Chiral Yukawa models on the lattice and decoupling

J. L. Alonso
Departamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain

Ph. Boucaud
Laboratoire de Physique Théorique et Hautes Energies, Université Paris XI, 91405 Orsay CEDEX, France

J. L. Cortés
Departamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain

E. Rivas
Physics Department, University of California at Los Angeles, Los Angeles, California 90024-1547
(Received 29 April 1991)

A new formulation on the lattice of a theory with a chiral fermion-scalar coupling and minimal field content is proposed. The arbitrariness in the regularization is used in order to decouple the replica fermions from the scalar. A continuum limit with just one fermion coupled to the scalar is obtained in perturbation theory and a Golterman-Petcher-like symmetry related to the decoupling of the replica fermions is identified.

I. INTRODUCTION

The only sector, if any, of the standard electroweak model (SM) in which one has to go beyond the perturbative treatment based on the renormalizability of the theory, is in the scalar-fermion sector. The formulation of the model on the lattice is an appropriate framework where one can study the possibility of having a strong scalar self-coupling and/or a strong Yukawa coupling. The relevance of the implications of such studies such as the limitations on the masses of the scalars and fermions as well as the possible influence of heavy fermions on our present understanding of the Higgs mechanism is on the basis of the recent activity in this area [1–3].

In order to obtain upper bounds on the masses of the scalars and fermions in the standard model through numerical lattice simulations (going beyond the upper bounds determined perturbatively) a definition of a Yukawa model on the lattice with a global chiral symmetry is required. The chiral Yukawa models considered in the literature go from the simplest toy model with one real scalar field and a $Z(2)$ global symmetry to the complete scalar sector of the standard electroweak model. Another way of classifying these models is according to the solution used for the doubling problem that one finds with fermion fields on the lattice.

Studies of a Yukawa model with one real scalar field coupled to staggered lattice fermions [4] have shown how a combination of analytic and numerical results can be used to determine the rich phase structure of the model and to put limits on the renormalized Yukawa and scalar couplings on the phase boundary connected with the perturbative region (small Yukawa coupling). Another interesting feature of this model is that while the continuum limit defined at the "perturbative phase boundary" does not depend on the details of the lattice regularization, the phase structure at large Yukawa couplings and the related possible nonperturbative continuum limit are sensitive to the type of lattice regularization used [4].

A different formulation of lattice chiral Yukawa models, which makes contact with perturbation theory, is based on the introduction of explicit mirror fermions in order to use the standard Wilson term [5] to avoid the fermion doubling and using the spontaneous symmetry breaking of the global symmetry to give large masses to the mirror fermions [6] or alternatively tune parameters in order to decouple the mirror fermions from the scalar field [7]. The introduction of new parameters and their required tuning make the study of the phase diagram and cutoff-dependent upper limit on the renormalized couplings quite difficult to determine.

The last class of lattice Yukawa models essentially rely on the Wilson method [8], now used in a manifestly chiral-invariant way, with the scalar field, in the so-called Wilson-Yukawa term. The main idea is that one can use the strong Yukawa coupling phase structure to get a mass for the replica fermions of the order of the cutoff and at the same time to approach to the continuum limit with a vanishing vacuum expectation value for the scalar field in lattice units. The phase structure of a $SU(2)\otimes SU(2)$ fermion scalar model which correspond to the standard model with mass degenerated doublets has been studied by a combination of analytical and numerical methods and arguments in favor of the possibility to approach the continuum limit with a finite mass for the physical fermion while the doublers get a mass of the order of the cutoff have been given, both in the quenched [9] and unquenched approach [10,11]. But in contrast with the fermion-scalar sector of the perturbative standard model, the Yukawa coupling is a relevant parameter in the Wilson-Yukawa model. Also the crucial role played by the definition of the continuum limit on the phase boundary located in the strong Wilson-Yukawa coupling region makes the studies of nonperturbative effects quite involved, especially if one thinks of the SM as an effective theory with a cutoff $\Lambda \sim 1$ TeV, not much higher than the scale which generates all the masses (solution to the fine-tuning problem based on triviality). Note that in this case the meaning of the decoupling of the doublers would not be so clear.

One can ask whether the conclusions obtained so far (additional fields are required in order to make direct contact with perturbation theory which is the basis of our understanding of the SM) are general to any lattice formulation of the scalar-fermion sector of the SM or if they are a consequence of the Wilson formulation. In other words, the main question we address in this paper is the following. Can we define a lattice Yukawa chiral model with a minimal field content which allows us to define the continuum limit around the Gaussian fixed point with just one fermion for each fermion field coupled to the scalar field?

II. LATTICE CHIRAL-YUKAWA MODEL

In order to answer this question we will consider a lattice chiral Yukawa model, where we will handle the replica fermions in a different way from the Wilson method. Let us consider a theory with a fermion field Φ and a complex scalar field Ψ interacting through the lattice action

$$I(\Psi,\Phi)=I_B(\Phi)+I_F(\Psi)+I_Y(\Psi^{(1)},\Phi) , \tag{2.1}$$

where

$$I_B(\Phi)=-\sum_x \Phi_x^\dagger\Phi_x+\frac{k}{2}-\sum_{x,\mu}(\Phi_{x+\hat\mu}^\dagger\Phi_x+\Phi_x^\dagger\Phi_{x+\hat\mu}) -\lambda\sum_x(\Phi_x^\dagger\Phi_x-1)^2 \tag{2.2}$$

is the naive free fermion action with the replica fermions at $\theta_\mu=\pi$, θ being the momentum in lattice units, and

$$I_F(\Psi)=-\tfrac{1}{2}\sum_{x,\mu}(\overline{\Psi}_x\gamma_\mu\Psi_{x+\hat\mu}-\overline{\Psi}_{x+\hat\mu}\gamma_\mu\Psi_x) \tag{2.3}$$

is the naive free fermion action with the replica fermions at $\theta_\mu=\pi,\theta$ being the momentum in lattice units, and

$$I_Y(\Psi^{(1)},\Phi)=-y\sum_x(\overline{\Psi}^{(1)}_{Lx}\Phi_x\Psi^{(1)}_{Rx}+\overline{\Psi}^{(1)}_{Rx}\Phi_x^\dagger\Psi^{(1)}_{Lx}) \tag{2.4}$$

gives the coupling of the fermions and the scalars.

The way to implement the decoupling of the replica fermions is based [12] on the use of the component $\Psi^{(1)}$,

$$\Psi^{(1)}(\theta)=F(\theta)\Psi(\theta),\quad F(\theta)=\prod_\mu f(\theta_\mu) ,$$
$$f(\theta)=\cos\left[\frac{\theta}{2}\right],\quad \theta\in(-\pi,\pi] , \tag{2.5}$$

in the interaction term. In coordinate space the fields $\Psi_x^{(1)}$ corresponds to an average over the Ψ components defined in an elemental hypercube in the positive direction

$$\Psi_x^{(1)}\equiv\int_\theta \exp\left\{i\left[\theta\left[x+\tfrac{1}{2}\sum_\mu\hat\mu\right]\right]\right\}F(\theta)\Psi(\theta)$$
$$=\frac{1}{2^D}\left[\Psi_x+\sum_{n=1}^{D}\sum_{\mu_{i_1}<\cdots<\mu_{i_n}}\Psi_{x+\hat\mu_{i_1}+\cdots+\hat\mu_{i_n}}\right] , \tag{2.6}$$

and then the interaction does not spread out over the whole lattice. The arbitrariness in the regularization allows many other options for the $f(\theta)$ function, but the expression in (2.5) corresponds to the most local and rotational invariant possibility. The fermion-scalar vertex of the model in (2.1)–(2.5) is given by $V(\bar\theta,\theta)=yF(\bar\theta)F(\theta)$ and it vanishes for all replica fermions ($\theta_\mu=\pi$), due to the choice of $f(\theta)$ in (2.5). The model has a global U(1)⊗U(1) symmetry

$$\Psi_L\to e^{i\alpha_L}\Psi_L,\quad \Psi_R\to e^{i\alpha_R}\Psi_R ,$$
$$\Phi\to e^{i\alpha_L}\Phi e^{-i\alpha_R} \tag{2.7}$$

and could be immediately generalized to the fermion-scalar sector of the SM by adding appropriate fermion and scalar fields, but all the properties of this sector are already present in this simplified model.

The crucial point in order to prove the validity of the model in (2.1) as a way to solve the doubling problem, at the level of the scalar-fermion interaction, is to see whether quantum corrections do not destroy the decoupling of the replica fermions implemented at the classical level.

This problem can be studied in perturbation theory and the result is that, as a consequence of the extension to the lattice of the standard power-counting theorem [13], the continuum limit of any Green's function calculated with the model (2.1) coincides after renormalization with the Green's function of the Yukawa model in the continuum *with one fermion* [14]. The only thing one has to check is that all properties in order to establish the power-counting theorem on the lattice are satisfied by the lattice Yukawa model in (2.1). The reason why the proof does not work in the case of the naive fermions is that the denominator D_F of the fermion propagator is of order 1 at some points at the boundary of the Brillouin zone (doubling problem). The proof of the power-counting theorem [13] requires that, for lattice spacing a, smaller than some value a_0, $|D_F(k,a)|\geq A\bar k^2$, for some constant A, where $\bar k_\mu=(2/a)\sin(ak_\mu/2)$. For k_μ near π/a, the denominator D_F is of order one, while $A\bar k^2$ can be arbitrarily large for $a\to 0$. On the contrary, in the case of the model in (2.1), this proof works. The reason is that for any Feynman diagram, an internal propagator is always accompanied by $F(\theta)$ factors coming from the vertices, and then, in the power-counting arguments, the naive denominator is replaced by $\tilde D=D_F(k,a)/F^2(ak)$, which now can be bounded by $A\bar k^2$ in all the Brillouin zone. As an example, an explicit one-loop calculation can be found in the Appendix.

The former result (i.e., that the continuum limit of any

Green's function coincides after renormalization with the Green's function of the continuum Yukawa model *with one fermion*), based on the Reisz theorem [13], may be heuristically understood on the basis of a symmetry of the action (2.1) which is a consequence of the use of the component $\Psi^{(1)}$ in the interaction term I_Y. Actually, the action is invariant under the 2^D-1 transformation of the fermion field

$$\Psi_x \to \Psi_x + \epsilon_x^{(i)}, \quad \epsilon_{x+\hat\mu}^{(i)} = e^{i\theta_\mu^{(i)}}\epsilon_x^{(i)}, \quad i=2,\cdots 2^D , \tag{2.8}$$

where $\theta_\mu^{(i)}=0,\pi$ and at least one component is different from zero (momenta corresponding to the replica fermions).

The naive free term, I_F in (2.3), is invariant under this transformation, in fact it is also invariant under the transformations with $\epsilon_{x+\hat\mu}=\epsilon_x$. The invariance of the interaction term I_Y follows from the invariance of the $\Psi^{(1)}$ component under the transformation (2.8).

In order to see the implications of this symmetry, we consider the partition function with sources $\eta,\bar\eta,J$, and $J^\dagger$ coupled to the elementary fields

$$\begin{aligned} &Z(\eta,\bar\eta,J,J^\dagger)=\int D\Phi\, D\Psi\, D\bar\Psi \exp(I_J) , \\ &I_J=I(\Psi,\Phi)+\sum_x(\bar\Psi_x\eta_x+\bar\eta_x\Psi_x+J_x^\dagger\Phi_x+\Phi_x^\dagger J_x) . \end{aligned} \tag{2.9}$$

In order to avoid complicated nonlinear terms, it is also convenient to introduce sources $\bar\chi,\chi$ coupled to some composite operator $A_x=[P_L\Phi_x^\dagger+P_R\Phi_x]\Psi_x^{(1)}$ and its partner $\bar A_x$. Then our starting point is

$$Z(\eta,\bar\eta,J,J^\dagger;\chi,\bar\chi)=\int D\Phi\, D\Psi\, D\bar\Psi \exp(\tilde I_J) , \tag{2.10}$$

where $\tilde I_J=I_J+\bar\chi A+\bar A\chi$.

It is simpler to work in momentum space, where

$$I_F(\Psi)+I_Y(\Psi,\Phi)=\int_\theta \bar\Psi_\theta S_\theta^{-1}\Psi_\theta - y\int_{\bar\theta}\int_\theta \bar\Psi_{\bar\theta}F_{\bar\theta}Q^\Phi_{\theta-\bar\theta}F_\theta\Psi_\theta , \tag{2.11}$$

with

$$\begin{aligned} &S_\theta^{-1}=-i\sum_\mu \gamma_\mu \sin(\theta_\mu) , \\ &Q_\theta^\Phi=P_L\Phi_\theta^\dagger+P_R\Phi_{-\theta} , \end{aligned} \tag{2.12}$$

the Fourier transform for the scalar is defined through $\Phi_x=\int_\theta \exp i[\theta(x+\frac{1}{2}\sum_\mu\hat\mu)]\Phi_\theta$, and $\int_\theta=\int d^D\theta/(2\pi)^D$ for $\theta\in(-\pi,\pi]^D$.

If we consider the two independent transformations for the fermion fields Ψ and $\bar\Psi$,

$$\Psi(\theta)\to\Psi(\theta)+\epsilon(\theta), \quad \bar\Psi(\theta)\to\bar\Psi(\theta)+\bar\epsilon(\theta) , \tag{2.13}$$

we find the Ward identities

$$\begin{aligned} &\int D\Phi\, D\Psi\, D\bar\Psi\, e^{\tilde I_J}\left[\bar\Psi_\theta S_\theta^{-1}-yF_\theta\bar A_\theta+F_\theta\int\bar\chi_{\bar\theta}Q^\Phi_{\theta-\bar\theta}+\bar\eta_\theta\right]=0 , \\ &\int D\Phi\, D\Psi\, D\bar\Psi\, e^{\tilde I_J}\left[S_\theta^{-1}\Psi_\theta-yF_\theta A_\theta+F_\theta\int_{\bar\theta}Q^\Phi_{\bar\theta-\theta}\chi_{\bar\theta}+\eta_\theta\right]=0 . \end{aligned} \tag{2.14}$$

The reason for the introduction of the extra sources is that we can write the Ward identities in the form of a linear functional differential equation:

$$\begin{aligned} &\left[S_\theta^{-1}\frac{\partial}{\partial\eta_\theta}-yF_\theta\frac{\partial}{\partial\chi_\theta}-F_\theta\int_{\bar\theta}\bar\chi_{\bar\theta}Q^J_{\theta-\bar\theta}-\bar\eta_\theta\right]\ln Z=0 , \\ &\left[S_\theta^{-1}\frac{\partial}{\partial\bar\eta_\theta}-yF_\theta\frac{\partial}{\partial\bar\chi_\theta}+F_\theta\int_{\bar\theta}Q^J_{\bar\theta-\theta}\chi_{\bar\theta}+\eta_\theta\right]\ln Z=0 , \end{aligned} \tag{2.15}$$

where

$$Q_\theta^J=P_L\frac{\partial}{\partial J_\theta}+P_R\frac{\partial}{\partial J^\dagger_{-\theta}} . \tag{2.16}$$

These identities can be written in a more convenient form by introducing the generating functional Γ depending only on the classical fields $\Psi',\bar\Psi',\Phi',\Phi'^\dagger,A',\bar A'$:

$$\begin{aligned} \Gamma(\Psi',\bar\Psi',\Phi',\Phi'^\dagger;A',\bar A')=&\ln Z(\eta,\bar\eta,J,J^\dagger;\chi,\bar\chi) \\ &+\bar\eta\Psi'+\bar\Psi'\eta+J^\dagger\Phi' \\ &+\Phi'^\dagger J+\bar A'\chi+\bar\chi A' , \end{aligned} \tag{2.17}$$

with the relations

$$\begin{aligned} &\Psi'=-\frac{\partial\ln Z}{\partial\bar\eta}, \quad \bar\Psi'=\frac{\partial\ln Z}{\partial\eta\ln} , \\ &A'=-\frac{\partial\ln Z}{\partial\bar\chi}, \quad \bar A'=\frac{\partial\ln Z}{\partial\chi} , \\ &\Phi'=\frac{\partial\ln Z}{\partial J^\dagger}, \quad \Phi'^\dagger=\frac{\partial\ln Z}{\partial J} . \end{aligned} \tag{2.18}$$

The Ward identities are given in terms of Γ and the classical fields by

$$\begin{aligned} &S_\theta^{-1}\bar\Psi'_\theta-yF_\theta\bar A'_\theta+F_\theta\int_{\bar\theta}\frac{\partial\Gamma}{\partial A'_{\bar\theta}}Q^{\Phi'}_{\theta-\bar\theta}+\frac{\partial\Gamma}{\partial\Psi'_\theta}=0 , \\ &S_\theta^{-1}\Psi'_\theta-yF_\theta A'_\theta-F_\theta\int_{\bar\theta}Q^{\Phi'}_{\bar\theta-\theta}\frac{\partial\Gamma}{\partial\bar A'_{\bar\theta}}-\frac{\partial\Gamma}{\partial\bar\Psi'_\theta}=0 . \end{aligned} \tag{2.19}$$

Now if one takes $\theta=\theta^{(i)}$ for $i=2,\ldots,2^D$ one has

$$\frac{\partial\Gamma}{\partial\Psi'(\theta^{(i)})}=0, \quad \frac{\partial\Gamma}{\partial\overline{\Psi}'(\theta^{(i)})}=0 . \tag{2.20}$$

Then, for finite a, one finds that the inverse of the exact propagator vanishes for θ exactly equal to $\theta^{(i)}$ and therefore the position of the replica fermion in momentum space is not changed (see the Appendix for an explicit one-loop calculation). Also by taking the functional derivative of (2.19) with respect to the classical fields, one finds that any one-particle-irreducible amputated Green's function vanishes if at least one external fermion line has $\theta=\theta^{(i)}$. Nevertheless, the control of the limit $a\to 0$ requires one to come back to the former perturbative argument.

The decoupling symmetry (2.8) is in correspondence with a similar symmetry found [15] in the case of the Wilson-Yukawa formulation of the standard model which allows one to prove the decoupling of the right-handed neutrino in the continuum limit. The right-handed neutrino is always required in order to construct the Wilson term; but because it appears only through derivative terms, it decouples in the continuum limit for both the Smit-Swift model and the mirror-fermion model [16].

III. SUMMARY AND DISCUSSION

The model presented in this paper can be considered just as a nonperturbative formulation of chiral Yukawa theories or as a first step in a formulation of a chiral gauge theory such as the standard electroweak model.

From the first viewpoint (without any reference to gauge chiral models), we can summarize the discussion in this paper by saying that a formulation of chiral Yukawa models on the lattice has been found which allows a weak-coupling perturbative analysis showing, using power-counting arguments, that in this weak-coupling region the doublers are decoupled. The identification of the decoupling symmetry (2.8) can be used to argue that also in a nonperturbative regime the replica fermions decouple, if a sensible continuum limit exists with only the terms of the action (2.1) (i.e., if not unexpected divergences occur).

This model presents very different features from the most known lattice chiral models (mirror-fermions and Smit-Swift-Yukawa models); mainly, the way unphysical fermions are decoupled deals with light replica fermions instead of heavy doublers. According with our results, the model (2.1) has in common with Yukawa mirror fermions the fact that the weak-coupling region is sensible to define a continuum limit with one physical fermion, and with the Smit-Swift-Yukawa model it shares a minimal field content. Then, further numerical investigations should be interesting to figure out whether the model (2.1) works correctly (numerical evidence of the decoupling of the replica) and to compare with the results known for these other lattice Yukawa models [16].

The study of the corrections to the perturbative bounds on the renormalized Yukawa and self-scalar coupling obtained by going to the strong-coupling region and the study of the possibility to define a nonperturbative continuum limit requires one to determine the phase diagram of the model. This can be done with a mean-field approach [17]. The next step in the nonperturbative formulation of the standard electroweak model would be to introduce gauge interactions in the scalar-fermion model. There are two possibilities corresponding to having gauge invariance at the level of the regularization or asking only for gauge invariance in the continuum limit after a tuning of appropriate counterterms on the lattice action. The first possibility looks more natural, but when applied to our case it leads to a perturbatively nonrenormalizable model with similar characteristics to the model based on the Wilson-Yukawa term [8]. This option is essentially nonperturbative and therefore escapes the reach of this work. If we want the present formulation of chiral Yukawa models to be interpreted as a global symmetry limit of a chiral gauge theory for which a perturbative approach is possible, only the second possibility remains. The structure of counterterms required in order to recover gauge invariance (and Lorentz covariance) at the perturbative level has been considered in the Wilson-Yukawa model [7]. In this case, the consistency of the method at the one-loop level with an anomaly-free fermion field content has been established. A nonperturbative tuning associated with the divergent mass term for the gauge field is present due to the gauge noninvariance of the regularization and the structure of counterterms required to obtain a gauge-invariant continuum limit only makes sense in the presence of the gauge fixing and Faddeev-Popov term [7], so that one must keep such terms also at the nonperturbative level. The main difference between this Wilson-Yukawa case [7] and our method is that the fermion sector has a global chiral symmetry at the level of the regularization; this symmetry avoids the appearance of a mass counterterm for the fermion field.

It is an open question whether a sensible continuum limit can be defined along these lines for a formulation of a chiral gauge theory based on the model presented in this paper.

ACKNOWLEDGMENTS

One of us (E.R.) would like to thank M. Golterman and D. Petcher for some interesting discussions. This work was partially supported by the CICYT (proyecto AEN 90-0030), the Acción integrada entre España y Francia No. 72 (1990), the spanish F.P.U. grants, and a Proyecto Conjunto CNRS-CSIC.

APPENDIX

In this appendix, we perform an explicit one-loop computation of the fermion propagator to show how the mechanism described in this paper works. For simplicity, we work in the symmetric phase where $\langle\Phi\rangle=0$; no mass term is generated for the fermion and the one-loop vertex correction is 0. The mass of the scalar particle is called M. The one-loop self-energy is found to be

$$y^2|F(ap)|^2\mathcal{J}(p,M,a) , \tag{A1}$$

where

$$\mathcal{J}(p,M,a)=\frac{1}{a}\int_{-\pi}^{+\pi}\frac{d^d\mathbf{k}}{(2\pi)^d}|F(k)|^2\frac{-i\gamma_\mu\sin(k_\mu)}{\sum_\lambda\sin^2(k_\lambda)}\times\frac{1}{2\sum_\rho[1-\cos(k_\rho-ap_\rho)]+a^2M^2}\ . \tag{A2}$$

From this expression, we see that the mass renormalization $\mathcal{J}(0,M,a)$ vanishes, a direct consequence of the chiral symmetry of the action. Second, as the factor $|F(ap)|^2$ vanishes when the momentum p coincides with one of the naive replica, (A1) implies that the positions of the replica in momentum space are not affected by the quantum corrections. These F factors come from the vertices and will accompany all the external fermion legs of any given Green's function; they will prevent the coupling of the replica.

We study next the behavior of $\mathcal{J}$ when $a\to 0$ for a fixed value of p. We can first subtract and add the integrand at $p=0$ and change the variable k into ak; we then have

$$\begin{aligned}\mathcal{J}(p,M,a)=\mathcal{J}(0,M,a)&-i\gamma_\mu\frac{2}{a}\sin\left[\frac{ap_\rho}{2}\right]\int_{-\pi/a}^{+\pi/a}\frac{d^d\mathbf{k}}{(2\pi)^d}|F(ak)|^2\frac{\frac{1}{a}\sin(ak_\mu)}{\frac{1}{a^2}\sum_\lambda\sin^2(ak_\lambda)}\frac{2}{\frac{2}{a^2}\sum_\rho[1-\cos(ak_\rho)]+M^2}\\&\times\left[\frac{\frac{1}{a}\sin(ak_\rho-a\frac{p_\rho}{2})}{\frac{2}{a^2}\sum_\nu[1-\cos(ak_\nu-ap_\nu)]+M^2}-\frac{\frac{1}{a}\sin(ak_\rho)}{\frac{2}{a^2}\sum_\nu[1-\cos(ak_\nu)]+M^2}\right]\\&-i\gamma_\mu\frac{2}{a}\sin\left[\frac{ap_\mu}{2}\right]\frac{2}{d}\int_{-\pi}^{+\pi}\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{|F(k)|^2-1}{\left[2\sum_\nu[1-\cos(ak_\nu)]+a^2M^2\right]^2}\\&-i\gamma_\mu\frac{2}{a}\sin\left[\frac{ap_\mu}{2}\right]\frac{2}{d}\int_{-\pi}^{+\pi}\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{1}{\left[2\sum_\nu[1-\cos(ak_\nu)]+a^2M^2\right]^2}\ .\end{aligned} \tag{A3}$$

With the use of the Lebesgue's lemma, it is possible to show that the limit a goes to 0 and the integration can be exchanged for the first integral in Eq. (A3) which then converges toward its continuum limit; the expected logarithmic divergence lies in the last integral which can be studied with usual methods [18] and gives

$$\int_{-\pi}^{+\pi}\frac{d^4\mathbf{k}}{(2\pi)^2}\frac{1}{\left[\sum_\nu[1-\cos(ak_\nu)]+a^2M^2\right]^2}\to\frac{1}{16\pi^2}[F_{0000}-\gamma_E-\ln(a^2M^2)]\ , \tag{A4}$$

where γ_E is the Euler constant, and $F_{0000}\simeq 4.369$ is a constant defined in [18]. The limit of the second integral is trivial and gives a constant independent of M but dependent of the explicit choice made for the function F.

[1] P. Hasenfratz, in *Lattice '88,* Proceedings of the International Symposium, Batavia, Illinois, 1988, edited by A. S. Kronfeld and P. B. Mackenzie [Nucl. Phys. B (Proc. Suppl.) **9**, 3 (1988)].

[2] J. Jersák, HLRZ Jülich. Report No. 90-55 (unpublished).

[3] J. Smit, in *Lattice '89,* Proceedings of the International Symposium, Capri, Italy, 1989, edited by R. Petronzio *et al.* [Nucl. Phys. B (Proc. Suppl.) **17**, 3 (1990)].

[4] I. H. Lee, J. Shigemitsu, and R. E. Shrock, Nucl. Phys. **B330**, 225 (1990); **B334**, 265 (1990).

[5] K. G. Wilson, in *New Phenomena in Subnuclear Physics,* Proceedings of the International School of Subnuclear Physics, Erice, Italy, 1975, edited by A. Zichichi, Subnuclear Series Vols. 13A and 13B (Plenum, New York, 1975).

[6] I. Montvay, Phys. Lett B **199**, 89 (1987); **205**, 315 (1988).

[7] A. Borrelli, L. Maiani, G. L. Sisto, and M. Testa, Phys. Lett. B **221**, 360 (1989); Nucl. Phys. **B333**, 335 (1990).

[8] J. Smit, Acta Phys. Polonica **B17**, 531 (1986); Nucl. Phys.

B175, 307 (1980); in *Lattice '88* [1], p. 579; P. D. V. Swift, Phys. Lett **145B**, 256 (1984).

[9] W. Bock, A. K. De, K. Jansen, J. Jersák, T. Neuhaus, and J. Smit, Phys. Lett. B **232**, 486 (1989).

[10] W. Bock, A. K. De, K. Jansen, J. Jersák, T. Neuhaus, and J. Smit, Nucl. Phys. **B344**, 207 (1990).

[11] W. Bock and A. K. De, Report No. DESY 90-051 (HLRZ 90-029) (unpublished).

[12] J. L. Alonso, Ph. Boucaud, J. L. Cortes, and E. Rivas, in *Lattice '89* [3], p. 461; Mod. Phys. Lett. A **5**, 275 (1990).

[13] T. Reisz, Commun. Math. Phys. **116**, 81 (1988).

[14] T. Reisz, Commun. Math. Phys. **117**, 79 (1988); **117**, 639 (1988); **116**, 573 (1988).

[15] M. F. L. Golterman and D. N. Petcher, Phys. Lett B **225**, 159 (1989).

[16] M. F. L. Golterman, in *Lattice '90,* Proceedings of the Conference, Tallahassee, Florida, 1990, edited by U. M. Heller, A. D. Kennedy, and S. Sanielevici [Nucl. Phys. B (Proc. Suppl.) **20** (1990)].

[17] J. L. Alonso, Ph. Boucaud, and F. Lesmes (in preparation).

[18] A. Gonzales Arroyo and C. P. Korthals Altes, Nucl Phys. **B206**, 681 (1988).