Momentum-transfer contributions to the radiative corrections of the Dalitz plot of semileptonic decays of charged baryons with light or charm quarks

D. M. Tun and S. R. Juárez W.
Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edificio 9, Unidad Profesional "Adolfo Lopez Mateos," Col. Lindavista C.P. 07738 México, Distrito Federal, México

A. García
Departamento de Física, Centro de Investigación y de Estudios Avanzados, Instituto Politécnico Nacional, Apartado Postal 14-740, 07000 México, Distrito Federal, México

(Received 29 August 1990; revised manuscript received 21 March 1991)

We obtain an expression for the Dalitz plot of semileptonic decays of charged baryons, including radiative corrections with all the terms of the order α times the momentum transfer. The model dependence of the radiative corrections is kept in a general form which is suitable for model-independent experimental analysis. The bremsstrahlung contribution is given in two ways. The first one leaves the triple integration over the photon variables to be performed numerically and the second one is completely analytic. Our result is suitable for high-statistics decays of ordinary baryons as well as for medium-statistics decays of charm baryons.

I. INTRODUCTION

In a previous paper [1] we have obtained the radiative corrections to the Dalitz plot of semileptonic decays of charged and neutral hyperons. Those corrections were calculated within the approximation of neglecting terms of the order of $\alpha q/\pi M_1$ (with q the four-momentum transfer and M_1 the mass of the decaying baryon). This approximation makes those results reliable up to a theoretical precision of around 0.5% over most of the Dalitz plot of non-heavy-quark decaying baryons, e.g., $\Sigma^- \to n e \nu$, $\Xi^- \to \Sigma^0 e \nu$, etc.

Of course, those results must be improved for high-precision measurements, at, say, 1% experimental precision over most of the Dalitz plot or when the four-momentum transfer is large, and it can no longer be neglected in radiative corrections. This is the case with heavy-quark semileptonic decays such as in charm decays.

It is the purpose of this paper to improve our previous result. We will include all the terms of order $\alpha q/\pi M_1$ in the radiative corrections to the Dalitz plot of semileptonic decays of charged hyperons.

In the process of including all such terms, we encounter the difficulty that the model dependence of the radiative corrections appears. In the virtual part this can be handled by defining effective form factors in the uncorrected amplitude [2]. The effective form factors have a new dependence in the electron and emitted baryon energies other than the ones in the q^2 dependence of the original form factors. In the bremsstrahlung part the situation is more favorable. The Low theorem [3] allows us to know all the $\alpha q/\pi M_1$ terms. There is no model dependence if we know the experimental values of the electromagnetic static parameters of the baryons.

In Sec. II we deal with the virtual radiative corrections. We basically adapt previous results [2] for the Dalitz plot. In Sec. III we give the bremsstrahlung amplitude with all the $\alpha q/\pi M_1$ terms included. Also, we handle the infrared divergence, identifying carefully all the finite terms that come along with it. At this stage we arrive at a result ready for a triple numerical integration. In Sec. IV we integrate analytically over the photon variables, obtaining a closed expression for the bremsstrahlung part of the Dalitz plot. Section V is devoted to collecting our partial results into a final analytical expression. Therefore, we end with two choices for obtaining the radiative corrections to the Dalitz plot of the semileptonic decays of charged baryons. The first one leaves the triple numerical integration over the photon variables to be performed numerically, but with the infrared divergence and the accompanying finite terms already integrated. The second one is completely integrated into a closed analytic result.

II. VIRTUAL RADIATIVE CORRECTION

We shall begin this section by first introducing our notation and conventions, and next we shall obtain the virtual radiative corrections. The four-momenta and masses of the particles involved in baryon semileptonic decays

$$A \to B + e + \nu_e \tag{1}$$

will be denoted by $p_1=(E_1,\mathbf{p}_1)$, $p_2=(E_2,\mathbf{p}_2)$, $l=(E,\mathbf{l})$, and $p_\nu=(E_\nu^0,\mathbf{p}_\nu)$ and by M_1, M_2, m, and m_ν, respectively. We shall assume throughout this paper that $m_\nu=0$. p_2, l, and p_ν will also denote the magnitudes of the corresponding three-momenta when we specialize our calculations to the center-of-mass frame of A. There will be no confusion because in this case our expressions will not be manifestly covariant. The uncorrected transition amplitudes for process (1) is

$$M_0=\frac{G_V}{\sqrt{2}}\overline{U}_B W_\mu U_A \overline{U}_l O_\mu V_\nu \ , \quad (2)$$

where

$$W_\mu=f_1(q^2)\gamma_\mu+\frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu+\frac{f_3(q^2)}{M_1}q_\mu + \left[g_1(q^2)\gamma_\mu+\frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu+\frac{g_3(q^2)}{M_1}q_\mu\right]\gamma_5 \ , \quad (3)$$

$O_\mu=\gamma_\mu(1+\gamma_5)$, and $q=p_1-p_2$ is the four-momentum transfer. Our metric and γ-matrix conventions are those of I.

In order to obtain the virtual radiative correction to the Dalitz plot, we can follow the discussions and adapt the results of Secs. I and II of Ref. [2]. Thus there is no need to enter into details here. We only point out that the virtual radiative corrections can be separated into a model-independent part M_v and into a model-dependent part. M_v is given by Eqs. (7) and (8) of Ref. [2], and it is finite and calculable. The model-dependent part can be absorbed into M_0 through the definition of effective form factors. This will be denoted by putting a prime on M_0 (see below). The decay amplitude with virtual radiative corrections is given by

$$M_V=M_0'+M_v \ , \quad (4)$$

where

$$M_v=\frac{\alpha}{2\pi}[M_0\Phi(E)+M_{p_1}\Phi'(E)] \ . \quad (5)$$

The model-independent functions $\Phi(E)$ and $\Phi'(E)$ containing terms up to order $\alpha q/\pi M_1$ are explicitly given by

$$\Phi(E)=2\left[\frac{1}{\beta}\operatorname{arctanh}\beta-1\right]\ln\left[\frac{\lambda}{m}\right]-\frac{1}{\beta}(\operatorname{arctanh}\beta)^2+\frac{1}{\beta}L\left[\frac{2\beta}{1+\beta}\right]-\frac{1}{\beta}L\left[\frac{2\beta}{M_1/E-1+\beta}\right]$$
$$+\frac{1}{\beta}\operatorname{arctanh}\beta\left[1+\frac{E(1-\beta^2)}{M_1-2E}\right]+\frac{3}{2}\ln\left[\frac{M_1}{m}\right]-\frac{11}{8}-\frac{1}{\beta}\ln\left[1-\frac{2\beta}{M_1/E-1+\beta}\right]\left[\ln\left[\frac{M_1}{m}\right]-\operatorname{arctanh}\beta\right] \ , \quad (6)$$

$$\Phi'(E)=\frac{1-\beta^2}{\beta}\left[-\operatorname{arctanh}\beta\left[1+\frac{E}{M_1-2E}\right]+\frac{\beta E}{M_1-2E}\ln\left[\frac{M_1}{m}\right]\right] \ . \quad (7)$$

The second matrix element in Eq. (5) is given by

$$M_{p_1}=\frac{G_V}{\sqrt{2}}\frac{E}{mM_1}\overline{U}_B W_\lambda(p_1,p_2)U_A\overline{U}_l \not{p}_1 O_\lambda V_\nu \ . \quad (8)$$

Above, we use the definitions $\beta=l/E$, L as the Spence function, and λ as the infrared-divergence cutoff. This latter will be canceled by its counterpart in the bremsstrahlung contribution. Φ and Φ' go correspondingly to ϕ and ϕ' of I if we neglect terms of order $\alpha q/\pi M_1$ and $(\alpha q/\pi M_1)\ln(q/M_1)$.

The Dalitz plot with virtual radiative corrections is now obtained by standard trace calculations. We leave as the relevant independent variables the energies E_2 and E of the emitted baryon and electron, respectively. The decay rate is compactly given by

$$d\Gamma_V=d\Omega\left[A_0'+\frac{\alpha}{\pi}(B_1'\Phi+B_1''\Phi')\right] \ , \quad (9)$$

where

$$A_0'=B_1'-Q_5p_2^2ly_0(p_2+ly_0) \ , \quad (10)$$

$$B_1'=B_1''-Q_3l(p_2y_0+l)+Q_4E_\nu^0p_2ly_0 \ , \quad (11)$$

$$B_1''=Q_1EE_\nu^0-Q_2Ep_2(p_2+ly_0) \ , \quad (12)$$

$$d\Omega=\frac{G_V^2}{2}\frac{dE_2\,dE\,d\Omega_l\,d\varphi_2}{(2\pi)^5}2M_1 \ . \quad (13)$$

Here y_0 is the cosine of the angle between the electron and baryon B three-momenta and is given by

$$y_0=\frac{(E_\nu^0)^2-p_2^2-l^2}{2p_2l} \ . \quad (14)$$

where

$$E_\nu^0=M_1-E_2-E \ . \quad (15)$$

The Q_i $(i=1,\ldots,5)$ are long quadratic functions of the form factors. They are given explicitly in I in Eqs. (16)–(20). We will not repeat them here.

As it was pointed out in Ref. [2], if we consider contributions up to first order in q, the model dependence can be handled by defining effective form factors in M_0. These effective form factors are

$$f_1'(q^2,p_+\cdot l)=f_1(q^2)+\frac{\alpha}{\pi}a(p_+\cdot l) \ ,$$

$$g_1'(q^2,p_+\cdot l)=g_1(q^2)+\frac{\alpha}{\pi}a'(p_+\cdot l) \ ,$$

$$f_2'(q^2,p_+\cdot l)=f_2(q^2)+\frac{\alpha}{\pi}b \ ,$$

$$g_2'(q^2,p_+\cdot l)=g_2(q^2)+\frac{\alpha}{\pi}b' \ ,$$

$$f_3'(q^2,p_+\cdot l)=f_3(q^2)+\frac{\alpha}{\pi}c \ ,$$

$$g_3'(q^2,p_+\cdot l)=g_3(q^2)+\frac{\alpha}{\pi}c' \ ,$$

and within our approximations b, b', c, and c', are constant. Only a and a' are functions of $p_+ \cdot l = (p_1 + p_2)\cdot l$. These quantities are the only energy-dependent contributions of the model dependence to the virtual radiative corrections. In the rest frame of A, $p_+ \cdot l$ takes the form $p_+ \cdot l = (M_1 + E_2)E - p_2 l y_0$, which shows the direct dependence on E_2 and E and an indirect dependence through y_0. The primes in Eq. (9) will remind us that the above primed form factors are the ones that appear in it.

For applications, the emission of real photons must be added to Eq. (9). We turn to this in the following sections.

III. BREMSSTRAHLUNG AMPLITUDE AND INFRARED DIVERGENCE

In this section we turn to the emission of a real photon:

$$A \to B + e + \nu_e + \gamma \ . \tag{16}$$

We shall first give the amplitude of this process, and right afterwards we shall extract the infrared divergence. Next, we shall obtain a complete expression for the differential bremsstrahlung decay rate that together with Eq. (9) gives the Dalitz plot with radiative corrections of process (1).

What we want is the amplitude of process (16) with all the $\alpha q/\pi M_1$ terms. These terms can be obtained in a model-independent fashion by virtue of the Low theorem [3,4]. The amplitude for process (16) is given by Eq. (4) of Ref. [5] and can be split into three contributions:

$$M_B = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 \ , \tag{17}$$

with

$$\mathcal{M}_1 = eM_0 \left[\frac{\varepsilon\cdot l}{l\cdot k} - \frac{\varepsilon\cdot p_1}{p_1\cdot k} \right] , \tag{18}$$

$$\mathcal{M}_2 = \frac{eG_V}{\sqrt{2}} \varepsilon_\mu \bar{U}_B W_\lambda U_A \bar{U}_l \frac{\gamma_\mu \not{k}}{2l\cdot k} O_\lambda V_\nu \ , \tag{19}$$

$$\begin{aligned}\mathcal{M}_3 = \frac{G_V}{\sqrt{2}} \bar{U}_l O_\lambda V_\nu \varepsilon_\mu \bar{U}_B \Bigg[& \frac{eW_\lambda \not{k}\gamma_\mu}{2p_1\cdot k} - \kappa_1 W_\lambda \frac{\not{p}_1 + M_1}{2p_1\cdot k}\sigma_{\mu\nu}k_\nu + \kappa_2 \sigma_{\mu\nu}k_\nu \frac{\not{p}_2 + M_2}{2p_2\cdot k} W_\lambda \\ & + e\left[\frac{p_{1_\mu} k_\lambda}{p_1\cdot k} - g_{\mu\lambda} \right] \left[\frac{f_3 - f_2}{M_1} + \gamma_5 \frac{g_3 - g_2}{M_1} \right] \\ & + e\left[\frac{p_{1_\mu} k_\nu}{p_1\cdot k} - g_{\mu\nu} \right] (\sigma_{\lambda\nu} + g_{\lambda\nu}) \left[\frac{f_2 + g_2\gamma_5}{M_1} \right] \Bigg] U_A \ . \end{aligned} \tag{20}$$

κ_1 and κ_2 are the anomalous magnetic moments of A and B. M_0 and W_λ are given in Eqs. (2) and (3). ε_μ is the photon polarization four-vector, k_ν is its four-momentum, and ω its energy. The electromagnetic vertices of A and B are given (at zero momentum transfer in the form factors and at the order which interests us) by

$$\bar{U}_i \Gamma_\mu U_i = \bar{U}_i (e_i \gamma_\mu + \kappa_i \sigma_{\mu\nu} k_\nu) U_i \ , \tag{21}$$

with $i = A, B$. In terms of the observed total magnetic moments μ_i^T, the anomalous ones are given by

$$\kappa_i = \mu_i^T - \frac{e_i M_p}{e_p M_i} \ , \tag{22}$$

where κ_i and μ_i^T are in units of μ_N (the nuclear magneton). e_p and M_p are the charge and mass of the proton.

Equation (17) gives the complete amplitude of process (16) to order $\alpha q/\pi M_i$; it has no model dependence. The latter will appear only after terms of order $\alpha q^2/\pi M_1^2$ or higher are included.

Because we are interested in the radiative corrections to process (1) and not in the process (16) itself, we restrict ourselves to the three-body region of the Dalitz plot of (16) (see the Appendix also) defined by

$$E_2^{\min} \le E_2 \le E_2^{\max}$$

and

$$m \le E \le E_m \ ,$$

with

$$E_2^{\max,\min} = \frac{1}{2}(M_1 - E \pm l) + \frac{M_2^2}{2(M_1 - E \pm l)}$$

and

$$E_m = (M_1^2 - M_2^2 + m^2)/2M_1 \ .$$

As in I, we will use a coordinate frame in the rest system of A with the z axis along the electron three-momentum and the x axis oriented so that the final baryon three-momentum is in the first or fourth quadrants of the x-z plane.

It is easy to see that $\mathcal{M}_3$ of Eq. (20) is one order in q higher than $\mathcal{M}_1$ of Eq. (18) and $\mathcal{M}_2$ of Eq. (19). This is the reason why the M_B of I only contains $\mathcal{M}_1$ and $\mathcal{M}_2$. In addition to $\mathcal{M}_3$, we have another source of order $\alpha q/\pi M_1$ terms. The weak vertex W_λ contains, apart from the q^2 dependence in the form factors, terms of order $(q)^0$ and q.

In I we eliminated the order-q terms; in the present case we have to keep them and employ the complete expression Eq. (3) for W_λ both in $\mathcal{M}_1$ and $\mathcal{M}_2$. Nevertheless, because of the above reason, in $\mathcal{M}_3$ we can still neglect the terms proportional to f_2, f_3, g_2, and g_3 in W_λ.

In order to calculate the bremsstrahlung contribution to radiative corrections including terms up to order $\alpha q/\pi M_1$, we shall trace a close parallelism with the calculation of I. We shall first extract the infrared divergence, which is entirely contained in $\mathcal{M}_1$.

The square of M_B summed over spins can be split, after trace calculations, into the sum of three contributions,

$$\sum_{\text{spins}} |M_B|^2 = \sum_{\text{spins}} \mathcal{M}_1^2 + \sum_{\text{spins}} (2\mathcal{M}_1\mathcal{M}_2 + \mathcal{M}_2^2) + \sum_{\text{spins}} 2(\mathcal{M}_1\mathcal{M}_3 + \mathcal{M}_2\mathcal{M}_3) \ . \tag{23}$$

The term $\mathcal{M}_3^2$ will contribute to order $\alpha q^2/\pi M_1^2$ and higher, and thus it is not included in (23).

The first summand in (23) is explicitly given by

$$\sum_{\text{spins}} \mathcal{M}_1^2 = \frac{e^2 G_V^2}{2} \frac{4M_1}{M_2 m m_\nu} \frac{4\sum(\epsilon\cdot l)^2}{(2l\cdot k)^2} [Q_1 E E_\nu^0 - Q_2 E \mathbf{p}_2\cdot(\mathbf{p}_2+\mathbf{l}) - Q_3 \mathbf{l}\cdot(\mathbf{p}_2+\mathbf{l}) + Q_4 E_\nu^0 \mathbf{p}_2\cdot\mathbf{l}$$
$$- Q_1 E\omega - Q_2 E \mathbf{p}_2\cdot\mathbf{k} - Q_3 \mathbf{l}\cdot\mathbf{k} - Q_4 \omega \mathbf{p}_2\cdot\mathbf{l}] \ . \tag{24}$$

The coefficients Q_i, $i=1,\ldots,4$, are the same as in Sec. II. With respect to the corresponding result of I, the new terms that arise are those proportional to Q_2 and Q_4. They do not introduce new features. The first four summands of Eq. (24) contain all the infrared-divergent terms.

In order to handle the infrared-divergent terms separately from the convergent ones, let us write the differential decay rate of (16) as

$$d\Gamma_B = d\Gamma_B^{\text{ir}} + d\Gamma_B^{\text{I}} + d\Gamma_B^{\text{II}} + d\Gamma_B^{\text{III}} \ . \tag{25}$$

$d\Gamma_B^{\text{ir}}$ and $d\Gamma_B^{\text{I}}$ contain the infrared-divergent and convergent terms of (24), respectively. $d\Gamma_B^{\text{II}}$ and $d\Gamma_B^{\text{III}}$ contain the contributions from the second and third summands in (23).

$d\Gamma_B^{\text{ir}}$ can be dealt with in the same way as $d\omega_B^{\text{ir}}$ of I. There is no need to reproduce the details here, since they can be traced in exact parallelism with I. We only write the result

$$d\Gamma_B^{\text{ir}} = \frac{\alpha}{\pi} B_1 I_0(\alpha) d\Omega + d\Gamma_B^0 \ . \tag{26}$$

The argument α is evaluated at $\alpha = M_1(1+y_0)/\lambda$. Here $B_1 = B_1'$ and B_1' and $d\Omega$ are given in Eqs. (11) and (13), respectively. Explicitly, $d\Gamma_B^0$ is given by

$$d\Gamma_B^0 = \frac{\alpha}{\pi} \left[-\frac{B_1}{2\pi} \frac{\beta^2}{2} \int_0^{2\pi} d\varphi_k \int_{-1}^{1} dx \frac{1-x^2}{(1-\beta x)^2} \ln(M_1 f') \right.$$
$$\left. + B_1 C + p_2 l (Q_2 E + Q_3 - Q_4 E_\nu^0) \frac{\beta^2(1+y_0)}{2} \int_{-1}^{1} dx \frac{1-x^2}{(1-\beta x)^2} \right] d\Omega \ , \tag{27}$$

where f', I_0, and C are given by Eqs. (47), (51), and (57) of I, respectively.

After having extracted the infrared divergence and the finite terms that accompany it, we turn to the other terms in $d\Gamma_B$. To obtain $d\Gamma_B^{\text{I}}$, $d\Gamma_B^{\text{II}}$, and $d\Gamma_B^{\text{III}}$, we use Eq. (38) of I. The results for them are

$$d\Gamma_B^{\text{I}} = -\frac{\alpha}{\pi} d\Omega \frac{p_2 l}{2\pi} \int_{-1}^{1} dx \frac{\beta^2}{2} \frac{1-x^2}{(1-\beta x)^2} \int_{-1}^{y_0} dy \int_0^{2\pi} \frac{d\varphi_k}{D} \left[Q_1 E + Q_2 E (D - E_\nu^0 - l x) + Q_3 l x + Q_4 p_2 l y \right] \ . \tag{28}$$

$$d\Gamma_B^{\text{II}} = \frac{\alpha}{\pi} d\Omega \frac{p_2 l}{2\pi} \int_{-1}^{1} dx \int_0^{2\pi} d\varphi_k \int_{-1}^{y_0} \frac{F}{4D^2} dy |M'|^2 \ , \tag{29}$$

$$d\Gamma_B^{\text{III}} = \frac{\alpha}{\pi} d\Omega \frac{p_2 l}{2\pi M_1} \int_{-1}^{1} dx \int_0^{2\pi} d\varphi_k \int_{-1}^{y_0} dy \frac{F}{4D^2} |M''|^2 \ . \tag{30}$$

The squared matrix elements are rather long and tedious:

$$|M'|^2 = \frac{1}{l\cdot k} \left[(Q_1 E_\nu + Q_2 \mathbf{p}_2\cdot\mathbf{p}_\nu) \left[\omega + (1+\boldsymbol{\beta}\cdot\hat{\mathbf{k}})E - \frac{m^2\omega}{l\cdot k} \right] + (Q_3 \mathbf{p}_\nu + Q_4 E_\nu \mathbf{p}_2)\cdot \left[(E+\omega)\hat{\mathbf{k}} + \mathbf{l} - \frac{m^2}{l\cdot k}\mathbf{k} \right] \right] \tag{31}$$

and

$$|M''|^2=-(f_1-g_1)^2\beta^2\frac{1-x^2}{1-\beta x}\frac{EE_\nu}{\omega}+(f_1+g_1)^2E\left[-\frac{\beta x}{1-\beta x}\frac{D}{\omega}+\frac{E_\nu^0 lx+(\mathbf{p}_2+\mathbf{l})\cdot\mathbf{l}}{E(1-\beta x)}\frac{1}{\omega}\right]$$
$$+2f_1(f_3-f_2)E\left[-\frac{\beta x}{1-\beta x}\frac{D}{\omega}+\frac{E_\nu^0 lx+(\mathbf{p}_2+\mathbf{l})\cdot\mathbf{l}}{E(1-\beta x)}\frac{1}{\omega}+\left[\frac{1-\beta^2}{1-\beta x}-(1+\beta x)\right]\frac{E_\nu^0}{\omega}+\beta^2\frac{1-x^2}{1-\beta x}\right]$$
$$-2(f_1-g_1)^2E_\nu+(f_1^2-g_1^2)D-2f_1(f_3-f_2)(2E_\nu-D)$$
$$-\frac{2M_1\kappa_1}{e}\left[(g_1^2+f_1g_1)\left[-\frac{\beta x}{1-\beta x}D+\frac{2-\beta x}{1-\beta x}E_\nu^0-2\omega+\frac{p_2ly+l^2}{E(1-\beta x)}\right]-4f_1g_1E_\nu\right]$$
$$-\frac{2M_1\kappa_2}{e}\left[(g_1^2-f_1g_1)\left[-\frac{\beta x}{1-\beta x}D+\frac{2-\beta x}{1-\beta x}E_\nu^0-2\omega+\frac{p_2ly+l^2}{E(1-\beta x)}\right]+4f_1g_1E_\nu\right]$$
$$-2\left[(2f_1f_2-g_1g_2-g_1f_2)E_\nu^0+\frac{g_1(g_2-f_2)}{1-\beta x}(E_\nu^0+\beta p_2y+\beta l)\right.$$
$$\left.-\left[f_2(f_1+g_1)+\frac{g_1(g_2-f_2)}{1-\beta x}\right]D-2f_2(f_1-g_1)\omega\right]$$
$$-2E\left[(f_1f_2-g_1g_2-2g_1f_2)\frac{\beta^2(1-x^2)}{1-\beta x}\left[\frac{E_\nu^0}{\omega}-1\right]+(f_1f_2-g_1g_2+2f_2g_1)\frac{\beta}{\omega}\frac{(D-E_\nu^0)x-l-p_2y}{1-\beta x}\right]. \tag{32}$$

In all of these expressions, $E_\nu=E_\nu^0-\omega$ is the energy of the neutrino in process (16).

Our preceding partial results are still collected in Eq. (25), namely,

$$d\Gamma_B=d\Gamma_B^{\rm ir}+d\Gamma_B^{\rm I}+d\Gamma_B^{\rm II}+d\Gamma_B^{\rm III}, \tag{25}$$

where now $d\Gamma_B^{\rm ir}$, $d\Gamma_B^{\rm I}$, $d\Gamma_B^{\rm II}$, and $d\Gamma_B^{\rm III}$ are explicitly given in Eqs. (26)–(30).

We wish to stress that, despite its detailed length, the bremsstrahlung decay probability is organized in a rather easy-to-handle compact expression through Eq. (25). As it stands now, it is ready for numerical integration.

We want to point out that the infrared divergence is completely extracted. The coefficient of this divergence is opposite to that of the virtual part, as required for the cancellation of such divergence. The finite terms that accompany it are included analytically in $d\Gamma_B^{\rm ir}$ of Eq. (26), in the term B_1C. All of the remaining contributions [Eqs. (27)–(30)] can be evaluated numerically without infrared-divergence ambiguities. Nevertheless, the remaining integrations involved can also be performed analytically. We shall proceed to do this in the following section.

IV. ANALYTICAL INTEGRATIONS

Our result contained in Eqs. (25)–(30) is ready to be used in the bremsstrahlung contribution to the Dalitz plot of process (1). We can perform a numerical evaluation of the integrals contained in Eq. (25), but as before in I, its analytical integration presents no in-principle difficulty. All the integrals that appear are standard and can be explicitly performed. However, the major difficulty comes from the very long detailed expressions involved in Eqs. (25)–(32). Fortunately, a substantial part of the effort required has already been done in I, and what remains, although still not negligible, can be reasonably accomplished.

We shall choose the axis of coordinates as in Sec. III. Also, we shall endeavor to stay in an as close as possible parallelism with the corresponding calculation of I. The most delicate point to keep in mind is the pitfall represented by the double signs that appear in some expressions. These double signs arise because in some places it is necessary to take the positive square root of $(x-x_0)^2$.

The order of integration is, first, over the azimuthal angle of the photon φ_k, second, over the cosine of the polar angle of the final baryon y, and, third, over the cosine of the polar angle of the photon x.

The integrals over φ_k are the same that appear in I. They are five and are given by Eqs. (63)–(67) of I. We shall not repeat them here. With respect to the variable y, we have two more integrals in addition to the same integrals as in I [Eqs. (68)–(70) there]. The two new integrals are

$$\int_{-1}^{y_0}dy\,yk_1\equiv 2\pi\left[\xi_4(x)-\frac{x(E_\nu^0+lx)}{p_2}\xi_1(x)\right] \tag{33}$$

and

$$\int_{-1}^{y_0}F^2k_2dy\equiv 2\pi\{\xi_5(x)+4l^2[(E_\nu^0+lx)(1-3x^2)-2p_2y_0x]\xi_1(x)\}. \tag{34}$$

$\xi_1(x)$ is given by Eq. (72) in I. The other ξ's are

$$\xi_4(x)=-\frac{E_\nu^0+(l-p_2)x}{p_2^2}+\begin{cases}-\dfrac{E_\nu^0}{p_2^2}(x-x_0), & x\in(-1,x_0)\ ,\\ \dfrac{E_\nu^0}{p_2^2}(x-x_0), & x\in(x_0,1)\ ,\end{cases} \tag{35}$$

$$\xi_5(x)=4l^2\left[-\frac{3x\,[E_\nu^0+(l-p_2)x\,]}{p_2}-2(1+y_0)+\frac{(E_\nu^0)^2}{l^2}-\frac{(E_\nu^0)^2(x_0+a^+)^2}{2p_2l\,(x+a^+)}+\frac{(E_\nu^0)^2(x_0+a^-)^2}{2p_2l\,(x+a^-)}\right.$$
$$\left.\mp E_\nu^0\left[\frac{3x\,(x-x_0)}{p_2}+\frac{E_\nu^0(E_\nu^0+lx_0)}{p_2l^2}-\frac{E_\nu^0(x_0+a^-)^2}{2p_2l(x+a^-)}-\frac{E_\nu^0(x_0+a^+)^2}{2p_2l\,(x+a^+)}\right]\right]\ , \tag{36}$$

where

$$x_0=-\frac{p_2y_0+l}{E_\nu^0}\ ,$$

$$a^\pm=\frac{E_\nu^0\pm p_2}{l}\ .$$

In performing the y integration, it is necessary to take the positive square root of $(x-x_0)^2$, and because x_0 is always between -1 and 1, this leads one to divide the range of x as indicated in Eq. (35). Similarly, the upper sign in Eq. (36) must be chosen when $x\in(-1,x_0)$ and lower sign when $x\in(x_0,1)$.

The integrals over x contain the same ones as I, θ_i ($i=0,\ldots,9$) given by Eqs. (87), (88), and (80)–(83), as well as seven new integrals. The latter are

$$\theta_{10}=\int_{-1}^{1}dx\,x^2\xi_1(x)\ , \tag{37}$$

$$\theta_{13-n}=\int_{-1}^{1}dx\frac{\xi_4(x)}{(1-\beta x)^n},\quad n=0,1,2\ , \tag{38}$$

$$\theta_{14}=\int_{-1}^{1}dx\,x\xi_2(x)\ , \tag{39}$$

$$\theta_{15}=\int_{-1}^{1}dx\,\xi_3(x)\ , \tag{40}$$

$$\theta_{16}=\int_{-1}^{1}dx\frac{\xi_5(x)}{1-\beta x}\ . \tag{41}$$

To avoid repetitions the result of the explicit integration of Eqs. (37)–(41) will be given at the end of this section.

Once we have identified all the different integrals, we have to substitute them in Eqs. (25)–(32) of the preceding section. After some straightforward, albeit long and tedious algebraic steps, we obtain compact expressions for the several bremsstrahlung contributions to the Dalitz plot. The result is

$$d\Gamma_B=d\Gamma_B^{\rm ir}+d\Gamma_B^{\rm I}+d\Gamma_B^{\rm II}+d\Gamma_B^{\rm III}\ , \tag{42}$$

with

$$d\Gamma_B^{\rm ir}+d\Gamma_B^{\rm I}=\frac{\alpha}{\pi}d\Omega\sum_{i=0}^{13}H_i\theta_i\ , \tag{43}$$

$$d\Gamma_B^{\rm II}=\frac{\alpha}{\pi}d\Omega\sum_{i=0}^{15}H_{i+14}\theta_i\ , \tag{44}$$

$$d\Gamma_B^{\rm III}=\frac{\alpha}{\pi}d\Omega\sum_{i=0}^{16}H_{i+30}\theta_i\ . \tag{45}$$

The lengthy detailed expressions involved in Eqs. (43)–(45) now follow:

$$H_0=(Q_3-Q_4E_\nu^0)p_2l\ ,$$

$$H_1=B_1\ ,$$

$$H_2=\frac{p_2lE(1-\beta^2)}{2}[Q_1+Q_3-(Q_2+Q_4)(E_\nu^0+E)]\ ,$$

$$H_3=-\frac{p_2lE}{2}\{2(Q_1-Q_2E_\nu^0+Q_4E)+(3-\beta^2)[Q_3-Q_2E-Q_4(E_\nu^0+2E)]\}\ ,$$

$$H_4=\frac{p_2lE}{2}[Q_1+2Q_3-(Q_2+2Q_4)(E_\nu^0+2E)+Q_4E(1+\beta^2)]\ ,$$

$$H_5=-\frac{p_2l^2}{2}[Q_2E-Q_3+Q_4(E_\nu^0+2E)]\ ,\quad H_6=H_7=H_8=H_9=0\ ,$$

$$H_{10}=-\frac{l}{p_2}H_{13}\ ,\quad H_{11}=(1-\beta^2)H_{13}\ ,\quad H_{12}=-2H_{13}\ ,$$

$$H_{13}=Q_4p_2^2l^2/2\ ,\quad H_{14}=-\frac{H_0}{2}\ ,\quad H_{15}=H_{27}=0\ ,$$

$$H_{16}=\frac{p_2 l(1-\beta^2)}{2}[-(Q_1-Q_2E+Q_3)(E_\nu^0+E)+Q_4(E_\nu^0+2E)(E_\nu^0+E)+(Q_2+Q_4)p_2 l y_0+Q_2p_2^2] ,$$

$$H_{17}=\frac{p_2 l}{2}\left[(Q_1+Q_3)\left[\frac{1-\beta^2}{2}E+E_\nu^0+E\right]-Q_3E_\nu^0(1-\beta x_0)-Q_2\left[p_2 l y_0+p_2^2+E(E_\nu^0+E)+\frac{1-\beta^2}{2}E(E_\nu^0+3E)\right]\right.$$
$$\left.+Q_4\left[-E(E_\nu^0+E)+p_2\beta E_\nu^0 y_0-\frac{1-\beta^2}{2}E(5E_\nu^0+7E)\right]\right] ,$$

$$H_{18}=\frac{p_2 l}{2}\left[-Q_1E/2+Q_3(E_\nu^0-E/2)+Q_2\left[\frac{E(E_\nu^0+E)}{2}+\frac{(2-\beta^2)E^2}{2}\right]\right.$$
$$\left.+Q_4[-p_2 l y_0+E(E_\nu^0+E)/2+(4-3\beta^2)E^2/2-(E_\nu^0)^2]\right] ,$$

$$H_{19}=\frac{p_2 l^2}{4}[Q_1+Q_3+Q_2(E-E_\nu^0)+Q_4(E-5E_\nu^0)] ,$$

$$H_{20}=\frac{p_2 l(1-\beta^2)}{4}[Q_1+Q_3-(Q_2+Q_4)(E_\nu^0+E)] ,$$

$$H_{21}=-\frac{p_2 l}{4}\left[(Q_1+Q_3)\frac{2E-E_\nu^0}{E}+(Q_2+Q_4)[p_2\beta y_0-2(E_\nu^0+E)-E(1-\beta^2)]+Q_2p_2^2/E+Q_4(E_\nu^0+E)^2/E\right] ,$$

$$H_{22}=\frac{p_2 l}{4}[Q_1+Q_3-Q_2(E_\nu^0+2E)+Q_4(E_\nu^0-E)] ,$$

$$H_{23}=\frac{p_2\beta}{8}[-Q_1-Q_3+(Q_2+Q_4)(E_\nu^0+E)] ,$$

$$H_{24}=-\frac{p_2 l^3}{4}(Q_2+3Q_4) ,\quad H_{25}=-(1-\beta^2)H_{26} ,$$

$$H_{26}=\frac{p_2^2 l^2}{2}Q_4 ,\quad H_{28}=-\frac{p_2 l^2}{4}Q_2 ,\quad H_{29}=-\frac{p_2 l}{8}(Q_2+Q_4) ,$$

$$H_{30}=-p_2 lE\left[\frac{B^+}{2M_1}+\frac{A^+}{M_1}\right] ,\quad H_{31}=H_{32}=H_{36}=H_{39}=H_{41}=0 ,$$

$$H_{33}=\frac{p_2 lE(1-\beta^2)}{2M_1}[(E_\nu^0+E)B^- -EB^+]$$
$$+\frac{p_2 lE(1-\beta^2)}{M_1}\left[E_\nu^0[A^-+g_1(f_2-g_2)]-2Eg_1(f_2+g_2)+M_1\frac{h^+}{e}(E_\nu^0+2E)\right] ,$$

$$H_{34}=\frac{p_2 lE}{2M_1}[-(E_\nu^0+E)B^-+EB^++p_2\beta y_0C^-] ,$$
$$+\frac{p_2 lE}{M_1}\left[-M_1\frac{h^+}{e}[E_\nu^0+2E(1-\beta^2)]+M_1\frac{h^-}{e}\beta^2E+E_\nu^0[g_1(f_2+3g_2)-(f_1f_2+g_1g_2)(1+\beta x_0)]\right.$$
$$\left.+E[(1-\beta^2)(f_1f_2+4g_1g_2-g_1f_2)-f_1f_2+g_1(3f_2-2g_2)]\right] ,$$

$$H_{35}=\frac{p_2 l^2}{2M_1}[(E_\nu^0-E)B^-+EB^++E_\nu^0C^-]$$
$$+\frac{p_2 l^2}{M_1}\left[-M_1\frac{h^+}{e}(E_\nu^0+2E)-M_1\frac{h^-}{e}2E_\nu^0+E_\nu^0(2f_1f_2+3g_1g_2-g_1f_2)+2Eg_1(f_2+g_2)\right] ,$$

$$H_{37}=-\frac{p_2 l}{4M_1}E(1-\beta^2)B^-+\frac{p_2 l}{2M_1}\left[M_1\frac{h^+}{e}(E_\nu^0+\beta^2E+p_2\beta y_0)-E_\nu^0(1-\beta x_0)g_1(g_2-f_2)-E(1-\beta^2)A^-\right] ,$$

$$H_{38}=\frac{p_2 l}{4M_1}(E-2E_\nu^0)B^- +\frac{p_2 l}{2M_1}\left[\frac{E_\nu^0}{e}M_1(2h^- - h^+)-E_\nu^0[2f_1f_2-g_1(f_2+g_2)]+EA^-\right],$$

$$H_{40}=\frac{p_2 l^3}{2M_1}(2B^- + C^-)+\frac{p_2 l^3}{M_1}\left[-2M_1\frac{h^+}{e}-3M_1\frac{h^-}{e}+3f_1f_2+4g_1g_2-3g_1f_2\right],$$

$$H_{42}=\frac{p_2^2 l^2}{2M_1}B^+ +\frac{p_2^2 l^2}{M_1}\left[M_1\frac{h^+}{e}+A^+ - g_1(g_2-f_2)\right],$$

$$H_{43}=-\frac{p_2^2 l^2}{2M_1}C^- -\frac{p_2^2 l^2}{M_1}\left[M_1\frac{h^+}{e}+f_2(f_2+g_1)\right],$$

$$H_{44}=\frac{p_2 l^2}{2}\left[\frac{B^-}{2M_1}+\frac{A^-}{M_1}\right],$$

$$H_{45}=\frac{p_2 l}{4M_1}B^- +\frac{p_2 l}{2M_1}\left[-M_1\frac{h^-}{e}+f_2(f_1-g_1)\right],$$

$$H_{46}=\frac{p_2\beta}{4M_1}\left[-M_1\frac{h^+}{e}+g_1(g_2-f_2)\right].$$

We have used the definitions

$$A^{\pm}=f_1f_2-g_1g_2\pm 2g_1f_2\ ,\quad h^{\pm}=-g_1^2(\kappa_1+\kappa_2)\pm f_1g_1(\kappa_2-\kappa_1)\ ,$$

$$B^{\pm}=(f_1\pm g_1)^2+2f_1(f_3-f_2),\quad C^-=f_1^2-g_1^2+2f_1(f_3-f_2)\ .$$

The functions $\theta_0 \cdots \theta_9$ are given in Eqs. (95), (96), and (99) of I. The result of the integration of the other θ_i is

$$\theta_i=\frac{1}{p_2}(T_i^+ + T_i^-)\ , \tag{46}$$

with

$$T_{10}^{\mp}=\tfrac{1}{3}(x_0^3\mp 1)\ln(1\mp x_0)+\tfrac{1}{3}[(a^{\mp})^3\mp 1]\ln(1\mp a^{\mp})-\tfrac{1}{3}[x_0^3+(a^{\mp})^3]\ln[\mp(x_0+a^{\mp})]$$
$$+\tfrac{1}{6}(1-x_0^2)(a^{\mp}\pm 1)-\tfrac{1}{3}(x_0\pm 1)[1-(a^{\mp})^2]\ ,$$

$$T_{11}^+=T_{11}^-=\frac{1}{2p_2\beta}\{E_\nu^0[(1-\beta x_0)J_4-J_1]-(\beta E_\nu^0+l-p_2)I_4+(l-p_2)I_1\}\ ,$$

$$T_{12}^+=T_{12}^-=\frac{1}{2p_2\beta}[E_\nu^0(1-\beta x_0)J_1+2E_\nu^0 x_0+2(l-p_2)-(\beta E_\nu^0+l-p_2)I_1]\ ,$$

$$T_{13}^+=T_{13}^-=-\frac{1}{2p_2}E_\nu^0(1-x_0^2)\ ,$$

$$T_{14}^{\pm}=E_\nu^0[1+x_0^2+2a^{\pm}(x_0\mp 1)\pm a^{\pm}(x_0+a^{\pm})(I_2^{\pm}\pm J_2^{\pm})]\ ,$$

$$T_{15}^{\pm}=3E_\nu^0[2p_2(1+y_0)+l(1-x_0^2)]-(E_\nu^0)^2(x_0+a^{\pm})^2(J_3^{\pm}\pm I_3^{\pm})-2lE_\nu^0(x_0+a^{\pm})a^{\pm}(J_2^{\pm}\pm I_2^{\pm})\ ,$$

$$T_{16}^{\pm}=4l^2\left[\frac{3}{2\beta^2}[2(l-p_2+E_\nu^0 x_0)+\beta E_\nu^0(1-x_0^2)]+\left[-\frac{3(l-p_2+\beta E_\nu^0)}{2\beta^2}-p_2(1+y_0)+\frac{p_2(E_\nu^0)^2}{2l^2}\right]I_1\right.$$
$$\left.-\frac{(E_\nu^0)^2(x_0+a^{\pm})^2}{2l(1+\beta a^{\pm})}(\beta J_1+J_2^{\pm}\pm\beta I_1\pm I_2^{\pm})+\left[\frac{3E_\nu^0(1-\beta x_0)}{2\beta^2}+\frac{(E_\nu^0)^2(E_\nu^0+lx_0)}{2l^2}\right]J_1\right],$$

where I_1, $I_2^{\pm}$, $I_3^{\pm}$, I_4, J_1, $J_2^{\pm}$, $J_3^{\pm}$, and J_4 are given at the end of Sec. V of I.

With Eqs. (43)–(46) we have a full analytic result for the bremsstrahlung part of the Dalitz plot of the semileptonic decay of charged hyperons [Eq. (42)]. This result is model independent and contains all the $\alpha q/\pi M_1$ contributions. These properties are a direct consequence of the Low theorem.

V. FINAL RESULT AND CONCLUSIONS

Now we are in position to obtain the Dalitz plot of process (1) with radiative corrections up to order

$\alpha q/\pi M_1$. Our complete result is given by the sum of $d\Gamma_V$ and $d\Gamma_B$, namely,

$$d\Gamma(A\rightarrow Be\nu)=d\Gamma_V+d\Gamma_B \ . \tag{47}$$

$d\Gamma_V$ is given in Eq. (9) and $d\Gamma_B$ is given by Eqs. (25)–(32) or by Eqs. (42)–(45). As we have stressed all along in this paper, we have two choices for $d\omega_B$. We can perform numerically the triple integration involved in Eq. (25), or we can use the analytical results of Eqs. (43)–(45) in Eq. (42). As expected, the infrared divergence no longer appears in the sum of Eq. (47), and the finite terms that accompany it have been extracted analytically and are included both in Eq. (25) [in the terms B_1C of Eq. (26)] and in Eq. (42) [in $H_1\theta_1$ of Eq. (43)].

For easy reference we shall now collect our final analytical result for the complete radiatively corrected Dalitz plot. It is compactly summarized in the expression

$$d\Gamma(A\rightarrow Be\nu)=\left[A_0'+\frac{\alpha}{\pi}\left[H_1'(\Phi+\theta_1)+B_1''\Phi'+H_0'\theta_0+\sum_{i=2}^{16}H_i'\theta_i\right]\right]d\Omega \ , \tag{48}$$

with

$$H_i'=H_i+H_{i+14}+H_{i+30} \quad (i=0,\ldots,13) \ ,$$

$$H_i'=H_{i+14}+H_{i+30} \quad (i=14,15) \ ,$$

$$H_{16}'=H_{46} \ ,$$

where the H_i's were given in the previous section. Φ, Φ', A_0', B_1'', and $d\Omega$ are given in Eqs. (6), (7), (10), (12), and (13), respectively. The functions $\theta_{10},\ldots,\theta_{16}$ are given in Eq. (46). The others θ_i are given in I [θ_0, θ_1, and θ_i $(i=2,\ldots,9)$ are given in Eqs. (95), (96), and (99) of I].

Equation (48) improves the result obtained in Eq. (108) of I, because in it we have incorporated all the terms of order $\alpha q/\pi M_1$. Equation (48) has the same virtues as Eq. (108) of I. It has no infrared divergences, it does not contain an ultraviolet cutoff, and it is not compromised by any model dependence of radiative corrections. As discussed at the end of Sec. II, this model dependence is absorbed into the already existing form factors. In the case that $\alpha q/\pi M_1\rightarrow 0$, the model dependence amounts only to two constants that are respectively summed into f_1 and g_1, but if $\alpha q/\pi M_1\neq 0$, we have to sum a function of $p_+\cdot l=(p_1+p_2)\cdot l$ to f_1 and another function to g_1. The other form factors are modified only by additive constants.

Equation (48) is very useful for processes where the momentum transfer is not small and therefore cannot be neglected. To first order in q this leads to terms of order $\alpha q/\pi M_1$ in the radiative corrections. The expected error by the omission of higher-order terms is of the order of $\alpha q^2/\pi M_1^2\approx 0.0006$ in charm decay. If the accompanying factors amount to one order-of-magnitude increase, then we can estimate an upper bound to the theoretical uncertainty of 0.6%. This is acceptable even with an experimental precision of 2–3 %. This precision will not be attained experimentally in the immediate future in the case of charged charm baryon decays. We may then expect our result to remain useful for a long time.

We have limited our calculations of the bremsstrahlung part to the Dalitz plot of the nonradiative semileptonic decay. Because of the experimental error in the determination of the energies of emitted particles, the events of the purely radiative decay that lie outside but near the boundary of the Dalitz plot will not be experimentally discriminated. Since it is at this boundary that the infrared divergence appears, one may wonder if Eq. (25) or (42) can be used safely to account for those properly radiative events. The answer is in the affirmative. This is discussed in detail in the Appendix.

There are no other analytical results available in the literature to which we can compare our expressions (42)–(46). There is a numerical analysis available [6], though. In this respect we can say that our procedure is similar to that of Ref. [6], except that there the anomalous magnetic moments of particles A and B were ignored, while in this paper they have been included. A detailed comparison must be done numerically. We shall try to do this sometime in the near future.

ACKNOWLEDGMENTS

S.R.J.W. gratefully acknowledges partial support by Comisión de Operación y Fomento de Actividades Académicas (Instituto Politécnico Nacional).

APPENDIX

In this appendix we show that the bremsstrahlung differential decay rate in the three-body region (TBR), given by the Eq. (42), can be used in the four-body region (FBR) if we are near the boundary between them. A graph of the Dalitz plot showing clearly this boundary may be found in I.

First, we note that the bremsstrahlung amplitude M_B, given in Eq. (17), is the same for the TBR and FBR. Next, we note that the bremsstrahlung differential decay rate has, in both cases, the form

$$d\Gamma_B=\frac{M_2 m m_\nu}{(2\pi)^8}\,\frac{p_2 l\, dE_2 dE\, d\Omega_l\, d\varphi_2}{2}$$

$$\times\int_{-1}^{1}dx\int_0^{2\pi}d\varphi\int_{-1}^{Y}dy\frac{k^2}{2\omega D}\sum|M_B|^2 \ .$$

In the TBR the upper limit Y of the integral over the variable y is the y_0 given by Eq. (14), while in the FBR Y is one.

The infrared divergence in the TBR comes from a term of the form $\ln(y_0-y)$, which diverges when $y=y_0$. Within the FBR there is no infrared divergence because y_0 is greater than 1 and the range of y is restricted to be between -1 and 1. This divergence is only present on the boundary between the TBR and FBR, where y_0

equals 1, but this is just the infrared divergence of the TBR reached from the FBR side. If we did not reach the same divergence, it would amount to counting twice the boundary.

Since the rest of M_B is finite, one can easily convince oneself that the bremsstrahlung decay rate is connected continuously and smoothly in going from the TBR to the FBR. One can therefore use Eq. (42), which was specialized to the TBR, in the FBR as long as one stays close to boundary. The error introduced goes as the difference between the bremsstrahlung differential decay rates in the TBR and FBR, namely,

$$d\Gamma_B^{\mathrm{TBR}} - d\Gamma_B^{\mathrm{FBR}} = \frac{M_2 m m_\nu}{(2\pi)^8} \frac{p_2 l \, dE_2 \, dE \, d\Omega_l \, d\varphi_2}{2} \times \int_{-1}^{1} dx \int_0^{2\pi} d\varphi \int_{y_0}^{1} dy \frac{k^2}{2\omega D} \sum |M_B|^2 \, .$$

But as we approach the boundary from the FBR and y_0 approaches 1, this difference becomes negligible. In practice, this difference will be proportional to the experimental energy step of the bins that fall on the boundary. As long as this step is of the order of 10 or 15 MeV, the error introduced is smaller than our error of order $\alpha q^2/\pi M_1^2$.

[1] D. M. Tun, S. R. Juárez W., and A. García, Phys. Rev. D **40**, 2967 (1989), hereafter called I.

[2] A. García and S. R. Juárez W., Phys. Rev. D **22**, 1132 (1980); **22**, 2923(E) (1980).

[3] F. E. Low, Phys. Rev. **110**, 974 (1958).

[4] H. Chew, Phys. Rev. **123**, 377 (1961).

[5] S. R. Juárez W., A. Martinez V., and A. García, Phys. Rev. D **43**, 282 (1991).

[6] F. Glück and K. Tóth, Phys. Rev. D **41**, 2160 (1990).