$\gamma\gamma \to \pi^0\pi^0$ and $K_L \to \pi^0\gamma\gamma$ in the chiral quark model

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We consider the process $\gamma\gamma \to \pi^0\pi^0$ within the quark model. We find small corrections to the lowest-order result obtained from chiral loops. These corrections are very similar to those found in vector-meson-dominated models, and too small to be tested with the present data. We use this result, together with a simple pole model, to find the amplitude for $K_L \to \pi^0\gamma\gamma$ and use this to estimate the *CP*-conserving part of $K_L \to \pi^0 e^+e^-$. For this last process we find a branching ratio $\sim 2.3\times 10^{-13}$, significantly smaller than the *CP*-violating contribution.

I. INTRODUCTION

The framework of chiral perturbation theory [1–3] has proved extremely useful for analyzing low-energy processes involving the pseudoscalar-meson octet and photons. At low energies, the strong and electromagnetic interactions of these particles can be adequately described with a chiral Lagrangian with up to four derivatives. The most general chiral Lagrangian to this order has been written down by Gasser and Leutwyler [2]. It consists of two terms at leading order (p^2), and of ten more at the next order (p^4). All these 12 arbitrary coupling constants have been fixed from experiment [2].

In a similar way chiral Lagrangians can be used to relate different weak decays. In the standard model, the dominant $|\Delta S|=1$ operators in the effective weak Hamiltonian transform as $(8_L,1_R)$ or $(27_L,1_R)$ under chiral rotations. Empirically we know that transitions induced by the octet operators are significantly enhanced. We can then write a chiral representation for these leading operators. The lowest-order term encountered in chiral perturbation theory contains two derivatives and is unique. Hence, to this order there is only one unknown coupling constant which can be determined from a measurement of, say, $K\to\pi\pi$. Unfortunately the situation is much more complicated at the next order, where a very large number of operators and therefore of unknown coupling constants has been identified [4].

The interest of the process $\gamma\gamma\to\pi^0\pi^0$ lies in the fact that it receives *no* contributions from local operators up to order p^4 in an energy expansion. There is, however, a finite one-loop contribution to it at this order. This results in a unique prediction from chiral perturbation theory containing no unknown constants, which can be compared to experiment [5]. Tree-level contributions start at order p^6 and therefore expected to be smaller. Of course, in chiral perturbation theory we do not know the couplings that appear at order p^6 and hence we cannot explicitly confirm this conjecture. Within the context of specific models, however, we can predict the higher-order constants. Since there exists experimental information for this process [6], it is of interest to study the predictions of different models. In particular, the quark model has given surprisingly good results [7–10]. (There are many versions of the quark model, and many calculations; we list only a few recent ones.)

The case of $K_L\to\pi^0\gamma\gamma$ is even more important, because this mode, with the subsequent conversion of the photon pair into an e^+e^- pair, represents a potentially large CP-conserving background to the process $K_L\to\pi^0e^+e^-$ [11–15]. The CP-conserving contribution to the latter is estimated from the absorptive part of the two-photon intermediate state. The p^4 terms in $K_L\to\pi^0\gamma\gamma$ are such that their contribution to $K_L\to\pi^0e^+e^-$ is suppressed by powers of the electron mass. Hence, the main contribution is expected to come from the terms or order p^6. As has been emphasized in the literature, a complete calculation of $K_L\to\pi^0\gamma\gamma$ requires the knowledge of direct weak counterterms in addition to the usually considered pole diagrams [14]. A calculation of these counterterms within the quark model is beyond the scope of the present paper, and we will content ourselves with the usual pole model result as an estimate for the full amplitude.

In Sec. II we briefly review the chiral quark model, and we then apply it to the processes $\gamma\gamma\to\pi^0\pi^0$ and $K_L\to\pi^0\gamma\gamma$ in Secs. III and IV.

II. THE MODEL

The chiral quark model is a model that describes effective interactions between constituent quark fields and the octet of pseudo Goldstone bosons. The approach [7] consists of the description of strong interactions below the scale of chiral-symmetry breaking (~ 1 GeV) in terms of an $SU(3)_V$ multiplet of quark fields, the pseudoscalar-meson octet, ϕ, and perhaps gluons. These quark and gluon fields are not to be interpreted as the fundamental fields that appear in the QCD Lagrangian, but rather as a set of effective fields. Using the theory of nonlinear realizations of chiral symmetry one constructs the interactions of these effective fields, and organizes them as an expansion in powers of external momenta [16].

To do this one introduces the matrix $\xi = \exp[i(\boldsymbol{\lambda}\cdot\boldsymbol{\phi}/2F_\pi)]$, where $F_\pi = 93.3$ MeV, and defines U by the transformation properties of ξ under $SU(3)_L \times SU(3)_R$:

$$\xi \to L\xi U^\dagger = U\xi R^\dagger . \tag{2.1}$$

One can then adopt a fermion multiplet that transforms under the chiral group as

$$\Psi \to U\Psi \tag{2.2}$$

to find that the most general Lagrangian with up to one derivative is [3]

$$\begin{aligned} \mathcal{L} &= \bar{\Psi}[i\gamma_\mu(\partial^\mu + v^\mu - g_A a^\mu \gamma_5) - m]\Psi , \\ v_\mu &= \tfrac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) , \\ a_\mu &= \tfrac{1}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) . \end{aligned} \tag{2.3}$$

This contains the couplings of the pseudoscalar octet to a set of quarks with constituent mass m and effective axial vector coupling g_A. Typical values are $m \sim 320-360$ MeV [17]. Recently, Weinberg [18] has argued that these effective quarks should have $g_A = 1$ as ordinary fundamental quarks do. Furthermore, the analysis of $\pi-\pi$ scattering within this model showed that the results were not very sensitive to the value of g_A provided it did not deviate too much from 1 [8]. With this motivation, and since in practice it is much simpler to work with quarks that have $g_A = 1$, we will use that value for the rest of the paper. We can then write the neutral-meson–quark couplings that follow from Eq. (2.3):

$$\begin{aligned} \mathcal{L} = \frac{1}{2F_\pi} \Bigg[& \bar{u}\gamma_\mu\gamma_5 u \left[\partial_\mu \pi^0 + \frac{1}{\sqrt{3}} \partial_\mu \eta_8 \right] + \sqrt{2}\bar{d}\gamma_\mu\gamma_5 s \partial_\mu K^0 \\ & + \bar{d}\gamma_\mu\gamma_5 d \left[-\partial_\mu \pi^0 + \frac{1}{\sqrt{3}} \partial_\mu \eta_8 \right] \\ & + \sqrt{2}\bar{s}\gamma_\mu\gamma_5 d \partial_\mu \bar{K}^0 - \frac{2}{\sqrt{3}} \bar{s}\gamma_\mu\gamma_5 s \partial_\mu \eta_8 \Bigg] . \end{aligned} \tag{2.4}$$

This then determines all the couplings such as Fig. 1(a). Notice that our couplings do not have SU(3) breaking.

To introduce electromagnetic couplings we require the Lagrangian to be gauge invariant under electromagnetism, which is accomplished as usual with the introduction of covariant derivatives $\partial_\mu \to \partial_\mu + ieQ_q A_\mu$ in the quark kinetic term and $\partial_\mu \to \partial_\mu + ieA_\mu[Q,\ ]$ in the terms v_μ, a_μ containing the pseudoscalars (Q is the diagonal matrix with elements Q_u, Q_d, Q_s).

One can easily convince oneself that the couplings that would appear in the vertices of Figs. 1(b) and 1(c) vanish for the case of the π^0 (as well as a possible coupling with two π^0 and one photon). Similarly, there are no direct couplings of photons to the neutral pseudoscalars.

A more familiar version of the quark model is obtained by choosing a set of fermions with left- and right-handed components transforming separately under the chiral group as $(3_L, 1_R)$ or $(1_L, 3_R)$. With this set of fermions, one obtains a Lagrangian with no derivative couplings provided $g_A = 1$. This is why in practice calculations are simpler when $g_A = 1$. We give the relevant couplings in this case, noting that with this choice, the coupling of Fig. 1(c) no longer vanishes:

$$\begin{aligned} \mathcal{L} = -i\frac{m}{F_\pi} \Bigg[& \bar{u}\gamma_5 u \left[\pi^0 + \frac{1}{\sqrt{3}} \eta_8 \right] + \sqrt{2}\bar{d}\gamma_5 s K^0 \\ & + \bar{d}\gamma_5 d \left[-\pi^0 + \frac{1}{\sqrt{3}} \eta_8 \right] \\ & + \sqrt{2}\bar{s}\gamma_5 d \bar{K}^0 - \frac{2}{\sqrt{3}} \bar{s}\gamma_5 s \eta_8 \Bigg] \\ & + \frac{m}{2F_\pi^2} \pi^0\pi^0(\bar{u}u + \bar{d}d) . \end{aligned} \tag{2.5}$$

The two realizations of chiral symmetry ought to give the same results (this is because we are interested in a nonanomalous process) and one is free to choose one or the other for convenience. We have verified our results using both sets of fermions.

III. $\gamma\gamma \to \pi^0\pi^0$

The amplitude for this process starts at order p^4 in the energy expansion and, at this order, it only gets contributions from pseudo-Goldstone-boson loops. The general form of the amplitude as required by gauge invariance for the process $\gamma(q_1, \epsilon_1)\gamma(q_2, \epsilon_2) \to \pi^0(p)\pi^0(p')$ is

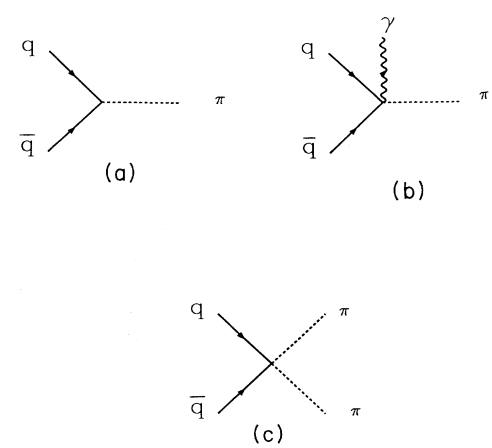

FIG. 1. Quark-model couplings: (a) to one pion (b) to one pion and one photon; (c) to two pions.

$$
\begin{aligned}
M &= \epsilon_1^{\mu}(q_1)\epsilon_2^{\nu}(q_2)M_{\mu\nu}\,,\\
M_{\mu\nu} &= A(s,t,u)\left[-\frac{s}{2}g_{\mu\nu}+q_2^{\mu}q_1^{\nu}\right]\\
&\quad+\frac{B(s,t,u)}{m^2}\left[-\frac{(m_\pi^2-t)(m_\pi^2-u)}{2}g_{\mu\nu}-sp_\mu p_\nu\right.\\
&\qquad\left.+(m_\pi^2-t)q_{2\mu}p_\nu+(m_\pi^2-u)p_\mu q_{1\nu}\right],
\end{aligned}
\tag{3.1}
$$

where s,t,u are the usual Mandelstam variables. Notice that the second form factor B only appears at order p^6.

The result from kaon and pion loops has been obtained in the literature [5]:

$$
\begin{aligned}
A_{\rm CL}(s,t,u) &= \frac{1}{16\pi^2}\frac{4e^2}{F_\pi^2}\left[\frac{s-m_\pi^2}{s}F\left[\frac{s}{m_\pi^2}\right]+\tfrac{1}{4}F\left[\frac{s}{m_K^2}\right]\right],\\
B_{\rm CL}(s,t,u) &= 0\,,
\end{aligned}
\tag{3.2}
$$

where the function $F(x)$ is

$$
F(x)=\begin{cases}1-\dfrac{4}{x}\left[\arcsin\dfrac{\sqrt{x}}{2}\right]^2, & x\le 4\,,\\[2ex] 1+\dfrac{1}{x}\left[\ln\dfrac{1-\sqrt{1-4/x}}{1+\sqrt{1-4/x}}+i\pi\right]^2, & x>4\,.\end{cases}
\tag{3.3}
$$

The kaon loop contribution represents a small correction to the pion loops.

To obtain the tree-level contributions that appear in the quark model at order p^6 and higher, consider the diagrams of Fig. 2. The result at order p^6 is given by

$$
\begin{aligned}
A_{\rm QM}(s,t,u) &= -\frac{e^2N_c}{16\pi^2F_\pi^2}\frac{5}{27}\left[\frac{s-4m_\pi^2}{m^2}\right],\\
B_{\rm QM}(s,t,u) &= -\frac{e^2N_c}{16\pi^2F_\pi^2}\frac{10}{27}\,,
\end{aligned}
\tag{3.4}
$$

where $N_c=3$ is the number of colors and we will always take $m=350$ MeV.

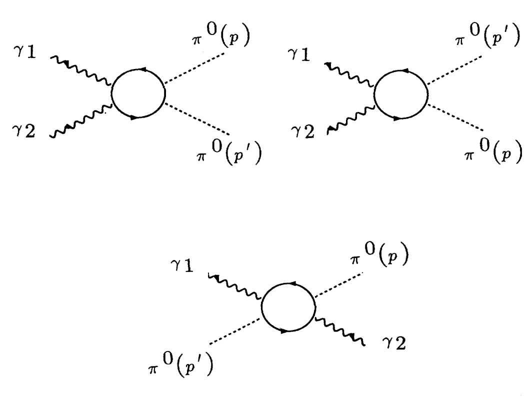

FIG. 2. Loop diagrams giving $\gamma\gamma\to\pi^0\pi^0$ in the quark model. (There are three more diagrams such as these).

This result can also be obtained from more general methods where the quark fields are formally integrated out to produce an effective action. We have checked our result with this method without obtaining the full effective action but just those terms that contribute to the process at hand. Using the techniques described by Ball [19] we find the effective Lagrangian with two photons and two pions:

$$
\mathcal{L}=\frac{-5e^2N_c}{432F_\pi^2m^2}(\partial_\alpha\pi^0\,\partial^\beta\pi^0\,F^{\alpha\nu}F_{\beta\nu}-\tfrac{1}{2}\partial_\mu\pi^0\,\partial^\mu\pi^0\,F_{\alpha\beta}F^{\alpha\beta})\,.
\tag{3.5}
$$

From Eq. (3.5), one can reproduce the A and B amplitudes of Eq. (3.4). However, we disagree with the result of Ref. [19] (see Ref. [20]).

For comparison, we also quote the result obtained in a vector-meson-dominance model:

$$
\begin{aligned}
A_{\rm VMD}(s,t,u) &= -\frac{e^2 8h_V^2}{F_\pi^2}\frac{5}{9}\left[\frac{2m}{m_V}\right]^2\left[\frac{s-4m_\pi^2}{m^2}\right],\\
B_{\rm VMD}(s,t,u) &= -\frac{e^2 8h_V^2}{F_\pi^2}\frac{10}{9}\left[\frac{2m}{m_V}\right]^2,
\end{aligned}
\tag{3.6}
$$

where $M_V=M_\rho$ and $h_V=3.7\times10^{-2}$ is extracted from the decays $V\to\pi\gamma$.

The different predictions are compared with each other and the experimental results in Fig. 3. We have used the same angular cut of the experimental results. The energy scale is extended only to $\sqrt{s}=600$ MeV, since π-π

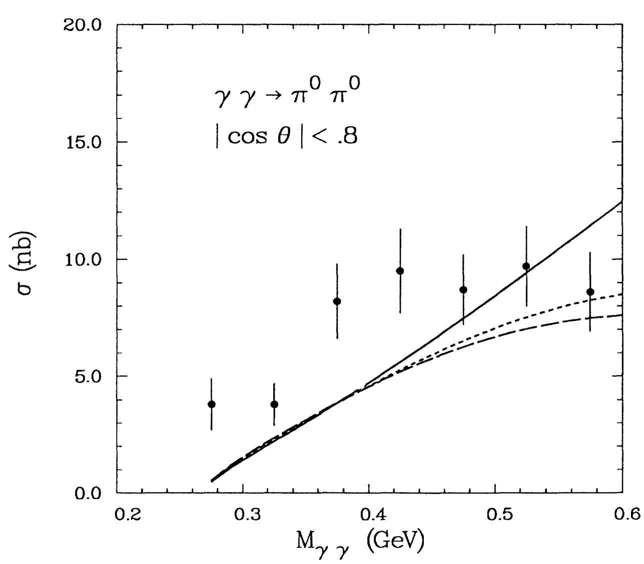

FIG. 3. Cross section for $\gamma\gamma\to\pi^0\pi^0$ at low energies. The experimental numbers are from Ref. [6]. The solid line gives the predictions of chiral perturbation theory to order p^4, Eq. (3.2); the dotted line includes also the quark-model results, Eqs. (3.2) and (3.4); and finally the dashed line represents the results of a vector-dominance model, Eqs. (3.2) and (3.6). We have used the same angular cut of Ref. [6].

scattering data tell us that chiral perturbation theory is breaking down by this scale. The data are quite flat for $\sqrt{s} \sim 350-600$ MeV and none of the predictions have this feature. However, the prediction of chiral perturbation theory for $\gamma\gamma \to \pi^0\pi^0$ has no free parameters and so it is gratifying that it is within 2σ of the data. The corrections due to quark loops are quite similar to the contributions from vector-dominance models, but the difference between the two is too small to be tested with the current data. (A different treatment of this process can be found in Ref. [21].)

IV. $K_L \to \pi^0\gamma\gamma$

We begin by computing the amplitude for an off-shell pion, $\pi^0(p)^*$ with $p^2 \neq m_\pi^2$, to decay to $\pi^0\gamma\gamma$. The amplitude for $\pi^0(p) \to \pi^0(p-q_1-q_2)\gamma(q_1,\epsilon_1)\gamma(q_2,\epsilon_2)$ is restricted by gauge invariance to be of the form (again the form factor B appears only at order p^6)

$$\begin{aligned} M &= \epsilon_1^\mu(q_1)\epsilon_2^\nu(q_2)M_{\mu\nu} , \\ M_{\mu\nu} &= A(x_1,x_2)(-m^2 s_{12} g_{\mu\nu} + q_2^\mu q_1^\nu) \\ &\quad + B(x_1,x_2)(-m^2 x_1 x_2 g_{\mu\nu} - s_{12} p_\mu p_\nu \\ &\qquad + x_1 q_{2\mu} p_\nu + x_2 p_\mu q_{1\nu}) , \end{aligned} \tag{4.1}$$

where we have now adopted the notation of Ref. [12],

$$x_i = \frac{p\cdot q_i}{m^2} , \quad s_{12} = \frac{q_1\cdot q_2}{m^2} , \quad r_K = \frac{p^2}{m^2} , \tag{4.2}$$

but we have used the constituent quark mass m for normalization. We can easily obtain $A(x_1,x_2)$ and $B(x_1,x_2)$ in the quark model to order p^6:

$$\begin{aligned} A(x_1,x_2) &= -\frac{e^2 N_c}{16\pi^2 F_\pi^2}\frac{10}{27}(x_1+x_2-2r_K) , \\ B(x_1,x_2) &= -\frac{e^2 N_c}{16\pi^2 F_\pi^2}\frac{20}{27} . \end{aligned} \tag{4.3}$$

The full result for the lowest-order (p^4) amplitude $K_L \to \pi^0\gamma\gamma$, obtained from chiral loops, is [22]

$$\begin{aligned} A_{\rm CL}(x_1,x_2) &= \frac{e^2}{4\pi^2}G_8\left[F\left[\frac{2q_1\cdot q_2}{m_\pi^2}\right]\left[1-\frac{m_\pi^2}{2q_1\cdot q_2}\right] + F\left[\frac{2q_1\cdot q_2}{m_K^2}\right]\left[\frac{m_K^2+m_\pi^2}{2q_1\cdot q_2}-1\right]\right], \\ B_{\rm CL}(x_1,x_2) &= 0 . \end{aligned} \tag{4.4}$$

To this lowest-order result we will add the next-order terms that appear in the quark model contributing through pole diagrams only. This pole model for $K_L \to \pi^0\gamma\gamma$ consists of considering the diagrams of Fig. 4(a) and of ignoring possible direct weak counterterms depicted schematically in Fig. 4(b). These counterterms are in general expected, and their effect on the overall amplitude can be significant, as is the case in the "weak deformation model" of Ref. [14].

A nice way to include all the poles is to simultaneously diagonalize all the terms bilinear in meson fields appearing in the lowest-order strong plus weak Lagrangian. (See the first paper in Ref. [22].) In terms of K_L and ignoring CP violation this amounts to

$$\begin{aligned} \pi^0 &\to \pi^0 + 2\frac{m_K^2 F_\pi^2}{m_K^2-m_\pi^2}G_8 K_L , \\ \eta_8 &\to \eta_8 - 2\left[\frac{1}{3}\right]^{1/2}\frac{m_K^2 F_\pi^2}{m_\eta^2-m_K^2}G_8 K_L , \\ K_L &\to K_L - 2\frac{m_\pi^2 F_\pi^2}{m_K^2-m_\pi^2}G_8\pi^0 + 2\left[\frac{1}{3}\right]^{1/2}\frac{m_\eta^2 F_\pi^2}{m_\eta^2-m_K^2}G_8\eta_8 , \end{aligned} \tag{4.5}$$

where $G_8 = G_8^* = 9.1\times10^{-6}$ GeV^{-2} if there is no CP violation.

In the quark model we obtain a cancellation between the π^0 and η_8 poles for the u-quark loop. The results for the d- and s-quark loops, as well as the K pole diagrams, combine to give (with degenerate u, d, and s quarks):

$$\begin{aligned} A_{\rm QM}(x_1,x_2) &= -\frac{e^2 N_c}{16\pi^2}G_8\frac{8}{27}(x_1+x_2-2r_K) , \\ B_{\rm QM}(x_1,x_2) &= -\frac{e^2 N_c}{16\pi^2}G_8\frac{16}{27} , \end{aligned} \tag{4.6}$$

where we have used the Gell-Mann–Okubo mass formula $3(m_\eta^2-m_K^2) = m_K^2-m_\pi^2$.

For comparison, the result obtained from vector-dominance models is

$$\begin{aligned} A_{\rm VMD}(x_1,x_2) &= -e^2 G_8\frac{(2m)^2}{m_V^2}\frac{64h_V^2}{9}(x_1+x_2-2r_K) , \\ B_{\rm VMD}(x_1,x_2) &= -e^2 G_8\frac{(2m)^2}{m_V^2}\frac{128h_V^2}{9} . \end{aligned} \tag{4.7}$$

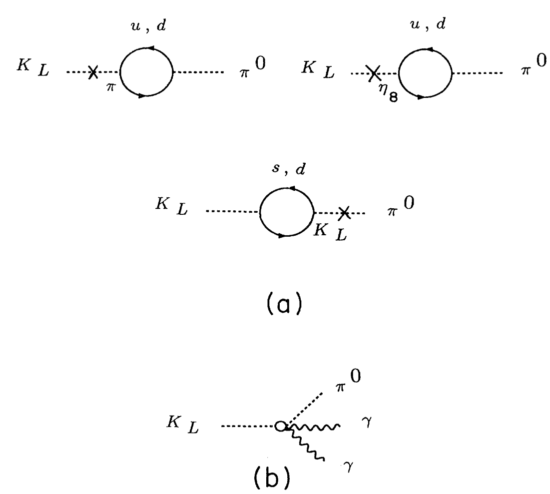

FIG. 4. Diagrams contributing to $K_L \to \pi^0\gamma\gamma$. (a) Pole diagrams (the photons are attached in all possible ways to the quark lines), and (b) possible direct weak counterterms.

We again compare these results with experiment [23] in Fig. 5. In this figure we have normalized the predictions to the experimental branching ratio. The shape of the distribution is affected very little by the small p^6 corrections. The corrections (both in the quark model and in the vector-dominance model) tend to increase the number of events to be expected for the lower values of the photon pair invariant mass. This, however, is a very small effect, and is difficult to check due to the different acceptance the experiment has for different regions of $M_{\gamma\gamma}$ [23].

It is interesting to notice that the quark-model and the vector-meson-dominance results have exactly the same form at lowest order. If we require the two models to give the same result we can use the vector-dominance model to predict the ratio m/g_A [to lowest order the effect of keeping $g_A\neq 1$ is simply to multiply the form factors $A(x_1,x_2)$ and $B(x_1,x_2)$ by g_A^2] or we can use this ratio from the quark model to predict the strength of the coupling h_V:

$$h_V=\frac{g_A}{8\pi\sqrt{2}}\left[\frac{m_V}{2m}\right]. \tag{4.8}$$

Taking $m_V=770$ MeV, $g_A=1$, and $m=350$ MeV gives $h_V=3.1\times10^{-2}$.

We find a total branching ratio for $K_L\to\pi^0\gamma\gamma$ of

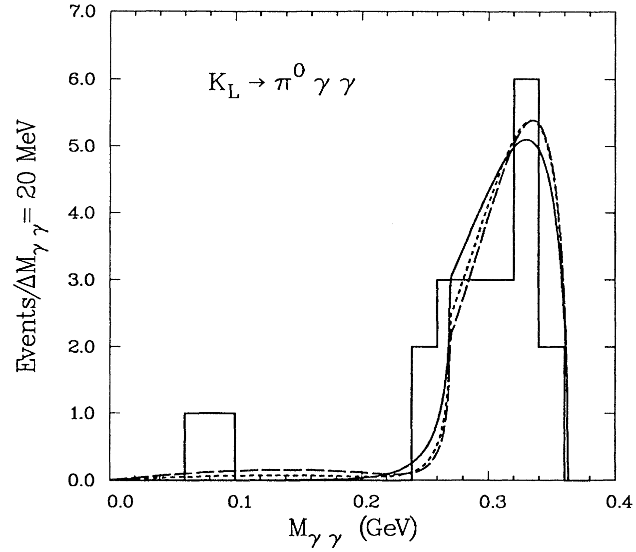

FIG. 5. Rate for $K_L\to\pi^0\gamma\gamma$. The experimental result is taken from Ref. [23]. The solid line is the lowest-order prediction from chiral loops; the dotted line includes the results of the quark model; and the dashed line contains the result of the VMD model for comparison. The predictions have been normalized to give the same branching ratio as the data.

$$B(K_L\to\pi^0\gamma\gamma)=\begin{cases}6.7\times10^{-7} & \text{for pion loops only,}\\ 6.0\times10^{-7} & \text{for pion plus quark loops,}\\ 6.0\times10^{-7} & \text{for pion loops plus VMD .}\end{cases} \tag{4.9}$$

If we put a cut on the invariant mass of the photon pair, $M_{\gamma\gamma}>280$ MeV (which is the cut imposed on the experimental results), then we find

$$B(K_L\to\pi^0\gamma\gamma)=\begin{cases}5.7\times10^{-7} & \text{for pion loops only,}\\ 5.2\times10^{-7} & \text{for pion plus quark loops,}\\ 5.1\times10^{-7} & \text{for pion loops plus VMD .}\end{cases} \tag{4.10}$$

These numbers must be compared with the experimental branching ratio of $(2.1\pm0.6)\times10^{-6}$ for $M_{\gamma\gamma}>280$ MeV from the CERN group and $(1.86\pm0.6\pm0.6)\times10^{-6}$ from Fermilab [23]. The predictions are smaller than the data, although they are not inconsistent with errors.

V. $K_L\to\pi^0e^+e^-$

Finally, we can use the previous answer to estimate the rate for the *CP*-conserving part of the $K_L\to\pi^0e^+e^-$ amplitude. As usual [11–14], we will simply give the contribution from the absorptive part of the two-photon intermediate state. The contribution from $A(x_1,x_2)$ is suppressed by m_e and can be neglected. Using Eq. (4.6) we find a simple result *if* $B(x_1,x_2)$ is constant or if it only depends on (x_1+x_2):

$$A_{\rm ABS}(K_L(p)\to\pi^0e^+(k')e^-(k))=\frac{\alpha}{6}\frac{B}{m^2}p\cdot(k-k')\bar{u}\not{p}v\ .$$

After squaring and integrating over phase space, this gives a lower limit for the branching ratio from the *CP*-conserving amplitude:

$$B_{CP}(K_L\to\pi^0e^+e^-)\geq2.31\times10^{-13}\ . \tag{5.1}$$

This is of course very close to the VMD prediction of Ref. [14], since we saw that the two models gave similar amplitudes for $K_L\to\pi^0\gamma\gamma$. It is also considerably smaller than the *CP*-violating contribution.

VI. CONCLUSIONS

We have computed the order p^6 contributions to the rates for $\gamma\gamma\to\pi^0\pi^0$ and $K_L\to\pi^0\gamma\gamma$ in the quark model. Both processes are dominated by the pion loops as is expected in chiral perturbation theory. The next-to-leading-order corrections found in the quark model are quite small so it is not possible to test them against present data. Their size, however, gives us confidence in the predictions from chiral perturbation theory for these two processes. More significantly, the corrections are very similar in size (and have the same form) as those obtained in vector-meson-dominance models. As an application we estimated the *CP*-conserving rate for $K_L \to \pi^0 e^+ e^-$ and found a result that suggests that this mode will be predominantly *CP* violating.

We conclude by comparing again our results with those found in the literature. In Ref. [22] the p^4 contribution to $K_L \to \pi^0\gamma\gamma$ was calculated in chiral perturbation theory and found to be unique. The same applies for Ref. [5] and $\gamma\gamma \to \pi^0\pi^0$. These are the lowest-order predictions for these processes and should always be included. The pion rescattering calculation of Ko and Rosner [15] takes the leading part of the chiral-perturbation-theory (ChPT) result, the pion loops, and treats it phenomenologically. Doing so they have included some higher-order terms. Since the lowest-order $\pi^+\pi^-\gamma\gamma$ amplitude is a very good approximation to the experimental result, they reach essentially the same conclusions of ChPT. The next-to-leading-order terms for both processes, p^6, contain free parameters that ChPT cannot determine. They can be fixed from a vector-meson model treated in the $SU(3)_V$ limit (all masses equal). This is what we and the the authors of Ref. [14] have done, and we agree with their result. They can also be fixed within the chiral quark model, and this is our main new result. Our calculation shows that both models yield essentially the same prediction. The p^6 predictions (in both models) can be supplemented with additional assumptions (such as nonet symmetry) to include the effects of the η' and η-η' mixing, which are formally of higher order, but can be phenomenologically important. However, as it was mentioned in Ref. [14], one can think of several other effects that occur at higher order. The inclusion of these effects is straightforward but very model dependent; since it would parallel the discussion of Ref. [14], we have chosen not to include them. The papers of Ko and Sehgal use a more phenomenological approach that does not separate the different contributions. They include $SU(3)_V$-breaking effects into the pole model, such as η' and η-η' mixing. They also include some other higher-order terms by keeping the complete vector-meson propagators. If one studies their amplitudes carefully, one notes that the terms of order p^6 in the strong vertex (in the pole model) are essentially the same as those of Ref. [14]. Their different results follow mainly from the way they have introduced the η' and the η-η' mixing, since the amplitude for $K_L \to \pi^0\gamma\gamma$ is very sensitive to these effects.

Note added in proof. Recently, Morgan and Pennington [24] have explained why the lowest-order chiral perturbation theory prediction for $\gamma\gamma \to \pi^0\pi^0$ is not accurate. Their method allows for an improved prediction of the rate using dispersion relations. Our main conclusions, that the tree-level $O(p^6)$ terms are unimportant in the low-energy region and that the quark model and vector dominance models give remarkably similar predictions, are not contradicted by this work.

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$$A(s,t,u) = -\frac{e^2 N_c}{16\pi^2 F_\pi^2}\frac{5}{27}\left[\frac{s-8m_\pi^2}{2m^2}\right],$$

$$B(s,t,u) = -\frac{e^2 N_c}{16\pi^2 F_\pi^2}\frac{5}{9}.$$

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