Electric dipole transitions of $\psi(3770)$ and S-D mixing between $\psi(3686)$ and $\psi(3770)$

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Various $E1$ transition rates for $\psi(3770)$ and other $c\bar{c}$ states are calculated in a QCD-motivated potential model. Relativistic corrections are found to be substantial. While a good agreement between theory and experiment is achieved for the rates of $\psi(2S)\to\gamma\chi_{cJ}$ and $\chi_{cJ}\to\gamma\psi(1S)$ $(J=0,1,2)$, the calculated rates for $\psi(3770)\to\gamma\chi_{cJ}$ are smaller by a factor of ~ 2 than their experimental values obtained by the Mark III Collaboration. The S-D mixing effects are further considered and are found to be important. The mixing angle $\theta=-10°$ is favored and $\theta=30°$ is ruled out. While most observed transition rates may be explained in the theory, the disagreement between theory and the $\psi(3770)\to\gamma\chi_1$ data still remains to be clarified.

The study of electric dipole transitions of heavy quarkonium may provide useful information on the interquark forces. In particular, some dipole transitions of charmonium are sensitive to the relativistic corrections which depend mainly on the spin-dependent forces of quarks through the wave-function corrections. Dipole transitions of the $\psi(3686)$ and the $\psi(3770)$ are particularly interesting, because they are sensitive not only to the relativistic corrections but also to the $2S$-$1D$ mixing. Indeed, the study of these transitions may be helpful in determining both the sign and size of the $2S$-$1D$ mixing angle. In this paper, we investigate the effects of both the relativistic corrections and the $2S$-$1D$ mixing on the dipole transitions of $\psi(3686)$ and, in particular, $\psi(3770)$. We will first concentrate on the relativistic corrections to the wave functions, and then discuss the $2S$-$1D$ mixing.

Recently, the electric dipole ($E1$) transitions for the charmonium $^3D_1$ state, $\psi(3770)$, into ^3P_J states, χ_0, χ_1, and χ_2, have been measured by the Mark III Collaboration [1] and branching ratios of $(2.0\pm0.5)\%$, $(1.7\pm0.7)\%$, and $\leq 0.2\%$ (90% C.L.) are obtained respectively. With the total width of (25 ± 3) MeV for $\psi(3770)$ [2], the partial widths for the above $E1$ decays are 500 ± 200 keV, 430 ± 180 keV, and ≤ 500 keV.

It is well known that the QCD-motivated potential model of heavy quarkonium has been successful in describing the mass spectra and many transition processes for the charmonium and bottomonium families [3]. According to the generally accepted point of view, the interaction between heavy quarks can be considered as a short-range one-gluon-exchange potential (Lorentz vector) plus a long-range confining potential (Lorentz scalar). There are many phenomenologically successful potentials in the literature. As a simple choice, we prefer to use the following scalar potential $S(r)$ and vector potential $V(r)$ [4]:

$$S(r)=kr\ , \tag{1}$$

$$V(r)=\frac{8\pi}{25}\frac{1}{r\ln\Lambda r}\frac{1-\Lambda r}{1+\Lambda r}\ . \tag{2}$$

This $V(r)$ has the following characteristics. When $r\to 0$,

$$V(r)\to\frac{8\pi}{25}\frac{1}{r\ln\Lambda r}\ ,$$

which represents a Coulomb potential with a running coupling constant; and when r increases up to the range comparable to the scale of the ψ family, the coefficient of the Coulomb potential approaches a fixed constant, i.e., $V(r)\to -\beta/r$, and $\beta\approx 0.50$ (e.g., for $\Lambda\sim 0.47$ GeV, $\beta\approx 0.50$ in the range $r\sim 1$–$5\ \mathrm{GeV}^{-1}$).

Adopting such a potential is based on the following consideration. The calculations in the lattice QCD show that the interquark potential is indeed the sum of a Coulomb potential and a linear potential when $r\geq 0.5\ \mathrm{GeV}^{-1}$. Under the approximation which neglects the dynamical effects of the light quarks, the coefficient of the Coulomb potential is about 0.2–0.3 [5], which will increase after these effects are further considered [6]. It is possible that the vacuum-polarization effects of the light-quark pairs make the coefficient increase up to about 0.5 [7]. The Coulomb potential adopted by us embodies just the characteristic that the Coulomb potential with a running coupling constant at short distances turns gradually into a Coulomb potential with a constant coefficient at long distances. When r increases up to, say, $10\ \mathrm{GeV}^{-1}$, the coefficient of our Coulomb potential will slightly reduce and then $\beta\approx 0.42$ (for $\Lambda=0.47$ GeV). Since at such a large distance the screening effects of the light quarks to the potential between the heavy quarks are important, the assumption of a Coulomb potential plus a linear one itself will be a very rough approximation. We believe that the Coulomb potential as a whole should be dominated by a Lorentz vector, in spite of the possibility that a part of the Coulomb potential at large distances might arise from a Lorentz scalar, i.e., the transverse zero-point oscillations of the string. We also note that our Coulomb potential in the range $r \sim 1$–$5\ \mathrm{GeV}^{-1}$ almost coincides with that used by the Cornell group [8].

Under the nonrelativistic approximation, the Hamiltonian of the heavy-quarkonium system can be expanded in powers of $\mathbf{p}^2/m^2$ (m and $\mathbf{p}$ are respectively the mass and the momentum of the quarks in the center-of-mass frame), and reads

$$H = H_0 + H_1 + \cdots , \tag{3}$$

$$H_0 = \frac{\mathbf{p}^2}{m} + S(r) + V(r) , \tag{4}$$

$$H_1 = H_{\mathrm{SD}} + H_{\mathrm{SI}} , \tag{5}$$

$$H_{\mathrm{SD}} = \frac{1}{2m^2 r}(3V' - S')(\mathbf{S}_1 + \mathbf{S}_2)\cdot\mathbf{L} + \frac{2}{3m^2}\mathbf{S}_1\cdot\mathbf{S}_2\nabla^2 V - \frac{1}{3m^2}[3(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - \mathbf{S}_1\cdot\mathbf{S}_2]\left[V'' - \frac{V'}{r}\right] , \tag{6}$$

$$H_{\mathrm{SI}} = -\frac{\mathbf{p}^4}{4m^3} + \frac{1}{4m^2}\left[\frac{2}{r}V'\mathbf{L}^2 + [\mathbf{p}^2, V - rV'] + 2(V - rV')\mathbf{p}^2 + \frac{1}{2}\left[\frac{8}{r}V' + V'' - rV'''\right]\right] , \tag{7}$$

where H_{SD} and H_{SI} represent respectively the spin-dependent and the spin-independent Hamiltonian up to the first order. Note that in our model for the scalar confining potential only the Thomas precession term, which is also indicated by the lattice QCD result [9], is included in H_{SD}, and no other spin-independent terms are taken into consideration in H_{SI}. Indeed, theoretically it is still unclear how to deal with the spin-independent corrections arising from the confining interaction, and practically it is found [4] that those corrections to the mass spectrum are unreasonably large if naively treating the confining potential as a scalar particle exchange, unless the linear potential is compensated by a large negative constant term, as suggested in Ref. [10].

We first use H_0 to solve the zeroth-order Schrödinger equation and then treat H_1 as perturbation to calculate the first-order relativistic corrections to the energies and wave functions. When taking the parameters

$$k = 0.22\ \mathrm{GeV}^2,\quad \Lambda = 0.47\ \mathrm{GeV} , \tag{8}$$

$$m_c = 1.84\ \mathrm{GeV},\quad m_b = 5.17\ \mathrm{GeV} , \tag{9}$$

a good agreement between calculated (with relativistic corrections) and observed mass spectra for both $c\bar{c}$ and $b\bar{b}$ states can be obtained, which have been briefly reported in Ref. [4] and will be discussed in detail in another publication. Here only the $E1$ transition processes for the charmonium will be concerned.

Under the nonrelativistic approximation the $E1$ transition width and the matrix element are given by (see, e.g., Ref. [10] for detailed discussions)

$$\Gamma(E1) = \frac{4}{27}e_Q^2\alpha|\langle f|r|i\rangle|^2(2J_f+1)S_{if}k^3 , \tag{10}$$

$$\langle f|r|i\rangle = \int_0^\infty R_f(r)R_i(r)r^3 dr , \tag{11}$$

where k is the energy of the emitted photon, $R(r)$ is the radial wave function, and S_{if} is a statistic factor. The transition widths $\Gamma(E1)$ calculated in our model and the corresponding experimental values are shown in Table I, where the widths without parentheses are the results calculated by using the zeroth-order wave functions and the values in the parentheses are calculated by using the first-order relativistically corrected wave function. The experimental values for $\psi(2S)\to\gamma\chi_{cJ}$ and $\chi_{cJ}\to\gamma\psi(1S)(J=0,1,2)$ are taken from Ref. [2].

It can be seen from Table I that the widths for $\psi(2S)\to\gamma\chi_{cJ}$ and $\chi_{cJ}\to\gamma\psi(1S)$ with the relativistic corrections are in rather good agreement with data, which shows that the relativistic correction is not negligible for the $c\bar{c}$ system. Although the velocity of the quark in such a system is quite low ($v^2/c^2 \approx 0.2$), the relativistic corrections can be rather large, since the rates of some $E1$ transitions are very sensitive to the small variation of the wave functions. This is particularly evident for the transition $\psi(3686)\to\gamma\chi_{c0}(2\,^3S_1\to 1\,^3P_0)$. In addition to the attractive effects of the spin-independent part of the first-order Hamiltonian H_1, the spin-orbit term from one-gluon exchange contributes a strong attractive force to the $^3P_0$ state, which makes the radial wave function contract towards the origin, and thus suppresses the matrix element $\langle 2\,^3S_1|r|1\,^3P_0\rangle$ and the width of the dipole transition. Since the matrix element is sensitive to the overlap between the wave functions of the initial and the final states (in particular, when these wave functions have different numbers of nodes), the relativistic corrections to the widths could be substantial in some cases. Concerning the relativistic corrections for $2\,^3S_1\to 1\,^3P_J$ and $1\,^3P_J\to 1\,^3S_1$, our results are qualitatively consistent with other authors' [10], although the potential used by us as

TABLE I. The electric dipole transitions for the charmonium. The states represent the following physical states $1\,^3S_1\psi(3097)$; $2\,^3S_1\psi(3686)$; $1\,^3P_0\chi_{c0}(3415)$; $1\,^3P_1\chi_{c1}(3510)$; $1\,^3P_2\chi_{c2}(3555)$; $1\,^3D_1\psi(3770)$. The widths without parentheses are the results calculated by using the zeroth-order wave functions and the values in the parentheses are calculated by using the first-order relativistically corrected wave functions. Experimental values are taken from Refs. [1,2].

Process	k (MeV)	S_{if}	Γ(keV)	Expt. (keV)
$2\,^3S_1\to 1\,^3P_0$	261	1	42(25)	23 ± 4
$2\,^3S_1\to 1\,^3P_1$	172	1	36(28)	21 ± 4
$2\,^3S_1\to 1\,^3P_2$	128	1	25(22)	19 ± 4
$1\,^3P_0\to 1\,^3S_1$	303	1	141(104)	95 ± 37
$1\,^3P_1\to 1\,^3S_1$	389	1	299(216)	<350
$1\,^3P_2\to 1\,^3S_1$	429	1	401(282)	350^{+160}_{-120}
$1\,^3D_1\to 1\,^3P_0$	338	2	312(199)	500 ± 200
$1\,^3D_1\to 1\,^3P_1$	250	1/2	95(72)	430 ± 180
$1\,^3D_1\to 1\,^3P_2$	208	1/50	3.6(3.0)	≤ 500

well as the method to deal with the relativistic correction is different.

For the currently interesting transitions $1\,^3D_1 \to 1\,^3P_J$, the results in Table I show that the relativistic correction is again important. For the $^3D_1$ state, there exist both attraction and repulsion in the first-order Hamiltonian H_1. The spin-orbit term from one-gluon exchange, $\sim(3/2m^3)(V'/r)$, is an attraction and the Thomas precession force from the scalar confining potential, $\sim -(1/2m^2)(S'/r)$, is a repulsion. The former decreases more rapidly than the latter as the radius of the bound state increases. In the spin-independent corrections, in addition to some attraction terms, the repulsion term $(1/4m^2)(2/r)V'\mathbf{L}^2$ increases as the quantum number of the angular momentum increases. The relative enhancement of the repulsions in the $^3D_1$ state plays a part in balancing the attraction, so that the total relativistic correction has a less effect on the wave function of the $^3D_1$ state. However, for the $^3P_0$ state, the attraction is dominant in H_1 and the relativistic correction makes its wave function contract towards the origin evidently. Therefore, the overlap between the wave functions of $^3D_1$ and $^3P_0$ decreases after the relativistic correction is taken into account, which results in the suppression of the matrix element $\langle 1\,^3D_1|r|1\,^3P_0\rangle$ and the width. The calculated results show that the relativistically corrected widths for $1\,^3D_1 \to 1\,^3P_0$ and $1\,^3D_1 \to 1\,^3P_1$ are smaller by a factor of ~ 2 than their experimental values.

Of course, it should be noted that there exist some uncertainties in the theory. The first is concerned with the quark mass. The quark mass adopted by us is the same as by the Cornell model [8] and by the authors of Ref. [10]. When we take such a large quark mass and then make the relativistic correction to get further suppressions, the obtained transition widths for $\psi(2S)\to\gamma\chi_{cJ}$ and $\chi_{cJ}\to\gamma\psi(1S)$ become compatible with the data. If a smaller charm quark mass is taken, although the transition widths for $1\,^3D_1 \to 1\,^3P_J$ can increase, the widths for $2\,^3S_1 \to 1\,^3P_J$ and $1\,^3P_J \to 1\,^3S_1$ will accordingly increase. In fact, for many models [11,12] which use a smaller quark mass, the calculated widths for $\psi(2S)\to\gamma\chi_{cJ}$ are substantially larger than the data.

The second uncertainty is concerned with the higher-order relativistic corrections, for which to give a quantitative estimation is still very difficult at present. In any event, because of the rather low velocity ($v^2/c^2 \approx 0.2$) of the quark in the charmonium system, the first-order relativistic correction should make sense and it is not expected that the higher-order relativistic corrections could give an enhancing factor more than 2.

The third one is concerned with the coupling effects with the decay channels [8], which, in general, always reduce the proportion of $c\bar{c}$ and increase the proportion of the continuum states such as $D\bar{D}$, etc., in the physical states (if the $2S$-$1D$ mixing from the coupling-channel effects is not considered for the moment), and thus only play a role to suppress the $E1$ transition widths.

The last one is concerned with the $2\,^3S_1$-$1\,^3D_1$ mixing. The $\psi(3770)$ has an appreciable leptonic decay width $\Gamma_{ee}=0.26\pm0.05$ keV [2]. This indicates that $\psi(3770)$ must contain a component of the $^3S_1$ state. Considering $\psi(3686)$ and $\psi(3770)$ as two mixed states of $2\,^3S_1$ and $1\,^3D_1$, the interference between S and D states will strengthen some transition processes and weaken some others. These effects are not only dependent on the quantum numbers of the initial and the final states, but also dependent on the sign of the mixing angle. The degree of strengthening and weakening is dependent on the magnitude of the mixing angle. The sign and the magnitude of the mixing angle are dependent on the mixing mechanism, e.g., by the coupling channel or(and) by the tensor force. Evidently, the $E1$ transition processes for $\psi(3686)$ and $\psi(3770)$ will provide very useful information on the $2\,^3S_1$-$1\,^3D_1$ mixing, which is worth studying.

In the following we assume $\psi' \equiv \psi(3686)$ and $\psi'' \equiv \psi(3770)$ to be admixtures of $2\,^3S_1$ and $1\,^3D_1$ $c\bar{c}$ states and

$$|\psi'\rangle = |2^3S_1\rangle\cos\theta + |1^3D_1\rangle\sin\theta\ , \tag{12}$$

$$|\psi''\rangle = -|2^3S_1\rangle\sin\theta + |1^3D_1\rangle\cos\theta\ . \tag{13}$$

According to the expression of the $E1$ transition width,

$$\Gamma(i\to f+\gamma)=\tfrac{4}{3}\alpha e_Q^2 k^3|\langle f|\mathbf{r}|i\rangle|^2\ , \tag{14}$$

for the $\psi'\to\chi_J\gamma$ and $\psi''\to\chi_J\gamma$ transitions [χ_J being the ^3P_J state (J=0,1,2)] we find

$$\Gamma(\psi'\to\chi_0\gamma)=\tfrac{4}{27}\alpha e_Q^2 k^3(\cos^2\theta\langle 1\,^3P_0|r|2\,^3S_1\rangle^2 - 2\sqrt{2}\cos\theta\sin\theta\langle 1\,^3P_0|r|2\,^3S_1\rangle\langle 1\,^3P_0|r|1\,^3D_1\rangle + 2\sin^2\theta\langle 1\,^3P_0|r|1\,^3D_1\rangle^2)\ , \tag{15}$$

$$\Gamma(\psi'\to\chi_1\gamma)=\tfrac{4}{9}\alpha e_Q^2 k^3(\cos^2\theta\langle 1\,^3P_1|r|2\,^3S_1\rangle^2 + \sqrt{2}\cos\theta\sin\theta\langle 1\,^3P_1|r|2\,^3S_1\rangle\langle 1\,^3P_1|r|1\,^3D_1\rangle + \tfrac{1}{2}\sin^2\theta\langle 1\,^3P_1|r|1\,^3D_1\rangle^2)\ , \tag{16}$$

$$\Gamma(\psi'\to\chi_2\gamma)=\tfrac{20}{27}\alpha e_Q^2 k^3(\cos^2\theta\langle 1\,^3P_2|r|2\,^3S_1\rangle^2 - (\sqrt{2}/5)\cos\theta\sin\theta\langle 1\,^3P_2|r|2\,^3S_1\rangle\langle 1\,^3P_2|r|1\,^3D_1\rangle + \tfrac{1}{50}\sin^2\theta\langle 1\,^3P_2|r|1\,^3D_1\rangle^2)\ , \tag{17}$$

$$\Gamma(\psi''\to\chi_0\gamma)=\tfrac{4}{27}\alpha e_Q^2 k^3(2\cos^2\theta\langle 1\,^3P_0|r|1\,^3D_1\rangle^2 + 2\sqrt{2}\cos\theta\sin\theta\langle 1\,^3P_0|r|1\,^3D_1\rangle\langle 1\,^3P_0|r|2\,^3S_1\rangle + \sin^2\theta\langle 1\,^3P_0|r|2\,^3S_1\rangle^2)\ , \tag{18}$$

$$\Gamma(\psi''\to\chi_1\gamma)=\tfrac{4}{9}\alpha e_Q^2 k^3(\tfrac{1}{2}\cos^2\theta\langle 1\,^3P_1|r|1\,^3D_1\rangle^2 - \sqrt{2}\cos\theta\sin\theta\langle 1\,^3P_1|r|1\,^3D_1\rangle\langle 1\,^3P_1|r|2\,^3S_1\rangle + \sin^2\theta\langle 1\,^3P_1|r|2\,^3S_1\rangle^2)\ , \tag{19}$$

$$\Gamma(\psi''\to\chi_2\gamma)=\tfrac{20}{27}\alpha e_Q^2 k^3[\tfrac{1}{50}\cos^2\theta\langle 1\,^3P_2|r|1\,^3D_1\rangle^2 + (\sqrt{2}/5)\cos\theta\sin\theta\langle 1\,^3P_2|r|1\,^3D_1\rangle\langle 1\,^3P_2|r|2\,^3S_1\rangle + \sin^2\theta\langle 1\,^3P_2|r|2\,^3S_1\rangle^2]\ , \tag{20}$$

where the definition of $\langle f|r|i\rangle$ is as in Eq. (11). From (14) to (20) we see that for most transitions the S-D interference terms which are proportional to $\cos\theta\sin\theta$ make substantial contributions. Therefore the sign of the mixing angle θ will be very important. Because the value of the matrix element $\langle 1P|r|2S\rangle$ is negative and $\langle 1P|r|1D\rangle$ is positive, for $\theta>0$ the rates of $\psi'\to\chi_0\gamma$ and $\psi''\to\chi_1\gamma$ would be enhanced, while that of $\psi'\to\chi_1\gamma$ and $\psi''\to\chi_0\gamma$ would be suppressed; whereas for $\theta<0$ the rates will be shifted in the opposite direction.

For the mixing angle we have limited information only at present. If naively assuming the leptonic decay width of $\psi(3770)$ is entirely due to its S-wave component, we would get

$$\tan^2\theta=\frac{\Gamma(\psi''\to e^+e^-)}{\Gamma(\psi'\to e^+e^-)}\approx 0.12 \tag{21}$$

$$\text{and } |\theta|\approx 19^\circ\ . \tag{22}$$

If further considering the D-wave component contribution to the leptonic widths, some calculations [13] give two solutions:

$$\theta\approx -10^\circ \quad\text{or}\quad \theta\approx 30^\circ\ . \tag{23}$$

Note that $\theta\approx -10^\circ$ is consistent with the result of coupled channel models [8,14], whereas the mixing caused purely by the tensor forces is much smaller [8].

We may also calculate the mixing angle in our model by virtue of the nonrelativistic formulas for the leptonic decay widths [15]

$$\Gamma(\psi'\to e^+e^-)=4\alpha^2\left[\frac{2}{3}\right]^2\frac{\left|\cos\theta R_{2S}(0)+\sin\theta\frac{5}{2\sqrt{2}m_c^2}R''_{1D}(0)\right|^2}{M(\psi')^2}\ , \tag{24}$$

$$\Gamma(\psi''\to e^+e^-)=4\alpha^2\left[\frac{2}{3}\right]^2\frac{\left|\sin\theta R_{2S}(0)-\cos\theta\frac{5}{2\sqrt{2}m_c^2}R''_{1D}(0)\right|^2}{M(\psi'')^2}\ , \tag{25}$$

where $R_{2S}(0)$ is the radial wave function at origin of the $2S$ state, and $R''_{1D}(0)$ the second derivative of the radial wave function at origin of the $1D$ state. With (1), (2), (8), (9), we find for the zeroth-order wave functions

$$R_{2S}(0)=0.892\ \text{GeV}^{3/2}\ , \tag{26}$$

$$R''_{1D}(0)=0.202\ \text{GeV}^{7/2}\ . \tag{27}$$

Because of large QCD corrections, (24) and (25) cannot be used to determine the absolute values of leptonic widths, but they are useful in determining the ratio of the two leptonic widths. With the experimental value for the ratio of $\Gamma(\psi'\to e^+e^-)=2.15\pm0.21$ keV to $\Gamma(\psi''\to e^+e^-)=0.26\pm0.05$ keV [2], we then find two solutions for the mixing angle:

$$\theta=-13^\circ \quad\text{or}\quad \theta=26^\circ\ . \tag{28}$$

Comparing the values in (26) and (27) with that calculated in the Cornell model [13],

$$R_{2S}(0)=0.963\ \text{GeV}^{3/2},\quad R''_{1D}(0)=0.314\ \text{GeV}^{7/2}\ ,$$

we see that our $R''_{1D}(0)$ is significantly smaller than the Cornell model, and this difference is due to the fact that our interquark potential is asymptotically free at short distances and is therefore less singular at origin than the typical Coulomb potential used in the Cornell model. With a smaller D wave contribution, the obtained values for the mixing angle will shift from the Cornell value $\theta=-10^\circ$ or $\theta=30^\circ$ given in (23) towards $\theta=-19^\circ$ or $\theta=19^\circ$ given in (22) by entirely neglecting the D-wave contribution. This is expected to be a rather general tendency for a wide range of interquark potentials which are softer at short distances than the Coulomb potential. Therefore we may view the Cornell value $\theta=-10^\circ$ or $\theta=30^\circ$ as two rather extreme values for the mixing angle determined by the ratio of leptonic widths in the $S-D$ mixing model for ψ' and ψ''.

We choose $\theta=0^\circ$, $\theta=-10^\circ$, and $\theta=30^\circ$ as three tentative values for the mixing angle and use (14) to (20) to calculate the $E1$ transition widths of $\psi(3686)$ and $\psi(3770)$. The results are shown in Table II. The transition matrix elements are calculated by using the zeroth-order wave functions without relativistic corrections. Comparing the results with experimental data, we find

(1) $\theta=30^\circ$ should be definitely ruled out, because it would give an excessively large value ($=135$ keV) for the $\psi'\to\chi_0\gamma$ width.

(2) $\theta=-10^\circ$ works well for the required suppression of $\psi'\to\chi_0\gamma$. It makes this transition rate close to the data even without any relativistic corrections. $\theta=-10^\circ$ also enhances $\psi''\to\chi_0\gamma$, which is also favored by the preliminary data. However, $\theta=-10^\circ$ will enhance $\psi'\to\chi_1\gamma$ and suppress $\psi''\to\chi_1\gamma$, making these rates deviate further from the data.

In any event we cannot explain the entire data by just setting $\theta=-10^\circ$ although it is favored. We emphasize that $\psi'\to\chi_0\gamma$ is the process which is particularly sensitive to the S-D mixing because the coefficient of the S-D interference term is very large in this transition. Therefore the study of $\psi'\to\chi_0\gamma$ will provide useful information on the S-D mixing. In addition, $\psi'\to\chi_0\gamma$ is also particularly sensitive to the relativistic corrections. Obviously, a combined study of S-D mixing with relativistic corrections is needed.

With both S-D mixing and relativistic corrections considered, if keeping $\theta=-10^\circ$, a smaller quark mass, say, $m_c\sim1.5$ GeV will be expected to fit the data of $\psi'\to\chi_0\gamma$, because a larger quark mass such as $m_c=1.8$ GeV would oversuppress the rate of $\psi'\to\chi_0\gamma$ when both $\theta=-10^\circ$ and relativistic corrections are taken into account. Accordingly, with $\theta=-10^\circ$ and $m_c\sim1.5$ GeV, the disagreement between theory and data for $\psi''\to\chi_0\gamma$ may then be removed. But the disagreement for $\psi' \to \chi_1 \gamma$ and $\psi'' \to \chi_1 \gamma$ may still remain.

The conclusions of the paper are as follows. In the framework of our charmonium model, considering the relativistic effect but neglecting the $2\,^3S_1 - 1\,^3D_1$ mixing, it is very difficult to make the experimentally observed transition widths for $\psi(2S) \to \gamma \chi_{cJ}$ and $\chi_{cJ} \to \gamma \psi(1S)$ consistent with the one for $\psi(3770) \to \gamma \chi_{cJ}$.

If we desire that the theory can give transition widths for $\psi(2S) \to \gamma \chi_{cJ}$ and $\chi_{cJ} \to \gamma \psi(1S)$ which one compatible with the data, then the predicted widths for $\psi(3770) \to \gamma \chi_{cJ}$ will be a factor of ~ 2 smaller than their experimental values obtained by the Mark III Collaboration. When further considering the S-D mixing, we find substantial effects of the S-D interference on the transition rates. $\theta = 30°$ is ruled out, and $\theta = -10°$ or a slightly smaller value is favored by the $\psi' \to \gamma \chi_0$ and $\psi'' \to \gamma \chi_0$ data. However, even with both S-D mixing and relativistic corrections, the calculated excessively large rate of $\psi' \to \gamma \chi_1$ and the excessively small rate of $\psi'' \to \gamma \chi_1$ are still incompatible with data. The coupled-channel effects may suppress the $\psi' \to \gamma \chi_1$ rate but are hard to enhance the $\psi'' \to \gamma \chi_1$ rate, even if the χ_1 state has a large component of virtual open charm-meson pairs. We believe that further theoretical studies of these problems are needed. At the same time we suggest making more accurate measurements of the electromagnetical transitions for $\psi(3770)$ on the Beijing Electron-Positron Collider.

Note added in proof. Very recently the E760 Collaboration has presented $p\bar{p}$ annihilation results for precise measurements of the total widths of $\chi_{c1}(3510)$ and $\chi_{c2}(3555)$ [see S. Palestini, talk given at the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, Geneva, Switzerland, 1991 (unpublished)]:

$$\Gamma_{\text{tot}}(\chi_1) = 0.88 \pm 0.11 \pm 0.08 \text{ MeV} ,$$

$$\Gamma_{\text{tot}}(\chi_2) = 1.98 \pm 0.17 \pm 0.07 \text{ MeV} .$$

TABLE II. The electric dipole transition widths for $\psi(3686)$ and $\psi(3770)$ in the S-D mixing model, where $\psi' \equiv \psi(3686)$, $\psi'' \equiv \psi(3770)$. The widths are calculated by using the zeroth-order wave functions for three cases $\theta = 0°, -10°$, and $30°$. The experimental values are taken from Refs. [1,2].

Process	Γ (keV) $\theta = 0°$	Γ (keV) $\theta = -10°$	Γ (keV) $\theta = 30°$	Expt. (keV)
$\psi' \to \chi_0 \gamma$	42	19	135	23 ± 4
$\psi' \to \chi_1 \gamma$	36	47	6.0	21 ± 4
$\psi' \to \chi_2 \gamma$	25	22	23	19 ± 4
$\psi'' \to \chi_0 \gamma$	312	363	110	500 ± 200
$\psi'' \to \chi_1 \gamma$	95	60	188	430 ± 180
$\psi'' \to \chi_2 \gamma$	3.6	13	12	≤ 500

Together with the observed branching ratios [2]

$$B(\chi_1 \to \gamma J/\psi) = (27.3 \pm 1.6)\%$$

and

$$B(\chi_2 \to \gamma J/\psi) = (13.5 \pm 1.1)\% ,$$

one gets

$$\Gamma(\chi \to \gamma J/\psi) = (240 \pm 40) \text{ keV} ,$$

$$\Gamma(\chi_x \to \gamma J/\psi) = (267 \pm 30) \text{ keV} .$$

Compared with all other model calculations [3,10,11,12], our results, i.e., relativistically corrected values for these radiative widths of χ_1 χ_2, as well as χ_0 (see Table I) are in good agreement with data. This indicates that relativistic corrections for transitions $1\,^3P_J \to 1\,^3S_1 (J = 0,1,2)$ are rather substantial.

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