the sum over the contributions of the mass eigenstates. However, for nonrelativistic neutrinos, this cannot be done and one expects a significant correction from the production and detection amplitudes of the nonrelativistic mass eigenstates. Therefore, the probability given in Eq. (16) cannot be directly applied to experiment, but its being different from the usual probability given in Eq. (17) should serve as a warning signal that the case of nonrelativistic neutrinos must be handled with special care. We have also shown that, if one of the mass eigenstates is nonrelativistic, it may dominate the constant flavor-changing probability. For a complete treatment of the nonrelativistic case, which requires a detailed study of the production and detection interaction processes, the simple quantum-mechanical treatment presented here must be extended by use of the quantum field theory of weak interactions. This will be discussed elsewhere [8].

ACKNOWLEDGMENTS

The authors would like to thank G. Feldman for his interest in this work and for many helpful discussions. This work was supported in part by the Fondazione Angelo Della Riccia (Italy) and the Korea Science and Engineering Foundation. One of the authors (C.W.K.) wishes to thank members of the Center for Theoretical Physics, Seoul National University, for their warm hospitality.

[1] S. M. Bilenky and B. Pontecorvo, Phys. Rep. **41**, 225 (1978).
[2] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).
[3] B. Kayser, Phys. Rev. D **24**, 110 (1981).
[4] R. G. Winter, Lett. Nuovo Cimento **30**, 101 (1981).
[5] W. P. Lam (private communication).
[6] S. Nussinov, Phys. Lett. **63B**, 201 (1976).

[7] H. Anada and H. Nishimura, Phys. Rev. D **37**, 552 (1988).
[8] C. Giunti, C. W. Kim, and U. W. Lee (unpublished).
[9] P. H. Frampton and P. Vogel, Phys. Rep. **82**, 339 (1982).
[10] F. Boehm and P. Vogel, *Physics of Massive Neutrinos* (Cambridge University Press, Cambridge, England, 1987).
[11] A. Bottino, C. W. Kim, Jewan Kim, and W. P. Lam, Mod. Phys. Lett. A **4**, 1799 (1989).