Path-integral derivation of the superconformal anomaly for the $N=1$ supersymmetric Yang-Mills theory

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(Received 28 June 1991)

Fujikawa's method of evaluating the supercurrent and the superconformal current anomalies, using the heat-kernel regularization scheme, is extended to theories with gauge invariance, in particular, to the off-shell $N=1$ supersymmetric Yang-Mills (SSYM) theory. The Jacobians of supersymmetry and superconformal transformations are finite. Although the gauge-fixing term is not supersymmetric and the regularization scheme is not manifestly supersymmetric, we find that the regularized Jacobians are gauge invariant and finite and they can be expressed in such a way that there is no one-loop supercurrent anomaly for the $N=1$ SSYM theory. The superconformal anomaly is nonzero and the anomaly agrees with a similar result obtained using other methods.

The supercurrent and the superconformal current anomalies for the on-shell Wess-Zumino model were evaluated using Fujikawa's method [1] with heat-kernel regularization [2] in Ref. [3] (from now on referred to as III). In this paper we extend this method to theories with local gauge invariance and evaluate the supercurrent and the superconformal anomaly for the on-shell $N=1$ supersymmetric Yang-Mills (SSYM) [4] theory.

It is known that there is no manifestly supersymmetric gauge-invariant regularization scheme. Hence it is not obvious that a gauge-invariant regularization scheme will preserve supersymmetry. In this paper we use a gauge-invariant regularization scheme to evaluate the superanomalies for the $N=1$ SSYM theory. Since we are studying a gauge theory, a gauge choice has to be made. A convenient choice is one for which the vacuum amplitude is manifestly covariant under the background gauge transformations [5]. However such a gauge-fixing condition is not invariant under supersymmetry (SUSY) transformations. To ensure that the SUSY-transformed variables satisfy the same gauge-fixing condition as the untransformed variables, the SUSY transformation laws will be modified by an appropriate field-dependent gauge transformation. Since the theory is gauge invariant, we can always do so. Further, the effective action also involves the Faddeev-Popov determinant associated with our choice of the gauge-fixing term, which will not be invariant under SUSY transformations. This will give an additional contribution to the SUSY transformation Jacobians.

We find that although the regularization scheme is not manifestly supersymmetric the Jacobian for the SUSY transformations is finite and can be expressed as the SUSY variation of a gauge-invariant local counterterm plus a total divergence. Thus there is no supercurrent anomaly for the on-shell $N=1$ SSYM theory. The superconformal current anomaly obtained from the SUSY anomaly calculation is nonzero but finite. The expression for the anomaly so obtained agrees with similar results obtained using other methods [6].

In Sec. I we go through the background gauge method and derive the SUSY Ward identity. The relevant regulators are evaluated in Sec. II, which are then used to evaluate the Jacobian for SUSY transformations in Sec. III. The superconformal anomaly is evaluated in Sec. IV. Finally the conclusions are given in Sec. V.

I. THE BACKGROUND GAUGE METHOD

The $N=1$ SSYM theory [4] consists of a Weyl fermion λ^a and a gauge boson $\mathbb{A}^a_\mu$, in the adjoint representation of the gauge group under study. Here a is the gauge group index. The $N=1$ SSYM action

$$S_0=\int d^4x\left(-\tfrac{1}{4}\mathbb{F}^a_{\mu\nu}\mathbb{F}^{\mu\nu a}+i\overline{\Psi}{}^a\mathbb{D}^{ab}\Psi^b\right) \tag{1}$$

is invariant under the gauge transformations

$$\delta\mathbb{A}^a_\mu=\partial_\mu\Lambda^a+f^{abc}\mathbb{A}^b_\mu\Lambda^c,\quad \delta\Psi^a=-f^{abc}\Lambda^b\Psi^c\ . \tag{2}$$

Here Λ^a are the gauge transformation parameters and f^{abc} are the structure constants.

As was done for the scalar multiplet in III, the fields $\mathbb{A}_\mu$ and Ψ can be split up into the classical background fields A_μ and λ and the fluctuation fields a_μ and ψ as

$$\mathbb{A}^a_\mu=A^a_\mu+a^a_\mu,\quad \Psi^a=\lambda^a+\psi^a\ . \tag{3}$$

The gauge transformations (2) can now be expressed in two different ways. One choice is the so-called background gauge transformations [5] where

$$\begin{aligned}&\delta A^a_\mu=\partial_\mu\Lambda^a+f^{abc}A^b_\mu\Lambda^c=(D_\mu\Lambda)^a,\quad \delta a^a_\mu=(a_\mu\times\Lambda)^a\\&\delta\lambda^a=-(\Lambda\times\lambda)^a,\quad \delta\psi^a=-(\Lambda\times\psi)^a\ .\end{aligned} \tag{4}$$

Here we have used the notation

$$f^{abc}u^bv^c=(u\times v)^a,\quad f^{abc}u^av^bw^c=u\times v\cdot w\ ,$$

$$(D_\mu u)^a=\partial_\mu u^a+(A_\mu\times u)^a\ .$$

The transformations (4) have the property that the fluctuation fields transform homogeneously and their transformations do not involve the background fields. Alternatively, the gauge transformations (2) can be split up as the quantum gauge transformation, which leave the background fields unchanged and are given by

$$\delta A^a_\mu=0,\quad \delta a^a_\mu=(D_\mu\Lambda)^a+(a_\mu\times\lambda)^a\ ,$$
$$\delta\lambda^a=0,\quad \delta\psi^a=-[\Lambda\times(\lambda+\psi)]^a\ . \tag{5}$$

The exponential of the effective action in the presence of the background fields A^a_μ and λ^a is given by

$$W[A,\lambda]=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar\psi]\mathcal{D}[\psi]\times\exp\{iS_0[\mathbb{A},\lambda]+iS_G[\mathbb{A}]+\ln[\mathrm{Det}(-D^2)]\}\ . \tag{6}$$

Here $S_G=-\frac{1}{2}(D_\mu a^\mu)^2$ is the background gauge-fixing term, chosen such that (6) is invariant under the background gauge transformations (4). The $[\mathrm{Det}(-D^2)]$ is the corresponding Faddeev-Popov determinant.

The SUSY transformations which leave the action (1) invariant are

$$\delta_\varepsilon\mathbb{A}^a_\mu=i(\bar\varepsilon\gamma_\mu\Psi^a-\bar\Psi^a\gamma_\mu\varepsilon),\quad \delta_\varepsilon\Psi^a=\Sigma_{\mu\nu}\mathbb{F}^{\mu\nu a}\varepsilon\ . \tag{7}$$

Here $\Sigma_{\mu\nu}=\frac{1}{4}[\gamma_\mu,\gamma_\nu]$ and rest of the notation is as defined in III. In terms of the background and the fluctuation fields (3), the SUSY transformations (7) are

$$\delta_\varepsilon A^a_\mu=i(\bar\varepsilon\gamma_\mu\lambda^a-\bar\lambda^a\gamma_\mu\varepsilon),\quad \delta_\varepsilon a^a_\mu=i(\bar\varepsilon\gamma_\mu\psi^a-\bar\psi^a\gamma_\mu\varepsilon)\ ,$$
$$\delta_\varepsilon\lambda^a=\Sigma_{\mu\nu}F^{\mu\nu a}\varepsilon,\quad \delta_\varepsilon\psi^a=\Sigma_{\mu\nu}(2(D_\mu a_\nu)^a+(a_\mu\times a_\nu)^a)\varepsilon\ . \tag{8}$$

To obtain the SUSY Ward identity, the SUSY variation of the effective action (6) has to be computed. The variation of the measure, according to Fujikawa's interpretation, will then give the anomaly as the Jacobians of the SUSY transformations. However, these Jacobians have to be regularized, which can be done only in Euclidean space. Hence we first continue the effective action (6) to the Euclidean space and then compute its variation. As in III, this is done by letting $t\to -it$. The exponential of the Euclidean effective action is defined as

$$W^E[A,\lambda]=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar\psi]\mathcal{D}[\psi]\times\exp\{S^E_0+S^E_G+\ln[\mathrm{Det}(D^2)]\}\ , \tag{9}$$

where

$$S^E_0=\int d^4x[-\tfrac{1}{4}\mathbb{F}^a_{\mu\nu}\mathbb{F}^a_{\mu\nu}+\bar\psi^a(\not{\mathbb{D}}\Psi)^a] \tag{10}$$

and

$$S^E_G=-\int d^4x\tfrac{1}{2}(D_\mu a_\mu)^2\ . \tag{11}$$

Here $\bar\Psi$ and Ψ are independent Dirac spinors, unlike in Minkowski space [7,8]. The Euclidean metric is $\delta_{\mu\nu}$ and the Euclidean γ matrices are Hermitian. The Euclidean gauge transformations are the same as (4) and (5), and the Euclidean SUSY transformations corresponding to (8) are

$$\delta_\varepsilon A^a_\mu=-(\bar\varepsilon\gamma_\mu\lambda^a-\bar\lambda^a\gamma_\mu\varepsilon),\quad \delta_\varepsilon a^a_\mu=-(\bar\varepsilon\gamma_\mu\psi^a-\bar\psi^a\gamma_\mu\varepsilon)\ ,$$
$$\delta_\varepsilon\lambda^a=-\Sigma_{\mu\nu}F^a_{\mu\nu}\varepsilon,\quad \delta_\varepsilon\psi^a=-\Sigma_{\mu\nu}(2D_\mu a_\nu+a_\mu\times a_\nu)^a\varepsilon\ . \tag{12}$$

Change in W^E under the SUSY variation of the background fields is given by

$$\delta_\varepsilon W^E[A_\mu,\lambda]=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar\psi]\mathcal{D}[\psi]\times\exp[S^E_0+S^E_G+\ln(\mathrm{Det}D^2)]\times\left[\delta_\varepsilon A_\mu\frac{\delta}{\delta A_\mu}+\delta_\varepsilon\lambda\frac{\delta}{\delta\lambda}\right]\times[S^E_0+S^E_G+\ln(\mathrm{Det}D^2)]\ .$$

Making the change of integration variables,

$$a^a_\mu\to(a^a_\mu)'=a^a_\mu+\delta_\varepsilon a^a_\mu\ ,$$
$$\psi^a\to(\psi^a)'=\psi^a+\delta_\varepsilon\psi^a$$
$$\bar\psi^a\to(\bar\psi^a)'=\bar\psi^a+\delta_\varepsilon\bar\psi^a\ ,$$

with δ_ε as in (12), and retaining terms which are at most linear in ε gives

$$\delta_\varepsilon W^E=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar\psi]\mathcal{D}[\psi]\exp[S^E_0+S^E_G+\ln(\mathrm{Det}D^2)]\times\{\delta_\varepsilon[S^E_0+S^E_G+\ln(\mathrm{Det}D^2)]+J^E(\varepsilon)\}\ . \tag{13}$$

Here $J^E(\varepsilon)$ is the change in the measure to be evaluated. Similar to the Wess-Zumino model calculation, here too the Euclidean action S^E_0 is not invariant under rigid SUSY transformations. The change in the action under rigid SUSY transformations is proportional to $\bar\Psi(\delta_\varepsilon A)\Psi$ and involves four Dirac fermions. This is nonzero because, unlike in Minkowski space, $\bar\Psi$ and Ψ are independent fermions in Euclidean space. (See, however Ref. [8], where these problems do not arise.) However, as the Euclidean Ward identity will eventually be continued to Minkowski space, where such a term vanishes by the Fierz rearrangement, it will not be written explicitly. In this sense S^E_0 is invariant under rigid SUSY transformations. However, the gauge-fixing term is not invariant under the SUSY transformations even in Minkowski space. Therefore we will now modify the SUSY transformation laws such that they leave the gauge-fixing term invariant. This is done by adding a field-dependent quantum gauge transformation (5) to the SUSY transformation laws (12). The modified SUSY transformations denoted by $\tilde\delta_\varepsilon$ are

$$\tilde\delta_\varepsilon a_\mu=\delta_\varepsilon a_\mu+\partial_\mu\Lambda+(A_\mu+a_\mu)\times\Lambda$$
$$=\delta_\varepsilon a_\mu+D_\mu\Lambda+a_\mu\times\Lambda\ ,$$
$$\tilde\delta_\varepsilon\psi=\delta_\varepsilon\psi+(\lambda+\psi)\times\Lambda,\quad \tilde\delta_\varepsilon A_\mu=\delta_\varepsilon A_\mu,\quad \tilde\delta_\varepsilon\lambda=\delta_\varepsilon\lambda\ . \tag{14}$$

The gauge transformation parameter Λ is chosen such that $\tilde\delta_\varepsilon(D_\alpha a_\alpha)=0$; that is,

$$D_\alpha a_\alpha=[D_\alpha-(\bar{\varepsilon}\gamma_\alpha\lambda-\bar{\lambda}\gamma_\alpha\varepsilon)]\times[a_\alpha-(\bar{\varepsilon}\gamma_\alpha\psi-\bar{\psi}\gamma_\alpha\varepsilon)+D_\alpha\Lambda+a_\alpha\times\Lambda]=0$$
$$\Longrightarrow -(\bar{\varepsilon}\gamma_\alpha\lambda-\bar{\lambda}\gamma_\alpha\varepsilon)\times(D_\alpha\lambda+a_\alpha\times\Lambda)+D_\alpha(a_\alpha\times\lambda)+D^2\Lambda=-[-D_\alpha(\bar{\varepsilon}\gamma_\alpha\psi-\bar{\psi}\gamma_\alpha\varepsilon)-(\bar{\varepsilon}\gamma_\alpha\lambda-\bar{\lambda}\gamma_\alpha\varepsilon)\times a_\alpha+O(\varepsilon^2)]\ . \tag{15}$$

This equation has to be solved for Λ. As is evident, every term in the solution will contain ε. Since, at most order-ε terms are needed, terms of the type $\varepsilon\Lambda$ can be ignored. The solution of Λ so obtained will be at least of order a_μ or ψ. Further, for our purposes we need to keep the terms which are at most quadratic in the fluctuation fields. Therefore we need to keep the terms which are at most linear in the fluctuation fields a_μ or ψ in the transformed gauge-fixing condition. Thus Eq. (15) for Λ reduces to

$$(D^2)^{ab}\Lambda^b=[D_\alpha(\bar{\varepsilon}\gamma_\alpha\psi-\bar{\psi}\gamma_\alpha\varepsilon)+(\bar{\varepsilon}\gamma_\alpha\lambda-\bar{\lambda}\gamma_\alpha\varepsilon)\times a_\alpha]^a\ .$$

This equation can be solved for $\Lambda^a(x)$ to yield

$$\Lambda^a(x)=\int d^4x'G^{ab}(x,x')[D_\alpha(\bar{\varepsilon}\gamma_\alpha\psi-\bar{\psi}\gamma_\alpha\varepsilon)+(\bar{\varepsilon}\gamma_\alpha\lambda-\bar{\lambda}\gamma_\alpha\varepsilon)\times a_\alpha]^b\ , \tag{16}$$

where the Green's function $G^{ab}(x,x')$ satisfies the relation

$$(D^2)^{ab}_x G^{bc}(x,x')=\delta^{ac}(x-x')\ . \tag{17}$$

Since S_0^E is invariant under the quantum gauge transformations,

$$\tilde{\delta}_\varepsilon S_0^E=\delta_\varepsilon S_0^E=\int(\bar{Q}_\mu\partial_\mu\varepsilon+\partial_\mu\bar{\varepsilon}Q_\mu)d^4x\ ,$$

where $Q_\mu=-\Sigma\cdot\mathrm{F}^a\gamma_\mu\Psi^a$ is the Euclidean supercurrent. As required $\tilde{\delta}_\varepsilon S_G^E=0$. Thus the Ward identity (13) becomes

$$\delta_\varepsilon W^E=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar{\psi}][\mathcal{D}\psi]\exp[S_0^E+S_G^E+\ln(\mathrm{Det}D^2)]\times\left[\int(\partial_\mu\bar{Q}_\mu\varepsilon+\bar{\varepsilon}\partial_\mu Q_\mu)d^4x+\delta_\varepsilon\ln(\mathrm{Det}D^2)+\tilde{J}^E(\varepsilon)\right]\ , \tag{18}$$

where $\tilde{J}^E(\varepsilon)$ is the Jacobian of the modified SUSY transformations (14). After evaluating $\tilde{J}^E(\varepsilon)$ and $\tilde{\delta}_\varepsilon\ln[\mathrm{Det}D^2]$ in a regularized way, the result can be continued back to Minkowski space to obtain the required SUSY Ward identity:

$$\delta_\varepsilon W=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp[iS_0+iS_G+\ln\mathrm{Det}(-D^2)]\times\left[i\int(\bar{Q}^\mu\partial_\mu\varepsilon+\partial^\mu\bar{\varepsilon}Q_\mu)d^4x+\tilde{J}(\varepsilon)+\delta_\varepsilon\ln\mathrm{Det}(-D^2)\right]\ . \tag{19}$$

II. EVALUATION OF THE REGULATORS

Our task now is to evaluate the Jacobians and the variation of the Faddeev-Popov determinant. For this purpose we need the appropriate regulators. As in case of the Wess-Zumino model, to obtain the necessary regulators, we define a quantity S^Q, which contains all the terms in $(S_0^E+S_G^E)$ which are bilinear in the fluctuation fields a_μ, ψ, and $\bar{\psi}$. The double functional differentiation of S^Q with respect to the relevant variables gives the regulators. Using (3) S^Q can be extracted from (10). Thus

$$S^Q=\int d^4x\{\tfrac{1}{2}a_\mu^a[(D^2)^{ab}\delta_{\mu\nu}+2F^c_{\mu\nu}f^{acb}]a_\nu^b+\bar{\psi}^a(\not{D}\psi)^a+\bar{\lambda}^a(\not{a}\times\psi)^a+\bar{\psi}^a(\not{a}\times\lambda)^a\}\ . \tag{20}$$

The last two terms are off diagonal in the boson-fermion pair and would lead to boson-fermion mixed regulators, which are inconvenient to handle. Hence (20) is diagonalized using the new variables:

$$\begin{aligned}a'^a_\mu(x)&=a^a_\mu(x)\ ,\\ \psi'^a(x)&=\psi^a(x)+\int d^4y\,\tilde{G}^{ab}(x,y)(\not{a}\times\lambda)^b(y)\ ,\\ \bar{\psi}'^a(x)&=\bar{\psi}^a(x)+\int d^4y(\bar{\lambda}\times\not{a})^b(y)\tilde{G}^{ba}(y,x)\ ,\end{aligned} \tag{21}$$

where the Green's function $\tilde{G}$ satisfies the condition

$$\not{D}^{ab}_x\tilde{G}^{bc}(x,y)=\tilde{G}^{cb}(x,y)\overleftarrow{\not{D}}^{ba}_y=\delta^4(x-y)\delta^{ac} \tag{22}$$

with

$$\overleftarrow{\not{D}}^{ab}=-\overleftarrow{\not{\partial}}\delta^{ab}+f^{acb}\not{A}^c\ .$$

In terms of the new variables (21), the diagonalized S^Q (20) becomes

$$S^Q=\int d^4x\left[\int d^4y\,a^a_\mu(x)\{\tfrac{1}{2}[(D^2)^{ab}\delta_{\mu\nu}+2F^{ab}_{\mu\nu}]\delta(x-y)-\bar{\lambda}^{ac}(x)\gamma_\mu\tilde{G}^{cd}(x,y)\gamma_\nu\lambda^{db}(y)\}a^b_\nu+\tfrac{1}{2}\bar{\psi}'^aD^{ab}\psi'^b\right]\ , \tag{23}$$

where we have use the notation $f^{acb}X^c \equiv X^{ab}$.

The bosonic Jacobian to be discussed in Sec. III will be regulated by the operator $e^{tB_{\mu\nu}}$, where $B_{\mu\nu}$ is obtained by differentiating S^Q with respect to $a_\mu^a(x)$ and $a_\nu^b(y)$, and is given by

$$B_{\mu\nu}^{ab}(x,y)=B_{\mu\nu}^{\prime ab}(x,y)-\bar{\lambda}^{ac}(x)\gamma_\mu \tilde{G}^{cd}(x,y)\gamma_\nu \lambda^{db}(y) -\bar{\lambda}^{bc}(y)\gamma_\nu \tilde{G}^{cd}(x,y)\gamma_\nu \lambda^{da}(x) , \tag{24}$$

where

$$B'_{\mu\nu}(x,y)=[(D^2)^{ab}\delta_{\mu\nu}+2F_{\mu\nu}^{ab}]\delta(x-y) \tag{25}$$

is the local part of the bosonic regulator $B_{\mu\nu}$.

Unlike the case of the Wess-Zumino model discussed in III, the operator $\not{D}$ has a definite Hermiticity property $\not{D}^\dagger=-\not{D}^\dagger$. Hence ψ' and $\bar{\psi}'$ can be regulated by the *same* regulator $\exp(t\mathcal{F})$, with

$$\mathcal{F}^{ab}=(\not{D}^2)^{ab}=(D^2)^{ab}+\Sigma\cdot F^{ab} . \tag{26}$$

The SUSY variation of the ghost determinant can be naturally regulated using the operator $\exp(tC)$, where

$$C^{ab}=(D^2)^{ab} . \tag{27}$$

In terms of the new variables $\bar{\psi}'$ and ψ', all the regulators are diagonal in the boson-fermion pair. Further, since ψ' and $\bar{\psi}'$ are regulated using the same regulator $\mathcal{F}$ (26) we need evaluate only the ψ Jacobian, which depends on $\bar{\varepsilon}$. For the bosonic Jacobian and the ghost determinant also we shall evaluate only the $\bar{\varepsilon}$-dependent contributions. The ε-dependent part of the total Jacobian (in Minkowski space) is then obtained by taking the Hermitian conjugate of the $\bar{\varepsilon}$-dependent part. Hence in the following calculation, only the $\bar{\varepsilon}$-dependent contributions will be displayed explicitly.

In terms of the diagonalizing variables, the SUSY transformation of a_μ in (14), with Λ as in (16), is

$$\tilde{\delta}_\varepsilon a_\mu^a(x)=\int d^4y \left[D_\nu^{ab}G^{bc}(x,y)\left[(\bar{\varepsilon}\gamma_\nu\lambda)^{cd}(y)a_\nu^d(y)-\int d^4u\, D_\mu^{cd}[\bar{\varepsilon}\gamma_\nu\tilde{G}^{de}(y,u)\not{a}^{ef}(u)\lambda^f(u)]\right] +\bar{\varepsilon}(x)\gamma_\mu\tilde{G}^{ab}(x,y)\not{a}^{bc}(y)\lambda^c(y)\right]+(a_\mu\text{-independent terms}) , \tag{28}$$

where all the terms which do not contain the variable being transformed, i.e., a_μ, are dropped because, as explained in III, such terms do not contribute anything to the Jacobian up to order ε. Carrying out the covariant differentiation D_ν in the second term in (28) and using (22) gives

$$\tilde{\delta}_\varepsilon a_\mu^a=2D_\mu G\bar{\varepsilon}\gamma_\nu\lambda a_\nu-D_\mu G(\bar{\varepsilon}\overleftarrow{\not{\partial}})\tilde{G}\gamma_\nu\lambda a_\nu-\bar{\varepsilon}\gamma_\mu\tilde{G}\gamma_\nu\lambda a_\nu , \tag{29}$$

where we have suppressed all the indices and spacetime arguments for notational convenience. All the repeated indices and spacetime arguments are contracted between neighboring objects and summed or integrated, as is obvious by comparing (28) with (29).

The SUSY transformation law for ψ', obtained using (14) and (16), is

$$\tilde{\delta}_\varepsilon\psi'=GD_\mu(\bar{\varepsilon}\gamma_\mu\psi')\lambda+\tilde{G}\gamma_\mu\lambda(\bar{\varepsilon}\gamma_\mu\psi'-D_\mu GD_\nu(\bar{\varepsilon}\gamma_\nu\psi')) +(\psi'\text{-independent term}) .$$

Here, too, we will not worry about the terms which do not contain ψ' since they will not contribute anything to the Jacobian. Integrating the third term in the above equation by parts and using (22) yields

$$\tilde{\delta}_\varepsilon\psi'=\tilde{G}\gamma_\mu\lambda(\bar{\varepsilon}\gamma_\mu\psi')+\tilde{G}(\not{D}\lambda)GD_\mu(\bar{\varepsilon}\gamma_\mu\psi') . \tag{30}$$

Note that, in the present notation,

$$(\not{D}\lambda)\equiv(\not{D}\lambda)^{ab}\equiv f^{abc}(\not{\partial}\lambda^b+f^{bde}\not{A}^d\lambda^e) .$$

To get the regularized Jacobians, the SUSY transformation laws (29) and (30) are regulated using the heat kernels

$$h_{\mu\nu}(x,y,t)=\langle y|e^{tB_{\mu\nu}}|x\rangle , \tag{31}$$

and

$$\tilde{h}(x,y,t)=\langle y|e^{t\mathcal{F}}|x\rangle , \tag{32}$$

respectively, with the regulator operators $B_{\mu\nu}$ and $\mathcal{F}$ as in (24) and (26). The regularized transformation laws corresponding to (29) and (30) are

$$\tilde{\delta}_\varepsilon a_\mu=2D_\mu G\bar{\varepsilon}\gamma_\nu\lambda h_{\nu\alpha}a_\alpha-D_\mu G(\bar{\varepsilon}\overleftarrow{\not{\partial}})\tilde{G}\gamma_\nu\lambda h_{\nu\alpha}a_\alpha -\bar{\varepsilon}\gamma_\mu\tilde{G}\gamma_\nu\lambda h_{\nu\alpha}a_\alpha \tag{33}$$

and

$$\tilde{\delta}_\varepsilon\psi'=\tilde{G}\gamma_\mu\lambda(\bar{\varepsilon}\gamma_\mu\tilde{h}\psi')+\tilde{G}(\not{D}\lambda)GD_\mu(\bar{\varepsilon}\gamma_\mu\tilde{h}\psi') . \tag{34}$$

Thus the regularized Jacobians corresponding to the SUSY transformations (33) and (34) are

$$\operatorname{Det}\left[\frac{\delta(a_\mu+\tilde{\delta}_\varepsilon a_\mu)}{\delta a_\nu}\right]=1+2\operatorname{Tr}D_\mu G\bar{\varepsilon}\gamma_\nu\lambda h_{\nu\mu} +\operatorname{Tr}D_\mu G(\bar{\varepsilon}\overleftarrow{\not{\partial}})\tilde{G}\gamma_\nu\lambda h_{\nu\mu} -\operatorname{Tr}\bar{\varepsilon}\gamma_\mu\tilde{G}\gamma_\nu\lambda h_{\nu\mu} \tag{35}$$

and

$$\operatorname{Det}\left[\frac{\delta(\psi'+\tilde{\delta}_\varepsilon\psi')}{\delta\psi'}\right]=1+\operatorname{Tr}D_\mu(\bar{\varepsilon}\gamma_\mu\tilde{h})\tilde{G}(\not{D}\lambda)G +\operatorname{Tr}\bar{\varepsilon}\gamma_\mu\tilde{h}\tilde{G}\gamma_\mu\lambda , \tag{36}$$

where, in the present notation,

$$\mathrm{Tr}\bar{\varepsilon}\gamma_\mu\tilde{h}\tilde{G}\gamma_\mu\lambda \equiv \mathrm{tr}\int\int d^4x\, d^4y\, \bar{\varepsilon}(x)\gamma_\mu\tilde{h}^{ab}(x,y) \times\tilde{G}^{bc}(y,x)\gamma_\mu\lambda(x) ,$$

and similarly for other terms. In (35) and (36), the last terms arise due to the SUSY transformations, whereas all the rest of the terms are due to the modifications of the SUSY transformation laws, which were made so as to leave the gauge-fixing term invariant. Using (22), (26), and (32) we see that $\tilde{G}$ and $\tilde{h}$ are made of the same operator D. Hence $\tilde{G}\tilde{h}=\tilde{h}\tilde{G}$. Making this change in the second term in (34) gives

$$D_\mu(\bar{\varepsilon}\gamma_\mu\tilde{h})\tilde{G}(\not{D}\lambda)G=\bar{\varepsilon}\gamma_\mu\tilde{G}\tilde{h}\not{D}\lambda G\overleftarrow{D}_\mu$$
$$=(\bar{\varepsilon}\overleftarrow{\not{\partial}})\tilde{G}\tilde{h}\not{D}\lambda G+\bar{\varepsilon}\tilde{h}\not{D}\lambda G . \qquad (37)$$

The change in the ghost determinant in (18) due to the SUSY transformations (8) of the background fields is

$$\delta_\varepsilon \ln \mathrm{Det}D^2=\mathrm{Tr}[(D^2)^{-1}\delta_\varepsilon D^2] . \qquad (38)$$

We shall rewrite this equation in terms of the Green's function G, defined as

$$(D^2)^{ab}G^{bc}(x,y)=\delta^{ac}\delta^4(x-y) . \qquad (39)$$

Further (38) is regulated using the heat kernel

$$h(x,y,t)=\langle y|e^{tD^2}|x\rangle . \qquad (40)$$

Equation (38) thus becomes

$$\delta_\varepsilon \ln(\mathrm{Det}D^2)=\mathrm{Tr}G[D_\alpha(\bar{\varepsilon}\gamma_\alpha\lambda)]h-2\,\mathrm{Tr}G\overleftarrow{D}_\alpha\bar{\varepsilon}\gamma_\alpha\lambda h . \qquad (41)$$

Collecting the Jacobians from (35)–(37) and the variation of the ghost determinant (41) gives

$$\mathcal{J}^E(\varepsilon)+\delta_\varepsilon \ln(\mathrm{Det}D^2)=\mathrm{Tr}[\bar{\varepsilon}\gamma_\mu\tilde{h}\tilde{G}\gamma_\mu\lambda-\bar{\varepsilon}\gamma_\mu\tilde{G}\gamma_\nu\lambda h_{\nu\mu}]+\mathrm{Tr}[2D_\mu G\bar{\varepsilon}\gamma_\nu\lambda h_{\nu\mu}-2G\overleftarrow{D}_\mu\bar{\varepsilon}\gamma_\mu\lambda]$$
$$+\mathrm{Tr}[\bar{\varepsilon}\overleftarrow{\not{\partial}}\tilde{G}\tilde{h}\not{D}\lambda G+G(\bar{\varepsilon}\overleftarrow{\partial}_\mu)\tilde{G}\gamma_\nu\lambda h_{\nu\mu}\overleftarrow{D}_\mu]+\mathrm{Tr}[\bar{\varepsilon}\tilde{h}\not{D}\lambda G]+\mathrm{Tr}[G(D_\alpha(\bar{\varepsilon}\gamma_\alpha\lambda))h] . \qquad (42)$$

These expressions have to be evaluated in the limit $t\to 0$. The pair of terms in each pair of square brackets will be calculated together, so that in the limit $t\to 0$ the cancellation of divergences between various traces is transparent and the calculations simplified. We shall use the short-distance expansion of the heat kernel and the Green's function involved. The short-distance expansion for a generic heat kernel is discussed in III and the relevant expansion coefficients evaluated in Appendix A of III. As in case of the Wess-Zumino model, the bosonic regulator $B_{\mu\nu}$ in (24) has a nonlocal contribution. In order to deal with such a regulator operator we define a heat kernel $h'_{\mu\nu}(x,y;t)$ from the local part $B'_{\mu\nu}$ (25) of the bosonic regulator $B_{\mu\nu}$ (24) as

$$h'(x,y;t)=\langle y|e^{tB'_{\mu\nu}}|x\rangle . \qquad (43)$$

For a generic regulator with a nonlocal contribution the procedure for separating the contribution to the Jacobian from the nonlocal part is developed in Appendix B of III. Substituting for $B_{\mu\nu}$ and $B'_{\mu\nu}$ from (24) and (25) in Eq. (B10) of III yields

$$h_{\mu\nu}(x,y,t)=h'_{\mu\nu}(x,y,t)-\int d^4u\frac{t}{2}\{h'_{\mu\alpha}(x,u,t)[\bar{\lambda}(u)\gamma_\alpha\tilde{G}(u,y)\gamma_\nu\lambda(y)-\bar{\lambda}(y)\gamma_\nu\tilde{G}(y,u)\gamma_\alpha\lambda(u)]$$
$$+[\bar{\lambda}(x)\gamma_\mu\tilde{G}(x,u)\gamma_\alpha\lambda(u)-\bar{\lambda}(u)\gamma_\alpha\tilde{G}(u,x)\gamma_\mu\lambda(x)]h'_{\alpha\nu}(u,y;t)\} . \qquad (44)$$

The contribution from the nonlocal part of the regulator is entirely contained in the curly brackets in (44). We will first evaluate the contribution from the local part, i.e., replace $h_{\mu\nu}$ by $h'_{\mu\nu}$ in (42).

As in the case of the Wess-Zumino model discussed in III, the heat kernels $\tilde{h}$, $h'_{\mu\nu}$, and h will be expanded in powers of t for small t. The regulator operators $B'_{\mu\nu}$, $\mathcal{F}$, and C defined in (25)–(27) have the form of a generic regulator R, Eq. (33) of III:

$$R=\Box+2X\cdot\partial+\partial\cdot X+X^2+Y . \qquad (45)$$

Using Eqs. (25)–(27) it is easy to see that

$$X_\mu=A_\mu \qquad (46)$$

for all the regulator operators and

$$Y=2F_{\mu\nu},\quad Y=\Sigma\cdot F,\quad \text{and } Y=0 \qquad (47)$$

for $B'_{\mu\nu}$, $\mathcal{F}$, and C, respectively. For small t the generic heat kernel $h(x,y;t)$ has an asymptotic expansion

$$h(x,y;t)=\frac{e^{-(x-y)^2/4t}}{16\pi^2t^2}\sum_{n=0}a_n(x,y)t^n . \qquad (48)$$

Here a_n are the expansion coefficients. The heat kernels (31), (32), (40), and (43) have a similar expansion with the corresponding expansion coefficients denoted by $(a_n)_{\mu\nu}$, $\tilde{a}_n$, a_n and $(a'_n)_{\mu\nu}$, respectively. The relevant heat coefficients and their derivatives at coincident point are evaluated in Appendix A of III for the generic regulator (45). Substituting for X_μ and Y from (46) and (47) in the results obtained there yields the required heat coefficients. These are tabulated below. The derivatives with respect to the first or the second argument of the heat coefficient are denoted by ∂_μ or $\bar{\partial}_\mu$, respectively. The coefficients $(a_n)_{\mu\nu}$ are expressed in terms of $(a'_n)_{\mu\nu}$ using Eq. (44) and hence we need tabulate only $(a'_n)_{\mu\nu}$. The coincident point values of the zeroth heat coefficients are

$$\tilde{a}_0|_{y=x}=a_0|_{y=x}=1, \quad (a'_0)_{\mu\nu}|=\delta_{\mu\nu} . \tag{49}$$

All the derivatives of a_0 are equal to those of $\tilde{a}_0$ and derivatives of $(a'_0)_{\mu\nu}$ are equal to $\delta_{\mu\nu}$ times derivatives of a_0; hence, we will tabulate only the derivatives of a_0:

$$\begin{aligned}
&\partial_\mu a_0|=-A_\mu, \quad \partial_\mu\partial_\nu a_0|=\tfrac{1}{2}(-\partial_\mu A_\nu-\partial_\nu A_\mu+\{A_\mu,A_\nu\}),\\
&\bar{\partial}_\mu a_0|=A_\mu, \quad \bar{\partial}_\mu\bar{\partial}_\nu a_0|=\tfrac{1}{2}(\partial_\mu A_\nu+\partial_\nu A_\mu+\{A_\mu,A_\nu\}),\\
&\tilde{a}_1|=\Sigma\cdot F, \quad \partial_\mu\tilde{a}_1|=\tfrac{1}{2}(\partial_\mu\Sigma\cdot F-\{\Sigma\cdot F,A_\mu\})+\partial_\mu Y_0,\\
&\bar{\partial}_\mu\tilde{a}_1|=\tfrac{1}{2}(\partial_\mu\Sigma\cdot F+\{A_\mu,\Sigma\cdot F\})-\partial_\mu Y_0, (a'_1)_{\mu\nu}|=2F_{\mu\nu},\\
&\partial_\alpha(a'_1)_{\mu\nu}|=\tfrac{1}{2}(\partial_\alpha F_{\mu\nu}-\{F_{\mu\nu},A_\alpha\})+\partial_\alpha Y_0\delta_{\mu\nu},\\
&\bar{\partial}_\alpha(a'_1)_{\mu\nu}|=\tfrac{1}{2}(\partial_\alpha F_{\mu\nu}+\{A_\alpha,F_{\mu\nu}\})-\partial_\alpha Y_0\delta_{\mu\nu},\\
&a_1|=0, \quad \partial_\mu a_1|=\partial_\mu Y_0, \quad \bar{\partial}_\mu a_1|=-\partial_\mu Y_0 .
\end{aligned} \tag{50}$$

Here Y_0 contains $\partial\cdot A$ and A^2, and as will become clear later, the contributions from such terms cancel off between different Jacobians. Hence Y_0 need not be evaluated explicitly.

The Green's functions $\tilde{G}$ and G defined in (22) and (39) can be expressed in terms of the expansion coefficients $\tilde{a}_n$ and a_n, respectively. The procedure is similar to that followed in III. Thus

$$\begin{aligned}
G(y,x)&=\lim_{\tau\to\infty}\int_0^\tau dt\,\tilde{h}(y,x,t)\\
&=\frac{1}{4\pi^2}\left[\frac{a_0(y,x)}{z^2}-\frac{1}{4}\ln\left[\frac{z^2}{4\tau}\right]a_1(y,x)\right]
\end{aligned} \tag{51}$$

(where $z_\mu=y_\mu-x_\mu$ as usual). The Green's function $\tilde{G}$ which satisfies $\not{D}\tilde{G}(x,y)=\delta^4(x,y)$ can be obtained from the Green's function G', defined as $\not{D}^2G'(x,y)=\delta^2(x,y)$, as

$$\begin{aligned}
G'(y,x)&=\lim_{\tau\to\infty}\int_0^\tau dt\,\tilde{h}(y,x,t)\\
&=\frac{1}{4\pi^2}\left[\frac{\tilde{a}_0(y,x)}{z^2}-\frac{1}{4}\ln\left[\frac{z^2}{4\tau}\right]\tilde{a}_1(y,x)\right] ;
\end{aligned}$$

hence,

$$\begin{aligned}
\tilde{G}(y,x)&=\not{D}\tilde{G}'\\
&=-\frac{1}{4\pi^2}\left[\frac{2\not{z}}{z^4}\tilde{a}_0+\frac{\not{z}}{2z^2}\tilde{a}_1-\frac{\not{D}\tilde{a}_0}{z^2}+\frac{1}{4}\ln\frac{z^2}{4\tau}\not{D}\tilde{a}_1\right] .
\end{aligned} \tag{52}$$

Using these expressions for G and $\tilde{G}$ and the coincident point values for the expansion coefficients, the Jacobians in (42) can now be evaluated

III. EVALUATION OF THE JACOBIANS

We start with the first set of square brackets in (42) with only the local part of the bosonic heat kernel (44), i.e., $h_{\mu\nu}$ replaced by $h'_{\mu\nu}$:

$$\begin{aligned}
&\mathrm{Tr}[\bar{\varepsilon}(x)\gamma_\mu\tilde{h}(x,y)\tilde{G}(y,x)\gamma_\mu\lambda(x)-\bar{\varepsilon}(y)\gamma_\mu h'_{\nu\mu}(x,y)\tilde{G}(y,x)\gamma_\nu\lambda(x)]\\
&=\int d^4x\int d^4z\frac{e^{-z^2/4t}}{16\pi^2t^2}\mathrm{tr}\left\{\bar{\varepsilon}(x)\gamma_\mu\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left[\frac{(z\cdot\bar{\partial})^m}{m!}[\tilde{a}_n\delta_{\mu\nu}-(a'_n)_{\nu\mu}]|_{y=x}\right]\tilde{G}(x+z,x)\gamma_\mu\lambda(x)\right.\\
&\qquad\left.-\sum_{l=1}^{\infty}\frac{(z\cdot\partial)^l}{l!}\bar{\varepsilon}(x)\gamma_\mu\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left[\frac{(z\cdot\bar{\partial})^m}{m!}(a'_n)_{\nu\mu}|\right]\tilde{G}(x+z,x)\gamma_\mu\lambda(x)\right\},
\end{aligned} \tag{53}$$

where all functions of y are expanded around x as $y=x+z$. Later z will be replaced by $2\sqrt{t}z'$ so as to have the coincident point limit when $t\to 0$. Since the second arguments of the heat kernel coefficients a_n are expanded around the first, we have derivatives with respect to the second argument, i.e., $\bar{\partial}$. Similarly all the a_n appearing in $\tilde{G}$, given by (52), when Taylor-expanded will have a derivative with respect to the first argument, i.e., ∂. After z' integration is done, all the terms with positive powers of t will vanish in the limit $t\to 0$ and hence are irrelevant. The terms with negative powers of t constitute the divergent contributions. In particular the first term in $\tilde{G}$ in (52) is proportional to z^{-3}, i.e., $t^{-3/2}$ and hence singular in the limit $t\to 0$. Since $t\sim x^2$, this will correspond to a cubic divergent contribution. In (45) we have seen that $(a'_0)_{\mu\nu}|=\delta_{\mu\nu}$ and $\tilde{a}_0|=1$. Thus the cubic divergent contributions in the first term in (53) cancel off between bosons and fermions. All the other terms in (53) are less singular than $t^{-3/2}$. The coincident point values of the a_n, tabulated in (49) and (50), and the expression for $\tilde{G}$ in (52) can now be substituted in (53). Performing the z' integration and retaining all the terms with nonpositive powers of t, the right-hand side of Eq. (53) becomes

$$\begin{aligned}
&-\frac{1}{4\pi^2}\int d^4x\ \mathrm{tr}[\tfrac{1}{16}\bar{\varepsilon}\gamma_\nu D_\alpha(2F_{\nu\mu}+\Sigma\cdot F\delta_{\nu\mu})\gamma_\alpha\\
&\quad-\tfrac{1}{8}\partial_\alpha\bar{\varepsilon}(2\gamma_\nu F_{\nu\mu}\gamma_\alpha-\gamma_\mu\gamma_\alpha\Sigma\cdot F+F_{\alpha\nu}\gamma_\mu\gamma_\nu)]\gamma_\mu\lambda .
\end{aligned} \tag{54}$$

This can be simplified using the identity

$$[D_\alpha,F_{\nu\mu}]\gamma_\mu\gamma_\alpha\gamma_\nu=0\Longrightarrow\not{D}\Sigma\cdot F=-D_\alpha\Sigma\cdot F\gamma_\alpha$$

obtained from the Bianchi identity. Thus,

$$\mathrm{Tr}[\bar{\epsilon}\gamma_\mu \tilde{h}\tilde{G}\gamma_\mu\lambda-\bar{\epsilon}\gamma_\nu\tilde{G}\gamma_\mu\lambda h'_{\mu\nu}]$$
$$=\frac{C_v}{4\pi^2}\int d^4x\ \mathrm{tr}\left[\frac{\bar{\epsilon}}{8}(D_\mu F_{\mu\nu})^a\gamma_\nu-\frac{3}{4}\bar{\epsilon}\overleftarrow{\partial}\Sigma\cdot F^a+\frac{3}{4}\partial_\mu\bar{\epsilon}F^a_{\mu\nu}\gamma_\nu\right]\lambda^a\ , \qquad (55)$$

where we have used $F^{ab}_{\mu\nu}=f^{acb}F^c_{\mu\nu}$, and $f^{acb}f^{adb}=-C_v\delta^{ad}$. Here C_v is the Casimir constant of the adjoint representation of the gauge group.

Notice that all the divergent contributions have canceled off and the answer is finite. The first term in (55) can be expressed as a SUSY variation of $F^a_{\mu\nu}F^a_{\mu\nu}$ and the last two terms can be absorbed in the supercurrent as an improvement. This completes the calculation of the first set of square brackets of Eq. (42). Let us now evaluate the second set of square brackets in Eq. (42). Displaying all the spacetime arguments, these terms can be written as

$$2\int\int d^4x\ d^4y\ \mathrm{tr}[\bar{\epsilon}(x)\gamma_\alpha\lambda(x)]\times[-h(x,y,t)G(y,x)\overleftarrow{D}_\alpha+h_{\alpha\beta}(x,y,t)D_\beta G(y,x)]\ .$$

Substituting for G from (51) and for the heat coefficients from (49) and (50), this term takes the form

$$\frac{1}{2\pi^2}\int\int d^4x\ d^4z\frac{e^{-z^2/4t}}{16\pi^2t^2}\mathrm{tr}\bar{\epsilon}\gamma_\alpha\lambda\times\left[-\frac{t}{2z^2}D_\beta F_{\alpha\beta}+\frac{1}{z^2}\left[\frac{(z\cdot\partial)^2}{2}+z\cdot Az\cdot\partial\right](-a_0\overleftarrow{D}_\alpha+D_\alpha a_0)\right]. \qquad (56)$$

Using

$$\int d^4x\frac{e^{-z^2/4t}}{z^2}z_\mu z_\nu=\frac{1}{4}\int d^4z\ e^{-z^2/4t}\delta_{\mu\nu}\ ,$$

the last term in (56) can be written as

$$\tfrac{1}{8}(\Box+2A\cdot\partial)(-a_0\overleftarrow{D}_\alpha+D_\alpha a_0)=\tfrac{1}{8}[D^2(-a_0\overleftarrow{D}_\alpha+D_\alpha a_0)-A^2(-a_0\overleftarrow{D}_\alpha+D_\alpha a_0)]\ . \qquad (57)$$

The second term vanishes because $a_0\overleftarrow{D}_\alpha=0=D_\alpha a_0$. From equation (A11) of III and Eqs. (47) we see that the heat coefficients a_n satisfy the heat equation

$$(1+z\cdot D)a_1=D^2a_0\ .$$

Using this and (50), (57) reduces to

$$\tfrac{1}{8}\{(a_1\overleftarrow{D}_\alpha+D_\alpha a_1)+[D^2,D_\alpha]a_0\}=\tfrac{1}{8}D_\beta F_{\beta\alpha}\ . \qquad (58)$$

Substituting this for the second term in (56) and doing the $z=2\sqrt{t}z'$ integration gives

$$2\,\mathrm{Tr}\bar{\epsilon}\gamma_\alpha\lambda(-hG\overleftarrow{D}_\alpha+h_{\alpha\beta}D_\beta G)=\frac{C_v}{4\pi^2}[\tfrac{1}{4}\bar{\epsilon}(D_\alpha F_{\alpha\beta})^a\gamma_\beta\lambda^a]\ . \qquad (59)$$

This Jacobian is also finite and it can be expressed as a SUSY variation of $F^a_{\mu\nu}F^a_{\mu\nu}$.

Now we consider the third set of square brackets in (42):

$$\int\int\int d^4x\ d^4u\ d^4y\ G(y,u)(\bar{\epsilon}\overleftarrow{\partial\!\!\!/}(u))\tilde{G}(u,x)\times[\tilde{h}(x,y;t)\overleftarrow{D\!\!\!\!/}\lambda(y)+\gamma_\nu\lambda(x)h_{\nu\mu}(x,y,t)\overleftarrow{D}_\mu]\ . \qquad (60)$$

This involves a product of two Green's functions. The short-distance expansion of such an object can be obtained as follows. Define

$$M(y,x)=\int d^4u\ G(y,u)(\bar{\epsilon}\overleftarrow{\partial\!\!\!/}(u))\tilde{G}(u,x)\ . \qquad (61)$$

From the definitions of $\tilde{G}$ and G in (22) and (39), the following conditions can be derived for N:

$$D^2M(y,x)=\bar{\epsilon}\overleftarrow{\partial\!\!\!/}(y)\tilde{G}(y,x) \qquad (62)$$

and

$$M(y,x)\overleftarrow{D\!\!\!\!/}=G(y,x)\bar{\epsilon}\overleftarrow{\partial\!\!\!/}(x)\ . \qquad (63)$$

The right-hand sides of the above equations are known from (49) and (50) for y near x. M is then evaluated by writing all the possible terms that can appear in it and the coefficients of these terms fixed by demanding (62) and (63). From (61) we see that each term has to be of the form $(\bar{\epsilon}\overleftarrow{\partial\!\!\!/})$ times a mass dimension-3 object. The most singular terms in $\tilde{G}$ and G are A_μ independent and proportional to $z\!\!\!/z^{-4}$ and z^{-2}, respectively. Hence the most singular term in M has to be of the form $z\!\!\!/z^{-4}$. All the other terms will be less singular and will contain A_μ and its derivatives. After a lot of straightforward algebra we find that, up to order z,

$$M(y,x)=-\frac{\bar{\epsilon}(y)}{4\pi^2}\left[-\frac{z\!\!\!/}{2z^2}[1-z\cdot A+\tfrac{1}{2}(z\cdot A\overleftarrow{D}\cdot z)]-\frac{1}{32}\Sigma\cdot Fz\!\!\!/+\frac{1}{16}\ln\left[\frac{z^2}{4\tau}\right]z\!\!\!/\Sigma\cdot F\right] \qquad (64)$$

$$=-\frac{\bar{\epsilon}(y)}{4\pi^2}\left[-\frac{z\!\!\!/}{2z^2}\tilde{a}_0(y,x)-\frac{1}{32}\tilde{a}_1(y,x)z\!\!\!/+\frac{1}{16}\ln\left[\frac{z^2}{4\tau}\right]z\!\!\!/\tilde{a}_1(y,x)\right]\ . \qquad (65)$$

Note that we have evaluated M up to order z, i.e., $\sqrt{t}$. This is necessary because the Jacobian here has a term $h_{\nu\mu}\overleftarrow{D}_\mu$ which has a contribution proportional to $t^{-1/2}$. Plugging in (65) and the values of a_n, the Jacobians (60) become

$$-\frac{1}{4\pi^2}\int\int d^4x\, d^4z\frac{e^{-z^2/4t}}{16\pi^2t^2}\times \mathrm{tr}\bar{\varepsilon}\overleftarrow{\not\partial}\left[-\frac{\not z}{2z^2}\left(\tfrac{1}{2}\gamma_\alpha F_{\alpha\beta}\gamma_\beta\lambda+\tfrac{1}{2}z_\alpha\gamma_\beta\lambda F_{\beta\alpha}\right)-\frac{1}{32}\Sigma\cdot F\lambda\frac{z^2}{2t}\right]=0\ . \quad (66)$$

Thus the Jacobians (60) contribute nothing to (42). With this, we are left with evaluation of the fourth and the fifth terms in Eq. (42). These calculations are rather easy. The fourth term reads

$$\mathrm{Tr}\int d^4y\int d^4x\; G(x,y)\bar{\varepsilon}(y)\tilde{h}(y,x)\not{D}\lambda(x)\ . \quad (67)$$

Substituting the relevant expansion coefficients and the expression for G, and expanding y around x, this term yields

$$-\frac{C_v}{4\pi^2}\int \tfrac{1}{4}\bar{\varepsilon}\Sigma\cdot F^a(\not{D}\lambda)^a d^4x\ . \quad (68)$$

Consider now the last term in (42). It is easy to verify that

$$\mathrm{Tr}\int\int d^4x\, d^4y\; G(y,x)D_\alpha(\bar{\varepsilon}\gamma_\alpha\lambda)h(y,x)=0\ . \quad (69)$$

This is because $a_1|=0$ and the most singular term in G is proportional to z^{-2}. Hence only a_0 contributes, and a_0 contributions from G cancel with those from h.

With this we have evaluated all the fermionic Jacobians, the variation of the ghost determinant and the local part of all the bosonic Jacobians. We are left with evaluation of the contribution from the nonlocal part of the bosonic regulator in Eq. (44), to the bosonic Jacobians in (42). Consider the bosonic Jacobian in the second term in (42). Using (44) it yields the following nonlocal contribution:

$$\frac{t}{2}\int\int\int d^4x\, d^4y\, d^4u\; \mathrm{tr}\bar{\varepsilon}(y)\gamma_\mu\tilde{G}(y,x)\gamma_\nu\lambda(x)\{h'_{\nu\alpha}(x,u,t)[\bar{\lambda}(u)\gamma_\alpha\tilde{G}(u,y)\gamma_\mu\lambda(y)-\bar{\lambda}(y)\gamma_\mu\tilde{G}(y,u)\gamma_\alpha\lambda(u)]+[\lambda(x)\gamma_\nu\tilde{G}(x,u)\gamma_\alpha\lambda(u)-\bar{\lambda}(u)\gamma_\alpha\tilde{G}(u,x)\gamma_\nu\lambda(x)]h'_{\alpha\mu}(x,y,t)\}\ . \quad (70)$$

These terms again involve the product of two Green's functions. Since $\tilde{G}$ is of dimension 3, $N=\int[\tilde{G}][\tilde{G}]$ is of dimension 2. Thus the most singular term in the integrand of (70) is proportional to t^{-1}. Since the integral is already multiplied by t, the most singular term is the only term which gives nonzero contribution in the limit $t\to 0$. Comparing the expression for $\tilde{G}$ in (52) and that of $\tilde{G}$ in Eq. (46) of III, we see that the most singular term, i.e., the first term in (52), is $-\frac{1}{2}$ times the first term in Eq. (46) of III. Hence N, the product of two $\tilde{G}$ here, is $-\frac{1}{4}$ times a similar N obtained in equations (53) of III. Substituting these expressions, (70) becomes

$$\frac{t}{4\pi^2}\int\int d^4x\, d^4y\frac{e^{-z^2/4t}}{16\pi^2t^2}\mathrm{tr}\bar{\varepsilon}\gamma_\mu\left[\frac{z_\alpha}{2z^2}-\frac{z_\alpha\not z}{z^4}\right]\times\gamma_\nu\lambda[\bar{\lambda}(\gamma_\nu\gamma_\alpha\gamma_\mu+\gamma_\mu\gamma_\alpha\gamma_\nu)\lambda] \quad (71)$$

$$=\frac{1}{4\pi^2}\frac{1}{8}\int d^4x\;\bar{\varepsilon}\gamma_\mu\lambda\bar{\lambda}\gamma_\mu\bar{\lambda}=0\ . \quad (72)$$

The last step follows due to Fierz rearrangement of the Minkowski-space value of (71). Thus there is no contribution from the nonlocal part of the bosonic regulator to the second term in (42).

It will now be shown that the other two bosonic Jacobians in (42) also receive zero contribution from the nonlocal part of the bosonic regulator. Consider the third term in (42): $D_\mu G\bar{\varepsilon}\gamma_\nu\lambda h_{\nu\mu}=\bar{\varepsilon}\gamma_\nu\lambda(D_\mu G h_{\nu\mu})$. From dimensional arguments, Lorentz invariance, and the form of the nonlocal portion in (44), it is obvious that the coefficient of $\bar{\varepsilon}\gamma_\nu\lambda$ has to be proportional to $\bar{\lambda}\gamma_\nu\lambda$. Hence this term would have to be proportional to $\bar{\varepsilon}\gamma_\nu\lambda\bar{\lambda}\gamma_\nu\lambda$ which is zero in Minkowski space due to Fierz rearrangement. Now consider the sixth term in (42). Since it already contains $(\bar{\varepsilon}\overleftarrow{\not\partial})$, from dimensional arguments and the form of the nonlocal term, it is clear that this term can contribute expressions with positive powers of t alone, which vanish in the limit $t\to 0$. Thus the nonlocal part of the bosonic regulator does not contribute anything to the entire set of Jacobians.

Thus all the terms in Eq. (42) have been evaluated. Collecting the results from (55), (59), (66), (68), and (69) into (42) gives

$$[\tilde{J}^E(\varepsilon)+\delta_\varepsilon\ln\mathrm{Det}D^2]=-\frac{C_v}{4\pi^2}\int d^4x\{\tfrac{1}{8}\bar{\varepsilon}[(D_\mu F_{\mu\nu})^a\gamma_\nu\lambda^a+2(\Sigma\cdot F)^a(\not{D}\lambda)^a]+\tfrac{3}{4}\partial_\mu\bar{\varepsilon}(\gamma_\mu\Sigma\cdot F^a-F^a_{\mu\nu}\gamma_\nu)^a\lambda\}\ . \quad (73)$$

Continuing this back to Minkowski space using $t\to it$, $A_\mu B_\mu\to -A_\mu B^\mu$, and $\gamma_\mu\partial_\mu\to +i\not\partial$ leads to

$$[\tilde{J}(\varepsilon)+\delta_\varepsilon \ln \mathrm{Det}(-D^2)]=i\frac{C_v}{4\pi^2}\int d^4x\left[\left[\frac{i}{8}\bar{\varepsilon}[(D^\mu F_{\mu\nu})^a\gamma^\nu\lambda^a+2(\Sigma\cdot F)^a(\not{D}\lambda)^a]+\frac{3i}{4}\partial^\mu\bar{\varepsilon}(\gamma_\mu\Sigma\cdot F^a-F^a_{\mu\nu}\gamma^\nu)^a\lambda^a\right]+\mathrm{H.c.}\right] . \tag{74}$$

Since $\bar{\psi}$ and $\bar{\psi}'$ are treated identically, the ϵ-dependent part of the total Jacobian is obtained by taking the Hermitian conjugate of the $\bar{\varepsilon}$-dependent part, and such a contribution has been added to Eq. (74). Integrating by parts the second term in the right-hand side of (74) and using the SUSY transformation laws (8) yields

$$[\tilde{J}(\varepsilon)+\delta_\varepsilon \ln \mathrm{Det}(-D^2)]=i\frac{C_v}{4\pi^2}\int d^4x\left[\frac{3i}{4}\partial^\mu\bar{\varepsilon}\left[\gamma_\mu\Sigma\cdot F^a-F^a_{\mu\nu}\gamma^\nu\lambda^a-\frac{1}{3}\Sigma\cdot F^a\gamma_\mu\right]\lambda^a+\mathrm{H.c.}-\frac{3}{16}\delta_\varepsilon(F^a_{\mu\nu}F^{\mu\nu a})\right] . \tag{75}$$

This expression can now be substituted in the SUSY Ward identity (19). To obtain an effective action invariant under SUSY transformations, we define a modified action

$$S_0'=S_0+\frac{3C_v}{64\pi^2}F^a_{\mu\nu}F^{a\mu\nu} , \tag{76}$$

and a modified supercurrent

$$Q_\mu'=Q_\mu+\frac{3iC_v}{16\pi^2}(\gamma_\mu\Sigma\cdot F^a-F^a_{\mu\nu}\gamma^\nu-\tfrac{1}{3}\Sigma\cdot F^a\gamma_\mu)\lambda^a . \tag{77}$$

With these definitions it is easy to see that the modified effective action

$$W'[A,\lambda]=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\times\exp[i(S_0'+S_G)+\ln \mathrm{Det}(-D^2)] \tag{78}$$

satisfies the Ward identity corresponding to (19):

$$\delta_\varepsilon W'[A,\lambda]=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\times\exp[i(S_0'+S_G)+\ln \mathrm{Det}(-D^2)]\times\left[i\int d^4x(\partial^\mu\bar{\varepsilon}Q_\mu'+\bar{Q}_\mu'\partial^\mu\varepsilon)\right] . \tag{79}$$

Thus the Jacobian of the SUSY transformations can be expressed as a SUSY variation of a local term and a term proportional to ε times a total divergence. These contributions can be absorbed in the classical action and in the supercurrent to obtain an improved effective action which is invariant under rigid SUSY transformations. Hence there is no one-loop SUSY anomaly for the on-shell $N=1$ SSYM theory. The absence of an anomaly can be understood as follows. Since our regularization scheme, although not manifestly supersymmetric, is gauge covariant and parity preserving, any contribution to the regularized Jacobian has to be Lorentz, gauge, and parity invariant and of mass dimension $4-\frac{1}{2}$ (ε has dimension $-\frac{1}{2}$). For rigid SUSY transformations, there are only two independent terms of this form: $\bar{\varepsilon}\gamma_\mu\lambda D_\mu F_{\mu\nu}$ and $\bar{\varepsilon}\Sigma\cdot F\not{D}\lambda$. These can be written as $\delta_\varepsilon(F^a_{\mu\nu}F^{\mu\nu a})$ and $\delta_\varepsilon(\bar{\lambda}\not{D}\lambda)$. Hence there cannot be any supercurrent anomaly for the $N=1$ SSYM theory.

IV. SUPERCONFORMAL ANOMALY FOR THE $N=1$ SSYM THEORY

Let us now evaluate the superconformal anomaly for the $N=1$ SSYM theory. The action (1) is invariant under the superconformal transformations

$$\delta^c_\varepsilon\mathbb{A}_\mu=i(\bar{\varepsilon}'\gamma_\mu\Psi-\bar{\Psi}\gamma_\mu\varepsilon'),\quad \delta^c_\varepsilon\Psi=\Sigma\cdot\mathbb{F}\varepsilon' \tag{80}$$

with $\varepsilon'=-i\gamma_\mu x^\mu\varepsilon$. Further

$$\delta^c_\varepsilon S_0[\mathbb{A},\Psi]=\int(\partial_\mu\bar{\varepsilon}S^\mu+\bar{S}_\mu\partial^\mu\varepsilon)d^4x ,$$

where

$$S_\mu=(-i\not{x})Q_\mu=(\not{x})\Sigma\cdot\mathbb{F}^a\gamma_\mu\Psi^a \tag{81}$$

is the superconformal current. The superconformal Ward identity, obtained in the same way as the supersymmetry Ward identity (19), is

$$\delta^c_\varepsilon W[A,\lambda]=\int\mathcal{D}[a_\mu]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp[i(S_0+S_G)+\ln \mathrm{Det}(-D^2)]\times\left[i\int d^4x(\bar{S}_\mu\partial^\mu\varepsilon+\partial_\mu\bar{\varepsilon}S^\mu)+\tilde{k}(\varepsilon)+\delta^c_\varepsilon\ln \mathrm{Det}(D^2)\right] . \tag{82}$$

When the SUSY transformation parameter $\bar{\varepsilon}$ is replaced by $\bar{\varepsilon}'=i\bar{\varepsilon}\not{x}$ in the result obtained in the earlier section for the SUSY anomaly we get the Jacobian $\tilde{k}(\varepsilon)$ of the superformal transformations and the variation of the Faddeev-Popov determinant. Making this change in (75) yields

$$[\tilde{k}(\varepsilon)+\delta^c_\varepsilon\ln \mathrm{Det}(-D^2)]=\frac{iC_v}{4\pi^2}\int d^4x\left[\left[-\tfrac{3}{2}\bar{\varepsilon}\Sigma\cdot F^a\lambda^a+\frac{3i}{4}(\partial^\mu\bar{\varepsilon})(i\not{x})(\gamma_\mu\Sigma\cdot F^a-F^a_{\mu\nu}\gamma^\nu-\tfrac{1}{3}\Sigma\cdot F^a\gamma_\mu)\lambda^a\right]+\mathrm{H.c.}+\frac{3}{16}\delta^c_\varepsilon(F^a_{\mu\nu}F^{\mu\nu a})\right] . \tag{83}$$

In terms of the modified action (76), the modified effective action (78), and the modified superconformal current,

$$S'_\mu = (-i\not{x})Q'_\mu \ , \tag{84}$$

where Q'_μ is as in (77), the superconformal Ward identity becomes

$$\delta^c_\varepsilon W'[A,\lambda] = \int \mathcal{D}[a_\mu]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi] \times \exp[i(S'_0 + S_G) + \ln \mathrm{Det}(-D^2)] \times \left\{ i\int d^4x(\bar{S}'_\mu \partial^\mu \varepsilon + \partial^\mu \bar{\varepsilon} S'_\mu) - \frac{3iC_v}{8\pi^2}\int d^4x(\bar{\varepsilon}\Sigma\cdot F^a\lambda^a + \mathrm{H.c.}) \right\} . \tag{85}$$

Thus the superconformal symmetry is anomalous and the anomaly is given by the last term in this equation.

Note that unlike the dilatation anomaly calculations [1], the Jacobian for the SUSY and the superconformal transformations do not have any infinite contributions and the results are finite.

V. CONCLUSIONS

The method established in III for evaluation of the superanomalies for the on-shell Wess-Zumino model, using Fujikawa's method and heat-kernel regularization, has been extended to a theory with a local gauge symmetry, namely, the $N=1$ SSYM theory. Here an additional problem of noninvariance (under SUSY transformations) of the gauge-fixing term and the corresponding Faddeev-Popov determinant was encountered. We have used a regularization scheme which is manifestly gauge invariant but not supersymmetric. This was done by adding a field-dependent gauge transformation to the SUSY transformations so that the modified SUSY transformations left the gauge-fixing term invariant. The Jacobian of these transformations and the SUSY variation of the Faddeev-Popov determinant were calculated in a gauge-invariant, regularized fashion. The resultant Jacobian was gauge invariant and finite and could be absorbed in the action and the supercurrent such that, the modified supercurrent was gauge invariant and conserved. Thus we have seen explicitly in this paper that although our regularization scheme is not manifestly supersymmetric there is no one-loop supercurrent anomaly for the on-shell $N=1$ SSYM theory. The superconformal anomaly was then obtained from this calculation using the same trick as the one used for the Wess-Zumino model. The expression for finite superconformal anomaly so obtained agrees with results obtained using different methods [6]. The method established here can be taken over to the case of extended SSYM theories in a natural fashion [9].

ACKNOWLEDGMENTS

I thank Romesh Kaul for suggesting this problem, innumerable discussions and help. Part of this work was supported by the CSIR, New Delhi, India.

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[1] K. Fujikawa, Phys. Rev. Lett. **44**, 1733 (1980); **42**, 1195 (1979); Phys. Rev. D **21**, 2848 (1980); **23**, 2262 (1981); for a review, see K. Fujikawa, in *Super Field Theories,* Proceedings of the NATO Advanced Study Institutes, Vancouver, Canada, 1986, edited by H. C. Lee *et al.*, NATO ASI, Series B: Physics, Vol. 160 (Plenum, New York, 1986).

[2] N. K. Nielsen, M. T. Grisaru, H. Romer, and P. van Nieuwenhuizen, Nucl. Phys. **B140**, 477 (1978); B. S. DeWitt, Phys. Rev. **162**, 1239 (1967); S. Hawking, Commun. Math. Phys. **55**, 133 (1977); P. Ramond, *Field Theory: A Modern Primer* (Benjamin/Cummings, Reading, MA, 1981).

[3] M. R. Mehta, Phys. Rev. D **44**, 1228 (1991).

[4] M. F. Sohnius, Phys. Rep. **128**, 39 (1985), and references given in Ref. [3].

[5] B. DeWitt, Phys. Rev. Lett. **13**, 114 (1964); Phys. Rev. **162**, 1195 (1967); L. F. Abbott, Nucl. Phys. **B185**, 189 (1981).

[6] T. Curtright, Phys. Lett. **71B**, 185 (1977); L. F. Abbott, M. T. Grisaru, and H. J. Schnitzer, Phys. Rev. D **16**, 2295 (1977); H. Inagaki, Phys. Lett. **77B**, 56 (1978); P. Majumdar, E. C. Poggio, and H. Schnitzer, *ibid.* **93B**, 321 (1980).

[7] K. Osterwalder and R. Schrader, Helv. Phys. Acta. **46**, 277 (1973).

[8] M. R. Mehta, Phys. Rev. Lett. **65**, 1983 (1991); **66**, 522(E) (1991); Mod. Phys. Lett. A (to be published). An alternative Euclideanization procedure is given here such that, unlike Ref. [7], the procedure is continuous, the fermionic degrees of freedom are not doubled in Euclidean space, and the $N=1$ SSYM theory is supersymmetric in Euclidean space. Hence the four-fermion term vanishes even in Euclidean space.

[9] M. R. Mehta, I.I. Sc. Report (unpublished).