Production cross sections for the multi-Higgs-boson process $e^+e^- \to H_i^0 H^+ H^-$

M. H. Nous
Faculty of Education, Ain-shams University, Cairo, Egypt

M. El-Kishen and T. A. El-Azem
Faculty of Science, Cairo University, Cairo, Egypt
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The production cross sections for the process $e^+e^- \to H_i^0 H^+ H^-$ are calculated in the two-Higgs-doublet model. We consider the situations in which H_i^0 could be produced either from the propagator exchange or from the initial legs or the final legs. At $\sqrt{s} = 300$ GeV, this process can be detected if the mass of the Higgs boson H_i^0 is smaller than M_Z. Our calculations show that the situation in which the propagator exchange is ZZ becomes the dominant one, while the other situations are negligible.

I. INTRODUCTION

The theory of supersymmetry (SUSY) in particle physics has a prominent status due to its amelioration of the theoretical "hierarchy" and "fine-tuning" problems of the standard model (SM). These problems arise as a consequence of quadratic divergences for the normalized scalar (Higgs) particle mass, combined with the prejudice that the SM Higgs particle should be "light," $\lesssim 1$ TeV, so as to achieve correct electroweak breaking and to avoid violation of tree-level perturbative unitarity. SUSY removes the quadratic divergences. Since global SUSY is not phenomenologically satisfactory, it is natural to consider a theory of local SUSY which includes gravity through a local symmetry between fermions and bosons. Since SUSY is manifestly not a low-energy symmetry, it must be broken. The idea of breaking supergravity at the Planck scale in a "hidden sector" has become very popular in the last few years.

Minimal $N=1$ supergravity models have been reviewed by several authors [1]. The absence of experimental evidence for supersymmetry implies lower bounds on the SUSY partner masses and the SUSY-breaking scale [2].

II. TWO-HIGGS-DOUBLET MODEL

In the general CP-conserving two-Higgs-doublet model, three of the eight original scalar degrees of freedom become the longitudinal components of the $W^\pm$ and Z via the Higgs mechanism [3]. The five remaining physical degrees of freedom manifest themselves as three neutral Higgs bosons H_1^0, H_2^0, H_3^0 and a pair of charged Higgs bosons $H^\pm$. We assume the model is CP conserving, in which case H_1^0 and H_2^0 are CP even (scalar), while H_3^0 is CP odd (pseudoscalar) with respect to coupling to the SM fermions.

There are seven independent parameters in the Higgs sector. These are the two vacuum expectation values (VEV's) v_1 and v_2, the mixing angle α that results from diagonalization of the H_1^0-H_2^0 mass matrix, and the masses m_1, m_2, m_3, and $m_\pm$ of the Higgs particles H_1^0, H_2^0, H_3^0, and $H^\pm$, respectively. The rms value of the VEV's is chosen to generate the correct masses of the W and Z bosons [4], leaving six undetermined parameters. One of these is conventionally chosen to be the VEV ratio v_2/v_1, and renamed $\tan\beta$, with β obviously restricted to $0 \le \beta \le \pi/2$. The minimal SUSY model has two Higgs doublets and additional constraints [5,6]

$$m_3^2 + M_Z^2 = m_1^2 + m_2^2 \, , \tag{1}$$

$$m_\pm^2 = m_3^2 + M_W^2 \, , \tag{2}$$

$$0 \le m_2 \le M_Z \le m_1 \, . \tag{3}$$

From these constraints it also follows that

$$m_2 \le m_3 \le m_1, \quad m_\pm \ge (M_W, m_3) \, , \tag{4a}$$

$$m_1 \gtrless m_\pm \quad \text{if } m_2 \gtrless M_Z \sin\theta_W \, , \tag{4b}$$

where θ_W is the standard weak mixing angle. Importantly, these SUSY relations guarantee that the neutral Higgs particle H_2^0 exists with a mass less than that of the Z. The two angles β and α are fixed in terms of the Higgs-boson masses:

$$\cos 2\alpha = -\cos 2\beta \left[\frac{m_3^2 - M_Z^2}{m_1^2 - M_Z^2} \right] , \qquad \sin 2\alpha = -\sin 2\beta \left[\frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \right] , \tag{5}$$

The angle α can be taken to lie in the interval $-\pi/2 \le \alpha \le 0$.

III. MULTI-HIGGS-BOSON PRODUCTION

One reaction which produces $H_i^0 H^+ H^-$ ($i = 1, 2, 3$) from e^+e^- annihilation is

$$e^+e^- \to H_i^0 H^+ H^- \, , \tag{6}$$

where H_i^0 could be produced either from the propagator exchange or from the initial legs or the final legs. The situation in which H_i^0 is emitted from the propagator exchange contains 18 Feynman diagrams. The situation in which H_i^0 is emitted from the initial or final legs contains ten Feynman diagrams for the process (6) when it is mediated by the Z^0 boson and 20 diagrams when it is mediated by the Higgs boson.

IV. THE MATRIX ELEMENTS VIA THE PROPAGATOR EXCHANGE

The 18 Feynman diagrams which contribute to process (6) for the situation in which H_i^0 is emitted from the propagator exchange are shown in Fig. 1. The matrix elements for each diagram are given as

$$M_{1-2}(a)=(2^{3/4}iG_F^{3/2}M_Z^4\cos2\theta_W)\bar{v}(P_+)\gamma_\mu F U(P_-)\times[(P_+ +P_-)^2-M_Z^2]^{-1}R_j\times[(K_+ +K_-)^2-M_Z^2]^{-1}(K_+ +K_-)\mu$$

where

$$F=1-4\sin^2\theta_W-\gamma_5 \; .$$

$R_1=\cos(\beta-\alpha)$, $R_2=\sin(\beta-\alpha)$ are the Feynman rules for the $H_j^0Z^0Z^0$ vertex; notice the absence at tree level of an $H_3^0Z^0Z^0$ vertex [7],

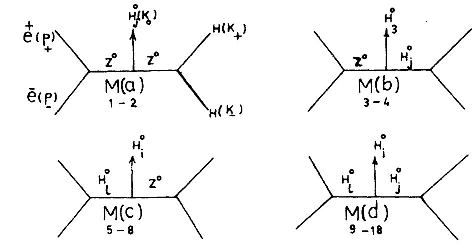

FIG. 1. Feynman diagrams for the process $e^+e^-\rightarrow H_i^0H^+H^-$ ($l,i=1,2,3$ and $j=1,2$) via the different propagators exchange. There are 18 Feynman diagrams.

$$M_{3-4}(b)=(2^{-1/4}iG_F^{3/2}M_Z^4)\bar{v}(P_+)\gamma_\mu F U(P_-)\; ,\times[(P_+ +P_-)^2-M_Z^2]^{-1}(1-R_j^2)^{1/2}\times(P_+ +P_-)_\mu[(K_+ +K_-)^2-m_j^2]^{-1}B_j \; .$$

Notice the absence of a $Z^0H_1^0H_2^0$ vertex due to CP invariance, while Bose symmetry forbids vertices of the kind $Z^0H_i^0H_i^0$ [8],

$$B_1=\cos^2\theta_W\cos(\beta-\alpha)-\tfrac{1}{2}\cos2\beta\cos(\beta+\alpha)\; ,$$

$$B_2=\cos^2\theta_W\sin(\beta-\alpha)+\tfrac{1}{2}\cos2\beta\sin(\beta+\alpha)\; ,$$

where CP invariance forbids vertices such as $H^+H^-H_3^0$ [8,9]:

$$M_{5-8}(c)=(2^{3/4}m_eG_F^{3/2}M_Z^2\cos2\theta_W)\bar{v}(P_+)A_iU(P_-)[(P_+ +P_-)^2-m_l^2]^{-1}\times(1-R_j^2)^{1/2}(K_+ +K_-)[(K_+ +K_-)^2-M_Z^2]^{-1}(K_+ +K_-)\; ,$$

where A_i are Yukawa couplings [6] given as

$$A_1=\frac{\cos\beta}{\cos\alpha},\quad A_2=-\frac{\sin\beta}{\cos\alpha},\quad A_3=i\gamma_5\tan\alpha\; ,$$

$$M_{9-18}(d)=(-2^{3/4}im_eM_Z^4G_F^{3/2})\bar{v}(P_+)A_iU(P_-)[(P_+ +P_-)^2-m_l^2]^{-1}F_{lij}[(K_+ +K_-)^2-m_j^2]^{-1}B_j$$

where

$$F_{111}=3\cos2\alpha\cos(\beta+\alpha),\quad F_{222}=3\cos2\alpha\sin(\beta+\alpha)\; ,$$

$$F_{331}=\cos2\beta\cos(\beta+\alpha),\quad F_{332}=-\cos2\beta\sin(\beta+\alpha)\; ,$$

$$F_{121}=F_{211}=F_{112}=2\sin2\alpha\cos(\beta+\alpha)+\cos2\alpha\sin(\beta+\alpha)\; ,$$

$$F_{221}=F_{122}=F_{212}=-2\sin2\alpha\sin(\beta+\alpha)+\cos2\alpha\cos(\beta+\alpha)\; .$$

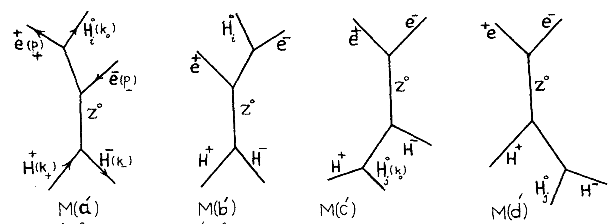

FIG. 2. Feynman diagrams for the process $e^+e^-\rightarrow H_i^0H^+H^-$, mediated by the Z^0 boson. There are 10 diagrams.

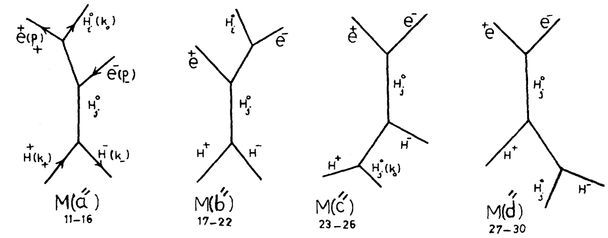

FIG. 3. Feynman diagrams for the process $e^+e^-\rightarrow H_i^0H^+H^-$, mediated by the neutral Higgs boson. There are 20 diagrams.

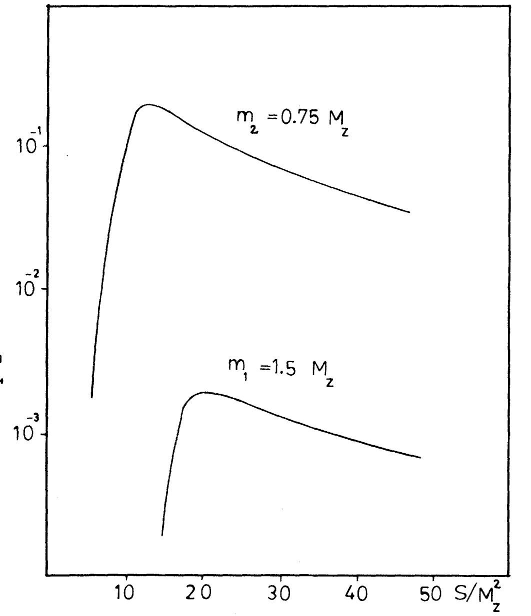

FIG. 4. The cross section for the process $e^+e^- \to H^+H^-H^0_{1,2}$ (through Z^0Z^0 exchange) as a function of s/M_Z^2 for $m_1 = 1.5M_Z$ and $m_2 = 0.75M_Z$.

Notice that for $H_i^0 H_j^0 H_j^0$ vertices ($i \neq j$), *CP* invariance forbids vertices where H_3^0 occurs singly [7–11].

V. THE MATRIX ELEMENTS VIA Z^0 BOSON

The Feynman diagrams for the process (6), mediated by the Z^0 boson are shown in Fig. 2. The matrix elements for each diagram are given as

$$M_{1-3}(a') = C\bar{v}(P_+)A_i D_1 \gamma_\mu F U(P_-) \times [(K_+ + K_-)^2 - M_Z^2]^{-1}(K_+ + K_-)_\mu ,$$

$$M_{4-6}(b') = C\bar{v}(P_+)\gamma_\mu F D_2 A_i U(P_-) \times [(K_+ + K_-)^2 - M_Z^2]^{-1}(K_+ + K_-)_\mu ,$$

$$M_{7-8}(c') = (2iM_Z^2 C/m_e)\bar{v}(P_+)\gamma_\mu F U(P_-) \times [(P_+ + P_-)^2 - M_Z^2]^{-1}(P_+ + P_-)_\mu \times [(K_+ + K_0)^2 - m_+^2]^{-1} B_j ,$$

$$M_{9-10}(d') = (2iM_Z^2 C/m_e)\bar{v}(P_+)\gamma_\mu F U(P_-) \times [(P_+ + P_-)^2 - M_Z^2]^{-1}(P_+ + P_-)_\mu \times [(K_- + K_0)^2 - m_-^2]^{-1} B_j ,$$

where

$$C = -2^{-1/4} m_e M_Z^2 G_F^{3/2} \cos 2\theta_W ,$$

$$D_1 = (i\gamma_l P_{l+} + m_e)/[(P_+ - K_0)^2 - m_e^2] ,$$

$$D_2 = (i\gamma_r P_{r-} + m_e)/[(P_- - K_0)^2 - m_e^2] .$$

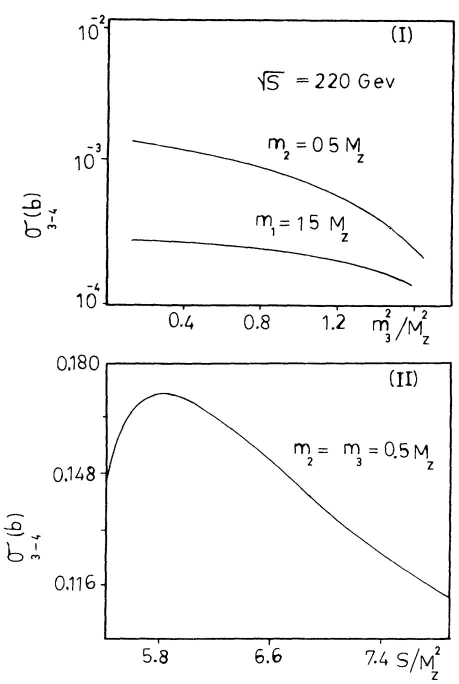

FIG. 5. The cross section for the process $e^+e^- \to H^+H^-H_3^0$ (through $Z^0H^0_{1,2}$ exchange): (I) as a function of m_3^2/M_Z^2 at $\sqrt{s} = 220$ GeV; (II) as a function of s/M_Z^2 for $m_2 = m_3 = 0.5M_Z$.

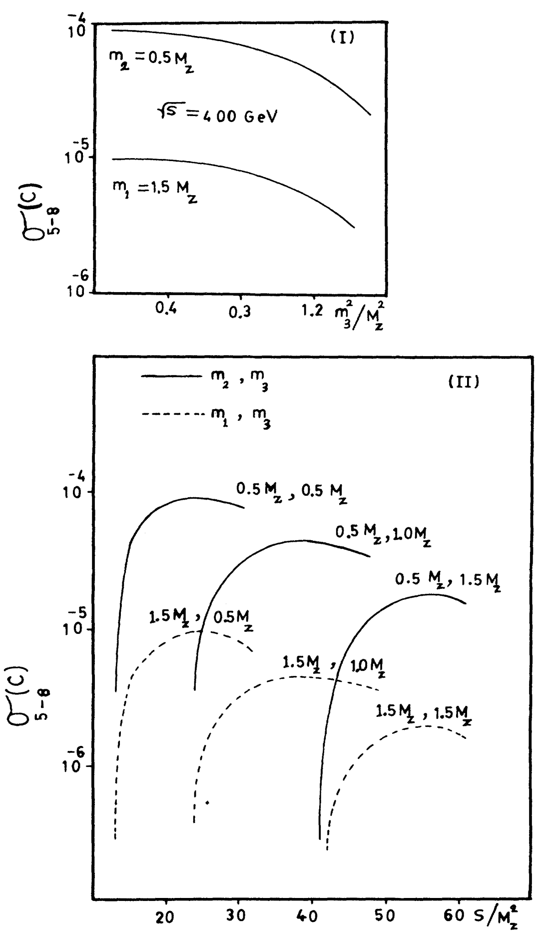

FIG. 6. The cross section for the process $e^+e^- \to H_i^0H^+H^-$ (through $H_i^0Z^0$ exchange): (I) as a function of m_3^2/M_Z^2 at $\sqrt{s} = 400$ GeV; (II) as a function of s/M_Z^2.

VI. THE MATRIX ELEMENTS VIA NEUTRAL HIGGS BOSON

The Feynman diagrams for the process (6), mediated by the Higgs boson H_j^0 $(j=1,2)$ are shown in Fig. 3. The matrix elements for each diagram are given as

$$
\begin{aligned}
M_{11-16}(a'') &= (2m_e C/\cos 2\theta_W)\bar{v}(P_+)A_i D_l A_j U(P_-) \\
&\quad \times [(K_+ + K_-)^2 - m_j^2]^{-1} B_j \,, \\
M_{17-22}(b'') &= (2m_e C/\cos 2\theta_W)\bar{v}(P_+)A_j D_2 A_i U(P_-) \\
&\quad \times [(K_+ + K_-)^2 - m_j^2]^{-1} B_j \,, \\
M_{23-26}(c'') &= (8iM_Z^2 C/\cos\theta_W)\bar{v}(P_+)A_j U(P_-) \\
&\quad \times [(P_+ + P_-)^2 - m_j^2]^{-1} \\
&\quad \times B_j [(K_+ + K_0)^2 - m_+^2]^{-1} B_j \,, \\
M_{27-30}(d'') &= (8iM_Z^2 C/\cos\theta_W)\bar{v}(P_+)A_j U(P_-) \\
&\quad \times [(P_+ + P_-)^2 - m_j^2]^{-1} \\
&\quad \times B_j [(K_- + K_0)^2 - m_-^2]^{-1} B_j \,,
\end{aligned}
$$

where $A_j = A_1, A_2$ and m_j is equal to the masses of the two neutral scalars.

VII. CROSS-SECTION CALCULATIONS

In general the cross-section for the $e^+e^- \to H_i^0 H^+ H^-$ process can be written in the form

$$
\sigma = \int \frac{|\overline{M}|^2}{16s(2\pi)^5} \delta^4(P_+ + P_- - K_0 - K_+ - K_-) \times \frac{d^3K_+}{E_+} \frac{d^3K_-}{E_-} \frac{d^3K_0}{E_0} \,,
$$

where $s = (P_+ + P_-)^2 = 2(P_+ P_-)$ and $|M|^2$ are the matrix elements given before. The integration is performed using a simple approximation obtained by an improved [12] Weizsäcker-Williams procedure. In all our calculations we assume the following values for the vector-boson masses suggested by [13] recent collider runs: $M_W = 80.0$ GeV, $M_Z = 91.17$ GeV, and $G_F = 1.166 \times 10^{-5}$ GeV^{-2}. We can take $\tan\beta$ and one of the Higgs-boson masses, for instance, that of H_3^0, as the unknown parameters. Different authors [14–18] prefer very different values for these parameters. We take $\tan\beta = 0.45$ and $0.5M_Z \le m_3 \le 1.5M_Z$, which are the ones commonly used [19,20].

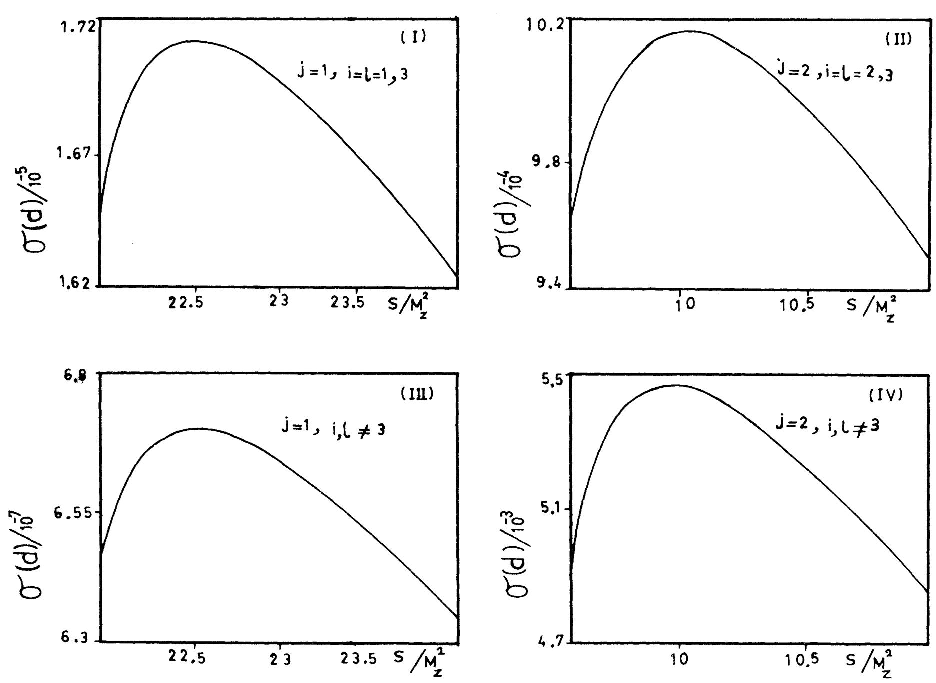

FIG. 7. The cross section for the process $e^+e^- \to H_i^0 H^+ H^-$ (through $H_i^0 H_j^0$ exchange) as a function of s/M_Z^2: (I) for $j=1$ and $i=l=1,3$; (II) for $j=2$ and $i=l=2,3$; (III) for $j=1$ and $i,l \neq 3$; (IV) for $j=2$ and $i,l \neq 3$.

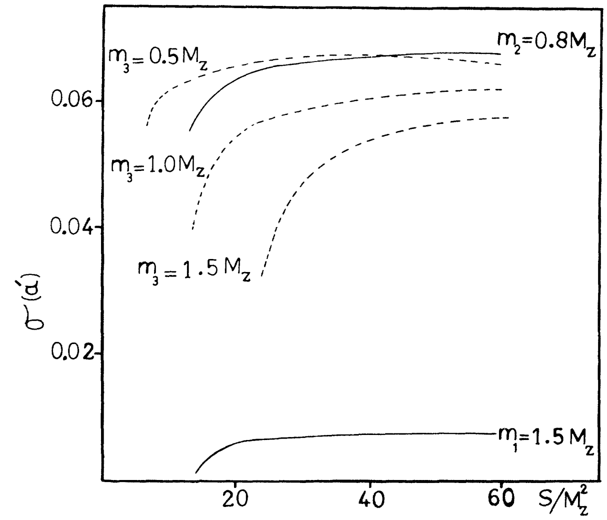

FIG. 8. The cross section $\sigma(a')$, where the process is mediated by Z^0 boson and H_i^0 is emitted from the e^+ line, as a function of s/M_Z^2 for different values of the mass m_i.

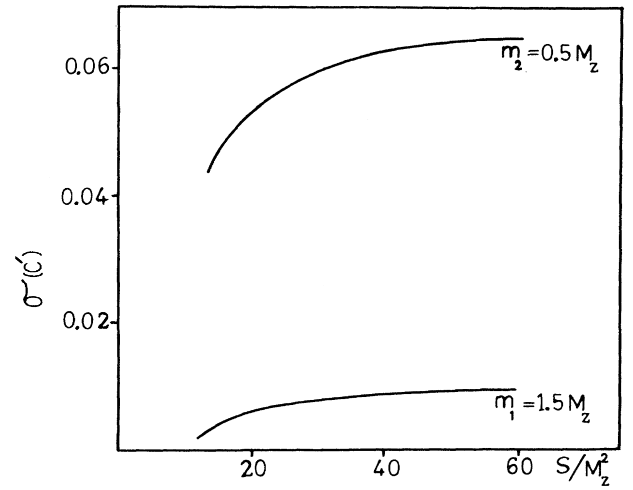

FIG. 9. The cross section $\sigma(c')$, where the process is mediated by Z^0 boson and H_j^0 is emitted from H^+ line, as a function of s/M_Z^2 for fixed m_1 and m_2.

VIII. DISCUSSION

The results of the calculated cross sections for the process $e^+e^- \to H_i^0 H^+ H^-$ for the different situations are presented in Figs. 4–11 as a function of s/M_Z^2. Figure 4, for the situation of Z^0Z^0 exchange, shows that $H^+H^-H_{1,2}^0$ production will be very difficult to observe unless the Higgs scalar is light and $\sqrt{s} \approx 300$ GeV. For the situation of $Z^0H_j^0$ exchange, Fig. 5(II) shows that the

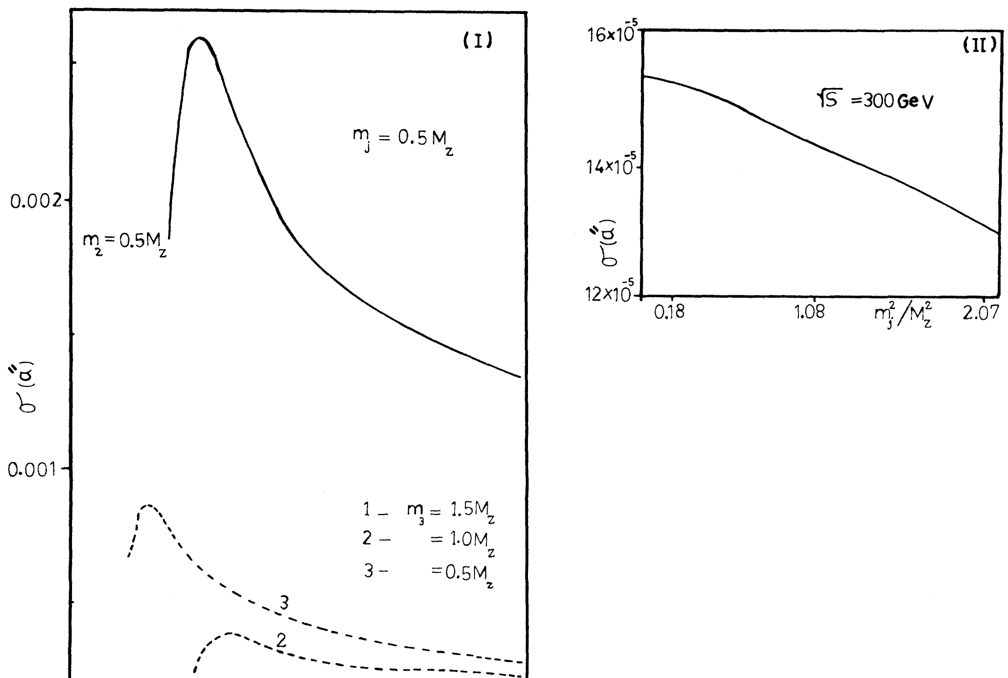

FIG. 10. The cross section $\sigma(a'')$ (where the process is mediated by Higgs boson H_j^0 and H_i^0 is emitted from e^+ line): (I) as a function of s/M_Z^2 for different values of the mass m_i; (II) as a function of m_j^2/M_Z^2 at $\sqrt{s} = 300$ GeV.

product $H^+H^-H_3^0$ is observable at $\sqrt{s} \approx 220$ GeV, while Fig. 5(I) shows that the cross section decreases with increasing the pseudoscalar mass m_3. The case for $H_l^0 Z^0$ exchange is exhibited in Figs. 6(I) and 6(II). From these figures we see that for a light Higgs boson and small values of $\sqrt{s}$, a large cross section is obtained. Figures 7(I)–7(IV) represent the case for $H_l^0 H_j^0$ exchange; in this case, the cross sections have a very low value.

Figure 8 for the situation in which H_i^0 is emitted from the initial legs [where the process (6) is mediated by the Z^0 boson] shows that the cross section $\sigma(a')$ rises sharply up to $\sqrt{s} = 450$ GeV and becomes flat for larger $\sqrt{s}$. Note that the cross sections for the process (6) where H_i^0 is emitted from either e^+ or e^- are equivalent. The same equivalence is obtained for the emission of H_j^0 either from H^+ or H^-. Figure 9 represents the cross section $\sigma(c')$ for the situation in which H_j^0 is emitted from the final legs, where process (6) is still mediated by the Z^0 boson. The behavior of $\sigma(c')$ is similar to that of $\sigma(a')$ and it is approximately 1.5–3 times smaller than $\sigma(a')$; this is mainly due to the propagator $[(K_+ + K_0)^2 - m_+^2]^{-1}$ of the Higgs boson in the amplitude $M(c')$. Figures 10(I) and 11(I) show that very low values for the cross sections are obtained for the situation in which H_i^0 is emitted from the different legs where process (6) is mediated by a neutral Higgs boson. It will be very difficult to observe the process unless the Higgs scalar is very light.

Finally Figs. 10(II) and 11(II) show that the cross sections are sensitive to the variation in the mass m_j of the neutral Higgs boson.

To conclude, we have calculated the cross sections for the process $e^+e^- \to H_i^0 H^+ H^-$ in the situations where H_i^0 is emitted either from the different propagators exchange or from the different legs. It seems to us that at $\sqrt{s} = 300$ GeV one can detect this process with a sizable cross section if the masses [21,22] of H_3^0 and H_2^0 are smaller than M_Z. H_1^0 and heavy H_3^0 can never be produced through this process. Our calculations have shown that the situation in which the propagator exchange is Z^0Z^0 becomes the dominant one, while the other situations are negligible. The production mechanism in this case will be $e^+e^- \to Z^0 \to ZH_i^0 \to H_i^0 H^+ H^-$. Although the production cross section in this case is not particularly big, this is the only process available and therefore it should receive proper attention in systematic searches for the Higgs bosons.

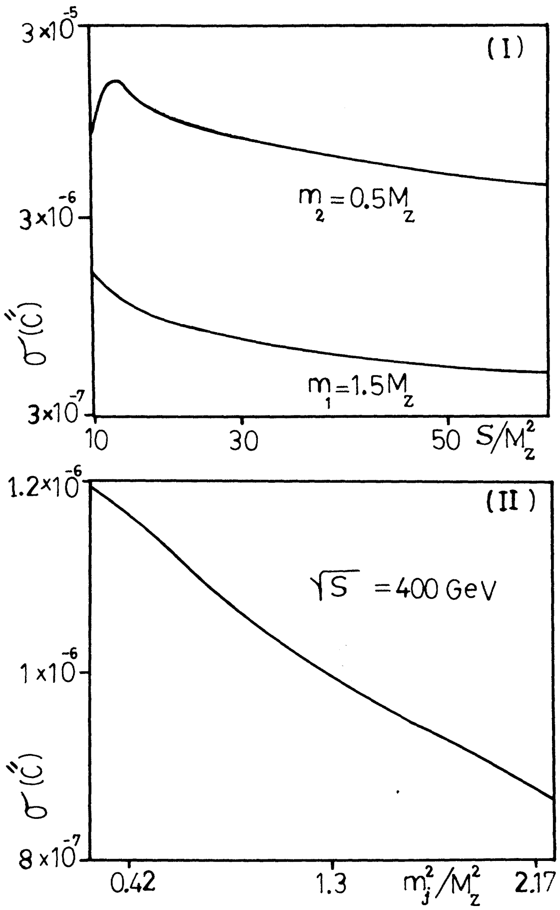

FIG. 11. The cross section $\sigma(c'')$ (where the process is mediated by Higgs boson H_j^0 and H_j^0 is emitted from H^+ line): (I) as a function of s/M_Z^2 for fixed m_1 and m_2; (II) as a function of m_j^2/M_Z^2 at $\sqrt{s} = 400$ GeV.

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