Model of a composite right-handed t quark: An alternative to the $t\bar{t}$ condensate model

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As an alternative to the composite Higgs-boson model with a $t\bar{t}$ condensate, a solvable model is built in which a right-handed t quark is a composite of left-handed quarks and elementary Higgs particles. This model leads us to virtually the same numerical prediction on low-energy parameters; e.g., the t-quark mass and the Higgs-boson mass as the composite Higgs-boson model, when the renormalization-group analysis is performed according to the prescription of Bardeen, Hill, and Lindner.

I. INTRODUCTION

The possibility of the Higgs doublet being a composite of the third-generation quarks was proposed by Nambu [1] and by Miransky, Tanabashi, and Yamawaki [2] in the Nambu–Jona-Lasinio model. In this model, breaking of electroweak and chiral symmetry is triggered by condensation of the t-quark field. We will therefore refer to this model as either the composite Higgs-boson model or the $t\bar{t}$ condensate model. The model was subsequently reanalyzed by Bardeen, Hill, and Lindner [3] using the renormalization-group method. Meanwhile, a closely related renormalization-group analysis was made by Marciano [4] with slightly different emphasis. In the renormalization-group analysis, the surge of the running Yukawa coupling of the t quark at a super-high-energy scale is taken as a signature of compositeness of the Higgs boson and $t\bar{t}$ condensate. However, since the renormalization-group equation takes the same form independent of particles being elementary or composite, one may wonder how unique the numerical prediction of the renormalization-group analysis [3] is to compositeness of the Higgs boson and condensation of the t-quark field. Is there not a completely different model that leads to the same numerical prediction? What happens if, for instance, the right-handed t quark t_R is a composite of the left-handed quarks and an elementary Higgs doublet? Such a model may not look aesthetically attractive because of the unnatural fine-tuning of an elementary spinless boson. However, the $t\bar{t}$ condensate model [1,2], as it stands now, shares the same unnaturalness. Therefore, it makes sense to study different composite models, for instance, a composite t_R-quark model, in order to have a better understanding of the $t\bar{t}$ condensate model and the implication of its numerical analysis. It is not that we favor a composite t_R-quark model over the $t\bar{t}$ condensate model. Rather, we wish to understand how well, as a matter of principle, the numerical prediction of the renormalization-group analysis [3] holds as a prediction unique to the composite Higgs-boson model and what alternative composite models can possibly give the same or a similar prediction on low-energy parameters.

In this paper we present and discuss a model of a composite right-handed t quark, which, if we follow the argument of Bardeen, Hill, and Lindner, leads us to the same renormalization-group equation and therefore to the same numerical prediction on the t-quark and Higgs-boson masses as that of the $t\bar{t}$ condensate model. Some aspect of our model is admittedly less attractive than the $t\bar{t}$ condenstate model, but there is no way to distinguish between the two models with low-energy phenomena. In Sec. II we first describe how to incorporate key ingredients into our model. Then we build our model step by step into the final form. In Sec. III, after fermion mass eigenstates are determined, the one-loop renormalization is computed in the leading N_F order, where $N_F=2$ in the real world. In Sec. IV we compare the renormalization-group equation of our model with that of the standard model. When the renormalization-group analysis is made for our model according to the prescription of Bardeen, Hill, and Lindner, the model produces the same numerical prediction as the $t\bar{t}$ condensate model. Several remarks are made in Sec. V, in particular, on some amusing conceptual similarity with *particle democracy* of the 1960s.

II. COMPOSITE MODEL OF t_R

We present here a solvable model of composite t_R. For the sake of keeping a close parallel with the $t\bar{t}$ condensate model, the model incorporates only the third-generation quarks as fermions, but with a large number of flavors, $N_F(\rightarrow\infty)$; i.e., the symmetry of the Lagrangian is $SU(3)_C\times SU(N_F)\times U(1)$. The large-$N_F$ limit is chosen to justify the chain-diagram approximation in solving for a bound state in an explicit form. For simplicity, we consider the model in which, among quarks, only the t quark acquires mass after $SU(N_F)\times U(1)$ symmetry breaking. Let us start with the particle content of the model.

A. Particle spectrum

Our model consists of elementary left-handed quarks $\psi_L^T=(t_L,b_L,\ldots)$ and an elementary Higgs multiplet $\Phi=(\phi_1,\phi_2,\ldots)$ which transform under $SU(3)_C\times SU(N_F)\times U(1)$ as

$$\psi_L=(3,N_F,Y/2) \quad \text{and} \quad \Phi=(1,\bar{N}_F,Q_t-Y/2)\;, \tag{2.1}$$

where $Y/2$ is the U(1) charge of ψ_L and Q_t is the electric charge of the t quark. The gauge bosons of $SU(3)_C \times SU(N_F) \times U(1)$ are later added to the model. In our model a right-handed t quark

$$t_R = (3, 1, Q_t) \tag{2.2}$$

is formed as an $SU(N_F)$-singlet bound state of $(\Phi\psi_L)$. Unless it is generated as a Goldstone fermion of broken supersymmetry, a composite fermion appears in both handednesses, in general. In our case the composite $\Phi\psi_L$ state is a Dirac fermion with its left-handed partner

$$\xi_L = (3, 1, Q_t) , \tag{2.3}$$

provided the bound-state mass not be tuned *exactly* to zero [5]. Since the standard model does not have a light fermion with the quantum numbers of ξ_L, we must remove this ξ_L state from our model in order to simulate the standard model. The simplest trick is to introduce an elementary right-handed singlet "quark"

$$\eta_R = (3, 1, Q_t) , \tag{2.4}$$

and to let ξ_L and η_R form a supermassive Dirac fermion of mass $O(\Lambda)$, where Λ represents a compositeness scale. Then, as we will see below, the right-handed t quark turns out to be mostly the composite $\Phi\psi_L$ state with a tiny mixture $[=O(m_t^2/\Lambda^2)]$ of η_R. Therefore, the low-energy particle spectrum of our model becomes identical to that of the standard model.

B. Binding force

In the standard model there is no binding force strong enough to form a *tight* bound state of quarks at the electroweak energy scale. As the precision of the experimental test for the standard model has increased steadily, the room to accommodate a new strong binding force has been narrowed down. For instance, the ρ parameter of the W and Z masses has become so accurate as to be potentially in conflict with some of the technicolor-type models, aside from the long-standing problem of flavor-changing neutral interaction. The original $t\bar{t}$ condensate model introduces in an *ad hoc* manner a new singular interaction among four quarks as the origin of the binding force and raises the compositeness scale to superhigh energies. This strategy has one bad and one good feature among others: A bad feature is the unnaturalness of fine-tuning in the Higgs-boson mass, i.e., the old issue of the hierarchy problem; a good feature is that, being strong only at a super-high-energy scale, the new strong interaction has practically no effect on the low-energy phenomena of our interest in the foreseeable future. When more than one generation of quarks is introduced in the $t\bar{t}$ condensate model, a new fundamental interaction is likely to contain flavor-changing neutral forces. After a composite Higgs doublet is formed, the residual four-quark interaction is of the order $1/\Lambda^2$ in strength and therefore evades the stringent experimental constraints on the flavor-changing neutral interaction. The problem appears more serious when more than one composite Higgs doublet is formed [6]. In passing, we would like to call attention to the recent extension of the $t\bar{t}$ condensate model to incorporate supersymmetry [7–10]. In those models the fine-tuning problem is solved, but the flavor-changing neutral interaction may become more a serious issue. In building our model, we adopt the fine-tuning aspect of the $t\bar{t}$ condensate model. Namely, our model Lagrangian introduces an unrenormalizable binding force that allows us to fine-tune a composite quark mass.

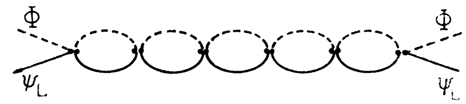

FIG. 1. Forming composite $\xi_{L,R}$ with ψ_L and Φ through infinite iteration of the $\Phi\psi_L$ loops. N_F flavors go around each $\Phi\psi_L$ loop.

Let us introduce our model step by step. We want our model Lagrangian to be explicitly solvable in the leading N_F expansion. The right-handed t quark is a composite state of Φ and ψ_L, which we denote as ξ_R for a while. Considering the matching of handedness, we expect the composite fields ξ_L and ξ_R to be as

$$\xi_L \sim \Phi_a \psi_L^a \quad \text{and} \quad \xi_R \sim (\partial\!\!\!/ \Phi_a)\psi_L^a , \tag{2.5}$$

to the leading N_F order, where the flavor index a is summed over from 1 to N_F. The singular interaction

$$L_{\text{int}} = -i(g_t/M^2)[\bar{\psi}_{La}\Phi^{\dagger a}(\partial\!\!\!/\Phi_b)\psi_L^b] + \text{H.c.} \tag{2.6}$$

provides one of the simplest binding forces that lead to a bound state $\xi_{L,R}$ through the diagram depicted in Fig. 1. The mass parameter M in Eq. (2.6) is defined to be the scale of compositeness of our model. This interaction is of the current-current form and may be generated as an effective Lagrangian through exchange of superheavy vector bosons. Here, however, we do not explore for its origin, but simply postulate it in an *ad hoc* manner. *To the leading order in* N_F, L_{int} of Eq. (2.6) can be transformed into an equivalent form by introducing auxiliary composite fields ξ_L and ξ_R with their Dirac mass term. Suppressing the flavor indices, we can write the equivalent Lagrangian

$$\begin{aligned} L_{\text{int}} &= -M[\bar{\xi}_L + (f/M)\bar{\psi}_L\Phi^\dagger][\xi_R + i(f'/M^2)(\partial\!\!\!/\Phi)\psi_L] \\ &\quad + i(ff'/M^2)[\bar{\psi}_L\Phi^\dagger(\partial\!\!\!/\Phi)\psi_L] + \text{H.c.} \\ &= -M\bar{\xi}_L\xi_R - f\bar{\psi}_L\Phi^\dagger\xi_R - i(f'/M)\bar{\xi}_L(\partial\!\!\!/\Phi)\psi_L + \text{H.c.} , \end{aligned} \tag{2.7}$$

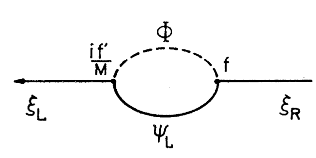

FIG. 2. Self-mass of $\xi_{L,R}$ through virtual dissociation into ψ_L and Φ.

where $g_t=-ff'$. Hereafter, we will continue to suppress flavor indices for notational simplicity. Prior to generation of the kinetic-energy terms through loop diagrams, this Lagrangian means that the $\xi_{L,R}$ fields are indeed of the form of Eq. (2.5) according to $\partial L_{\rm int}/\partial\xi_{R,L}=0$. The constant f corresponds to the *bare* Yukawa coupling at the cutoff scale Λ. We will choose Λ at $O(M)$ and call it also the compositeness scale. For the ξ mass, the *bare* Dirac mass M is $O(\Lambda)$, but the self-mass δm_ξ due to the virtual process $\xi_R\leftrightarrow\xi_L$ (Fig. 2) renormalizes M into $M+\delta m_\xi$, which we will later fine-tune to a value much smaller than Λ, e.g., the electroweak scale. It is important that the binding force $i(g_t/M^2)\bar\psi_L\Phi^\dagger(\partial\!\!\!/\Phi)\psi_L$ or its breakup $i(f'/M)\bar\xi_L(\partial\!\!\!/\Phi)\psi_L$ is singular enough to provide a large self-mass δm_ξ comparable to M.

The Lagrangian of Eq. (2.7), as it stands, leads to the unwanted light singlet quark ξ_L. Since ξ_L cannot have a Majorana mass without breaking $SU(3)_C$ symmetry, we must get rid of it by giving a large Dirac mass with another quark η_R of the same quantum numbers but opposite chirality. In order to realize this mechanism, we modify the Lagrangian of Eq. (2.7) into

$$\begin{aligned}L=&-M[\bar\xi_L+(f/M)\bar\psi_L\Phi^\dagger][\xi_R+i(f'/M^2)(\partial\!\!\!/\Phi)\psi_L+(f''/M)\rho\eta_R]+i(ff'/M^2)[\bar\psi_L\Phi^\dagger(\partial\!\!\!/\Phi)\psi_L]\\&+(ff''/M)\bar\psi_L\Phi^\dagger\rho\eta_R+\text{H.c.}-V(\rho,\Phi)\ ,\\=&-M\bar\xi_L\xi_R-f\bar\psi_L\Phi^\dagger\xi_R-[i(f'/M)\bar\xi_L(\partial\!\!\!/\Phi)\psi_L]-f''\bar\xi_L\rho\eta_R+\text{H.c.}-V(\rho,\Phi)\ ,\end{aligned}\tag{2.8}$$

where ρ is a superheavy singlet boson of mass $O(\Lambda)$ transforming as

$$\rho=(1,1,0)\ ,\tag{2.9}$$

under $SU(3)_C\times SU(N_F)\times U(1)$, and $V(\rho,\Phi)$ stands for the Higgs potential of ρ and Φ. The ρ part of the potential $V(\rho,\Phi)$ is responsible for producing a superheavy Dirac mass for $\xi_L-\eta_R$. In our model we may choose, for instance,

$$V(\rho,\Phi)=(M_\rho^2/2)\rho^2+(\kappa/M)(\partial^\mu\rho)(\partial_\mu\rho)\rho+V(\Phi)\ ,\tag{2.10}$$

which produces through the diagram of Fig. 3 a Dirac mass

$$M_\eta\approx\kappa f''^3(\Lambda^2/M)/(16\pi^2)^2\ ,\tag{2.11}$$

for $M_\rho\approx M$. With $M=O(\Lambda)$, this Dirac mass is large enough to remove the unwanted ξ_L from the low-energy spectrum. By contrast, η_R cannot have a Dirac mass with ψ_L by $SU(N_F)$ symmetry. Therefore, ψ_L remains massless until the electroweak symmetry is broken spontaneously.

III. LOOP CONTRIBUTIONS IN THE LEADING N_F ORDER

The bound state ξ is formed by the local interaction of Eq. (2.6) through an infinite series of the $\Phi\psi_L$ chains in Fig. 1. The mass and Yukawa coupling of ξ can be computed by this series. Since the off-mass-shell $\Phi\psi_L$ scattering has four invariant amplitudes, however, the series is not a trivial geometric series, unlike the one for the $\bar t_R\psi_L$ scattering in the $t\bar t$ condensate model. Working with the original Lagrangian Eq. (2.6) is therefore a little cumbersome. We will study our model with the equivalent Lagrangian Eq. (2.8) instead.

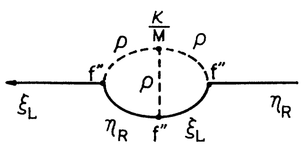

FIG. 3. Large Dirac mass between ξ_L and η_R.

With the effective Lagrangian of Eq. (2.8), we can identify light-fermion states and compute their renormalized parameters in the leading N_F order. To avoid unnecessary complications, we first consider the case where $\langle\Phi\rangle=0$ is a stable minimum of the Higgs potential $V(\Phi)$. For the time being, the gauge interactions are turned off. After loop corrections are made, the effective Lagrangian for ψ_L, ξ_R, ξ_L, η_R, and Φ can be put into

$$\begin{aligned}L=&Z_\psi\bar\psi_Li\partial\!\!\!/\psi_L+Z_{\xi R}\bar\xi_Ri\partial\!\!\!/\xi_R+Z_{\xi L}\bar\xi_Li\partial\!\!\!/\xi_L\\&+Z_\eta\bar\eta_Ri\partial\!\!\!/\eta_R+Z_\Phi|\partial^\mu\Phi|^2-m_\xi(\bar\xi_R\xi_L+\bar\xi_L\xi_R)\\&-M_\eta(\bar\eta_R\xi_L+\bar\xi_L\eta_R)-Z_\Gamma f(\bar\psi_L\Phi^\dagger\xi_R+\bar\xi_R\Phi\psi_L)\\&-V(\rho,\Phi)+\cdots\ ,\end{aligned}\tag{3.1}$$

where suppressed are terms of dimension five and above.

A. Quark mass eigenstates and low-energy Lagrangian

We first examine the mass spectrum. We can compute the $\Phi\psi_L$ scattering with the Lagrangian of Eq. (2.8) to find the renormalized ξ mass as a pole in the $\Phi\psi_L$ scattering amplitude. Alternatively, we may compute the self-energy diagram of Fig. 2 to obtain the equivalent result prior to the wave-function renormalization of $\xi_{L,R}$:

$$m_\xi=M[1-(N_Fff'\Lambda^2/16\pi^2M^2)+\cdots]\ ,\tag{3.2}$$

where we have kept only the most singular term with respect to the cutoff Λ. We will hereafter suppress nonleading terms in Λ. By tuning the coupling $N_Fff'(=-N_Fg_t)$, we force the Dirac mass m_ξ to be very small, i.e., many orders of magnitude smaller than Λ, say, the electroweak scale $v=(\sqrt{2}G_F)^{-1/2}$:

$$|m_\xi| = O(v) \ll M = O(\Lambda) . \quad (3.3)$$

The Dirac mass M_η between ξ_L and η_R is generated by the diagram of Fig. 3, and its magnitude is given by Eq. (2.11):

$$M_\eta \approx \kappa f''^3(\Lambda^2/M)/(16\pi^2)^2 = O(\Lambda) . \quad (3.4)$$

A precise value of M_η is not important here. All we need is that M_η is naturally $O(\Lambda)$ when M and M_ρ are $O(\Lambda)$. The mass eigenstates can then be read off from the mass matrix for the renormalized fields ($\xi_{L,R} \to \xi_{L,R}/\sqrt{Z_{\xi L,\xi R}}, \eta_R \to \eta_R/\sqrt{Z_\eta}$, $\psi_L \to \psi_L/\sqrt{Z_\psi}$):

$$(\bar\xi_R^c \bar\xi_L \bar\eta_R^c \bar\psi_L) \begin{bmatrix} 0 & m_\xi & 0 & 0 \\ m_\xi & 0 & M_\eta & 0 \\ 0 & M_\eta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_R \\ \xi_L^c \\ \eta_R \\ \psi_L^c \end{bmatrix} , \quad (3.5)$$

where the masses have been rescaled accordingly: $m_\xi \to \sqrt{Z_{\xi L} Z_{\xi R}}\, m_\xi$ and $M_\eta \to \sqrt{Z_{\xi L} Z_\eta}\, M_\eta$. The ψ_L state is obviously massless, which is a consequence of $SU(N_F)$ symmetry. The other massless eigenstate is the combination

$$t_R = \xi_R \cos\alpha - \eta_R \sin\alpha , \quad (3.6)$$

where

$$\tan\alpha = m_\xi / M_\eta = O(v/\Lambda) \ll 1 . \quad (3.7)$$

Since α is vanishingly small, t_R is made almost entirely of the composite state $\Phi\psi_L$. The remainder is a supermassive Dirac fermion made of ξ_L and $\eta_R \cos\alpha + \xi_R \sin\alpha$, with the mass term

$$-(M_\eta^2 + m_\xi^2)^{1/2} \bar\xi_L (\eta_R \cos\alpha + \xi_R \sin\alpha) + \text{H.c.} \quad (3.8)$$

Therefore, ignoring $O(m_\xi/M_\eta)$, we may express the low-energy effective Lagrangian as

$$L = Z_\psi \bar\psi_L i\partial\!\!\!/ \psi_L + Z_R \bar t_R i\partial\!\!\!/ t_R + Z_\Phi |\partial^\mu \Phi|^2 - Z_\Gamma f (\bar\psi_L \Phi^\dagger t_R + \bar t_R \Phi \psi_L) - V(\Phi) , \quad (3.9)$$

where $Z_R = Z_{\xi R}$ up to $O(v/\Lambda)$.

B. Renormalization and running coupling

We are now able to compute the running mass and Yukawa coupling in the leading N_F order. The running mass at scale μ is easily obtained from Eq. (3.2):

$$m_t(\mu) = m_t(0) + [M - m_t(0)]\mu^2/\Lambda^2 . \quad (3.10)$$

The renormalized Yukawa coupling is obtained from the Z factors defined in Eq. (3.9). The factors Z_ψ and Z_Φ correspond to the reciprocals of the wave-function renormalization constants for the elementary ψ_L and Φ fields, respectively. Z_Γ is also the reciprocal of the vertex renormalization constant. It is straightforward to compute them with the Lagrangian of Eq. (2.8):

$$\begin{aligned} Z_\psi &= 1 + (f^2/32\pi^2)\ln(\Lambda^2/\mu^2) \\ &\quad + (5f'^2/64\pi^2 M^2)(\Lambda^2 - \mu^2) + \cdots , \\ Z_\Phi &= 1 + (3f^2/8\pi^2)\ln(\Lambda^2/\mu^2) \\ &\quad - (3f'^2/8\pi^2 M^2)(\Lambda^2 - \mu^2) + \cdots , \\ Z_\Gamma &= 1 - (f^2/16\pi^2)\ln(\Lambda^2/\mu^2) \\ &\quad + (f'^2/16\pi^2 M^2)(\Lambda^2 - \mu^2) + \cdots , \end{aligned} \quad (3.11)$$

where the minus sign in front of the third term of Z_Φ is a peculiarity due to the derivative coupling of Φ. Since $M = O(\Lambda)$, only the running of the second terms logarithmic in Λ is important in the renormalization-group equation of our interest. However, none of the terms in Z_ψ, Z_Φ, and Z_Γ are proportional to N_F. By contrast, Z_R is given by

$$Z_R = (N_F f^2/32\pi^2)\ln(\Lambda^2/\mu^2) + \cdots , \quad (3.12)$$

where finite contributions are suppressed. The 1 is missing in Eq. (3.12) because the kinetic-energy term of t_R is generated only by loops apart from a negligibly small η_R component. The renormalized Yukawa coupling f_r at the scale μ is then obtained through $f_r(\mu)^2 = f^2 Z_\Gamma(\mu)^2/[Z_\psi(\mu) Z_R(\mu) Z_\Phi(\mu)]$. Expanding in N_F with $N_F f^2$ and $N_F f'^2$ kept fixed and keeping only the leading N_F terms, we find the renormalized coupling of the form

$$f_r(\mu)^2 = f^2/Z_R(\mu)[1 + O(1/N_F)] . \quad (3.13)$$

By use of Eq. (3.12),

$$f_r(\mu)^2 = 32\pi^2/[N_F \ln(\Lambda/\mu)^2] , \quad (3.14)$$

in the leading N_F order. As $\mu \to \Lambda$, $f_r(\mu)^2 \to \infty$, which signals compositeness at the scale Λ according to Bardeen, Hill, and Lindner [3] and Marciano [4]. It may be tempting to compare $f_r(\mu)^2$ of our model with the corresponding Yukawa coupling in the $t\bar t$ condensate model:

$$f_r(\mu)^2 = 16\pi^2/[N_C \ln(\Lambda/\mu)^2] \quad (t\bar t \text{ condensate model}) . \quad (3.15)$$

If one compares Eqs. (3.14) and (3.15) by substituting $N_F = 2$ and $N_C = 3$, one would find that the t-quark mass of our model is $\sqrt{3}$ time larger than that of the $t\bar t$ condensate model. If this argument were valid, our model would not be equivalent to the $t\bar t$ condensate model at low energies and would be clearly ruled out by the experimental constraint on the t-quark mass. However, Bardeen, Hill, and Lindner [3] argued that such a numerical estimate does not provide accurate values for low-energy parameters and that a correct numerical prediction must be made by the renormalization-group analysis. We will study the implication of our running coupling [Eq. (3.14)], along the line of the argument due to Bardeen, Hill, and Lindner.

IV. RENORMALIZATION-GROUP EQUATION

A. Uniqueness of numerical prediction

Before starting the renormalization-group analysis, we would like to point out an ambiguity involved in the Yukawa couplings (3.14) and (3.15). It was demonstrated by an example [11] that the coupling (3.15) can be modified if four-fermion interactions of higher dimension are added to the minimal $t\bar{t}$ condensate model [1–3] and therefore that the numerical prediction of Bardeen, Hill, and Lindner is not completely unique even for a fixed value of Λ. Recently, Hasenfratz *et al.* [12] made a thorough and systematic analysis on this ambiguity of higher-dimensional interactions and derived a relation of correspondence between the $t\bar{t}$ condensate and standard models. According to [12], if one adds appropriate higher-dimensional interactions to the minimal $t\bar{t}$ condensate model, one can reproduce as an effective low-energy theory the standard model having any values of the parameters in the Higgs-fermion sector. Therefore, in principle, the most general $t\bar{t}$ condensate model does not have real predictive power on low-energy parameters.

However, *if* $\ln(\Lambda/m_t)$ is much larger than unity, say, by an order of magnitude or more, *and if* dimensionless coupling constants of higher-dimensional interactions are of order 1 or less, then this ambiguity in choice of interactions does not lead to large numerical uncertainty in the values of low-energy parameters [13]. In renormalization-group language, the numerical uncertainty at large energy scales shrinks, thanks to its attraction to an infrared fixed point, as the renormalization-group equation evolves from $O(\Lambda)$ to low energies. Though the numerical prediction of Bardeen, Hill, and Lindner is, strictly speaking, not unique, it is the *most natural and almost unique* when the logarithmic running distance of renormalization group is very long. It may be said that the prediction is *saved by the infrared fixed point* [14].

The same is true for our composite t_R model. If we add binding interactions of higher dimension to our minimal Lagrangian (2.8), we can modify our Yukawa coupling (3.14). However, what is important to the renormalization-group program of Bardeen, Hill, and Lindner is the fact that the running Yukawa couplings (3.14) and (3.15) take large values ($\gg 1$) as energy rises to $O(\Lambda)$. Even with higher-dimensional interactions present, the Yukawa coupling (3.14) remains correct to leading order in $\ln\Lambda$ at energies below $O(\Lambda)$. Then, letting the renormalization-group equation evolve from large boundary values at $O(\Lambda)$ down to the electroweak scale, we can obtain the *most natural values,* if not truly unique values, of the low-energy parameters. We will argue below that the minimal $t\bar{t}$ condensate model is equivalent to the minimal composite t_R model. Our argument is applicable, within this uncertainty, to the nonminimal composite t_R and $t\bar{t}$ condensate models as well, provided higher-dimensional interactions are not *abnormally* large.

B. Comparison with the complete renormalization-group equation

The one-loop renormalization-group equation for the Yukawa coupling f is obtained from Eq. (3.14):

$$16\pi^2\mu df_r(\mu)/d\mu=(N_F/2)f_r(\mu)^3 \ . \tag{4.1}$$

This equation is valid in the leading N_F order before the gauge interactions are turned on. In comparison, the complete one-loop renormalization-group equation for the Yukawa coupling of the standard model with $SU(N_C)\times SU(N_F)\times U(1)$ symmetry reads

$$\begin{aligned}16\pi^2\mu df_r(\mu)/d\mu=&(N_C+N_F/2+\tfrac{1}{2})f_r(\mu)^3\\&-\{3[(N_C^2-1)/N_C]g_{N_C}(\mu)^2\\&+3[(N_F^2-1)/2N_F]g_{N_F}(\mu)^2\\&+3(Y^2/4+Q_t^2)g_1(\mu)^2\}f_r(\mu)\ .\end{aligned} \tag{4.2}$$

Our renormalization-group equation (4.1) is to be interpreted as the $N_F\to\infty$ limit of the complete equation (4.2) with N_C kept fixed and the gauge couplings turned off. If we introduce the gauge interactions in our model, our renormalization-group equation should turn into the complete equation. The one-loop-renormalized Yukawa couplings $f_r(\mu)^2$ of Eqs. (3.14) and (3.15) rise to infinity as the scale μ approaches the compositeness scale Λ. This rise is taken as a signature of compositeness of a particle involved in the Yukawa coupling f, i.e., the Higgs doublet in the case of the $t\bar{t}$ condensate model. Bardeen, Hill, and Lindner argued that this reasoning should apply to the solution of the complete renormalization-group equation as well and, therefore, that a correct numerical prediction of the $t\bar{t}$ condensate model on the low-energy parameters should be obtained by analyzing Eq. (4.2) with the real values for N_C $(=3)$ and N_F $(=2)$. If we follow this argument, we would also work with the solution of Eq. (4.2) for our model, starting with a large value of the Yukawa coupling at a high-energy scale $\mu\approx\Lambda$ and letting $f_r(\mu)$ evolve down to low energies. Then the numerical prediction of our model for $f_r(\mu)$ must be identical to that of the $t\bar{t}$ condensate model in the region $\mu\ll\Lambda$.

For the self-coupling λ of the Higgs bosons, the composite t_R model does not give $\lambda_r(\Lambda)=\infty$, since the Higgs bosons are elementary. In principle, the initial value $\lambda_r(\Lambda)$ is arbitrary in contrast with the $t\bar{t}$ condensate model, and so the ratio of the Higgs-boson mass to the t-quark mass cannot be determined uniquely. However, when the running distance from Λ to v is long enough, the ratio $\lambda_r(\mu)/f_r(\mu)^2$, which is attracted to its infrared fixed point, comes very close to it over a wide range of choice of the initial value $\lambda_r(\Lambda)/f_r(\Lambda)^2$. (See, for instance, the numerical calculation in Hill, Leung, and Rao [15]). Unless λ is abnormally smaller or larger than f^2 at the high-energy scale, the ratio $\lambda_r(v)/f_r(v)^2$ at the electroweak scale is insensitive to its initial value. We might be tempted to invoke some naturalness argument for $\lambda_r(\Lambda)/f_r(\Lambda)^2\sim 1$. It also should be recalled that when we move to the real world and solve the complete one-loop renormalization-group equation with $N_C=3$ and $N_F=2$, the running coupling $\lambda_r(\mu)$ blows up when $f_r(\mu)$ does. With this caveat it is very likely that the Higgs-boson mass determined by $\lambda_r(v)$ is virtually the same as in the $t\bar{t}$ condensate model.

The renormalized Yukawa couplings of our model and the $t\bar{t}$ condensate model [Eqs. (3.14) and (3.15)] are different, since the two models are solved in different large-N expansions, one in large-N_F expansion and the other in large-N_C expansion. The renormalization-group equation for the Yukawa coupling of the $t\bar{t}$ condensate model follows from Eq. (3.15).

$$16\pi^2\mu df_r(\mu)/d\mu = N_C f_r(\mu)^3 , \qquad (4.3)$$

for large N_C. The renormalization-group equations (4.1) and (4.3) are the $N_F\to\infty$ and $N_C\to\infty$ limits, respectively, of the complete equation (4.2) when the gauge couplings are turned off.

C. Case of spontaneously broken symmetry

We have so far discussed the symmetric phase of our model. For our model to be an electroweak model, the $SU(N_F)\times U(1)$ symmetry must be broken spontaneously by a vacuum condensate of the elementary Higgs field $|\langle\Phi\rangle| = v/\sqrt{2}$. It is trivial to extend our analysis to a broken-symmetry model. The only modification we should make is to incorporate self-consistently the t-quark mass and mass splitting of the Higgs doublet in the preceding calculation of one-loop diagrams. Since the symmetry is broken by the operator of dimension less than four, the renormalization-group equations, not only for $f_r(\mu)$ but also for all other couplings, are identical to those of the symmetric model, as far as we stay in the region $\mu \gg v$. Since the fine-tuning of the composite t-quark mass implies $\Lambda \gg v$, almost the entire running of $f_r(\mu)$ occurs in the region $v \ll \mu \ll \Lambda$. Then the renormalization-group analysis for a broken-symmetry model leads us to the same numerical prediction on low-energy parameters as that for the symmetric model. Therefore, we conclude that our composite t_R-quark model is indistinguishable from the $t\bar{t}$ condensate model at low energies ($\ll\Lambda$).

Before closing, we add one minor observation that is quantitatively incorrect, but instructive. After spontaneous symmetry breaking, the $t\bar{t}$ condensate model without the renormalization-group analysis [1,2] predicts $m_H=2m_t$ for the physical-Higgs-boson and t-quark masses. This *naive* prediction actually results from an infrared fixed point of the $N_C\to\infty$ limit of the renormalization-group equation for $f_r(\mu)^2/\lambda_r(\mu)$. Here $\lambda_r(\mu)$ is the renormalized Φ^4 coupling of $L_{\text{int}} = -\frac{1}{2}\lambda|\Phi^\dagger\Phi|^2$, to which the physical-Higgs-boson mass m_H is related through $m_H^2=\lambda_r(v)v^2$. With the gauge couplings turned off, the renormalization-group equation for $f_r^2(\mu)/\lambda_r(\mu)$ of the $t\bar{t}$ condensate model reads

$$16\pi^2\mu d\ln(f_r^2/\lambda_r)/d\mu = 4N_C f_r^2(f_r^2/\lambda_r - \tfrac{1}{2}) , \qquad (4.4)$$

for $N_C\to\infty$. As the couplings evolve from $\mu=\Lambda$ down to $\mu=v$, the ratio $f_r^2(\mu)/\lambda_r(\mu)$ moves toward the fixed point $\frac{1}{2}$. If the evolving distance is long enough, in other words, if only leading $\ln\Lambda$ terms are kept in diagrammatic calculation, this ratio of the couplings is indeed equal to $\frac{1}{2}$ at the low-energy scale v, which means

$$m_t^2/m_H^2 = f_r(v)^2/2\lambda_r(v) = \tfrac{1}{4} , \qquad (4.5)$$

for $\Lambda=\infty$. Let us apply this argument to our model. The $N_F\to\infty$ limit of the renormalization-group equation is

$$16\pi^2\mu d\ln(f_r^2/\lambda_r)/d\mu = N_F\lambda_r(f_r^2/\lambda_r - 2) . \qquad (4.6)$$

If the infrared fixed value $f_r^2/\lambda_r=2$ is reached in our model, it would predict

$$m_t^2/m_H^2 = 1 , \qquad (4.7)$$

instead of $\frac{1}{4}$. Namely, the physical-Higgs-boson and t-quark masses would be equal in the leading $\ln\Lambda$ approximation. However, as we have emphasized above, this naive prediction has little quantitative significance and must be replaced by the analysis of the complete renormalization-group equation. Then there is no way to distinguish between the two models by low-energy physics.

V. CONCLUDING REMARKS

We have presented one solvable model of a composite right-handed t quark. Without having supersymmetry to generate a Goldstone fermion, we must remove a left-handed singlet of composite quark (ξ_L) by introducing an extra right-handed elementary quark (η_R). This may be considered as small ugliness of our model. However, the right-handed t quark of our model is a composite only with a negligible mixture of the elementary quark (η_R). The extra quark, mostly consisting of η_R, is superheavy and has no effect at low energies. It is relevant to remark here that the elementary right-handed quark η_R with the same $SU(2)\times U(1)$ quantum numbers as t_R is actually needed to cancel the electroweak gauge anomaly.

Probably, one can build many other models that lead to the same low-energy physics as the $t\bar{t}$ condensate model and the composite t_R model. For instance, it is quite tempting to build a model in which ψ_L is a composite of elementary t_R and Φ. The only drawback for this type of model would be lack of a $1/N$ expansion to solve it explicitly. Otherwise, such a model can be easily built, following after our composite t_R model. Then a sharp rise of the running Yukawa coupling $f_r(\mu)$ toward $\mu=\Lambda$ should really be an indication of the fact that any one of ψ_L, t_R, and Φ is a composite of the others. Although the original scenario of the $t\bar{t}$ condensate and composite Φ may be the most attractive, low-energy physics, in particular the t-quark mass, cannot distinguish among three possibilities; Φ is composite, t_R is composite, or ψ_L is composite. To be precise, the composite t_R and composite ψ_L models have room to accommodate a Higgs-boson mass different from that of the $t\bar{t}$ condensate model. However, even the Higgs-boson mass is likely to come out to be virtually the same in all three cases if the running distance of the renormalization group is long enough.

It is amusing to compare our conclusion with the idea of *nuclear or particle democracy* advocated by Chew [16] in the 1960s. In particle democracy the compositeness scale and mass scale of constituent particles are comparable in magnitude. It asserted that there should be no way to tell which particles are elementary or composite. In fact, such a question is not even meaningful to ask in particle democracy. According to the line of argument presented here following Bardeen, Hill, and Lindner, elementariness or compositeness can never be answered for models with a high compositeness scale unless one has some means to probe the physics at a superhigh compositeness scale. This statement is not only very intuitive, but may be even trivial to many of us, when phrased in this way. In fact, Marciano alluded to this observation in his paper on renormalization-group analysis [4]. In this paper we have demonstrated this intuitive proposition in a solvable model.

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[1] Y. Nambu, in *New Theories in Physics,* Proceedings of XI International Symposium on Elementary Particle Physics, Kazimierz, Poland, 1988, edited by Z. A. Ajduk *et al.* (World Scientific, Singapore, 1989), p. 1; in *New Trends in Strong Coupling Gauge Theories,* Proceeding of 1988 International Workshop, Nagoya, Japan, 1988, edited by M. Bando *et al.* (World Scientific, Singapore, 1989), p. 2; in *Proceedings of the 1989 Workshop on Dynamical Symmetry Breaking,* Nagoya, Japan, 1989, edited by T. Muta and K. Yamawaki (Nagoya University, Nagoya, 1990), p. 2; University of Chicago Report No. EFI 89-08 (unpublished).

[2] V. A. Miransky, M. Tanabashi, and K. Yamawaki, Mod. Phys. Lett. A **4**, 1043 (1989); Phys. Lett. B **221**, 177 (1989).

[3] W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990).

[4] W. J. Marciano, Phys. Rev. Lett. **62**, 2793 (1989).

[5] Actually, even if exact tuning were made, the ξ_L state would still be there to interact with the gluons and the U(1) gauge boson.

[6] See, for instance, M. Suzuki, Phys. Rev. D **41**, 3457 (1990).

[7] T. E. Clark, S. Love, and W. A. Bardeen, Phys. Lett. B **237**, 235 (1990).

[8] M. Carena, T. E. Clark, C. E. Wagner, W. A. Bardeen, and K. Sasaki, Fermilab Report No. Fermilab-PUB-91/96-T (unpublished); PURD-TH-91-01, 1991 (unpublished).

[9] C. T. Hill, M. Luty, and E. A. Paschos, Phys. Rev. D **43**, 3011 (1991).

[10] Y. Okada, Y. Yamaguchi, and T. Yanagida, Phys. Lett. B **262**, 54 (1991).

[11] M. Suzuki, Mod. Phys. Lett. A **5**, 1205 (1990); in *Strong Coupling Gauge Theories and Beyond,* Proceedings of the Second International Workshop, Nagoya, Japan, 1990, edited by T. Muta and K. Yamawaki (World Scientific, Singapore, 1991), p. 53.

[12] A. Hasenfratz, P. Hasenfratz, K. Jansen, J. Kuti, and Y. Shen, University of California, San Diego/University of Arizona Report No. UCSD/PTH 91-06/AZPH-TH/91-12 (unpublished).

[13] W. A. Bardeen, Fermilab Report No. Fermilab 90/269/T (unpublished).

[14] H. Georgi, in *Strong Coupling Gauge Theories and Beyond* [11], p. 471.

[15] C. T. Hill, C. N. Leung, and S. Rao, Nucl. Phys. **B262**, 517 (1985).

[16] G. F. Chew, in *Strong Interaction Physics,* edited by M. Jacob and G. F. Chew (Benjamin, New York, 1964), p. 104.